useful analytical solutions in typical limiting cases

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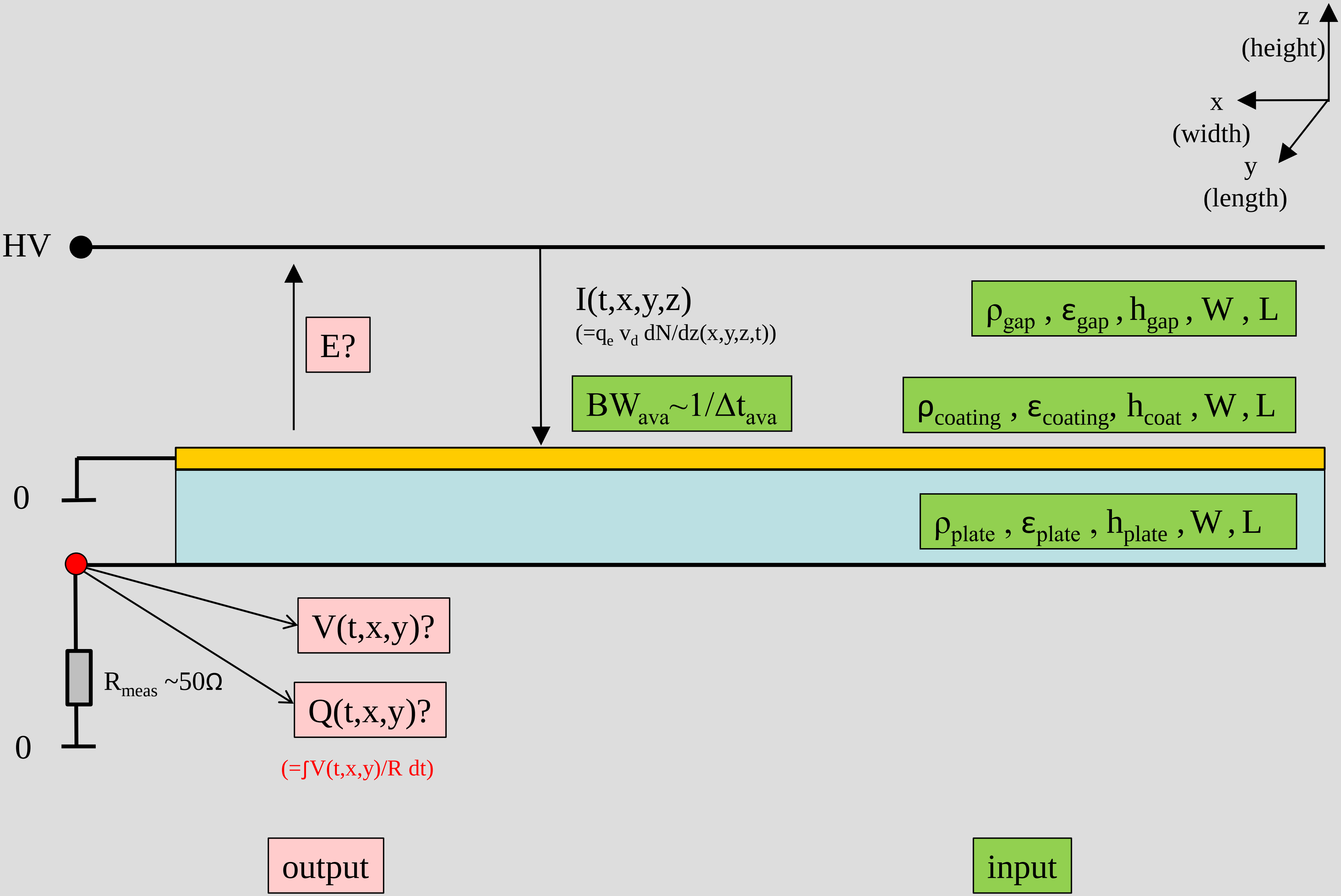
topical workshop on resistive electrodes
09-12-2015 , RD51 mini-week, CERN

1. Electric fields

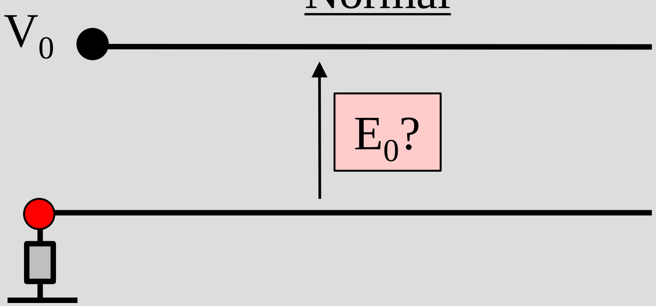


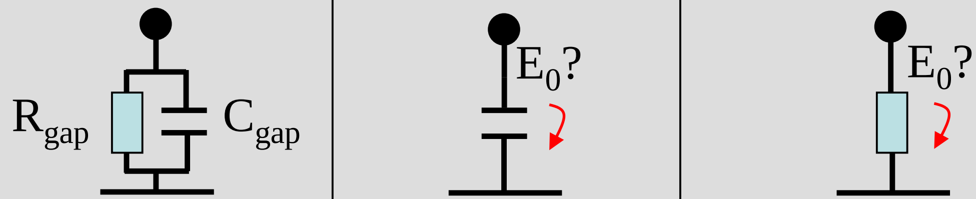
*In the following, it is assumed that length and width are enough so that no edge effects are present.

1) Field distribution (low rates)

t	t=0	t=∞ ($\gg R_{plate}(C_{gap}+C_{plate})$)

Normal

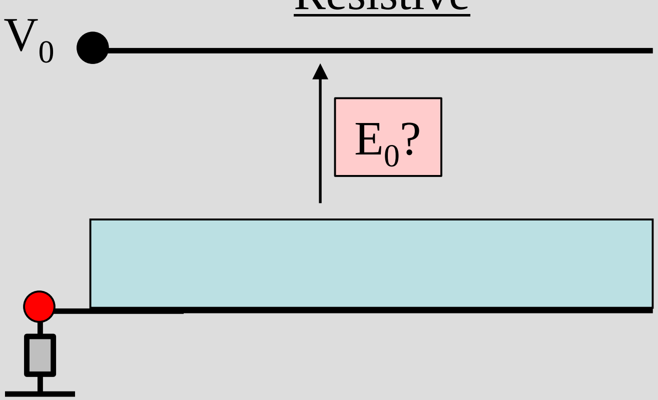


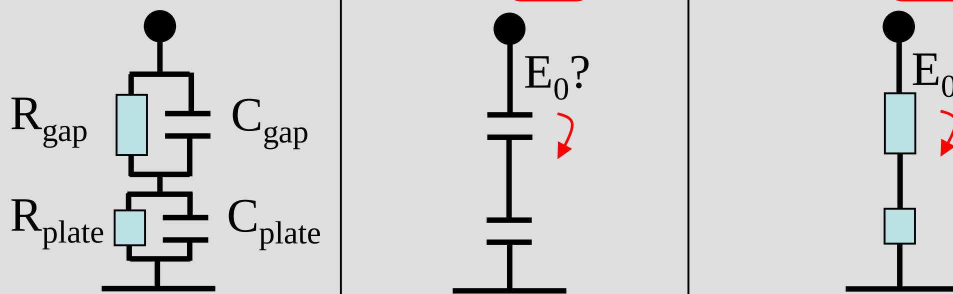


t=0: $E_0 = \frac{V_0}{h_{gap}}$

t=∞: $E_0 = \frac{V_0}{h_{gap}}$

Resistive

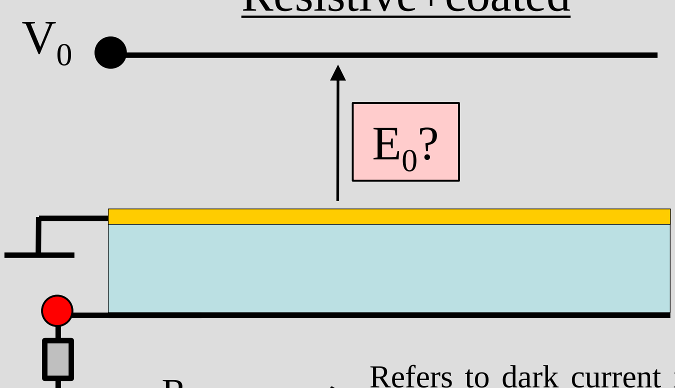


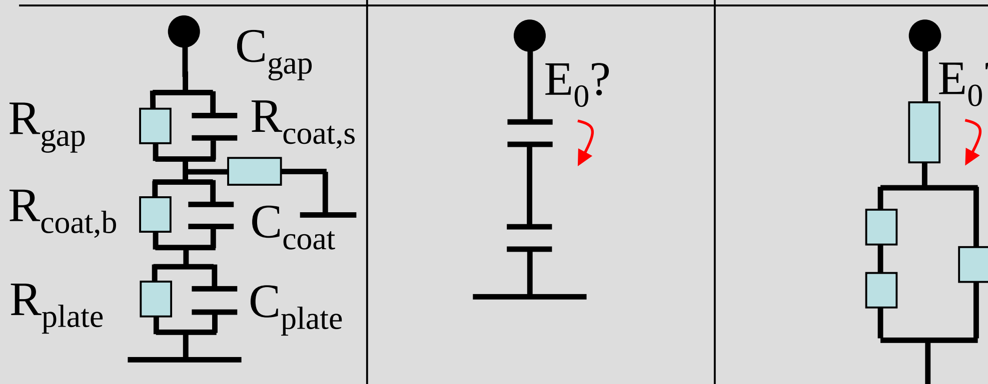


t=0: $E_0 = \frac{C_{plate}}{C_{plate}+C_{gap}} \frac{V_0}{h_{gap}}$

t=∞: $E_0 = \frac{R_{gap}}{R_{gap}+R_{plate}} \frac{V_0}{h_{gap}} \approx \frac{V_0}{h_{gap}}$

Resistive+coated





t=0: $E_0 \approx \frac{C_{plate}}{C_{plate}+C_{gap}} \frac{V_0}{h_{gap}}$

t=∞: $E_0 \approx \frac{R_{gap}}{R_{gap}+R_{plate}} \frac{V_0}{h_{gap}} \approx \frac{V_0}{h_{gap}}$

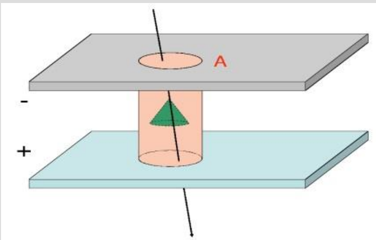
R_{gap} ⟶ Refers to dark current in the gap, whatever the origin, so $R_{gap} \sim V_0/I_{dark}$.

1) Field distribution (high rates)

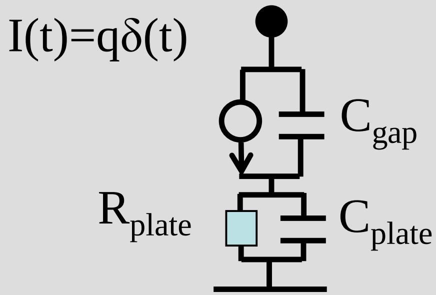
(Montecarlo results) **[Gon06]**

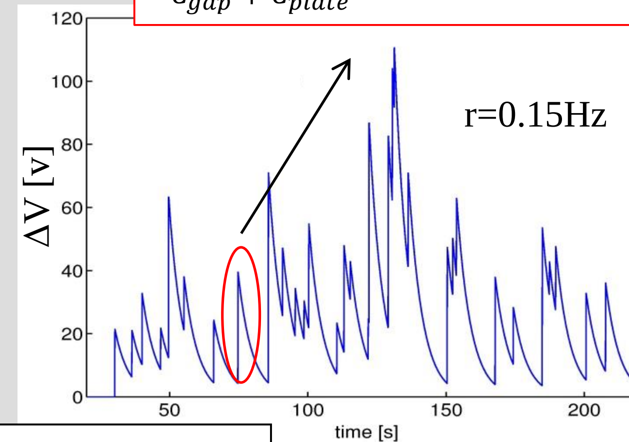
$$\frac{q}{C_{gap}+C_{plate}}\, e^{-R_{plate}(C_{gap}+C_{plate})t}$$

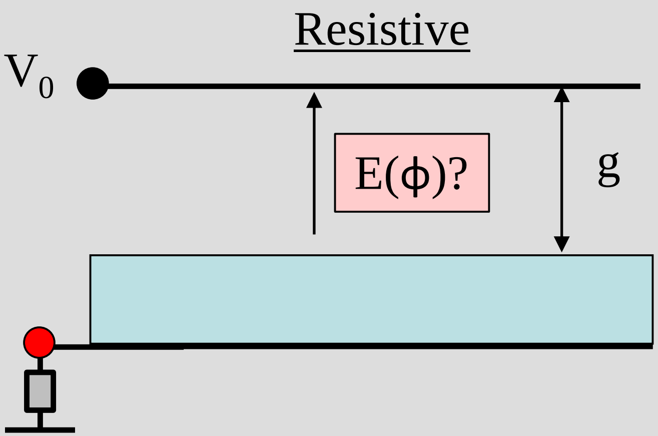
'cell model' **[Abb04]**

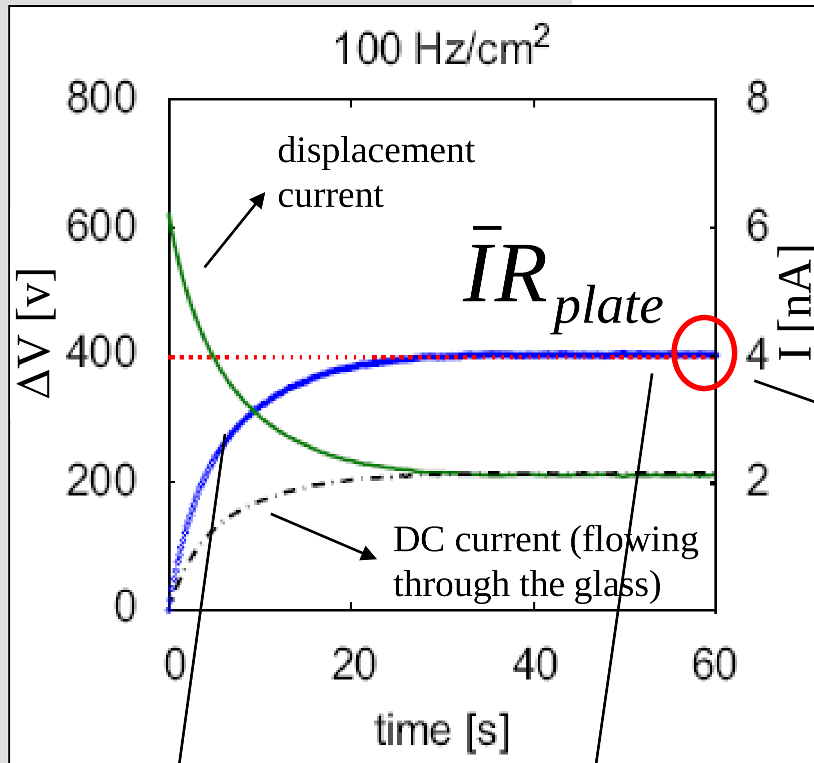


shot-noise circuit model



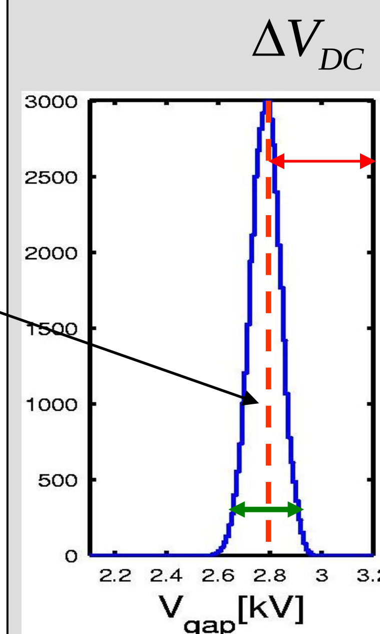






transient behavior, 'charging-up time'

voltage drop in stationary conditions (DC limit)



voltage fluctuations for each cell

1) Field distribution (high rates)

(analytical results) **[Gon06]**

average field

the steady-state ('DC') limit must satisfy:

$$\bar{E}(\phi) = \frac{1}{h_{gap}}(V_0 - \bar{I}R)$$

$$= \frac{1}{h_{gap}}(V_0 - \bar{q}\,\phi\rho h_{plate})$$

if we further assume that:

$$\bar{q} \sim a(\bar{V} - V_{th})$$

~true for saturated-avalanche RPC

$$\bar{E}(\phi) = \frac{V_{th}}{h_{gap}} + \frac{(V_0 - V_{th})}{h_{gap}(1 + a\phi\rho h_{plate})}$$

'DC model': detector response depends on average field. Good approximation for saturated-avalanche RPCs.

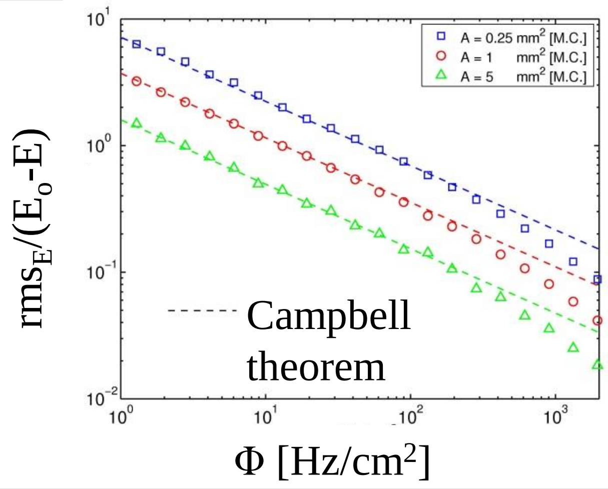
*A solution for exponential multiplication in μ-well given in **[Mor15]**, and it also seems to work!. Also at this workshop.*

fluctuations around the average field

Campbell theorem for shot noise (1910):

$$\frac{rms_E}{E_0 - \bar{E}(\phi)} = \sqrt{\frac{\left(1 + \frac{rms_q^2}{\bar{q}^2}\right)}{2\bar{N}}}$$

$$\bar{N} = A\tau\phi$$



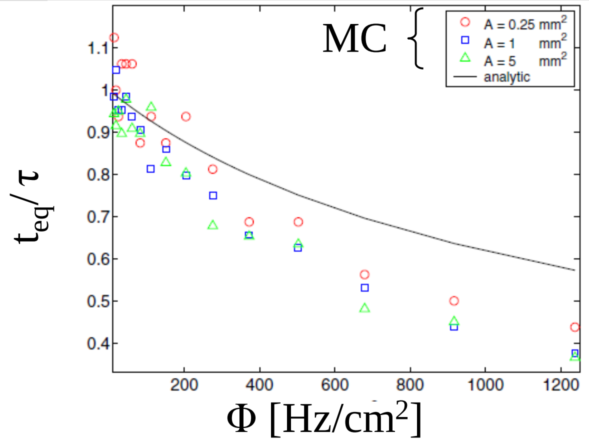
Attempts to generalize:
***[Lipp06]**: avoid circuit, use quasi-static approximation, epsilon f-independent.*

equilibration time

simple analytical estimate:

$$t_{eq} \sim \frac{\tau}{a\phi\rho h_{plate}} \ln(1 + a\phi\rho h_{plate})$$

$$\tau = R_{plate}(C_{gap} + C_{plate})$$



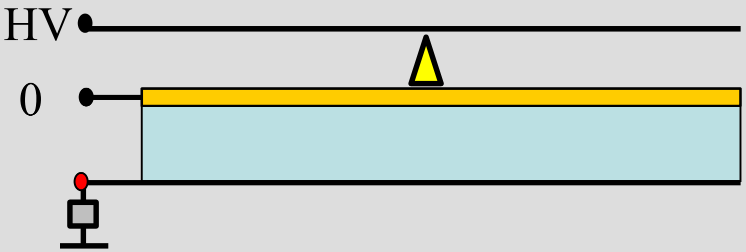
no dependence on the area A influenced by each avalanche!

More sophisticated attempts:

***[Bil09]:** solve electrostatic problem.*
***[Gon09]**: use glass response function.*

2. Induced signals

2) Induced signals

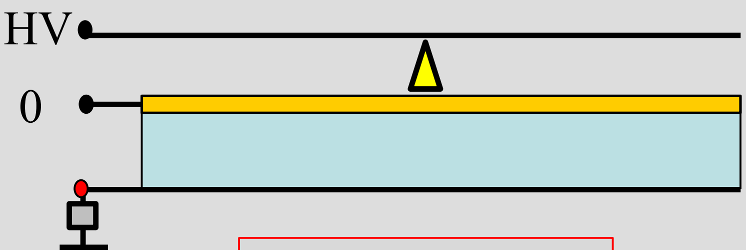


$\rho_{coating} = \rho_{plate}$

$\Delta t_{ava} << \rho_{plate}\varepsilon_{plate}$

(RPC, well and μ-well)

[Rie02a, 04]



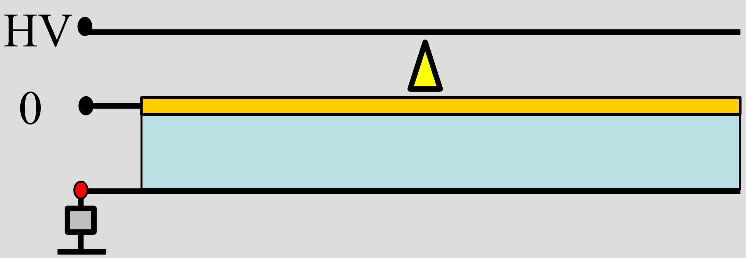
some particular limits

$\rho_{coating} \neq \rho_{plate}$

$\Delta t_{ava} \sim R_{\square}\varepsilon_0(h_{gap}+h_{plate})$

$\Delta t_{ava} << \rho_{plate}\varepsilon_{plate}$

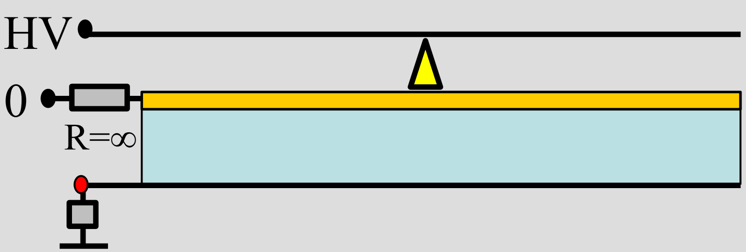
Resistive Micromegas



$\Delta t_{ava} << R_{\square}\varepsilon_0(h_{gap}+h_{plate})$

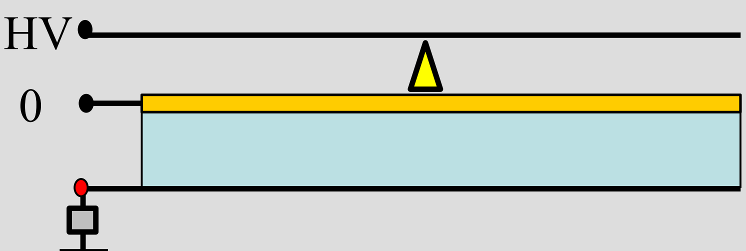
$\Delta t_{ava} << \rho_{plate}\varepsilon_{plate}$

Direct induction



$\Delta t_{ava} >> R_{\square}\varepsilon_0(h_{gap}+h_{plate})$

Capacitive coupling



$\Delta t_{ava} >> R_{\square}\varepsilon_0(h_{gap}+h_{plate})$

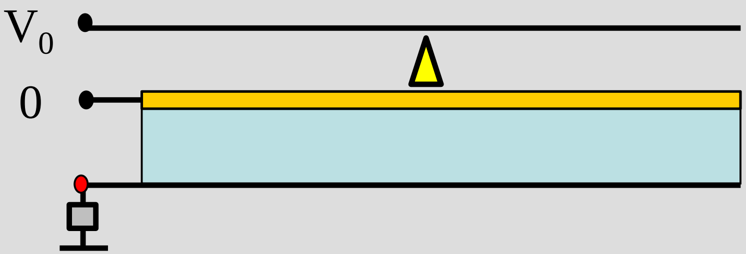
No pick-up

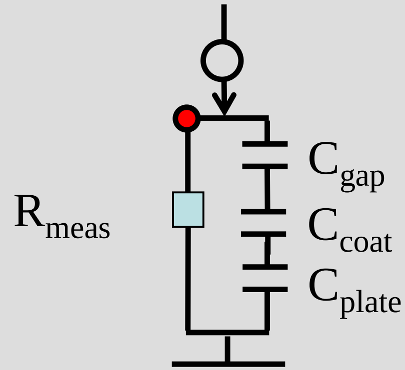
$R_{\square} = \rho_{coat}/h_{coat}\, L/W$

2) Induced signals. Limiting cases

Direct induction

$$\Delta t_{ava} << R_{\square}\,\varepsilon_0 (h_{gap}+h_{plate})$$





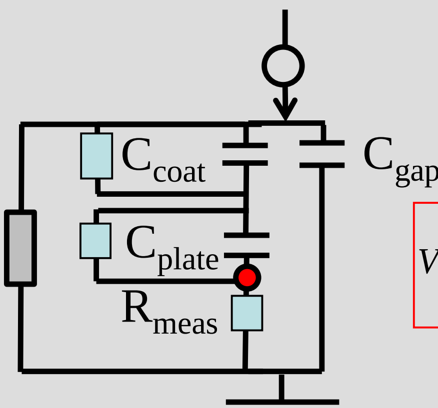
$$V_{meas}(t) = \frac{v_{drift}}{LW\varepsilon_0}\int_0^t \exp\left[\frac{t'-t}{R_{meas}C'}\right] N(t') q_e \, dt'$$

$$C' = \left(\frac{1}{C_{gap}} + \frac{1}{C_{coat}} + \frac{1}{C_{plate}}\right)^{-1}$$

solutions for $\Delta t_{ava} \sim R_{\square}\,\varepsilon_0 (h_{gap}+h_{plate})$ are scarce: **[Rie02a, 04, Dix06]**

position information preserved (e.g, if using strip readout)

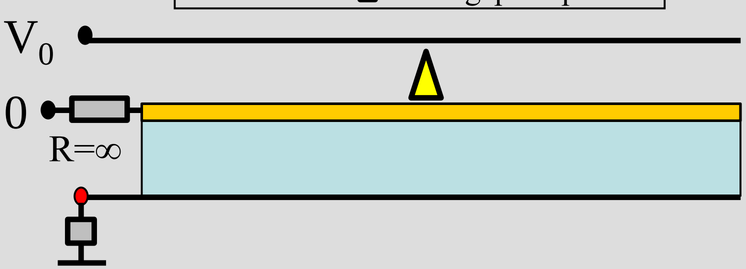
Direct induction, resistive charge spread and capacitive coupling take place simultaneously. Can be used advantageously to increase position resolution (P. Colas at this workshop).



$$V_{meas}(t) \simeq \frac{v_{drift}}{LW\varepsilon_0}\int_0^t \exp\left[\frac{t'-t}{R_{meas}C_{gap}}\right] N(t') q_e \, dt'$$

Capacitive coupling

$$\Delta t_{ava} >> R_{\square}\,\varepsilon_0 (h_{gap}+h_{plate})$$



position information destroyed

No net charge

3. Transmitted signals

3) Signal transmission (long structures*)

$$I_{meas,\,main}(t) \cong \left[\frac{T}{2}\right]\frac{1}{h_{gap}} v_d N(t) + reflections$$

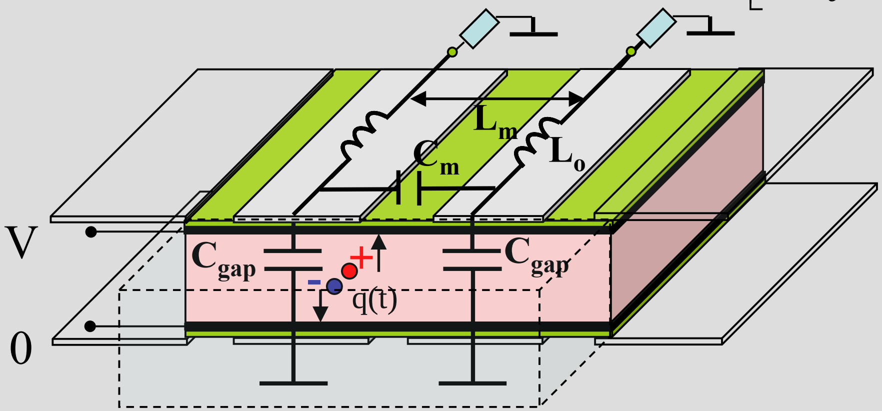
$$I_{meas.ct}(t) \cong \left[\frac{T}{2}\frac{R}{Z_c+R}\frac{C_m}{C_{gap}}\right]\frac{1}{h_{gap}} v_d N(t) + reflections$$

$\Delta t_{ava} << R_{\square}\varepsilon_0 h_{gap}$

*electrically long

$\Delta t_{ava} \leq L/c$

Normal



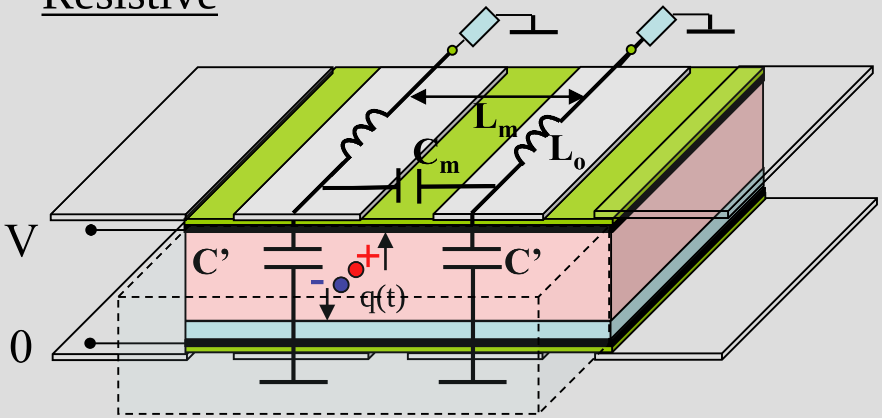
but compensation possible!

$$\frac{C_m}{C'+C_m} = \left.\frac{C_m}{C'+C_m}\right|_{vacuum} \quad = \quad \frac{L_m}{L_0} = \frac{C_m}{C'+C_m}$$

$$I_{means,main}(t) \cong \left[\frac{T}{2}\frac{R}{Z_c+R}\frac{C_m}{C'} f(N(t)) + \frac{T}{2}\right]\frac{1}{h_{gap}}\frac{C'}{C_{gap}} v_d N(t) + reflections$$

$$I_{meas,ct}(t) \cong \left[\frac{T}{2}\frac{R}{Z_c+R}\frac{C_m}{C'} + \frac{T}{2} f(N(t))\right]\frac{1}{h_{gap}}\frac{C'}{C_{gap}} v_d N(t) + reflections$$

Resistive



dispersive term due to the presence of insulators (modal dispersion).

[Wer02], [Gon11]

References

[Rie02a] W. Riegler, D. Burgarth, *Signal termination, propagation, cross-talk and losses in Resistive Plate Chambers* NIM A 481(2002)130.

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[Gon09] D. Gonzalez-Diaz, *The role of the resistive plate response function in bringing an RPC to a stationary situation*, NIMA 602(2009)713.

[Bil09] B. Bilki et al., *Measurement of the rate capability of resistive plate chambers*, JINST, 04(2009)P06003.

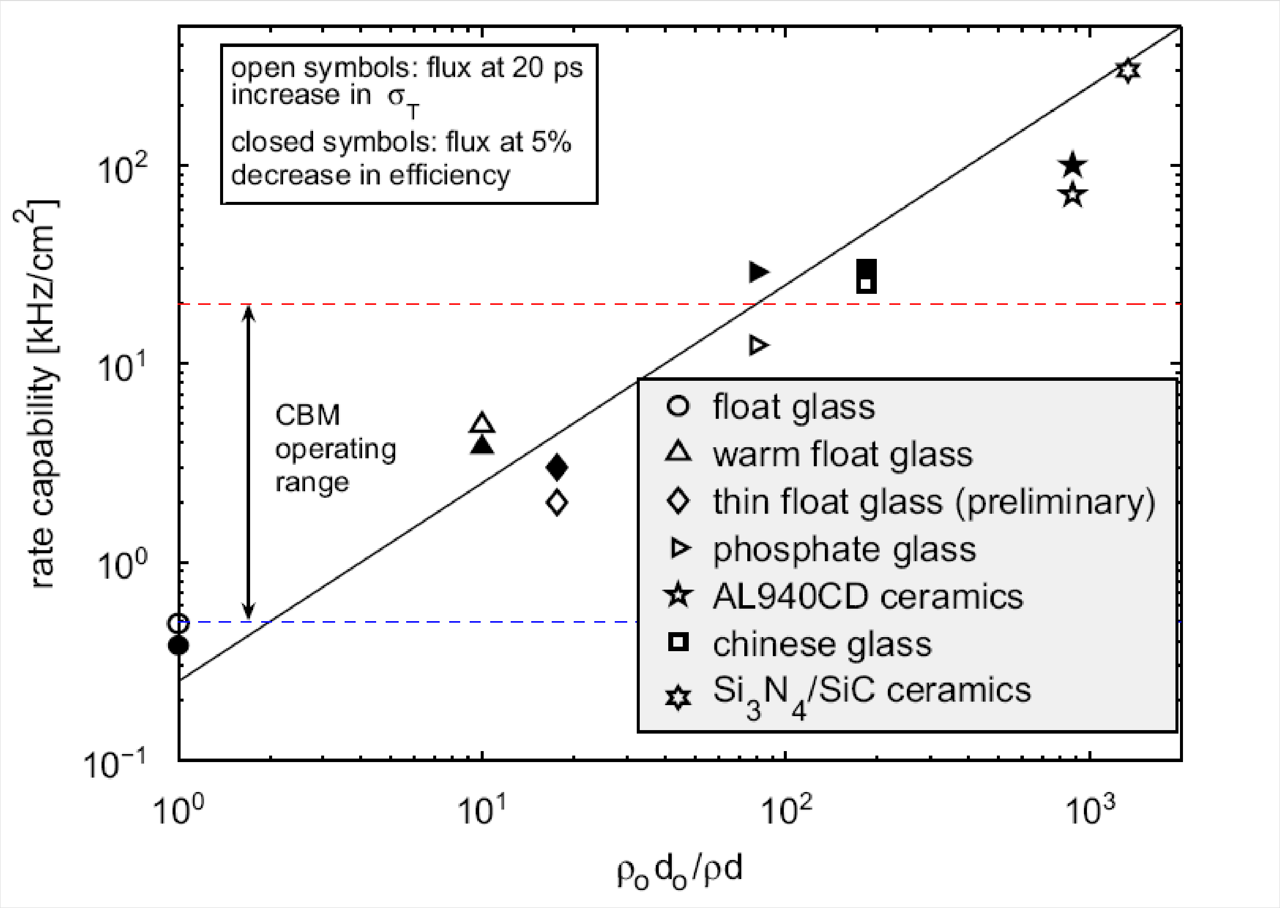
[Gon11] D. Gonzalez-Diaz, H. Chen, Y. Wang, *Signal coupling and signal integrity in Resistive Plate Chambers used for timing applications.* NIMA 648(2011)52.

[Dep12] I. Deppner et al, *The CBM time-of-flight wall*, NIMA 661(2012) 121.

[Mor15] G. Morello et al, JINST 10 (2015) 02, P02008 .

A popular way to make use of the 'DC model': RPCs.

$$\sigma_T = \sigma_0 + K_T \bar{q} \phi \rho h_{plate}$$
$$\varepsilon = \varepsilon_0 - K_\varepsilon \bar{q} \phi \rho h_{plate}$$



R decreasing

[Dep12]