Amplitude detuning measurements

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Many thanks to the Optics Measurement and Corrections team

- **Detuning with amplitude**
  $\longrightarrow$ dependence of tune on action ($J_{x,y}$) or CS-invariant ($\epsilon_{x,y} = 2J_{x,y}$)
  $\longrightarrow N\left[\sigma_{\text{nominal}}\right] = \sqrt{\frac{2J}{\epsilon_{nominal}}}$

$$Q_z(\epsilon_x, \epsilon_y) = Q_{z0} + \left(\frac{\partial Q_z}{\partial \epsilon_x}\epsilon_x + \frac{\partial Q_z}{\partial \epsilon_y}\epsilon_y\right) + \\ + \frac{1}{2!}\left(\frac{\partial^2 Q_z}{\partial \epsilon_x^2}\epsilon_x^2 + 2\frac{\partial^2 Q_z}{\partial \epsilon_x \partial \epsilon_y}\epsilon_x\epsilon_y + \frac{\partial^2 Q_z}{\partial \epsilon_y^2}\epsilon_y^2\right) + \ldots$$

Order	Source ($3 = \text{sextupole}$)
$\frac{\partial Q}{\partial \epsilon}$	$(K_3)^2$, K_4
$\frac{\partial^2 Q}{\partial \epsilon^2}$	$(K_3)^4$, $(K_3)^2 K_4$, $(K_4)^2$, $K_3 K_5$, K_6

- $\frac{\partial Q_x}{\partial \epsilon_x} \longrightarrow$ "Direct term"
- $\frac{\partial Q_y}{\partial \epsilon_x} \longrightarrow$ "Cross term"

Amplitude detuning from an octupole

$$\mathrm{H_n} = \frac{1}{B\rho}\,\mathrm{Re}\left[\frac{1}{n}\left[B_n(s) + iA_n(s)\right](x+iy)^n\right]$$

Normal octupole $\rightarrow \mathrm{H_4} = \frac{1}{4!}K_4 L\,(x^4 - 6x^2y^2 + y^4)$

In action-angle coordinates $\left(x, y = \sqrt{2J_{x,y}\beta_{x,y}}\cos\phi_{x,y}\right)$

$$\mathrm{H_4} = \frac{1}{4!}K_4 L\,(4J_x^2\beta_x^2\cos^4\phi_x - 24J_xJ_y\cos^2\phi_x\cos^2\phi_y + 4J_y^2\beta_y^2\cos^4\phi_y)$$

$$Q_x = \frac{1}{2\pi}\frac{\partial\langle\mathrm{H}\rangle}{\partial J_x} = \frac{1}{16\pi}K_4 L\left(J_x\beta_x^2 - 2J_y\beta_x\beta_y\right)$$

$$\frac{\partial Q_x}{\partial\epsilon_x} = \frac{1}{32\pi}\beta_x^2 K_4 L \qquad \frac{\partial Q_x}{\partial\epsilon_y} = -\frac{1}{16\pi}\beta_x\beta_y K_4 L \qquad \frac{\partial Q_y}{\partial\epsilon_y} = \frac{1}{32\pi}\beta_y^2 K_4 L$$

Equivalence of detuning cross terms

1st order detuning:

$$\frac{\partial Q_x}{\partial J_y} = \frac{1}{2\pi}\frac{\partial^2 \langle H \rangle}{\partial J_y \partial J_x} = \frac{\partial Q_y}{\partial J_x}$$

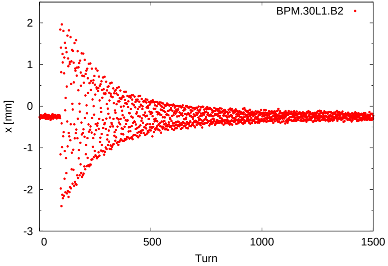
2nd order detuning:

$$\frac{\partial^2 Q_y}{\partial J_x^2} = \frac{1}{2\pi}\frac{\partial^3 \langle H \rangle}{\partial J_x^2 \partial J_y} = \frac{\partial^2 Q_x}{\partial J_x \partial J_y}$$

$$\frac{\partial^2 Q_x}{\partial J_y^2} = \frac{1}{2\pi}\frac{\partial^3 \langle H \rangle}{\partial J_x \partial J_y^2} = \frac{\partial^2 Q_y}{\partial J_x \partial J_y}$$

- Terms like $\frac{\partial^2}{\partial J_x \partial J_y}$ measured directly with diagonal kicks in H-V plane
- but from cross term equivalence actually determine all second order terms with pure H or V measurements
- Cross term equivalence gives good sanity check for data/fit quality

- **Traditional detuning measurement uses single kicks**
- Dipole kicker ramped up/down within single turn
- observe free betatron oscillations with turn-by-turn BPM data



- Oscillations **do not** decay
- Oscillations decohere
- **Measurement is destructive**
- Fresh beam for every kick

- BPM data post processed by **S**ingular **V**alue **D**ecomposition (**SVD**)

  R.Tomás & R.Calaga, *Statistical analysis of RHIC beam position monitors performance*, Phys.Rev.ST.AB,**7**,042801
- Identifies malfunctioning BPMs
- Removes uncorrellated noise from BPM signals

Action determined from mean peak-to-peak TbT data over BPMs

$$2J_{x,y} = \frac{\sum_{BPMs} \frac{\left(\frac{1}{2} Peak - to - Peak\right)^2}{\beta_{x,y}}}{N_{BPMs}}$$

- Various sources of uncertainty:
  beta-beat, coupling, BPM-scaling, BPM-nonlinearity, phase-space distortion from resonances

Tune determined via spectral analysis of TbT data

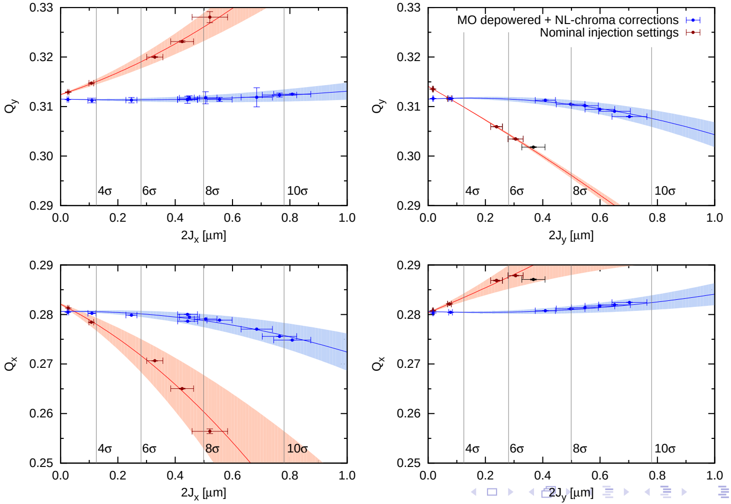
- Spectral analysis done via SUSSIX (interpolated FFT)
  R.Bartolini & F.Schmidt, CERN SL/Note 98-017(AP),
  'SUSSIX: a computer code for frequency analysis of non-linear betatron motion'
- Decoherence limits number of turns available for spectral analysis

- **Beams kicked to varying amplitudes for several angles in H-V plane** (at least pure kicks in H and V)
- **Limited by kicker strength, or machine / dynamic apertures**

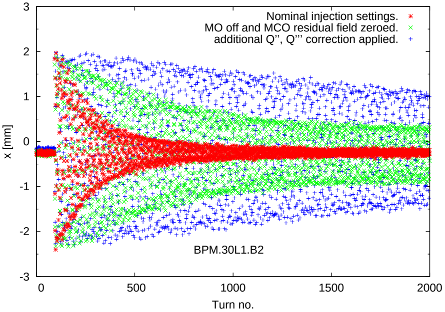
Traditional detuning measurements performed at injection in 2012

E.H.Maclean, R.Tomás, F.Schmidt, T.H.B.Persson. Phys.Rev.ST.AB,**18**,081002(2014)
Measurement of nonlinear observables in the Large Hadron Collider using kicked beams

- Nominal injection optics (Landau octupoles on)
- Landau octupoles off + beam-based correction of Q'' & Q'''

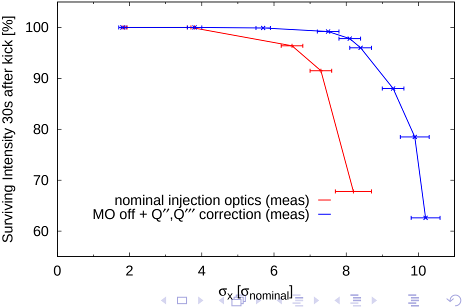


Beam-based correction also reduced decoherence and increased DA

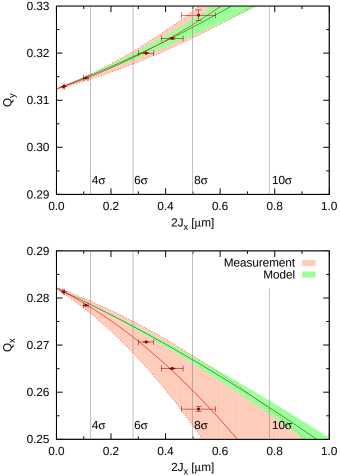


- Decoherence used as online check b_4 corrections worked

- In general Q'' correction wont correct detuning & DA
- Implies local correction



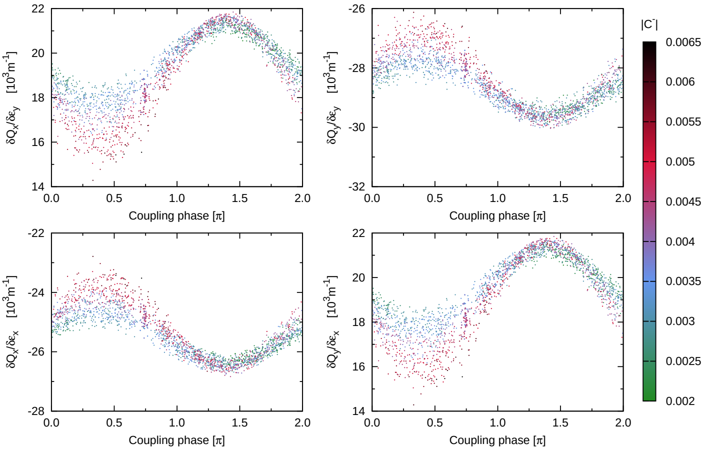
Comparison to LHC model



	[unit]	Meas'	± err	Model	± err
$\frac{\partial Q_x}{\partial \epsilon_x}$	$[10^3 \mathrm{m}^{-1}]$	−29	7	−27.0	0.8
$\frac{\partial Q_y}{\partial \epsilon_x}$		19	3	21	2
$\frac{\partial Q_x}{\partial \epsilon_y}$		24	4	21	2
$\frac{\partial Q_y}{\partial \epsilon_y}$		−32.8	0.4	−30.5	0.9
$\frac{\partial^2 Q_x}{\partial \epsilon_x^2}$	$[10^9 \mathrm{m}^{-2}]$	−60	30	−14	4
$\frac{\partial^2 Q_y}{\partial \epsilon_x^2}$		34	10	18	9

- Good agreement of 1st order detuning
- Qualitatively similar 2nd order

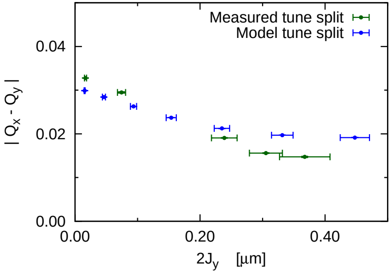
When comparing model and measurement must account for linear coupling

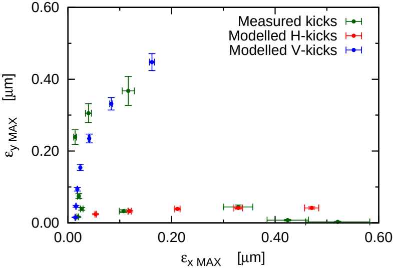


- In simulation linear coupling significantly affects the detuning... even far from the coupling resonance
- Not only δQ_{min} that's important: also phase of RDT
- Best option is a good correction at start of measurement

Coupling effect may also impinge upon detuning measurements

- Detuning with J_y moved tunes together
- Tune separation saturates
- Kicks at large J_y couple significantly into H-plane
- Observed in real LHC and simulation





- Behaviour associated with transverse planes becoming strongly coupled
- **Still very far from measured** $|C^-| = 0.0036$
- Implies existance of amplitude dependent δQ_{min}
- **Coupling stopband will distort detuning...**

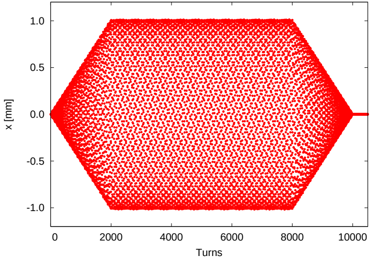
...but may have a nonlinear contribution

Single kick detuning measurements not possible at top energy in LHC

- Require new fill for every kick (destructive measurement)
- Machine protection

LHC is equiped with AC-dipole kickers

- Sinosoidally driven dipole kicker
- Driving frequency close (but not on!) natural tunes generates large response with little power, even at high energy
- If ramped up/down adiabatically kicks are non-destructive
- Routinely used for linear optics measurements throughout cycle



AC-dipole provides a tool to measure amplitude detuning at high energy

Actually rely on non-perfect adiabaticity
to excite natural tune lines in spectrum

AC-dipole modifies solutions to equations of motion:

$$x(s) = \sqrt{2J_x\beta_x(s)}\cos\phi_x(s) \quad \longrightarrow \quad x_D(s) = \sqrt{2J_x\beta_x(s)}\cos\phi_x(s) + \sqrt{2A_x\beta'_x(s)}\cos\phi_D(s)$$

· A, $\phi_D(s)$ are action angle variables of the driven oscillation

· β' is beta-function modified by the AC-dipole ($\beta' \approx \beta$)

Alters action-angle Hamiltonian & Q, eg octupole tune shift:

$$Q_x = \frac{1}{2\pi}\frac{\partial\langle \mathrm{H}\rangle}{\partial J_x} = \frac{1}{16\pi}K_4L\left(J_x\beta_x^2 - 2J_y\beta_x\beta_y\right)$$

$$\longrightarrow \quad \frac{1}{16\pi}K_4L\left(J_x\beta_x^2 + 2A_x\beta'_x\beta_x - 2J_y\beta_x\beta_y\right)$$

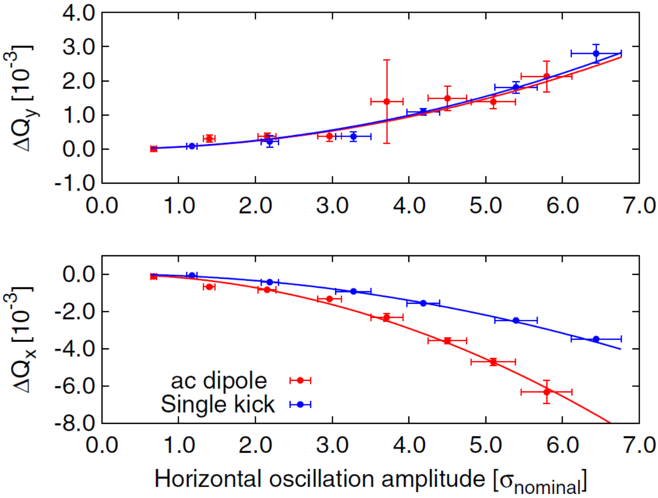
- $J_x << A_x \longrightarrow$ direct detuning $2\times$ expectation for free oscillations
- Detuning cross term unaffected
- Similar result for Q_y

In general:

Direct detuning terms from n^{th} order are $\frac{n}{2}$ larger when measured with AC-dipole than free oscillations. Cross terms are unaffected.

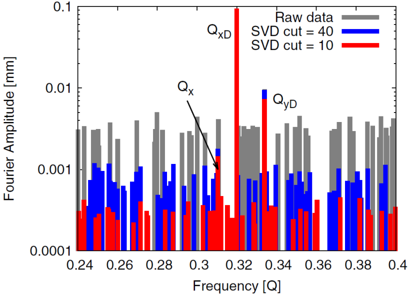
S.White, R.Tomás, E.H.Maclean, *'Direct amplitude detuning measurement with ac dipole'*, Phys.Rev.ST.AB,**16**,071002(2013)

Effect of AC-dipole on observed detuning was verified experimentally at injection



Measurement of natural tune variation with AC-dipole action is more challenging than with free oscillations

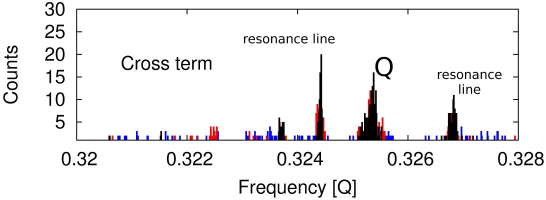
FFT of driven oscillations for different SVD cuts

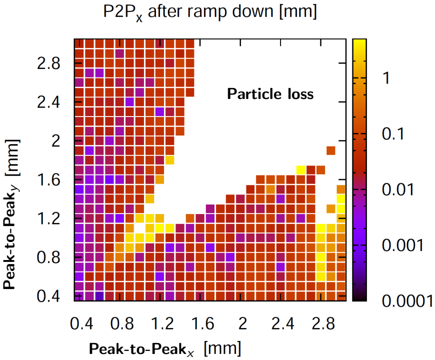


- Natural tune not a strong signal
- Need agressive SVD cleaning
- Additonal resonances

$$aQ_x + bQ_y = z$$
$$\rightarrow aQ_x + bQ_y + pQ_{ACx} + qQ_{ACy} = z$$

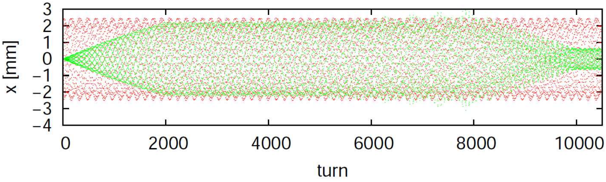
"Tune" identified by SUSSIX in ∼500 LHC BPMs





Watch out for dynamic aperture!

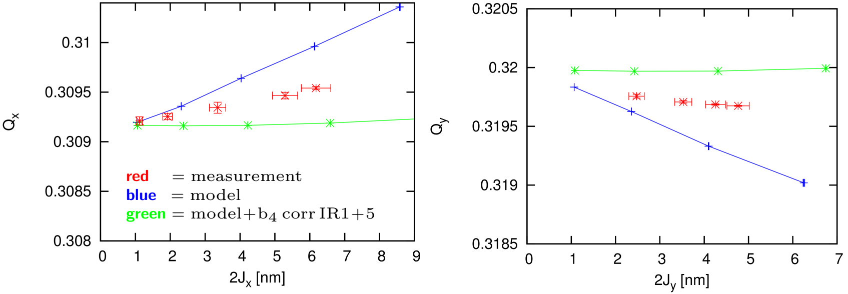
- **In general DA smaller than un-driven motion**
- **Kicking to DA will cause blow up and particle loss**
- **DA depends on Q_{AC}**



S.Monig et. al. *Short term dynamic aperture with AC dipoles*. CERN-ACC-NOTE-2015-0027

AC-dipole detuning measurements performed successfully at top energy

- Measured @ 6.5 TeV, $\beta^* = 0.4\,\mathrm{m}$ during 2015 MD
- Comparison of to MAD-X tracking simulations, **including AC-dipole**



Amplitude detuning measurements by A.Langner, comparison to simulation by S.Monig

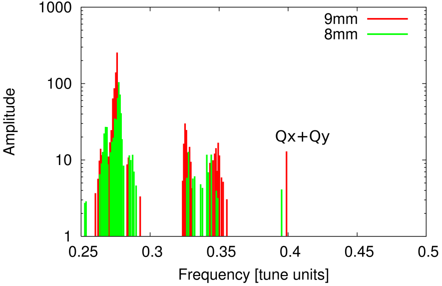
- For 0.4 m detuning dominated by b_4 errors in IR1+IR5
  (negligible contribution of arcs, which dominate Q'')
- **Implies** $\sim \frac{1}{2}$ **expected** b_4 **of IR1 + IR5**

Amplitude detuning is not the only probe available for NL-errors

- With AC-dipole detuning measurements gain spectral info for free
- Studied for Wire Excitation experiments in SPS

U.Dorda et. al. *Wire excitation experiments in the CERN SPS*, EPAC'08

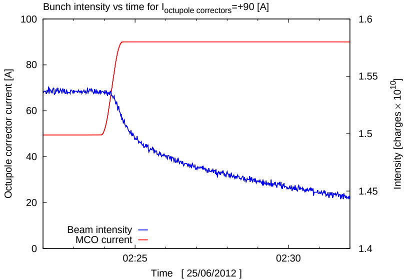
Vertical spectrum from simulations

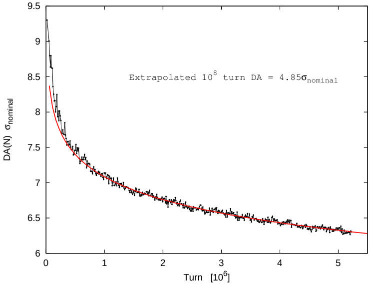


- Sextupole coupling line $Q_x + Q_y$
- Predict change in amplitude for change in beam-wire separation
- Qualitative agreement with observed spectrum

Amp' detuing
Single kicks
AC-dipole
Other method
Summary

- **Traditional DA measurement with single kicks used at injection**
  → Not viable at top energy
- **Alternative method is blow up emittance with transverse damper**
  → study long term DA via intensity loss & scaling laws
  → Demonstrated at injection, viable at top energy
  → being considered for optimization of NL-correctors in IR

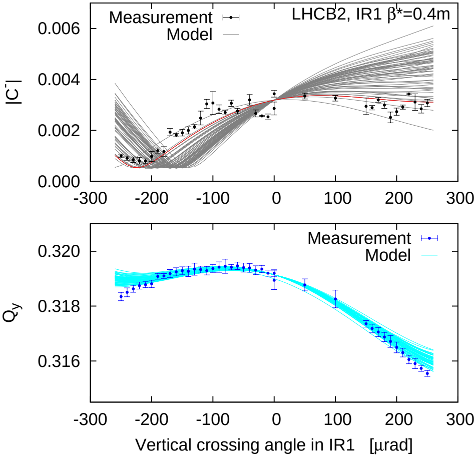




- possibilities for short-term DA measurement with AC-dipole

- **Currently study NL-errors in low-β^* IRs via feed-down**

E.H.Maclean, R.Tomás, M.Giovannozzi, T.H.B.Persson. Accepted to Phys.Rev.ST.AB
First measurement and correction of nonlinear errors in the experimental insertions of the CERN LHC



e.g. IR1 @ 0.4 m, 4 TeV

- $b_3 + a_4$ feed-down to $|C^-|$
- $a_3 + b_4$ feed-down to Q_y

- **Potentially quite useful in conjunction with other observables**

Conclusions

- **Traditional detuning measurement → single kicks**
- 1st & 2nd order detuning measured @ 450 GeV with single kicks
- Traditional measurement not viable at LHC top energy

- **AC-dipole measurment possible at LHC top energy**
- Theory predicts driven oscillations have different detuning
- Verified experimentally @ 450 GeV
- AC-dipole measurement tougher than single-kick
- Demonstrated at top energy → now routine

- **Various additional methods also available**
- long-term DA (ADT), short-term DA (AC-dipole), feed-down, spectra