

Time evolution of particle  
production in  $e^+e^-$  annihilation  
from Bose-Einstein  
Correlations

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# Questions and answers

*What is the aim of this work?*

To reconstruct the pion emission function.

*What is the emission function?*

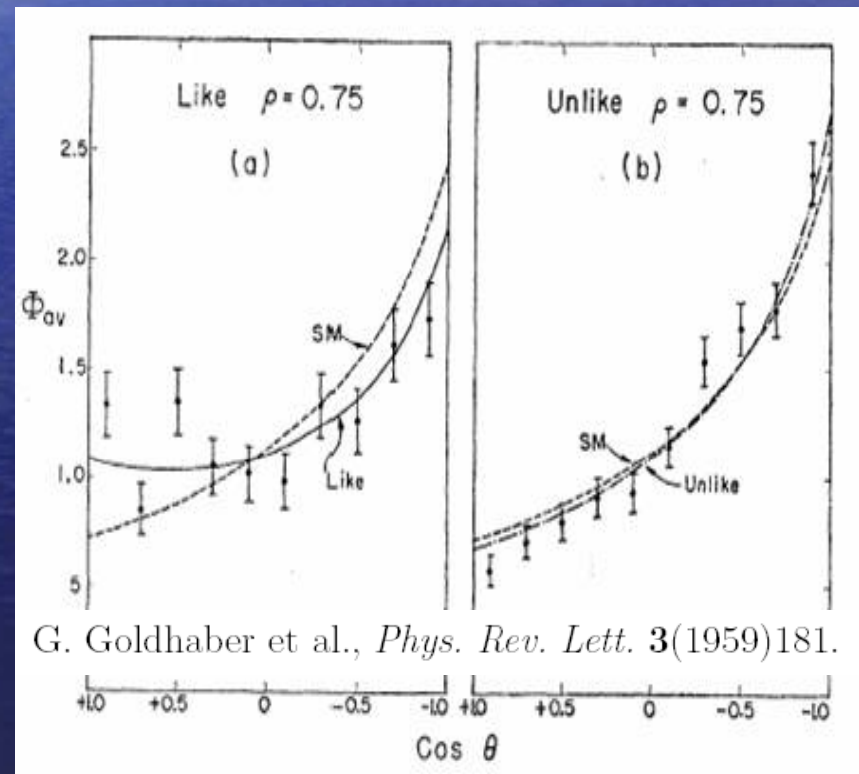
It says where and when and how many pions are produced. (See later)

*How to reach it?*

Using the Bose-Einstein Correlations. (See now)

# Bose-Einstein Correlations

- This topic is almost 50 years old in high energy physics (Intensity Interferometry)
- In multiple particle production BECs were discovered accidentally as a byproduct of an unsuccessful attempt to find the  $\rho$  meson.
- GGLP effect



# Bose-Einstein Correlation

- Analogous effect had been discovered earlier (HBT effect)

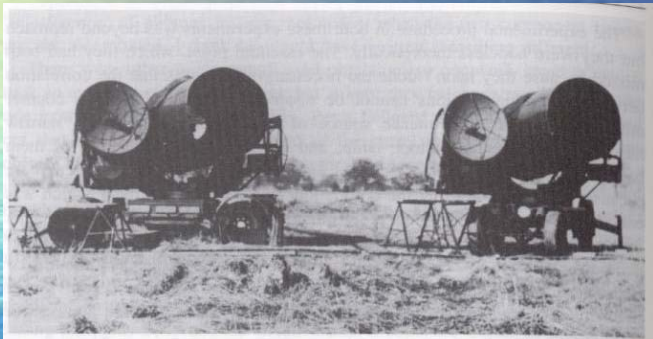


Figure 10.1 The first stellar intensity interferometer; the pilot model of the stellar intensity interferometer at Jodrell Bank in 1955. Two Army searchlights were used to make the first measurement of the angular diameter of a main sequence star (Sirius).



Figure 11.1 Narrabri Observatory in 1963, showing the circular railway track (618 foot diameter), the two optical reflectors (22 foot diameter), the central mast and control room, and the very large garage for the reflectors.

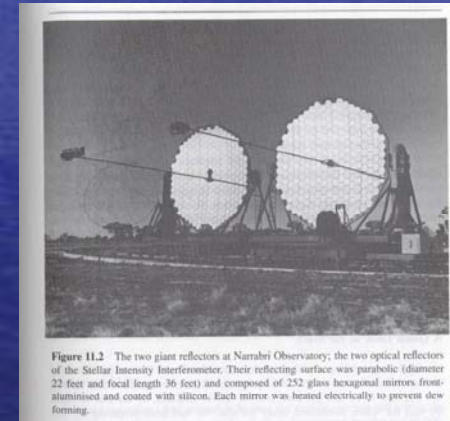


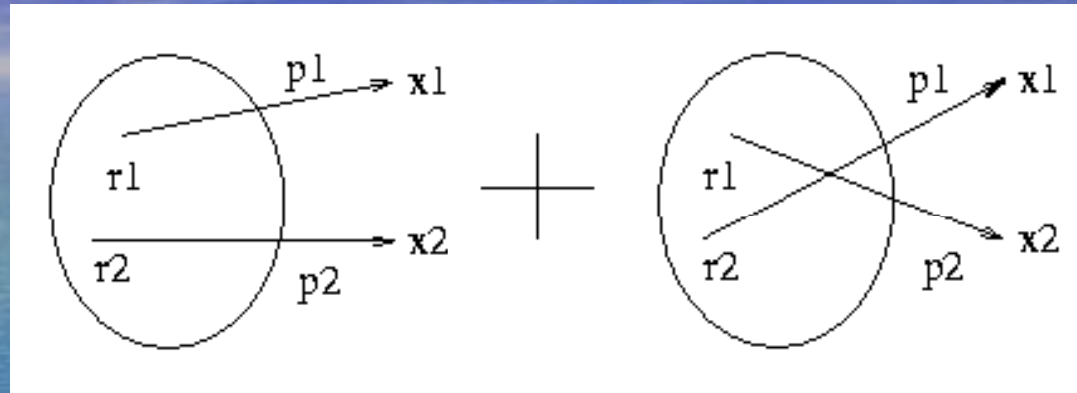
Figure 11.2 The two giant reflectors at Narrabri Observatory; the two optical reflectors of the Stellar Intensity Interferometer. Their reflecting surface was parabolic (diameter 22 feet and focal length 36 feet) and composed of 252 glass hexagonal mirrors front-aluminised and coated with silicon. Each mirror was heated electrically to prevent dew forming.

Determination of  
the angular radii  
of stars



Determination  
of the size of  
the source

# Bose-Einstein Correlation



In theory:

$$C_2(p_1, p_2) = \frac{\frac{d^6 n}{d^3 p_1 d^3 p_2}}{\frac{d^3 n}{d^3 p_1} \frac{d^3 n}{d^3 p_2}} = \frac{\int d^4 x_1 d^4 x_2 f(x_1) f(x_2) |\Psi(1, 2)|^2}{\int d^4 x_1 |\Psi(1)|^2 \int d^4 x_2 |\Psi(2)|^2} = 1 + |\tilde{f}(\Delta p)|^2$$

# Examples

The correlation function will be investigated as a function of  $Q$ , the invariant four-momentum difference. (See later)

Source function

Correlation function

Gaussian

Coherence parameter

$$R_0(Q) = 1 + \lambda \exp(-(RQ)^2)$$

Edgeworth expansion

$$R_2(Q) = 1 + \lambda \exp[-(RQ)^2] \left[ 1 + \frac{\kappa}{3!} H_3(RQ) \right]$$

Symmetric Lévy stable

$$R_2(Q) = 1 + \lambda \exp(-|RQ|^\alpha)$$

# Bose-Einstein Correlation

In experience:

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_0(p_1, p_2)}$$

Two particle density  
without BEC  
(Reference sample)

Two particle  
number density  
(Data)

# Puzzles in the late 80's

First, the measured correlation functions are consistent with the invariant  $Q$  dependence.

TASSO Collab., Althoff, M, Z. Phys. C30 (1986) 355–369.

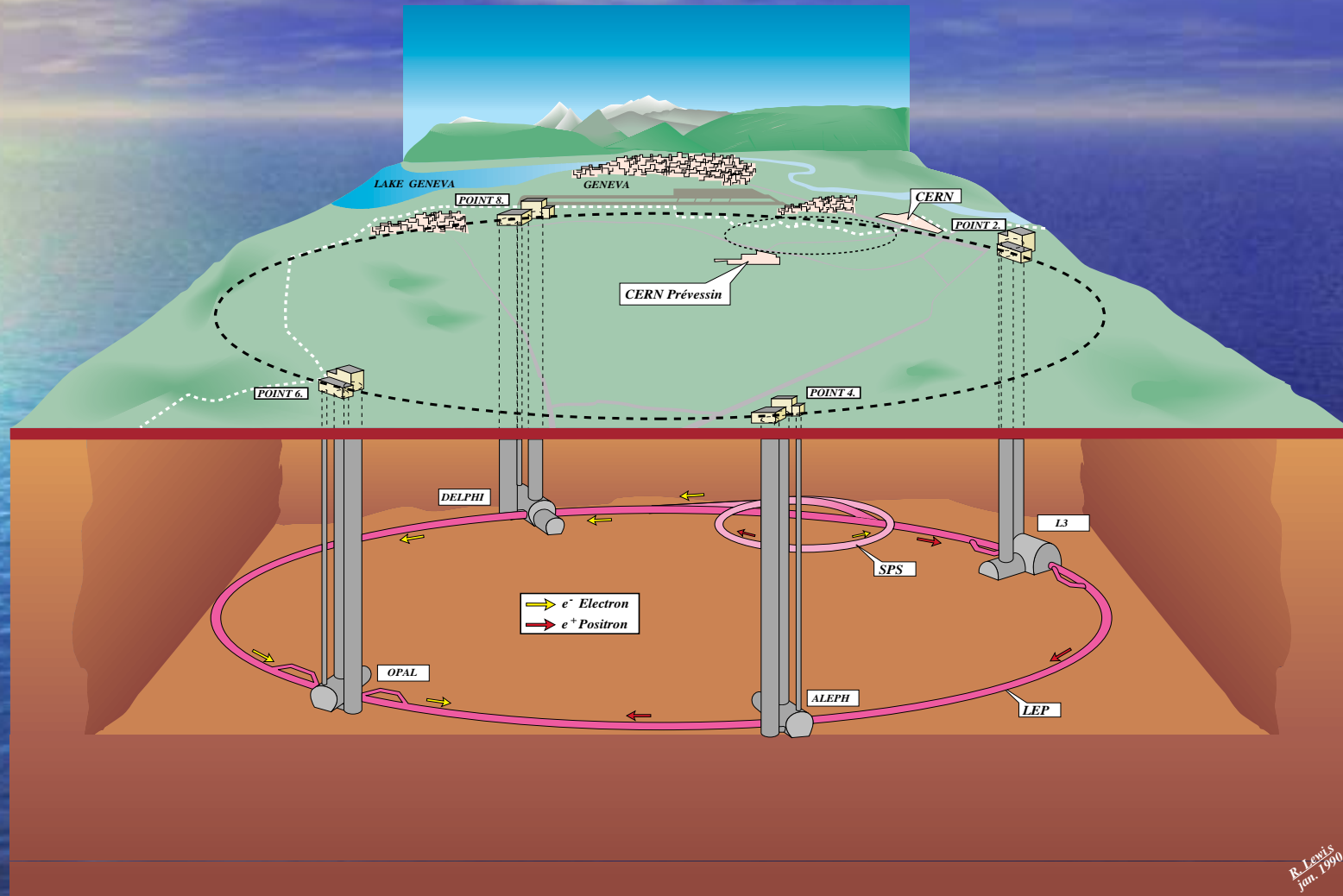
(the statistics were not large enough)

Second, the correlation function is more peaked than a Gaussian.

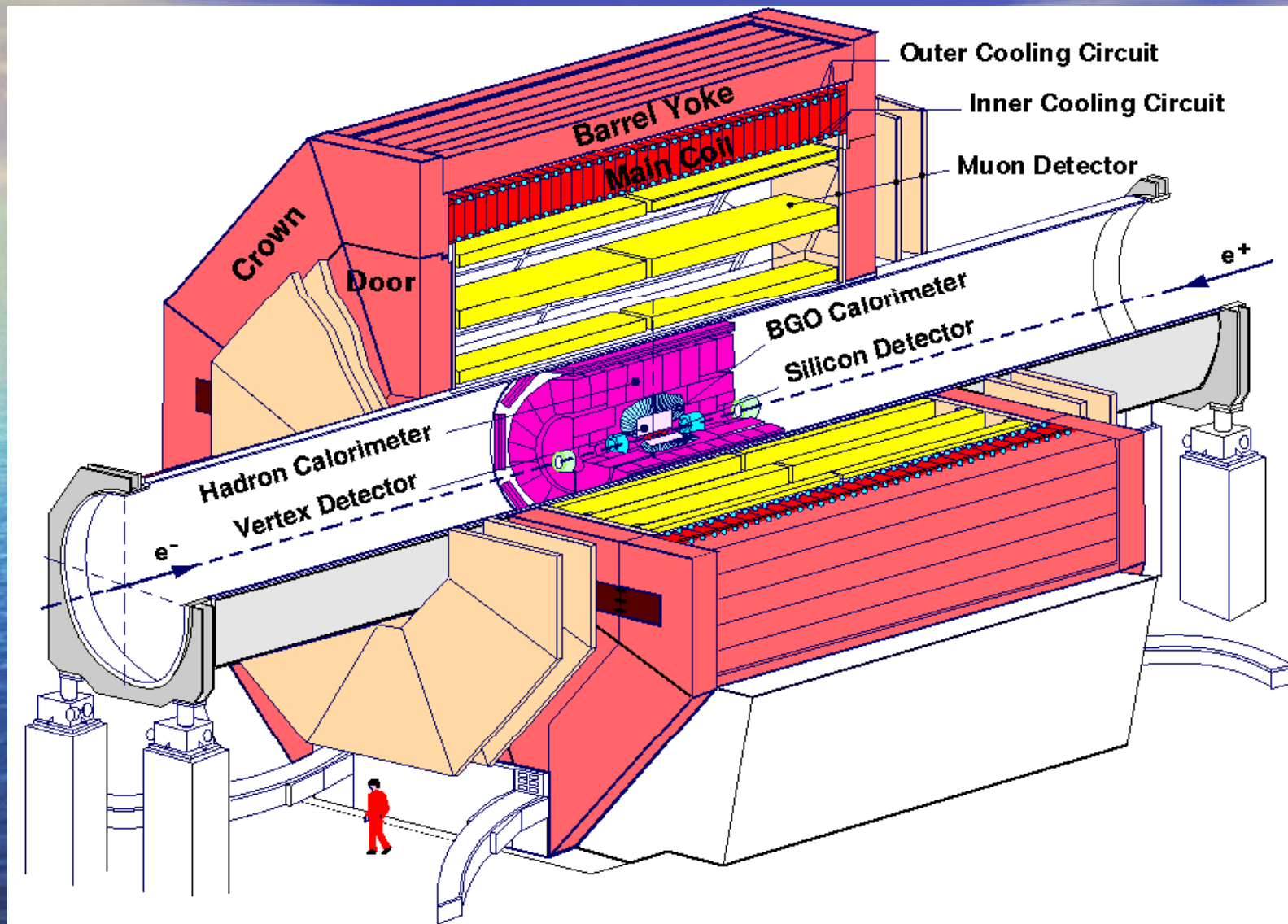
There was no theoretical model which predicted a mere  $Q$  dependence.



# Large Electron Positron (LEP) collider



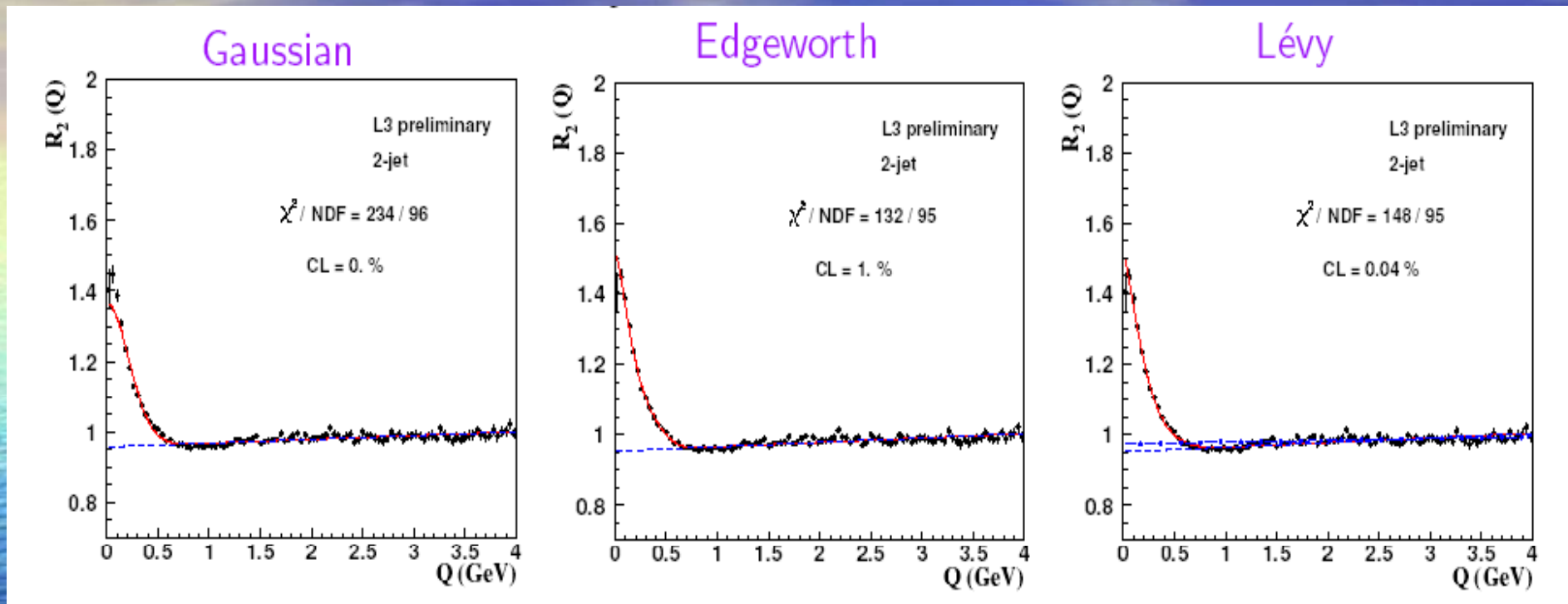
# The L3 detector



# Data Sample

- Hadronic Z decays using L3 detector
- 'Standard' event and track selection  $\approx$  *1 million events*
- Concentrate on 2-jet events using *Durham* algorithm  $\approx$  *0,5 million events*
- Correct distribution *bin-by-bin* by MC

# Beyond the Gaussian



Far from Gaussian       $K = 0,71 \pm 0,06$        $\alpha = 1,34 \pm 0,04$

Poor CLs. Edgeworth and Lévy better than Gaussian.  
Problem is the dip in the region  $0,6 < Q < 1,5$  GeV.

# The $\tau$ -model

It is assumed that the average production point of particles and the four momentum are strongly correlated.

This correlation is much narrower than the proper-time distribution.

In the plane-wave approximation, using the Yano-Koonin formula, one gets for two-jet events:

$$R_2(p_1, p_2) = 1 + \lambda \text{Re} \tilde{H}^2 \left( \frac{Q^2}{2m_t} \right)$$

T. Csörgő and J. Zimányi, Nucl. Phys. **A517** (1990) 588.

# Further assumptions

Assume a Lévy distribution for  $H(\tau)$ .

Since no particle production before the interaction,  $H(\tau)$  is one-sided.

Then

$$R_2(Q, \bar{m}_t) = \gamma \left[ 1 + \lambda \cos \left( \frac{\tau_0 Q^2}{\bar{m}_t} + \tan \left( \frac{\alpha \pi}{2} \right) \left( \frac{\Delta \tau Q^2}{2\bar{m}_t} \right)^\alpha \right) \exp \left( - \left( \frac{\Delta \tau Q^2}{2\bar{m}_t} \right)^\alpha \right) \right] (1 + \delta Q)$$

where  $\alpha$  is the index of stability

$\tau_0$  is the proper time of the onset of particle production

$\Delta \tau$  is a measure of the width of the dist.

$$R_2(Q, \overline{m}_t) = \gamma \left[ 1 + \lambda \cos \left( \frac{\tau_0 Q^2}{\overline{m}_t} + \tan \left( \frac{\alpha\pi}{2} \right) \left( \frac{\Delta\tau Q^2}{2\overline{m}_t} \right)^\alpha \right) \exp \left( - \left( \frac{\Delta\tau Q^2}{2\overline{m}_t} \right)^\alpha \right) \right] (1 + \delta Q)$$

Before fitting in two dimensions  $(Q, \overline{m}_t)$ , assume an 'average'  $\overline{m}_t$

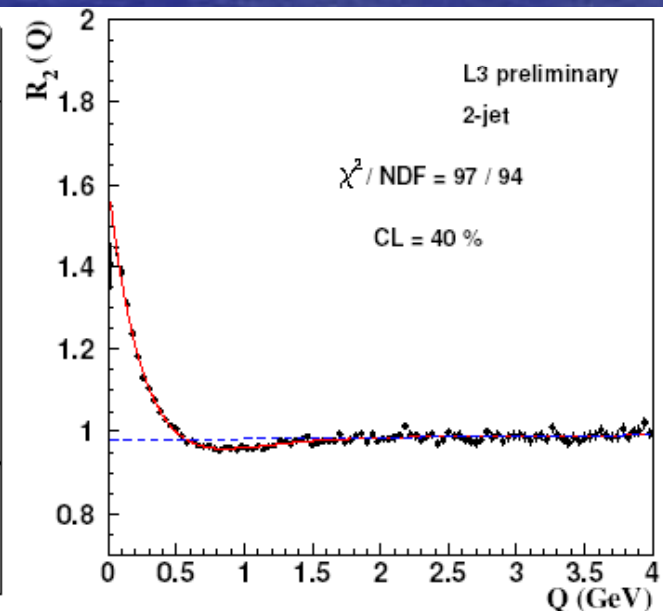
dependence by introducing effective radius  $R = \sqrt{\Delta\tau / (2\overline{m}_t)}$

Also assumed  $\tau_0 = 0$ . Then

$$R_2(Q) = \gamma \left[ 1 + \lambda \cos \left[ (R_a Q)^{2\alpha} \right] \exp \left( - (RQ)^{2\alpha} \right) \right] (1 + \delta Q)$$

$$R_a^{2\alpha} = \tan \left( \frac{\alpha\pi}{2} \right) R^{2\alpha}$$

parameter	$R_a$ free	$R_a^{2\alpha} = \tan \left( \frac{\alpha\pi}{2} \right) R^{2\alpha}$
$\alpha$	$0.42 \pm 0.02$	$0.42 \pm 0.01$
$\lambda$	$0.67 \pm 0.03$	$0.67 \pm 0.03$
$R$ (fm)	$0.79 \pm 0.04$	$0.79 \pm 0.03$
$R_a$ (fm)	$0.59 \pm 0.03$	—
$\delta$	$0.003 \pm 0.002$	$0.003 \pm 0.001$
$\gamma$	$0.979 \pm 0.005$	$0.979 \pm 0.005$
$\chi^2/\text{DoF}$	97/94	97/95
CL	40%	42%



$R_a$  free or not gives same results. – Good CL

$$R_2(Q, \bar{m}_t) = \gamma \left[ 1 + \lambda \cos \left( \frac{\tau_0 Q^2}{\bar{m}_t} + \tan \left( \frac{\alpha\pi}{2} \right) \left( \frac{\Delta\tau Q^2}{2\bar{m}_t} \right)^\alpha \right) \exp \left( - \left( \frac{\Delta\tau Q^2}{2\bar{m}_t} \right)^\alpha \right) \right] (1 + \delta Q)$$

Before fitting in two dimensions  $(Q, \bar{m}_t)$ , assume an 'average'  $\bar{m}_t$

dependence by introducing effective radius  $R = \sqrt{\Delta\tau / (2\bar{m}_t)}$

Also assumed  $\tau_0 = 0$ . Then

$$R_2(Q) = \gamma \left[ 1 + \lambda \cos \left[ (R_s Q)^{2\alpha} \right] \exp \left( - (RQ)^{2\alpha} \right) \right] (1 + \delta Q)$$

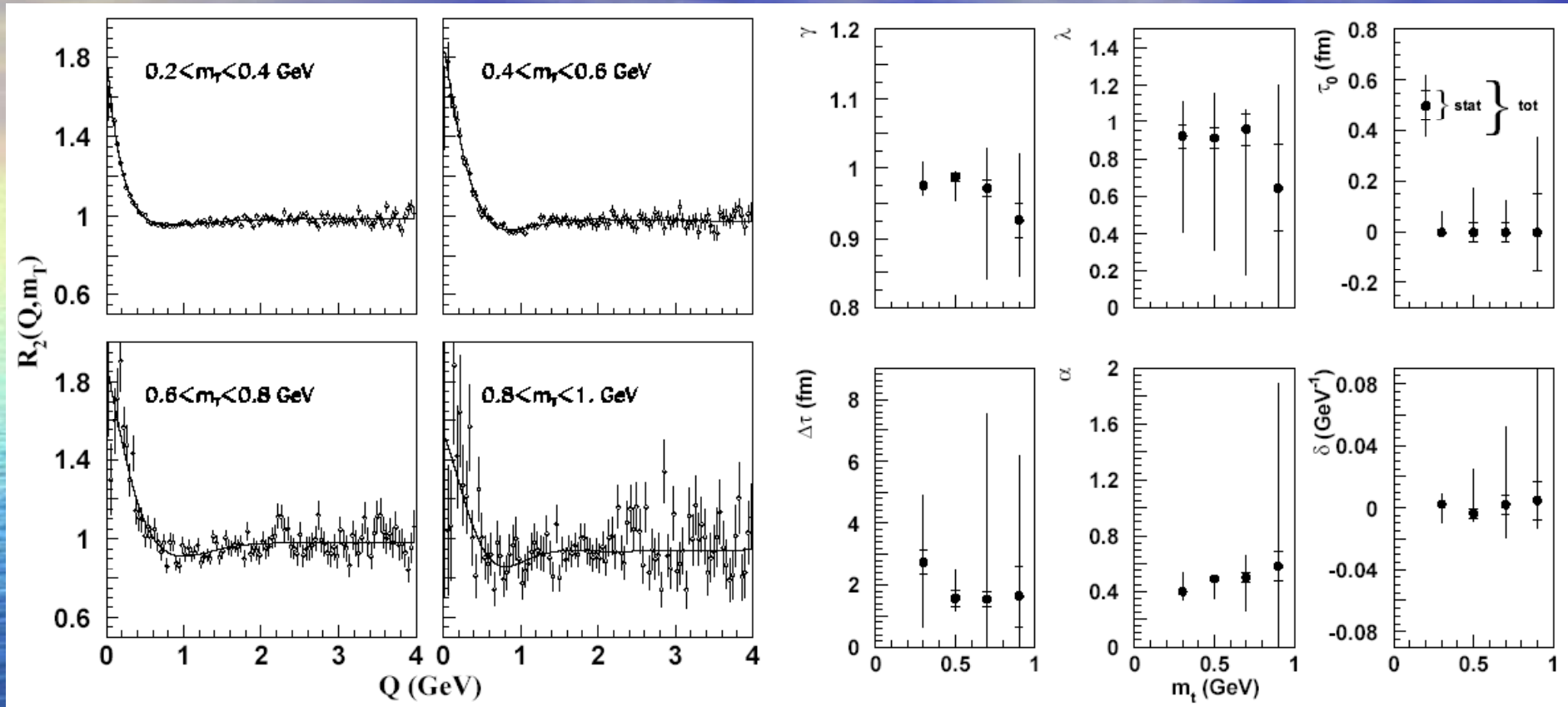
$$R_s^{2\alpha} = \tan \left( \frac{\alpha\pi}{2} \right) R^{2\alpha}$$

parameter	$R_s$ free	$R_s^{2\alpha} = \tan \left( \frac{\alpha\pi}{2} \right) R^{2\alpha}$
$\alpha$	$0.35 \pm 0.01$	$0.44 \pm 0.01$
$\lambda$	$0.84 \pm 0.04$	$0.77 \pm 0.04$
$R$ (fm)	$0.89 \pm 0.03$	$0.84 \pm 0.04$
$R_s$ (fm)	$0.88 \pm 0.04$	—
$\delta$	$-0.003 \pm 0.002$	$0.010 \pm 0.001$
$\gamma$	$1.001 \pm 0.005$	$0.972 \pm 0.001$
$\chi^2/\text{DoF}$	102/94	174/95
CL	27%	$10^{-6}$

For three-jet event



# Results



CLs are good. Parameters are approx. independent of  $m_\tau$ .

$$\tau_0 = 0,0 \pm 0,01 \text{ fm} \quad \alpha = 0,43 \pm 0,01 \quad \Delta\tau = 1,8 \pm 0,4 \text{ fm}$$

# Emission function of 2-jet events

In the  $\tau$ -model, the emission function is:

$$S(x) = \frac{d^4n}{d\tau d^3r} = \left(\frac{m_t}{\tau}\right)^3 H(\tau) \rho_1 \left(k = \frac{rm_t}{\tau}\right)$$

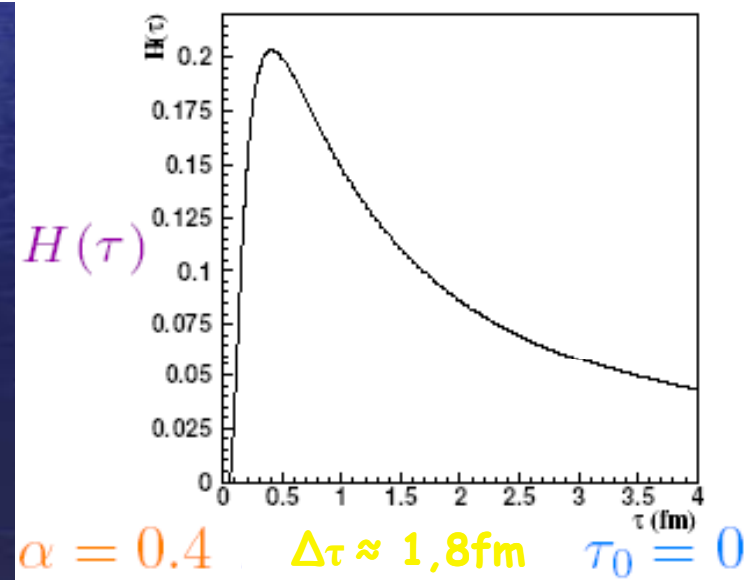
For simplicity, assume  $S(r, z, t) = G(\eta)I(r)H(\tau)$

where

$$G(\eta) = N_y(\eta) \quad I(r) = \left(\frac{m_t}{\tau}\right)^3 N_{p_t}(rm_t/\tau)$$

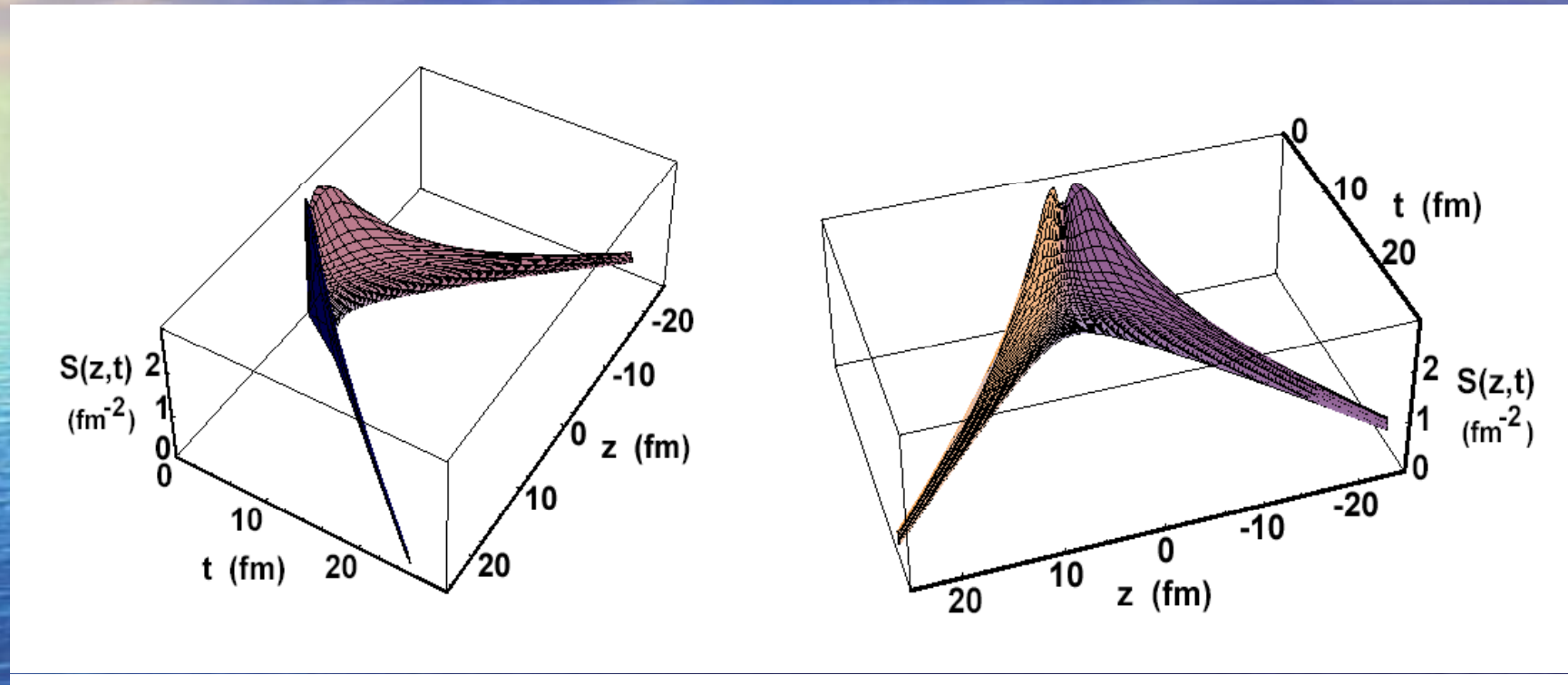
( $N_y, N_{p_t}$  are inclusive single-particle distributions)

So using experimental distributions and  $H(\tau)$  from BEC, we can reconstruct the emission function.



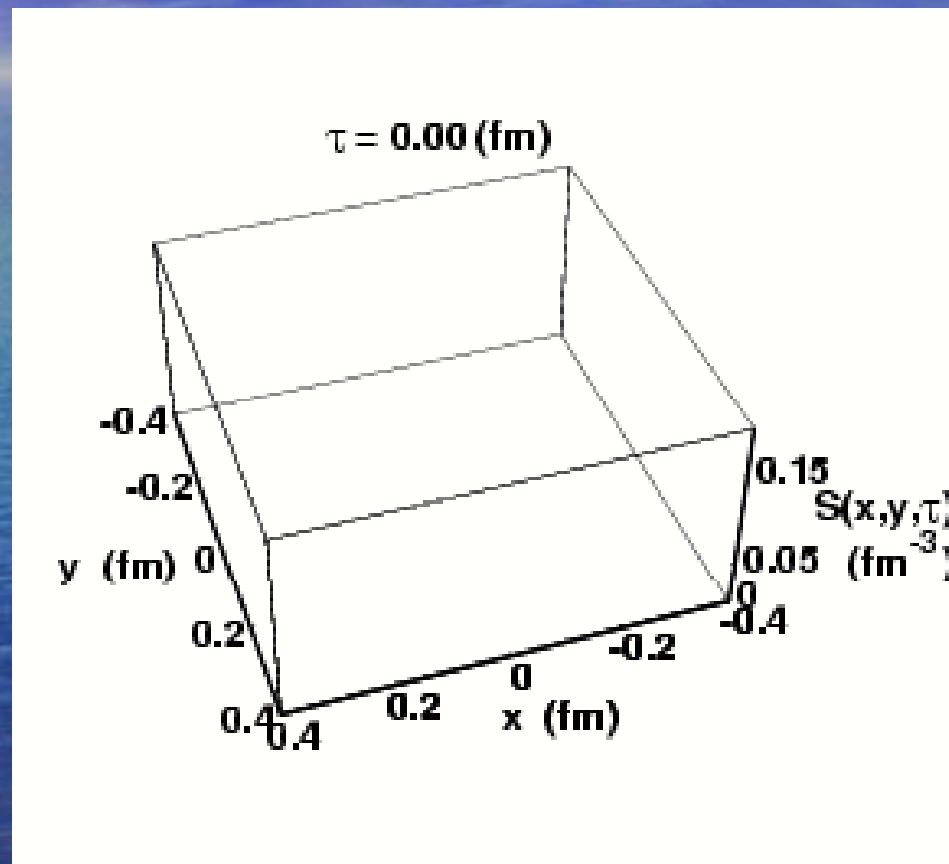
# The emission function

Integrating over  $r$



'Boomerang shape'; Particle production close to the light cone

# The shortest movie of nature



# Summary

- Parametrizing  $R_2$  as a function of  $Q$  only is a reasonably good approximation
- Symmetric Gaussian, Edgeworth, Lévy parametrizations of  $R_2$  do not fit well
- The  $\tau$ -model with a one-sided Lévy proper-time distribution leads to  $R_2(Q, m_t)$ , which successfully fits  $R_2$  for 2-jet events
- Emission function shaped like a boomerang in  $z-t$  and an expanding ring in  $x-y$   
Particle production is close to the light-cone

Thank you for your attention!