AdS/CFT correspondence

duality $\sim$ **equality**

is between

Quantum Field Theory (a special one) in $d = 4$

and

Classical Gravity in $d = 5$ (for $N_c >> 1, \; g_{YM}^2 N_c >> 1$)

(at finite temperature)

some classical gravity

Einstein-Hilbert with nonzero cosmological constant (d dimensions)

- $$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = -\Lambda_d g_{\mu\nu} \ , \quad \mu, \nu = 0, 1, ..., d-1$$
- ansatz:
$$ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + r^2 d\Omega^2_{(d-2)} \ , \quad g_{tt} = -\frac{1}{g_{rr}} = -f(r)$$
- $$R(1 - \frac{d}{2}) = -\Lambda_d d, \quad R_{\mu\nu} = \frac{2\Lambda_d}{d-2} g_{\mu\nu}$$

AdS gravity

- with

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + p g_{\mu\nu} = -\frac{\Lambda_d}{8\pi G_d} g_{\mu\nu}$$

follows

$$\epsilon_{vac} = -p_{vac} = \frac{\Lambda_d}{8\pi G_d} < 0$$

for $\Lambda_d < 0$ ...

Anti-deSitter

[Hawking and Page 83]

solution

- $$R_{tt} = \frac{f}{2}\frac{[r^{d-2}f']'}{r^{d-2}} = -\frac{2\Lambda_d}{d-2}f \ ,$$

$$f' = -\frac{4\Lambda_d}{(d-1)(d-2)}r \quad (+const) \ ,$$

$$f = 1 - \frac{2\Lambda_d}{(d-1)(d-2)}r^2 \equiv 1 + \frac{r^2}{L^2}$$

$r >> L \equiv R$ (radius of AdS space $=$ distance scale)

$$ds^2 = -\frac{r^2}{R^2}dt^2 + \frac{R^2}{r^2}dr^2 + \ldots$$

- AdS_5

$$ds^2 = \frac{r^2}{R^2}(-dt^2+d\vec{x}^2)+\frac{R^2}{r^2}dr^2 = \frac{R^2}{z^2}(-dt^2+d\vec{x}^2+dz^2) \ , \quad z = \frac{R^2}{r}$$

exercise

consider AdS_5 metric, but start from coordinates $y^\mu, \mu = 0, 1, ..., 5$:

$$(y^0)^2 - (y^1)^2 - .. - (y^4)^2 + (y^5)^2 = R^2$$

and change to

$$x^\mu = \frac{R}{r}(y^0, y^1, y^2, y^3) \ \ , \ \ r = y^4 + y^5 \ \ ,$$

$$y^4 - y^5 = \frac{R^2}{r} + \frac{r}{R^2}((x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2)$$

leads to

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}dr^2$$

AdS_5 has $SO(2,4)$ symmetry

Hawking temperature

black hole metric:

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + .. \, , \; g_{tt} = -\frac{1}{g_{rr}} = -[1 - (\frac{r_0}{r})^{(d-3)}]$$

expand around the horizon $r \approx r_0$:

$$g_{tt} \approx -\frac{d-3}{r_0}(r - r_0) = -\gamma_t(r - r_0) \; , \; g_{rr} \approx \frac{\gamma_r}{r - r_0}, \; \gamma_r = \frac{r_0}{d-3}$$

claim:

$$T_H = \frac{1}{4\pi}\sqrt{\frac{\gamma_t}{\gamma_r}} = \frac{d-3}{4\pi r_0}$$

and AdS_5: $\gamma_t = 1/\gamma_r = 4r_0/R^2$

$$T_H = \frac{r_0}{\pi R^2}$$

proof: with $\rho^2 = \frac{4r_0(r-r_0)}{d-3}$

$$ds^2 = \rho^2 (\frac{d-3}{2r_0})^2 d\tau^2 + d\rho^2 + ..., \quad euclidean \; d\tau^2 = -dt^2$$

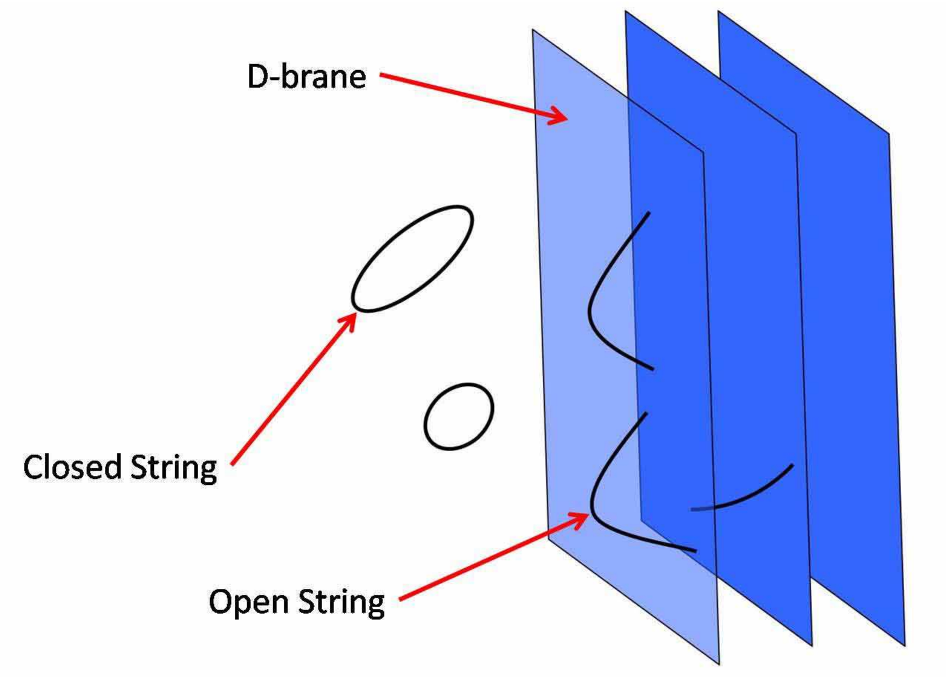
$$\longrightarrow$$

$$ds^2 = \rho^2 d\phi^2 + d\rho^2 \; , \quad \phi = \frac{d-3}{2r_0}\tau$$

requirement: periodicity $\phi \to \phi + 2\pi,$ i.e. NO conical singularity

$$2\pi = \frac{(d-3)}{2r_0}\beta \; , \quad \beta = \frac{1}{T_H}$$

$D-$branes



[from Myers and Vazquez 08]

dynamical walls on which strings can end:

theory of open strings living on $N_c - D3-$branes ($\mathcal{N} = 4$ SYM, $d = 4$) $\Longleftrightarrow$
gravity theory of fields living in the space curved by the branes ($AdS_5,\ d = 5$)

AdS_5 **metric at finite** T

generalization of the Einstein-Hilbert-Maxwell equation
- strong analogy to Reissner-Nordström black hole

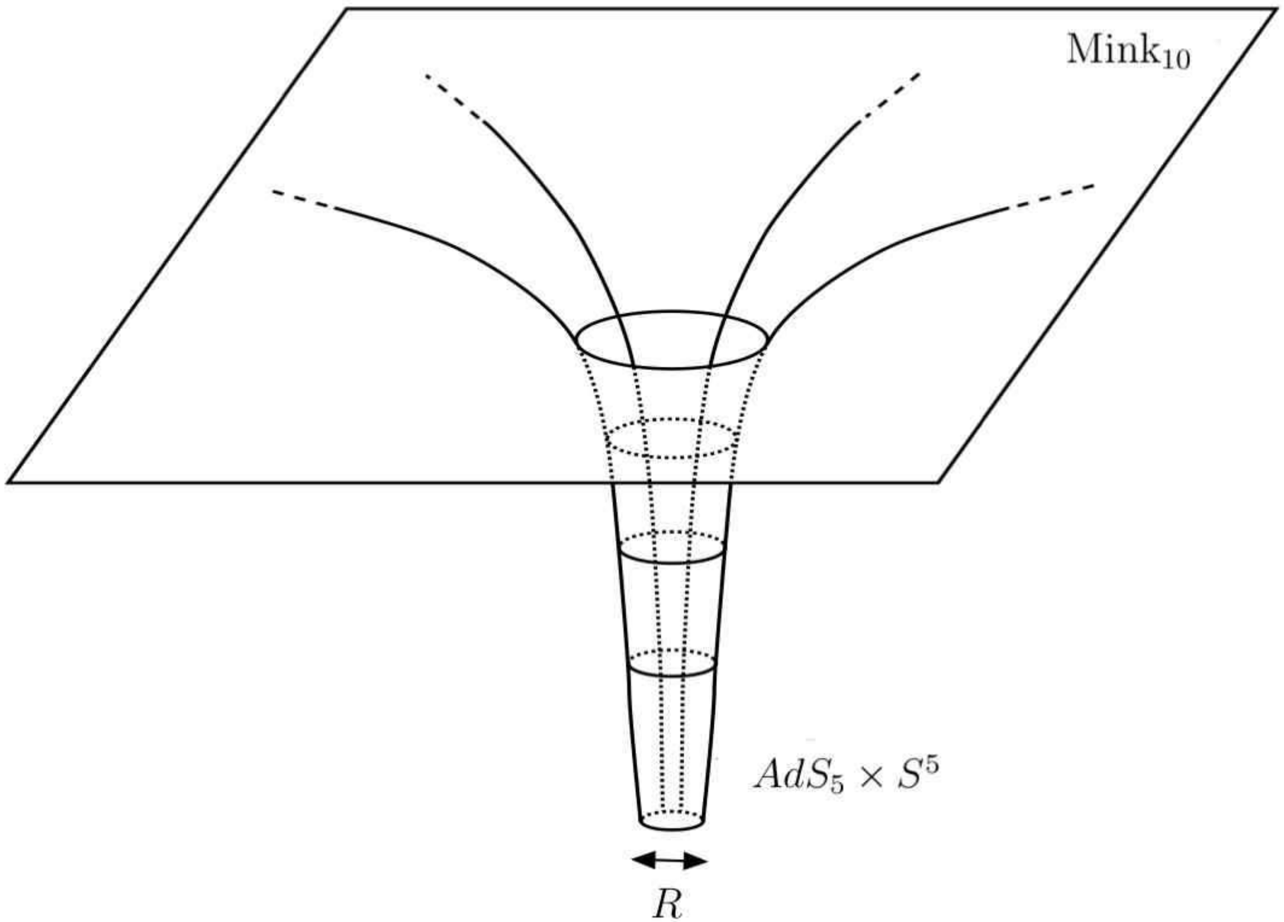
$$S = \frac{1}{16\pi G_{(10)}} \int d^{10}x \, \sqrt{-g} \, [R - \frac{F_5^2}{2\cdot 5!} + ...] \, ,$$

$$\int_{s^5} *F_5 \propto N_c \, , \; G_{(10)} = \frac{\pi^4 L^8}{2N_c^2}, \; H(r) = 1 + \frac{L^4}{r^4} \; , \quad f(r) = 1 - \frac{r_0^4}{r^4}$$

$$ds^2 = -\frac{f(r)}{\sqrt{H(r)}} dt^2 + \frac{1}{\sqrt{H(r)}} (dx_1^2 + dx_2^2 + dx_3^2) + \frac{\sqrt{H(r)}}{f(r)} dr^2 + r^2 \sqrt{H} d\Omega_5^2$$

near extremal black $D3-$ brane metric with horizon $r = r_0$
for $r_0 < r << L = R$, i.e. factorized metric for $AdS_5 \otimes S^5$

$$ds^2 = \frac{r^2}{R^2}(-f(r)dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f(r)} dr^2 + R^2 d\Omega_5^2$$



[from D. Mateos]

space-time around $D3-$branes

AdS/CFT [Maldacena 98]

strongly coupled quantized conformal gauge theory
in $d = 4$ dimensions ($\mathcal{N} = 4$ SYM with $8N_c^2$ (1 gauge and 6 scalar) bosons and ($4N_c^2$) Weyl fermions) [[NOT QCD !]]

$$\Longleftrightarrow$$

weakly coupled classical supergravity (type IIB)
in $d = 10$ dimensions (on $AdS_5 \times S^5$)
via holographic property: radial coordinate $r_0 \leq r < \infty$ with gauge theory on the boundary at ∞
in the limit:
't Hooft coupling $\lambda = g_{YM}^2 N_c$ is large, $N_c \to \infty, \quad g_{YM}^2 << 1$
i.e. string coupling $g_s = \frac{g_{YM}^2}{4\pi} << 1$ – NO LOOPS

and

small curvature $\frac{l_s^4}{R^4} = \frac{1}{\lambda} << 1$ – RADIUS R of CURVATURE is LARGE compared to the STRING SCALE $l_s = \sqrt{\alpha'}$

symmetries

duality maps the operators of QFT to the boundary values of the SUGRA fields

- QFT ($\mathcal{N} = 4$, $d = 4$ CFT)

$$S = -\frac{1}{g_{YM}^2} \int d^4x \, [\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} D_\mu \Phi^a_i D^\mu \Phi^a_i + ...]$$

.... massless Weyl fermions + interactions , $i = 1, .., 6, a = 1, ..., N_c^2$
has Poincaré and conformal symmetry $SO(2,4)$, e.g. $x^\mu \to \lambda x^\mu$, $\Phi^a_i(x) \to \Phi^a_i(\lambda x)$, etc, $\beta(\mu) = \mathbf{0}$, and $SU(\mathcal{N} = 4)$ symmetry and $SU(N_c)$

- $AdS_5 \otimes S^5$:

$$SO(2,4) \otimes SO(6) = SO(2,4) \otimes SU(4)$$

besides $SU(N_c)$ due to N_c parallel $D-3$ branes

relaxation phenomena/strong coupling limit

hydrodynamic transport coefficients by gauge/gravity duality:
compare with quite involved AdS/CFT-gravity calculations at Hawking temperature $T_H = \frac{r_0}{\pi R^2}$,
and for momentum $\omega, k << T_H$,
e.g. from sound channel dispersion up to $O(k^3)$, etc :

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad \tau_\pi = \frac{2 - \ln 2}{2\pi T}, \quad \kappa = \frac{\eta}{\pi T} = 2\lambda_1 \,,$$

$$\lambda_2 = -\frac{\ln 2}{2\pi T}\eta \,, \quad \lambda_3 = 0$$

conformal $\Pi^{\mu\nu}$

all 5 second order terms classified by conformal symmetry

constitutive relation (d= 4):

$$\begin{aligned}\Pi^{\mu\nu} \;=\;& -2\eta\sigma^{\mu\nu} + 2\eta\tau_\pi u^\lambda \mathcal{D}_\lambda \sigma^{\mu\nu} \\ & +4\lambda_1 \sigma^{<\mu}{}_\lambda \sigma^{\nu>\lambda} + 2\lambda_2 \sigma^{<\mu}{}_\lambda \Omega^{\nu>\lambda} + \lambda_3 \Omega^{<\mu}{}_\lambda \Omega^{\nu>\lambda} \\ & +2\kappa u_\alpha C^{\alpha\mu\nu\beta} u_\beta\end{aligned}$$

$C^{\alpha\beta\gamma\delta}$... Weyl tensor, $\Omega^{\alpha\beta}$... antisymmetric vorticity tensor

[Baier, Romatschke, Son, Starinets, Stephanov 07]

[Bhattacharyya, Hubeny, Minwalla, Rangamani 07, Loganayagam 08]

Bekenstein- Hawking entropy

thermodynamics: $dS = \frac{dE}{T}$

- Schwarzschild BH ($G = G_{(4)}, T_H = \frac{1}{4\pi r_0}$)

$$E = M = \frac{r_0}{2G} \; , \; dE = \frac{dr_0}{2G} \; , \; dS = \frac{4\pi}{2G} r_0 dr_0 \; ,$$

entropy

$$S = \frac{\pi r_0^2}{G} = \frac{A}{4G}$$

a universal relation with $A = 4\pi r_0^2$... area of the horizon

Bekenstein-Hawking entropy, cont.

- $AdS_5 \otimes S^5$: $T_H = \frac{r_0}{\pi R^2}$, $S = \frac{A_{(8)}}{4G_{(10)}}$

$$A_{(8)} = (\frac{r_0}{R})^3 V_3 R^5 \Omega_{(5)} = \pi^6 T_H^3 R^8 V_3 \ , \ G_{(10)} = \frac{\pi^4 R^8}{2N_c^2}$$

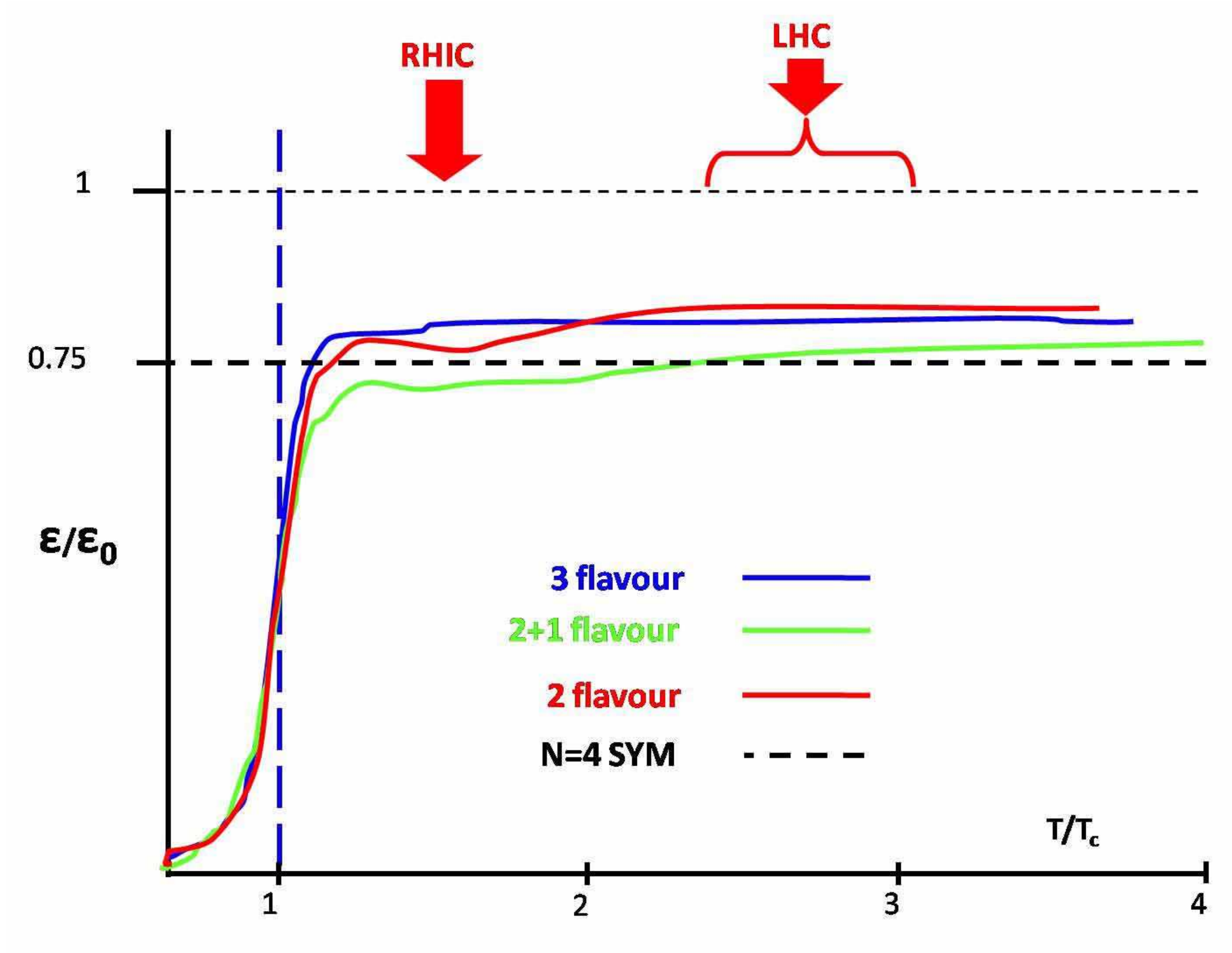
gives for the entropy density

$$s_{BH} = \frac{S}{V_3} = \frac{\pi^2 N_c^2}{2} T_H^3$$

NOTE:

$$s_{Boltzmann} = \frac{4p}{T} = \frac{4\pi^2 T^3}{90} [\frac{7}{4} \cdot 4 + 2 + 6] N_c^2 = \frac{2\pi^2 N_c^2}{3} T^3$$

$$s_{BH} = \frac{3}{4} s_{Boltzmann}$$



energy density of QCD and SYM - via BH entropy

[from Myers and Vazquez 08]

master formula for AdS/CFT

schematically in terms of coinciding partition functions:

$$\int e^{iS_{4d}^{gauge}+i\Phi_0 O} = \int e^{iS_{5d}[\Phi]} \simeq e^{iS^{classical}[\Phi_0]}$$

S_{5d} is computed with non-trivial boundary condition (holography)

$$\Phi(t,\vec{x},r) \stackrel{r->\infty}{=} \Phi_0(t,\vec{x})$$

$$\Longrightarrow$$

quantum correlation $=$ classical two-point function

$$G_R(x,y) = -i < O(x)O(y) >= -\frac{\delta^2 S^{classical}}{\delta\Phi_0(x)\delta\Phi_0(y)}|_{r=\infty}$$

gauge: $O = T_{\mu\nu}$ .. energy-momentum tensor
gravity: $\phi = g_{\mu\nu}$ .. graviton

correlators from gravity, $d = 4$

response of the fluid to small metric perturbations
example: $g_{xy} = h_{xy}(t, z, r) \neq 0,\ u^0 = 1,\ T =$ const
e.g. it leads from Christoffel symbols in the covariant derivatives to linear approximation:

$$\sigma^{xy} = \sigma_{xy} \approx \frac{1}{2}(\Gamma^{y}_{x0} + \Gamma^{x}_{y0}) \approx \frac{1}{2}\partial_t h_{xy}$$

and for the stress tensor

$$\delta\Pi^{xy} \approx -\eta\partial_t h_{xy} + \eta\tau_\pi\partial_t^2 h_{xy} - \frac{\kappa}{2}[\partial_t^2 h_{xy} + \partial_z^2 h_{xy}]$$

Fourier transform $h(t, z) = \exp\left(-i\omega t + ikz\right)\, h(\omega, k)$, etc.

$$\delta\Pi^{xy}(\omega, k) = -G_R^{xy,xy}(\omega, k) h_{xy}(\omega, k)$$

via linear response

correlators, cont.

retarded Green function:

$$G_R^{xy,xy}(\omega, k) = -i\eta\omega + \eta\tau_\pi\omega^2 - \frac{\kappa}{2}(\omega^2 + k^2) + \dots.$$

i.e. Kubo formula

$$\eta = \lim \frac{1}{\omega}\; iG_R^{xy,xy}(\omega, \vec{0})|_{\omega=0}$$

$$= \lim \frac{1}{\omega} \int\; dtd^3x\;\; \exp\left(i\omega t\right) < [T_{xy}(t, \vec{x}), T_{xy}(0, \vec{0})] > |_{\omega=0}$$

quasinormal mode

G_R from gravity action with AdS_5 metric:

$$S_{5d} = \frac{N_c^2}{8\pi^2 R^3} \int d^5x \, [\sqrt{-g} R_{5d} + ..] \ ,$$

factor from $\frac{1}{16\pi G_{(10)}} R^5 \Omega_{(5)}, \ \Omega_{(5)} = \pi^3$

and "dictionary" $G_{(10)} = 8\pi^6 g_s^2 l_s^8 \ , \ g_{YM}^2 N_c = 4\pi g_s N_c = R^4/l_s^4$

scalar mode and EoM (of Heun's type)

$$\sqrt{-g} R_{(5d)} \to -\frac{1}{2} \sqrt{-g} g^{\mu\nu} \ \partial_\mu \Phi \partial_\nu \Phi \ , \ \Phi \equiv h_{xy} \ :$$

$$\Phi(t, z, u = \tfrac{r_0^2}{r^2}) = \exp\left(-i\omega t + ikz\right) \Phi_k(u) \ , \ \Phi_k' = \tfrac{d}{du} \Phi_k \ ,$$

$$\Phi_k'' - \frac{1+u^2}{u(1-u^2)} \Phi_k' + \frac{1}{4\pi^2 T^2} \frac{\omega^2 - k^2(1-u^2)}{u(1-u^2)^2} \Phi_k(u) = 0$$

quasinormal mode, cont.

solution with ansatz $\Phi_k(u) = f_k(u)\Phi_0(k)\ ,\ f_k(0) = 1$ and boundary condition: only the incoming wave, the one which moves toward the horizon at $r- > r_0 \equiv u = 1$, i.e. nothing comes out the horizon

$$f_k(u) = (1-u^2)^{-i\omega/(4\pi T)} F_k(u) \approx 1 + i\frac{\omega}{4\pi T}u^2 + ...$$

note, near $u \sim 1$:

$$\exp\left(-i\omega t\right) f_k(u) \to \exp\left[-i\omega(t + r^*)\right]\ ,\ r^* = \frac{\ln(1-u)}{4\pi T}$$

moves from $u = 0$ at $t = 0$ to $u = 1$ at $t- > \infty$

classical gravity action

inserting the solution into $S_{(5d)}$

$$S_{(5d)} \approx \frac{N_c^2}{8\pi^2 R^3} \int d^4x \int_0^1 \, du \, \sqrt{-g}(-\frac{1}{2})g^{uu}\partial_u\Phi\partial_u\Phi$$

and keeping only the surface contribution at $u = 0$, $\Rightarrow$

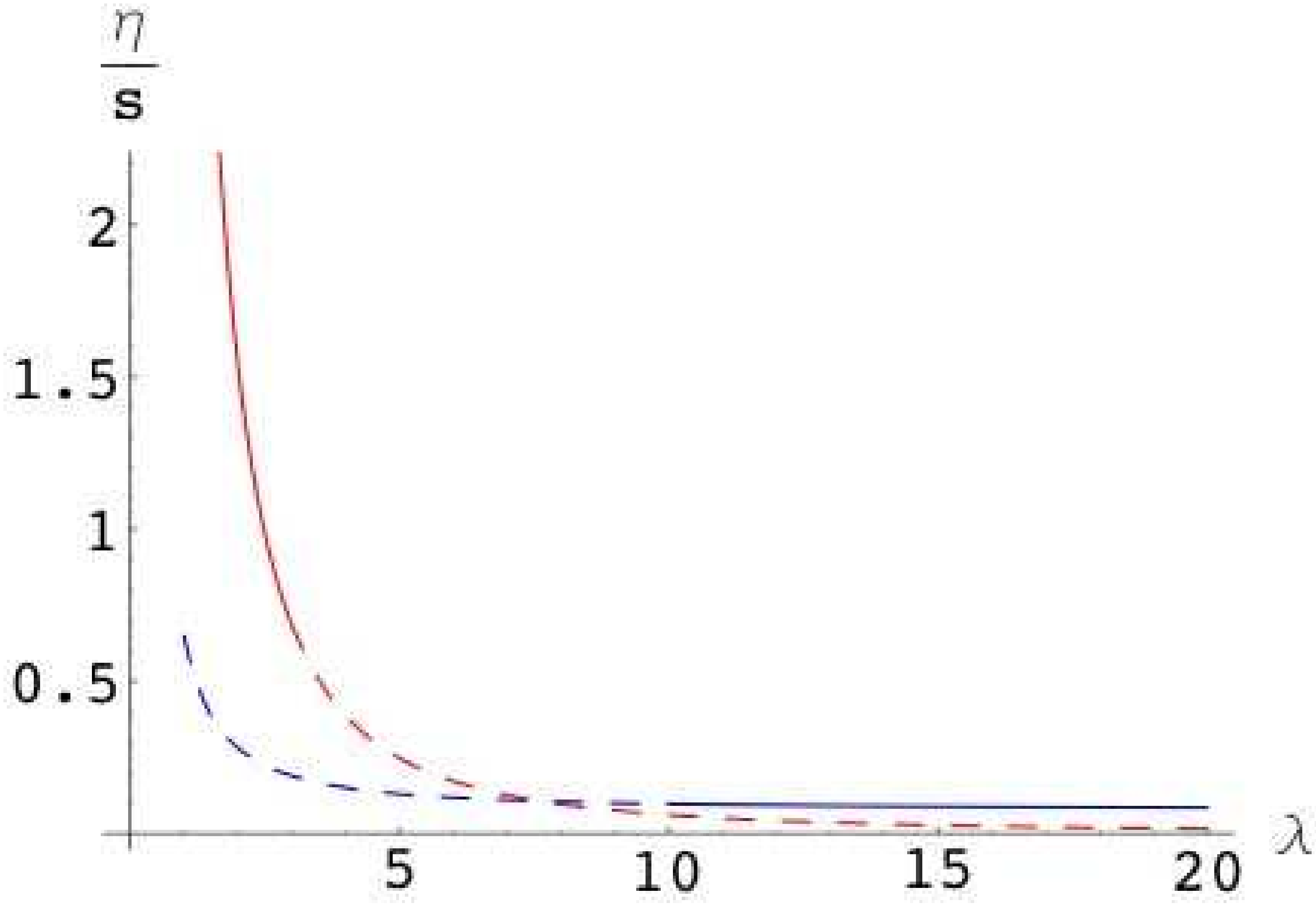
$$S^{classical} = -\frac{\pi^2 N_c^2 T^4}{8} \int \frac{d^4k}{(2\pi)^4} \, \Phi_0(-k)[\frac{(1-u^2)}{u} f_k'(u) f_k(u)]|_{u=0}\Phi_0(k)$$

and finally

$$G_R^{xy,xy}(\omega, k) = \frac{\pi N_c^2 T^3}{8}[-i\omega + \frac{1-2\ln 2}{2\pi T}\,\omega^2 - \frac{1}{2\pi T}\,k^2 \pm \ldots\ldots] \,,$$

$$\eta = \frac{\pi N_c^2 T^3}{8} \,, \;\tau_\pi = \frac{2-\ln 2}{2\pi T} \,, \;\kappa = \frac{\eta}{\pi T}$$

η/s



behaviour of η/s as a function of the 't Hooft coupling

shear viscosity η

near equilibrium: $\eta \;\simeq\; \epsilon \bar{v} \lambda_f, \quad entropy\ density \;\; s \simeq \epsilon / m \quad \rightarrow$

$$\frac{\eta}{s} \simeq m \bar{v} \lambda_f \simeq \hbar \, \frac{mean\ free\ path}{deBroglie\ wavelength}$$

- dilute system (QFT $->$ kinetic theory $->$ hydro):
  scale $\lambda_f \;\rightarrow \frac{\eta}{s} >> \hbar$, e.g. pQCD $(N_f = 0)$

$$\frac{\eta}{s} \simeq 3.8 \frac{1}{g^4 \ln(2.8/g)} \simeq O(1) \;\; for\ g = 2.5$$

  BUT with $\ln(2.8/g) \simeq O(1) : \frac{\eta}{s} \simeq 0.1 \quad \rightarrow$ sensitive to constant under the log !

  [Arnold, Moore and Yaffe 03]

- strongly coupled system (QFT $->$ hydro):
  scale $1/T \;\rightarrow \frac{\eta}{s} = \frac{\hbar}{4\pi} \simeq 0.08$

  [Policastro, Son and Starinets 01]

sound mode

consistency requirement from sound channel:

$$\omega_{1,2} = \pm c_s k - i\Gamma\, k^2 \pm \frac{\Gamma}{c_s}(c_s^2 \tau_\pi - \frac{\Gamma}{2})\, k^3 + \mathcal{O}(k^3)\ ,$$

with $\Gamma = \frac{2}{3}\frac{\eta}{\epsilon+p}$

from gravity perturbation $g_{tz} = h_{tz} << 1$

$$\omega_{1,2} \pm \frac{1}{\sqrt{3}}\, k - i\frac{1}{6\pi T}\, k^2 \pm \frac{3 - 2\ln 2}{24\pi^2 T^2 \sqrt{3}}\, k^3$$

gives sound velocity $c_s = \frac{1}{\sqrt{3}}$ and

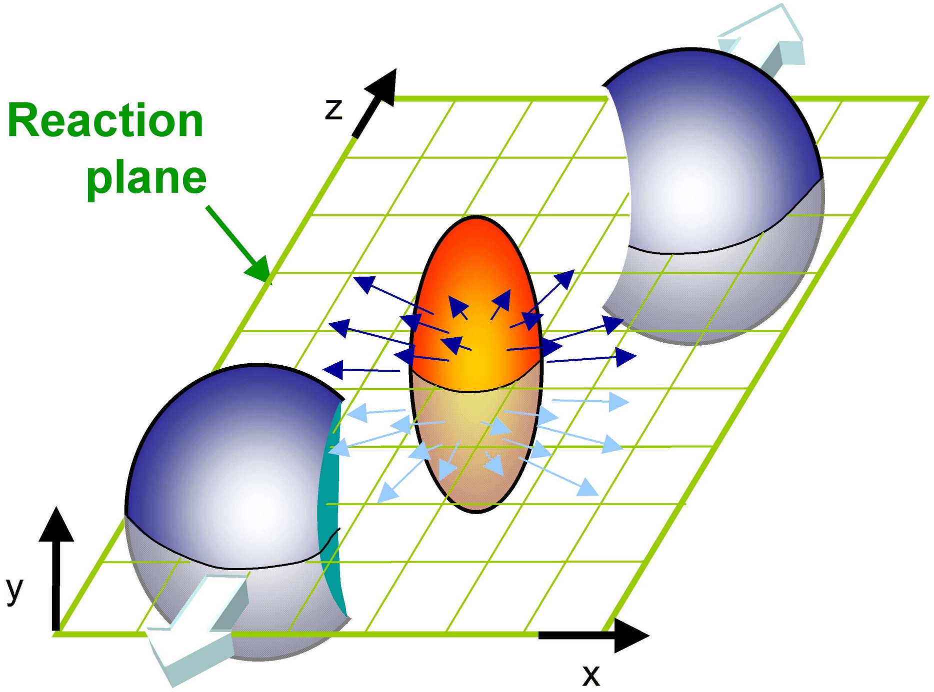
$$\frac{\eta}{s} = \frac{1}{4\pi}\ ,\ \ \tau_\pi = \frac{2 - \ln 2}{2\pi T}$$

hydro + RHIC

heavy-ion collisions require beyond hydrodynamics:

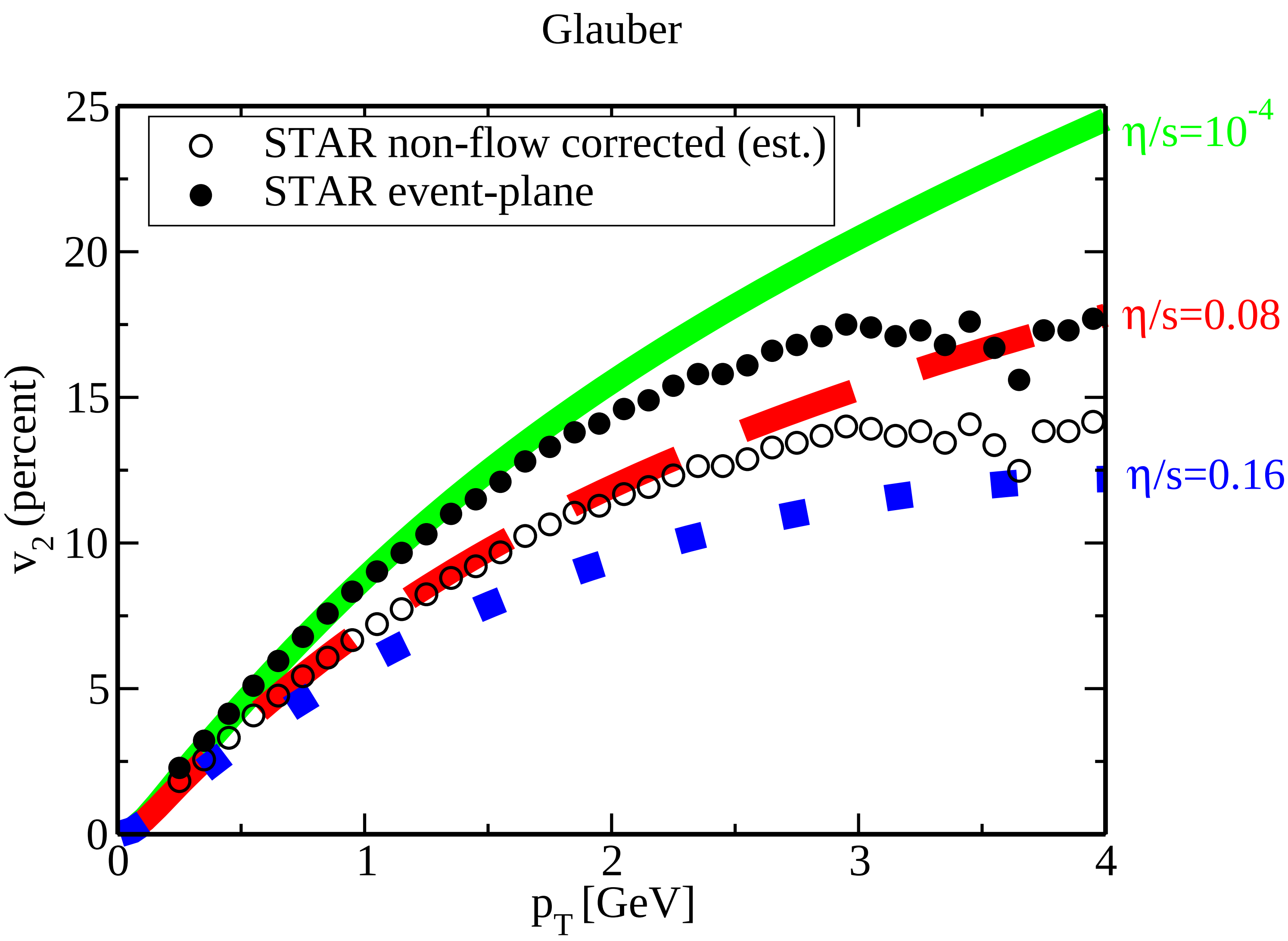
- hydrodynamics = differential equations
  initial conditions !
- initial = equilibration time
- distribution of energy density (Glauber? CGC ?)
- QCD equation of state
- hadronisation prescription (Cooper-Frye?)

as important example



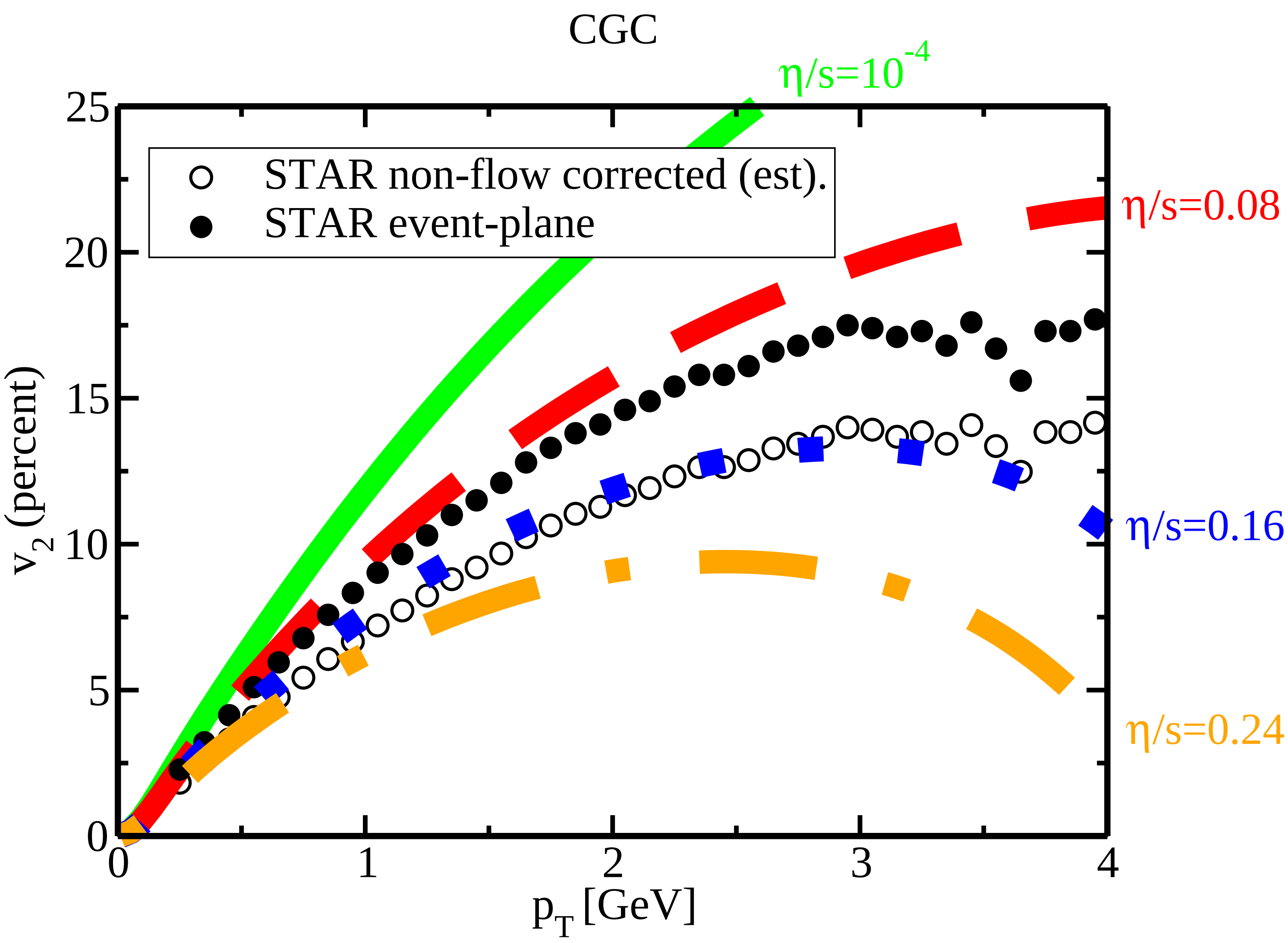
spatial asymmetry

- $$\frac{dN}{dy dp_\perp d\phi} = \langle \frac{dN}{dy dp_\perp d\phi} \rangle_\phi \left(1 + 2 v_2(p_\perp) \cos(2\phi) + \ldots\right)$$
- elliptic flow: $v_2(p_\perp)$



elliptic flow

[Luzum and Romatschke 08]



elliptic flow

[Luzum and Romatschke 08]

main results [Luzum and Romatschke 08]

- viscous hydrodynamics simulation give a good description of RHIC data for

$$\frac{\eta}{s} = 0.1 \pm 0.1(theory) \pm 0.08(experiment)$$

- modest estimate:

$$\frac{\eta}{s} < 0.5$$

- weak dependence on the values of the second-order parameters $\tau_\pi, \lambda_1, ..$

- early thermalisation time is questioned, but

$$\tau_0 < 2\ fm$$

transport coefficients

kinetic QCD theory (weak coupling):

$$\frac{\eta/s}{\tau_\pi T} \simeq 0.17\text{--}0.2,\ \lambda_1 \simeq -(2.0\text{--}2.2)\frac{\eta^2}{sT},\ \lambda_2 = -2\tau_\pi \eta,\ \lambda_3 = \kappa = 0$$

[York and Moore 08]

finite 't Hooft coupling $\lambda \equiv g_{YM}^2 N_c, \quad \lambda >> 1$ corrections to coefficients by gauge/gravity duality, e.g.:

[Buchel and Paulos 08]

$$\frac{\eta/s}{\tau_\pi T} = 0.383\ (1 - 3.52\ \lambda^{-3/2} + ...),$$

$$\kappa = \frac{\eta}{\pi T}\ (1 - \frac{145\zeta(3)}{8}\lambda^{-3/2} + ...)$$

excitement about gauge/gravity correspondence:

mainly to gain intuition into STRONG COUPLING

ASKING FOR MORE:

Is there an experiment whose outcome could cast strong doubts on the relevance of AdS/CFT to understand QCD ?

[P. Jacobs 08]

JET PHYSICS ? [Hatta, Iancu and Mueller 07 - 08]

MOST CHALLENGING TASK of the theory is to find the microscopic mechanism for the rather
RAPID EQUILIBRATION of matter in RHIC collisions

EXTRAS

	QCD	$\mathcal{N}$=4 SYM
T=0	$N_c=3=N_f$, confinement, discrete spectrum, scattering, ...	N_c large, N_f/N_c small, deconfined, conformal, supersymmetric, ...
	very different !!	
$T>T_c$	strongly-coupled plasma of gluons & fundamental matter deconfined, screening, finite corr. lengths, . . .	strongly-coupled plasma of gluons & adjoint and fundamental matter deconfined, screening, finite corr. lengths, . . .
	very similar !!	
$T>>T_c$	runs to weak coupling	remains strongly-coupled
	very different !!	

QCD and $\mathcal{N} = 4\ SYM$ as a function of temperature

[from Myers and Vazquez 08]

entropy current

Israel - Stewart 79: $s^\mu = (s - \frac{\tau_\pi}{4\eta T}\Pi_{\alpha\beta}\Pi^{\alpha\beta})u^\mu$

$\Rightarrow$ second law:

$$\nabla_\mu s^\mu = \frac{\Pi_{\alpha\beta}\Pi^{\alpha\beta}}{2\eta T} \geq 0$$

instead in causal viscous and conformal hydrodynamics
a more general current in terms of u^μ and its derivatives:

$$s^\mu = su^\mu + (\sharp\, \sigma^2 + \sharp\, \Omega^2)u^\mu + O(u\nabla^2 u)$$

with

$$\nabla_\mu s^\mu = \frac{\eta}{2T}\sigma^{\mu\nu}\sigma_{\mu\nu} + \frac{1}{4T}(\kappa - 2\lambda_1)\, \sigma^\mu_\nu \sigma^\nu_\lambda \sigma^\lambda_\mu$$

in $\mathcal{N} = 4$ SYM: $\kappa = 2\lambda_1$

[Loganayagam 08]

Bjorken flow

boost-invariant (irrotational) $1+1$ flow [Bjorken 83]
second-order equations (proper time τ, Φ... viscous flow):

$$\partial_\tau \epsilon = -\frac{4\epsilon}{3\tau} + \frac{\Phi}{\tau}$$

$$\tau_\pi \partial_\tau \Phi = \frac{4\eta}{3\tau} - \Phi - \frac{4\tau_\pi}{3\tau}\Phi - \frac{\lambda_1}{2\eta^2}\Phi^2$$

non-linear term NOT in MIS theory! [BRSSS 07]

compare with AdS/CFT calculation:

$$\frac{\lambda_1 T}{\eta} = \frac{1}{2\pi}\left[1 + \frac{215\,\zeta(3)}{8}\lambda^{-3/2} + \ldots\right]$$

[Janik, Peschanski, Heller 06; Buchel, Paulos 08]

Müller - Israel - Stewart theory

keeping esssentially one term in the derivative expansion
up to second order

$$\Pi^{\mu\nu} = -2\eta\sigma^{\mu\nu} + 2\eta\,\tau_\pi\,{}^{\langle} D\sigma^{\mu\nu\rangle} \quad , \quad D = u\cdot\nabla$$

remark: does not match with AdS/CFT $\mathcal{N}=4$ SYM

$$sound: \quad \tau_\pi = \frac{2-\ln 2}{2\pi T}$$

$$Bjorken\ flow: \quad \tau_\pi = \frac{1-\ln 2}{2\pi T}$$

$\Rightarrow$ all second-order terms consistent with
conformal symmetry have to be included !

equilibration time

REMINDER:

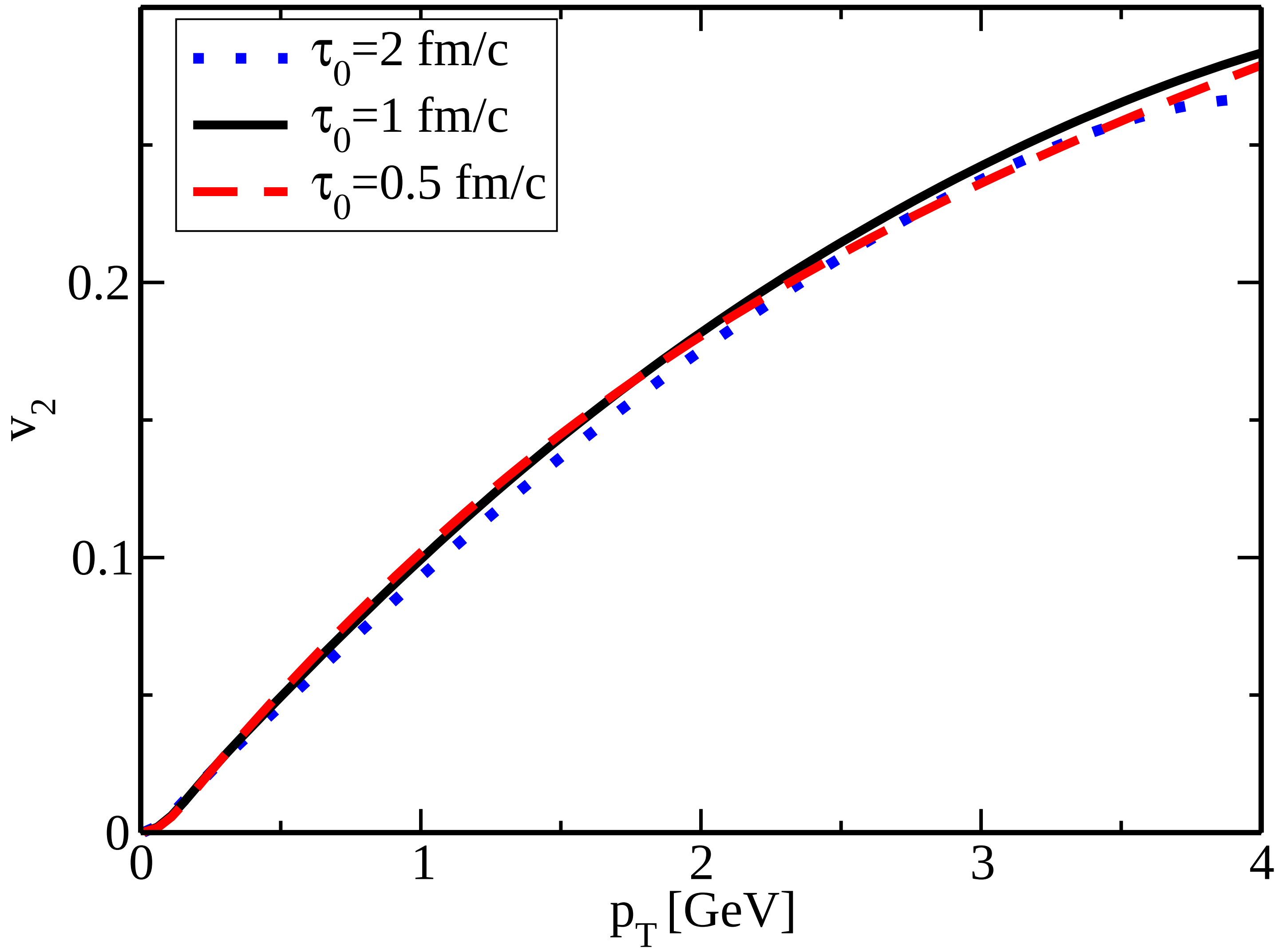
claim of short $\tau_0 \equiv \tau_{eq} \leq 0.5$ fm at RHIC

OPEN QUESTION:

$$CGC\ (\alpha_s << 1)\ \rightarrow\ sQGP\ (\alpha_s > O(1))$$

within a very short time < 0.5 fm ?

(b)



early thermalisation ?

[Luzum and Romatschke 08]

parametric pQCD estimate

for thermalisation in an expanding gluonic medium

near equilibrium at $T(\tau)$: Knudsen number *Kn*

$$\frac{1}{Kn} = \frac{longitudinal\ expansion\ time}{mean\ free\ path} = \frac{\tau}{\lambda_f} >> 1$$

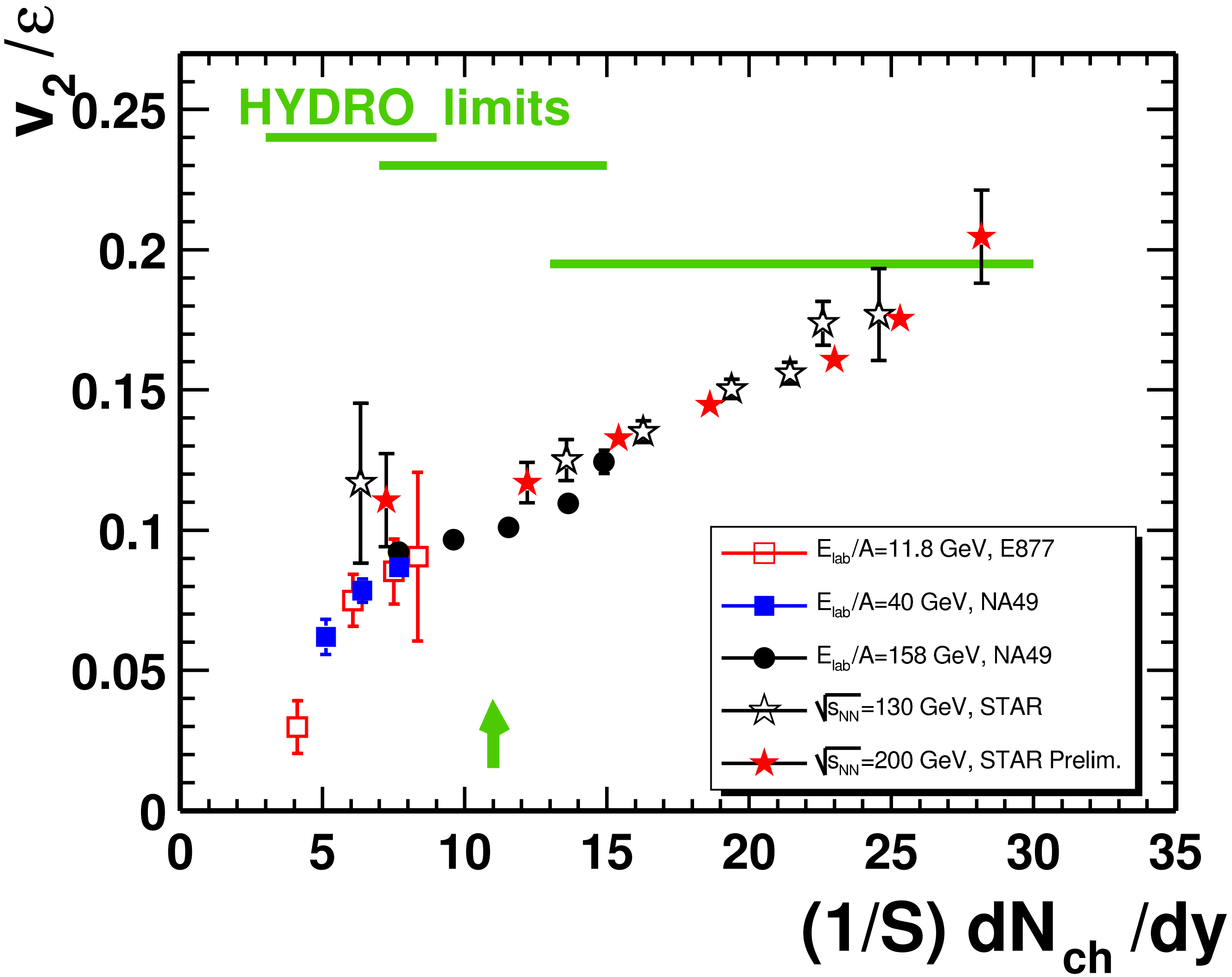
from many gluon interactions (including saturation):

Arnold et al.: $\tau_{eq} Q_s \geq \alpha_s^{-7/3}$

'bottom-up' [Baier, Son, Mueller and Schiff 01]

$$\tau_{eq} Q_s \geq \alpha_s^{-13/5}$$

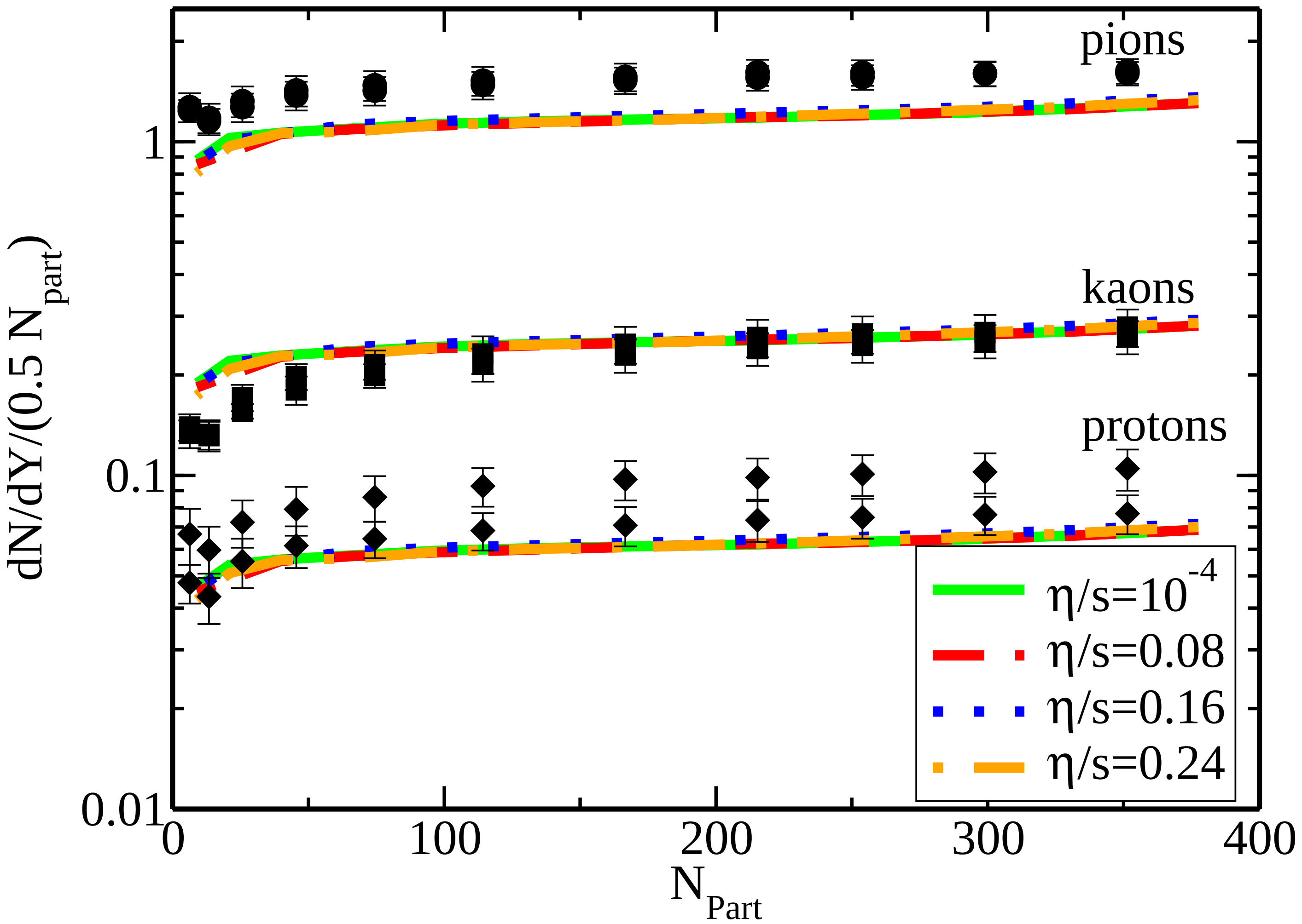
RHIC: $\tau_{eq} \geq 2 - 3$ fm



v_2 : experiment vs. perfect hydro

[Heinz 04]

CGC



multiplicity

[Luzum and Romatschke 08]