Shear viscosity of pure Yang-Mills theory at strong coupling

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Outlines

1. Introduction
   - An (almost) ideal fluid
   - Calculation of η/s
   - Status of the lower bound
2. Viscosity in strong coupling expansion
   - Euclidean formulation
   - Lattice discretization
   - Strong coupling expansion
   - Viscosity at finite temperature
3. The entropy
   - η/s ratio
4. Conclusions

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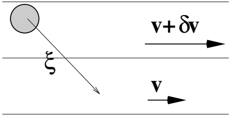
Message of RHIC: plasma is much more like a fluid than a gas!
Difference (at the same energy density) in fluid the mean free path is much shorter
Dynamical appearence: information propagation (transport) is much slower. Consider for example layers with different flow velocity. Assume diffusion equation for momentum equilibration



$$\varrho\dot{v} \sim \eta \triangle v$$

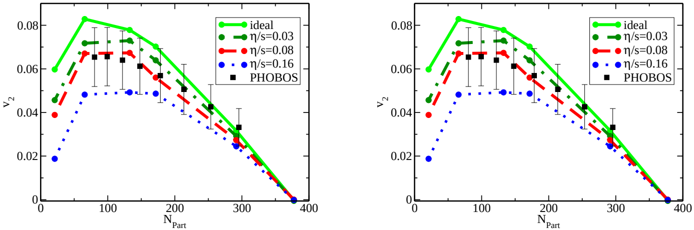
cf. Navier-Stokes equation
"diffusion constant": shear viscosity

Estimate η: insert typical lifetime (τ), path length (ℓ), velocity ($u = \ell/\tau$); σ cross section, ϵ the energy density

$$\varrho\frac{\delta v}{\tau} \sim \eta\frac{\delta v}{\ell^2} \quad \Rightarrow \quad \eta \sim \varrho u \ell \sim \epsilon\tau \sim \frac{mu}{\sigma},$$

ideal fluid: $\eta = 0$, ideal gas $\eta = \infty$.

Consequence: the larger the viscosity, the more extent the initial anizotropy is washed out



(P. Romatschke, U. Romatschke, Phys.Rev.Lett.99:172301,2007.)

$\Rightarrow$ flow seen by RHIC is very close to ideal hydro

$$\frac{\eta}{s} \lesssim 0.16$$

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Kubo formula

Linear response of the traceless spatial part of the energy-momentum tensor $T_{\mu\nu}$ to velocity inhomogeneities; response function:

$$C(x) = \frac{1}{10}\left\langle [\pi_{ij}(x), \pi_{ij}(0)] \right\rangle, \qquad \pi_{ij} = T_{ij} - \frac{1}{3}\delta_{ij} T_{kk}.$$

The viscosity from here (Kubo formula)

$$\eta = \lim_{\omega \to 0} \frac{C(\omega, \mathbf{k} = 0)}{\omega}$$

perturbation theory

- Perturbation theory is expansion around the ideal gas $\eta = \infty$
  $\Rightarrow$ small correction still yield large viscosity
- we expect $\sigma \sim g^4 \quad \Rightarrow \quad \eta/s \sim 1/g^4$
- more precise calculation needs resummation of ladder diagrams (cf. P.Arnold, G.D.Moore, L.G.Yaffe, JHEP 0305, 051 (2003))

$$\frac{\eta}{s} = \frac{\kappa}{g^4 \ln 1/g}$$

- taking into account higher order processes may lower the η/s ratio (Z. Xu, C. Greiner, H. Stoecker, Phys.Rev.Lett.101:082302,2008.)
- non-perturbative analytic methods: large N_f expansion (A. Gerts and J.M.M.Resco, hep-ph/0409090), strong coupling expansion (A.J., D. Nogradi, 0810.4181)

MC studies

- measure $\langle T_{12}(x) T_{12}(0) \rangle$ correlator on lattice $\Rightarrow$ Euclidean discrete time
- we need the spectral function, which is related to the correlator as

$$\int d^3\mathbf{x} \, \langle T_{12}(\tau, \mathbf{x}) T_{12}(0) \rangle = \int_0^\infty \frac{d\omega}{\pi} \, C(\omega, \mathbf{k}=0) \, \frac{\cosh(\omega(\beta/2 - \tau))}{\sinh \beta\tau/2}.$$

- invert this relation with the prior knowledge $C(\omega > 0) > 0$ Maximal Entropy Method, or ad hoc solutions
- too little sensitivity to small ω regime $\Rightarrow$ large systematical uncertainties; additional assumptions are needed
- best estimates $\eta/s = 0.102\,(56)$ at $T = 1.24\,T_c$ (H. B. Meyer, Phys. Rev. D 76, 101701 (2007))

AdS/CFT methods

- for theories with gravity dual, at $N_c \gg 1$, $\lambda = g^2 N_c \gg 1$ from graviton absorbtion in the dual theory:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

(P. Kovtun, D.T. Son, A.O. Starinets JHEP 0310, (2003) 064.)

- for weaker coupling we expect to increase the ratio; in fact $\mathcal{N} = 4$ SYM

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \frac{5\lambda^{1/2}}{16N^2} + \ldots\right] > \frac{1}{4\pi}$$

(R.C. Myers, M.F. Paulos, A. Sinha, arXiv:0806.2156)

- universal for a wide class of theories (A. Buchel, R.C. Myers, M.F. Paulos, A. Sinha, Phys.Lett.B669:364-370,2008.; M. Haack, A. Yarom, arXiv:0811.1794)
- QCD is not $\mathcal{N} = 4$ SYM theory! For other theories supplementary argumentations are needed: lower bound.

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generic arguments

(P. Kovtun, D.T. Son, A.O. Starinets PRL 94, 111601 (2005))

$$\eta \sim \epsilon\tau, \qquad s \sim n \quad \Rightarrow \quad \frac{\eta}{s} \sim E_{\mathrm{part}}\tau$$

ϵ: energy density, τ lifetime, n particle density, E_{part} particle energy
From uncertainty relation:

$$\frac{\eta}{s} \gtrsim \hbar$$

Coefficient can be found from gravity dual models

$$\frac{\eta}{s} \geq \frac{1}{4\pi}, \qquad \text{in } \frac{\hbar}{k_B} \text{ units.}$$

For real matters the bound is respected (RHIC quark matter near the bound; water: 380×, liquid He: 9× above bound)
At the bound tight packing: mean free path ∼ de Broglie wavelength!

caveats

- gravity side: in higher derivative gravity $\eta/s < 1/(4\pi)$ is possible (Y. Kats, P. Petrov, arXiv:0712.0743)
  unitarity? (M. Brigante, H. Liu, R.C. Myers, S. Shenker, S. Yaida, Phys.Rev.Lett.100:191601,2008.) – still allows $4\pi\eta/s > 16/25$.
- counterexample (T.D. Cohen, Phys.Rev.Lett.99:021602,2007.): nonrelativistic gas with lot of species with the same interaction: dynamics is unchanged, but entropy is enhanced by Gibbs mixing entropy η/s can approach zero!
  Controversal: metastability – are η and s both sensible quantities on the same physical size? (D T. Son, Phys.Rev.Lett.100:029101,2008)

Question is not settled, further investigations are going on

relativistic QFT

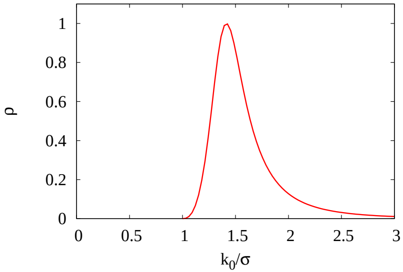
- KSS argumentation is based on quasiparticle picture
- in QFT it is not always hold. Spectral function (density of states) is a continuous function: very different from quasiparticle picture eg. near a threshold.
  Corresponds to “infinitely many” species in Cohen’s construction, without metastability.

results from strong coupling Yang-Mills theories

For details see later...
At strong coupling elementary excitation is the glueball, same quantum numbers as T_{ij} $\Rightarrow$ relevant for T_{ij} correlators.

Glueball spectral function:

- mass gap
- peak
- effective spectral function is multiplied by $e^{-k_0/T}$



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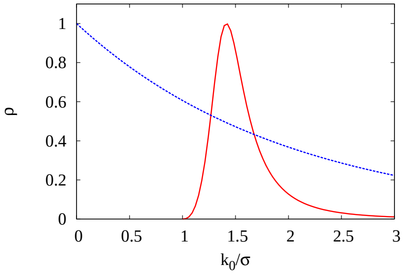
at high temperature small suppression



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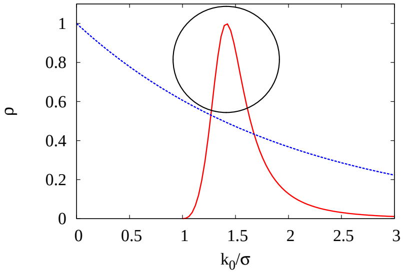
at high temperature small suppression

$\Downarrow$

quasiparticle peak dominates



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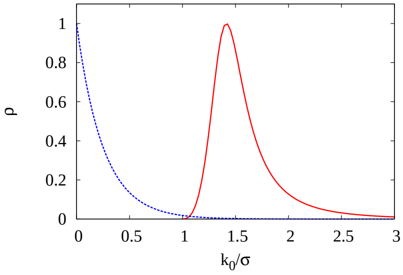

results from strong coupling Yang-Mills theories

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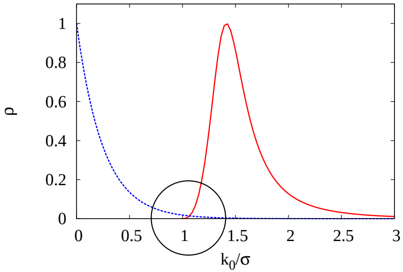
at small temperature large suppression

$\Downarrow$

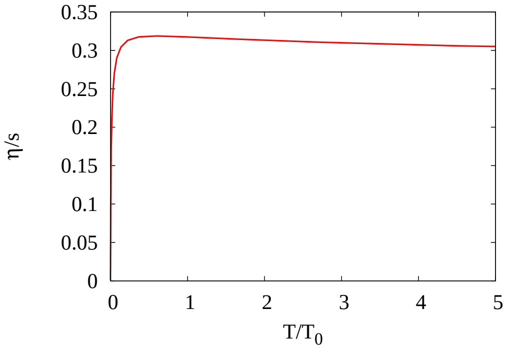
threshold dominates, not a quasiparticle system!!



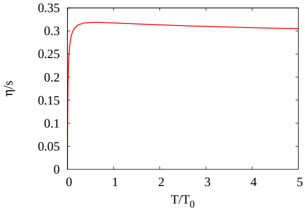
results from strong coupling Yang-Mills theories

We expect to respect at high temperature KSS bound, at low temperature something different.

End result of the calculation



results from strong coupling Yang-Mills theories



- $\eta/s \sim N^2$
- high T (but below T_c) regime: shallow, but decreasing curve (ca. 25% drop from maximum to minimum)
- value is larger than $1/(4\pi)$ (aware also systematic uncertainties in strong coupling expansion (see later))
- low T regime $\eta/s \to 0$
  threshold behavior, excitations are not quasiparticle

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We want to calculate in Euclidean time the quantity

$$M_{\mu\nu\,\varrho\sigma}(x) = \langle \mathrm{T}\,(T_{\mu\nu}(x)T_{\varrho\sigma}(0))\rangle \ ,$$

which has the properties $M_{\mu\nu\,\varrho\sigma} = M_{\nu\mu\,\varrho\sigma} = M_{\mu\nu\,\sigma\varrho} = M_{\varrho\sigma\,\mu\nu}$.
We are interested in the index combination

$$\mathcal{M}(x) = \frac{1}{10}\sum_{ij}\langle \mathrm{T}\,(\pi_{ij}(x)\pi_{ij}(0))\rangle \ = \frac{1}{10}\sum_{ij}\left[M_{ij\,ij} - \frac{1}{3}M_{ii\,jj}\right].$$

At $T = 0$ we use SO(4) symmetry of the spacetime to rotate to $\tau = (|x|, 0, 0, 0)$ position. As a result we obtain

$$\mathcal{M}(x) = \sum_a f_a(x_i^2/x_\mu^2)M_a(\tau)$$

$a = (0000, 0011, 0101, 1111, 1122)$, and f_a determinable functions.

We need the commutator of π_{ij}, not the Euclidean correlator: for measured values it is hard to achieve.
Now we will have analytic results, so we can perform analytic continuation:

$$\mathcal{M}(-ik_0+\varepsilon,\mathbf{k}) = i\mathcal{M}^{\mathrm{ret}}(k_0,\mathbf{k}), \qquad C(k) = -2\,\mathrm{Im}\,i\mathcal{M}^{\mathrm{ret}}(k_0+i\varepsilon,\mathbf{k}).$$

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To determine $M_a(\tau)$ we discretize the spacetime to a lattice with lattice spacing a. Yang-Mills theories in Wilson representation

$$Z = \int \mathcal{D}U e^{-S}, \qquad S = \sum_p S_p, \qquad S_p = \beta\left(1 - \frac{1}{N}\mathrm{Re\ Tr}\,U_p\right),$$

where $\beta = 2N/g_0^2$, p means plaquettes,
$U_p(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U^\dagger_\mu(x+a\hat{\nu})U^\dagger_\nu(x)$ plaquette variables,
$U_\mu(x) \in \mathrm{SU(N)}$ link gauge variables $U_\mu(x) = \mathcal{P}e^{-i\int_0^a dsA_\mu(x+s\hat{\mu})}$.
Expanding plaquettes to first order in lattice spacing yields $F_{\mu\nu}$, to second order yields $\sim T_{\mu\mu}$

So diagonal energy-momentum tensor elements can be expressed through plaquettes:

$$T_{\mu\mu} = \frac{\beta}{N}\left[-\sum_{\nu\neq\mu} P_{\mu\nu} + \sum_{\sigma,\nu\neq\mu;\sigma>\nu} P_{\sigma\nu}\right],$$

explicitely

$$\begin{aligned}
T_{00} &= (\beta/N)\left[-P_{01} - P_{02} - P_{03} + P_{21} + P_{31} + P_{32}\right]\\
T_{11} &= (\beta/N)\left[-P_{10} - P_{12} - P_{13} + P_{20} + P_{30} + P_{32}\right]\\
T_{22} &= (\beta/N)\left[-P_{20} - P_{21} - P_{23} + P_{10} + P_{30} + P_{31}\right]\\
T_{33} &= (\beta/N)\left[-P_{30} - P_{31} - P_{32} + P_{20} + P_{10} + P_{21}\right].
\end{aligned}$$

All needed M_a correlators can be expressed as plaquette-correlators, except $M_{01\,01}$.

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For small β values (large g_0^2), we expand e^{-S_p} in powers of β. For computational purposes it is better to write

$$e^{-S_p} = c(\beta)\left(1 + \sum_{R \neq 0} d_R\, a_R(\beta)\, \chi_R(U_p)\right) ,$$

where χ_R is the characters of the irreducible representation R of SU(N), d_R is the dimension of the representation and $c(\beta)$ and $a_R(\beta)$ are some coefficients. At leading order in β expansion

$$c(\beta) = 1, \qquad a_f(\beta) = \begin{cases} \frac{\beta}{N^2} & N = 2 \\ \frac{\beta}{2N^2} & N > 2 \end{cases} .$$

f is the fundamental representation.

To compute gauge observables, form closed surfaces, and assign to each surface a representation. Contribution survives, if unit representation is contained in the product. Expand result in powers of β.

Known properties of the strong coupling expansion

- describes confinement, chiral symmetry breaking; bound state spectrum and masses can be computed fairly well
- perturbation series converge
- difficulty in reaching continuum limit: asymptotic freedom requires $g_0 \to 0$, strong coupling
- towards continuum limit: roughening phase transition; infinite order phase transition for Wilson loops between area and perimeter law.
- renormalization? results not a-independent.
  Our strategy: identify robust results (a-independent); for a-dependent parts try to connect to physical observables.

To leading order we assign to each plaquettes the fundamental representation; after path integration each contributes

$$d_f^2 a_f(\beta) = N^2 a_f(\beta).$$

Plaquette-plaquette correlator along a lattice axis:

- opened tube contribution $\mathcal{G}^E(\ell)$ for length ℓ &
- factors coming from the orinentations of the end plaquettes:

$P_{10\,21}$	$P_{10\,32}$	$P_{10\,10}$	$P_{21\,21}$	$P_{10\,20}$	$P_{21\,31}$	$M_{01\,01}$
1	a_f^4	a_f^4	1	a_f^4	a_f^4	$< a_f^2$

where $P ij\, km$ is the correlator of ij and km plaquettes.

$\Rightarrow$ for calculation of $\mathcal{M}$ we need only $P_{21\,21}(\tau) = \mathcal{A}_N \mathcal{G}^E(|x|)$, where $\mathcal{A}_1 = 1$, $\mathcal{A}_{N>1} = 1/4$.

To leading order

$$\mathcal{M}(x) = \frac{2}{5}\frac{\beta^2 \mathcal{A}_N}{N^2}(15 - 10u - 4u^2)\,\mathcal{G}^E(|x|) \qquad u = \frac{x_i^2}{x_\mu^2}$$

angular and length dependence separated.
For Fourier transform at $\mathbf{p} = 0$ the u-dependence can be performed with angular integration

$$\mathcal{M}(p_0, \mathbf{p} = 0) = \frac{2\beta^2 \mathcal{A}_N}{N^2} \int d^4x e^{ip_0 t}\mathcal{G}^E(|x|) = \frac{2\beta^2 \mathcal{A}_N}{N^2}\, G^E(p_0, \mathbf{p} = 0),$$

where $G^E(x) = \mathcal{G}^E(|x|)$.

Straight line contribution

The minimal surface connecting two plaquettes is a straight tube: it containes $4 \times n$ plaquettes, where n is the length of the tube:

$$\mathcal{G}^E_{\rm straight}(\ell) = e^{-\sigma\ell}, \qquad \sigma = -\frac{4}{a} \ln a_f(\beta).$$

In strong coupling regime $a\sigma \gg 1$.
Fourier transformation

$$G^E(k) = \frac{\Delta}{(k^2/\sigma^2 + 1)^{5/2}}, \qquad \Delta = \frac{12\pi^2}{(a\sigma)^4}$$

Zig-zagging lines

Almost straight lines are just a little longer than straight lines, but there is a large number of such surfaces $\Rightarrow$ can compensate the small suppression

Contribution of zig-zagging lines with m breaking point is an m-fold convolution; in Fourier space $(G^E(k))^n$.

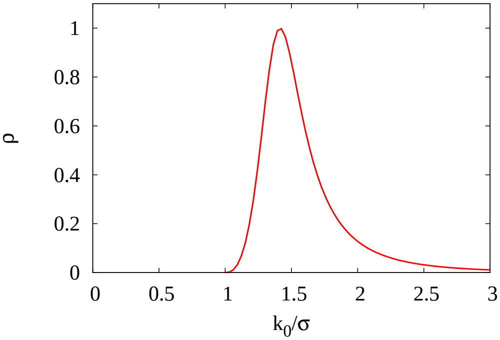
All zig-zagging lines:

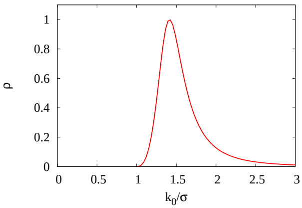
$$G^E_{\mathrm{zig-zag}}(k) = \frac{1}{G^E_{\mathrm{straight}}(k) - 1}$$

$\Rightarrow$ like self-energy expression: junction contribution is 1.

$G^E \to G^{\mathrm{ret}} \to \varrho$ with analytic methods:

$$\varrho(k) = \mathrm{sgn}(k_0)\Theta(k^2 - \sigma^2)\frac{2\Delta(k^2/\sigma^2 - 1)^{5/2}}{\Delta^2 + (k^2/\sigma^2 - 1)^5}$$





- mass gap until σ, then threshold behavior and a peak
- zig-zagging influences the vicinity of the threshold
- because mass gap $C(k_0 < \sigma) = 0 \quad \Rightarrow \quad$ zero temperature viscosity is zero $\quad \Rightarrow \quad$ superfluidity!
  Thermodynamics requires $s \to 0$ as $T \to 0 \quad \Rightarrow \quad$ for not-superfluids $\eta/s \to \infty$ at $T \to 0$
  Now we have $0/0$ contribution $\quad \Rightarrow \quad$ deeper investigations are needed!

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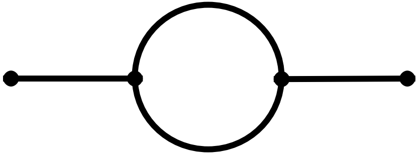
Finite temperature $\Rightarrow$ finite extent at τ-direction.
We assume that the Lorentz structure is not changing too much (at small temperatures)
Filling the gap in spectral function for small momenta $\Rightarrow$ like Landau damping; leading contribution comes from a loop diagram, where one tube wraps around the imaginary axis, the other goes straight:



The bubble contribution is the same as in perturbation theory, just the propagator is different:

$$\Sigma^{\rm ret}(p) = \frac{a^4}{a_f^2(\beta)} \int \frac{d^4 k}{(2\pi)^4} \left(\frac{1}{2} + n(k_0)\right) \varrho_0(k) G_0^{\rm ret}(p-k) \,.$$

The contribution of the diagram to the spectral function

$$\varrho_{1-\rm loop}(p_0 \approx 0, \mathbf{p} = 0) \approx (iG_0(0))^2 \operatorname*{Disc}_{p_0} \Sigma^{\rm ret}(p) = \Delta^2 \operatorname*{Disc}_{p_0} \Sigma^{\rm ret}(p).$$

After algebraic manipulations, and assuming $T \ll \sigma$, the viscosity reads:

$$\eta = \frac{\Delta^2 N^2 a^4}{2\pi^3 T} \int_0^\infty dk k^2 \int_{\omega_{\mathbf{k}}}^\infty dk_0 e^{-k_0/T} \varrho^2(k) \,,$$

where $\omega_{\mathbf{k}}^2 = \mathbf{k}^2 + \sigma^2$.

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The entropy density $s = -\frac{\partial f}{\partial T}$, f is the free energy density.
In strong coupling theory: closed surfaces.
The smallest surfaces (cube, double cube, etc.) do not feel the temperature $\Rightarrow$ yield zero entropy
First nontrivial surface which wraps around the temperature direction: we can express this contribution with help of the opened tube propagator

$$f_T = -6(aT)\left(G^E_T(0) - G^E_{T=0}(0)\right) .$$

With algebraic manipulations and assuming $T \ll \sigma$ from here

$$s = \frac{3a^5}{\pi^3 T} \int_0^\infty dk k^2 \int_{\omega_{\mathbf{k}}}^\infty dk_0\, k_0\, e^{-k_0/T}\, \bar{\varrho}(k) .$$

Zero temperature limit

The viscosity η and the entropy have a common form

$$F_{n,m} = \frac{3a^5}{2\pi^3 T}\int \frac{d^4k}{(2\pi)^4}\,\Theta(k_0)\Theta(k^2-\sigma^2)e^{-k_0/T}(ak_0)^n\varrho^m(k),$$

since

$$\eta = N^2\Delta^2 F_{0,2}, \qquad s = 2F_{1,1}.$$

After reducing the integrals

$$a^3F_{n,m} = C(a\sigma)^n\,(a^2\sigma T)^{3/2}e^{-\sigma/T}\int_0^\infty dz\,e^{-z}\left(\frac{2(wz)^{5/2}}{1+(wz)^5}\right)^m,$$

where $w = 2\Delta^{-2/5}T/\sigma$ rescaled temperature.
BOTH η and s goes to zero at zero temperature

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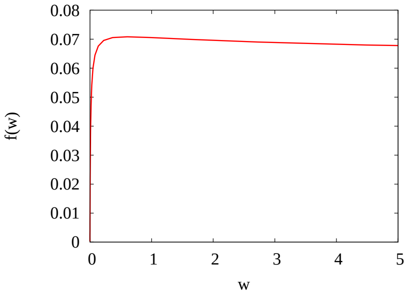
After simplifications

$$\frac{\eta}{s} = \frac{N^2}{2}\Delta^{9/4} f(\frac{2T}{\Delta^{2/5}\sigma}), \qquad f(w) = \frac{1}{3(12\pi^2)^{1/4}} \frac{\int_0^\infty dz\, e^{-z/w}\left(\frac{2z^{5/2}}{1+z^5}\right)^2}{\int_0^\infty dz\, e^{-z/w}\left(\frac{2z^{5/2}}{1+z^5}\right)}.$$



Asymptotic values can be computed analytically

$f(w \ll 1) \sim T^{5/2}$ and
$f(w \gg 1) = 0.056$

f does not contain lattice spacing $\Rightarrow$ robust prediction
To assess Δ:

- if $\Delta \ll 1$ $\Rightarrow$ strong coupling appr. good, cont. limit bad
- if $\Delta \gg 1$ $\Rightarrow$ strong coupling appr. bad, cont. limit good
- compromise: $\Delta \approx 1$.

To assess temperature scale: $\sigma \approx \sigma|_{T_c}$. Final result for SU(3):

$$\frac{\eta}{s} = 4.5 * f(T/M_{\mathrm{glueball}}(T_c)),$$

and

$$\left.\frac{\eta}{s}\right|_{T_c} \approx 0.25 \approx \frac{3.2}{4\pi}$$

Note that for $\Delta = 0.6$ we would obtain the KSS bound
$\Rightarrow$ large systematic uncertainties in the number prediction

Outlines

1. Introduction
   - An (almost) ideal fluid
   - Calculation of η/s
   - Status of the lower bound
2. Viscosity in strong coupling expansion
   - Euclidean formulation
   - Lattice discretization
   - Strong coupling expansion
   - Viscosity at finite temperature
3. The entropy
   - η/s ratio
4. Conclusions

Conclusions

we determined η/s for SU(N) Yang-Mills theories at strong coupling

- result is $\sim N^2$
- both η and s go to zero at zero temperature
- shape of the T-dependence is robust prediction
- high temperatures
  - quasiparticle approximation
  - curve slightly decreasing as we approach T_c
  - possible to accomodate to KSS bound $1/4\pi$
- very low temperatures $\Rightarrow$ threshold regime, $\eta/s \to 0$.