THERMALIZATION OF THE BAG CONSTANT

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VACUUM ENERGY DENSITY (VED)

The CJT effective potential approach for composite operators

J.M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. D **10** (1974) 2428

The gluon effective potential to leading order (log-loop level $\sim \hbar$) is

$$V(D) = \frac{i}{2}\int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\left\{\ln(D_0^{-1}D) - (D_0^{-1}D) + 1\right\}$$

where

$$iD_{\mu\nu}(q) = \left\{T_{\mu\nu}(q)d(-q^2,\xi) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

with ξ - gauge fixing parameter and

$$T_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} = g_{\mu\nu} - L_{\mu\nu}(q)$$

$iD_{\mu\nu}(q) \rightarrow iD^0_{\mu\nu}(q)$ when $d(-q^2,\xi) = 1$.

The normalization $V(D_0) = 0$.

$$Tr\ln(D_0^{-1}D) = 8\times 4\ln det(D_0^{-1}D) = 32\ln\left\{\frac{3}{4}d(-q^2,\xi) + \frac{1}{4}\right\}$$

and going over to Euclidean space, one obtains $[E_g = V(D)]$

$$E_g = -16\int\frac{d^4q}{(2\pi)^4}\left\{\ln[1+3\alpha_s(q^2)] - \frac{3}{4}\alpha_s(q^2) + a\right\}$$

$$a = (3/4) - 2\ln 2.$$

$d(q^2;\xi) \equiv \alpha_s(q^2)$ – eff. ("running") charge

Colorless, Transversal

The truly NP gluon effective charge

$$\alpha_s^{TNP}(q^2;\Delta^2) = \alpha_s(q^2,\Delta^2) - \alpha_s(q^2,\Delta^2 = 0)$$
$$= \alpha_s(q^2,\Delta^2) - \alpha_s^{PT}(q^2)$$

Δ^2 is the mass gap responsible for the NP dynamics in the QCD ground state (the so-called Jaffe-Witten (JW) mass gap).

THE BAG CONSTANT

$$B = VED^{PT} - VED = VED^{PT} - [VED - VED^{PT} + VED^{PT}] =$$

$$VED^{PT} - [VED^{TNP} + VED^{PT}] = -VED^{TNP} > 0$$

$$B_{YM} = 16 \int^{q^2_{eff}} \frac{d^4 q}{(2\pi)^4} \left\{ \ln[1 + 3\alpha_s^{TNP}(q^2)] - \frac{3}{4}\alpha_s^{TNP}(q^2) \right\}$$

G.G. Barnaföldi, V. Gogokhia, arXiv:0708.0163

$$P_g = E_g + B_{YM} = B_{YM} + P_{YM}$$

$$P_{YM} = -16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + 3\alpha_s(q^2)] - \frac{3}{4}\alpha_s(q^2) + a \right\}$$

$$\alpha_s(q^2) = \alpha_s^{TNP}(q^2) + \alpha_s^{PT}(q^2)$$

$$\alpha_s^{PT}(q^2) \to 1 + \alpha_s^{PT}(q^2)$$

$$\alpha_s^{PT}(q^2) = \frac{\alpha_s}{1 + \alpha_s b \ln(q^2/\Lambda^2_{QCD})}, \quad b = (11/4\pi),\ \alpha_s \equiv \alpha_s(M_z) = 0.1187$$

$$P_{YM} = P'_{YM} + P_{PT}$$

$$P'_{YM} = -16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + \frac{3}{4}\alpha_s^{TNP}(q^2)] - \frac{3}{4}\alpha_s^{TNP}(q^2) \right\}$$

$$P_{PT} = -16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln\left[1 + \frac{3\alpha_s^{PT}(q^2)}{4 + 3\alpha_s^{TNP}(q^2)}\right] - \frac{3}{4}\alpha_s^{PT}(q^2) \right\}$$

$$P_g = P_{NP} + P_{PT}$$

$$P_{NP} = B_{YM} + P'_{YM}$$

$$B_{YM} = 16 \int^{q^2_{eff}} \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + 3\alpha_s^{TNP}(q^2)] - \frac{3}{4}\alpha_s^{TNP}(q^2) \right\},$$

$$P'_{YM} = -16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + \frac{3}{4}\alpha_s^{TNP}(q^2)] - \frac{3}{4}\alpha_s^{TNP}(q^2) \right\},$$

Confining Ansatz for $\alpha_s^{TNP}(q^2)$

$$\alpha_s^{TNP}(q^2) = \frac{\Delta^2}{q^2}$$

exactly defined
uniquely defined
explicitly gauge-invariant
Wilson criteria –Area Law,
Linear rising potential between heavy quarks

Generalization to non-zero temperatures

In the imaginary time formalism these expressions can be easily generalized to non-zero temperatures T according to the prescription (there is already Euclidean signature)

$$\int \frac{dq_0}{(2\pi)} \to T \sum_{n=-\infty}^{+\infty}, \qquad q^2 = \mathbf{q}^2 + q_0^2 = \mathbf{q}^2 + \omega_n^2 = \omega^2 + \omega_n^2$$

$$\omega_n = 2n\pi T$$

i.e., each integral over q_0 of a loop momentum is to be replaced by the sum over Matsubara frequencies labelled by n, which obviously assumes the replacement $q_0 \to \omega_n = 2n\pi T$ for bosons (gluons). In frequency-momentum space

$$\alpha_s^{TNP}(q^2) = \alpha_s^{TNP}(\mathbf{q}^2, \omega_n^2) = \frac{\Delta^2}{\mathbf{q}^2 + \omega_n^2} = \frac{\Delta^2}{\omega^2 + \omega_n^2},$$

$$T^{-1} = \beta, \quad \omega = \sqrt{\mathbf{q}^2},$$

The derivation of $B_{YM}(T)$

$$B_{YM}(T) = 16 \int \frac{d^3q}{(2\pi)^3}\, T \sum_{n=-\infty}^{+\infty} \left[\ln[1 + 3\alpha_s^{TNP}(\mathbf{q}^2, \omega_n^2)] - \frac{3}{4}\alpha_s^{TNP}(\mathbf{q}^2, \omega_n^2) \right]$$

$$B_{YM}(T) = 16 \int \frac{d^3q}{(2\pi)^3}\, T \sum_{n=-\infty}^{+\infty} \left[\ln[\omega'^2 + \omega_n^2] - \ln[\omega^2 + \omega_n^2] - \frac{3}{4}\frac{\Delta^2}{\omega^2 + \omega_n^2} \right]$$

$$\omega' = \sqrt{\mathbf{q}^2 + 3\Delta^2} = \sqrt{\omega^2 + m'^2_{eff}}$$

$$m'_{eff} = \sqrt{3}\Delta.$$

$$\sum_{n=-\infty}^{+\infty} \frac{1}{\mathbf{q}^2 + \omega_n^2} = \sum_{n=-\infty}^{\infty} \frac{1}{\omega^2 + (2\pi T)^2 n^2} = (2\pi/\beta)^{-2} \sum_{n=-\infty}^{+\infty} \frac{1}{n^2 + (\beta\omega/2\pi)^2}$$

$$= (2\pi/\beta)^{-2}(2\pi^2/\beta\omega)\left(1 + \frac{2}{e^{\beta\omega} - 1}\right) = \frac{\beta}{2\omega}\left(1 + \frac{2}{e^{\beta\omega} - 1}\right).$$

The summation of logarithms

$$\sum_{n=-\infty}^{+\infty} \ln[3\Delta^2 + \mathbf{q}^2 + \omega_n^2] = \ln \omega'^2 + 2\sum_{n=1}^{\infty} \ln(2\pi/\beta)^2[n^2 + (\beta\omega'/2\pi)^2]$$

$$\sum_{n=-\infty}^{+\infty} \ln[\mathbf{q}^2 + \omega_n^2] = \ln \omega^2 + 2\sum_{n=1}^{\infty} \ln(2\pi/\beta)^2[n^2 + (\beta\omega/2\pi)^2].$$

$$L(\omega') = \sum_{n=1}^{\infty} \ln[n^2 + (\beta\omega'/2\pi)^2] = \sum_{n=1}^{\infty} \ln n^2 + \sum_{n=1}^{\infty} \ln\left[1 - \frac{x'^2}{n^2\pi^2}\right]$$

$$L(\omega) = \sum_{n=1}^{\infty} \ln[n^2 + (\beta\omega/2\pi)^2] = \sum_{n=1}^{\infty} \ln n^2 + \sum_{n=1}^{\infty} \ln\left[1 - \frac{x^2}{n^2\pi^2}\right].$$

$$x'^2 = -\left(\frac{\beta\omega'}{2}\right)^2, \quad x^2 = -\left(\frac{\beta\omega}{2}\right)^2.$$

$$L(\omega') - L(\omega) = \sum_{n=1}^{\infty} \ln\left[1 - \frac{x'^2}{n^2\pi^2}\right] - \sum_{n=1}^{\infty} \ln\left[1 - \frac{x^2}{n^2\pi^2}\right]$$

$$= \ln\sin x' - \frac{1}{2}\ln x'^2 - \ln\sin x + \frac{1}{2}\ln x^2,$$

$$L(\omega') - L(\omega) = -\frac{1}{2}\ln\left(\frac{x'^2}{x^2}\right) + \ln\left(\frac{\sin x'}{\sin x}\right).$$

$$x' = \pm i\left(\frac{\beta\omega'}{2}\right), \quad x = \pm i\left(\frac{\beta\omega}{2}\right),$$

$$L(\omega') - L(\omega) = -\frac{1}{2}\ln\left(\frac{\omega'^2}{\omega^2}\right) + \frac{1}{2}\beta(\omega' - \omega) + \ln\left(\frac{1 - e^{-\beta\omega'}}{1 - e^{-\beta\omega}}\right).$$

The explicit expressions for the integrals

$$B_{YM}(T, \omega_{eff}) = -\frac{6}{\pi^2}\Delta^2 B^{(1)}_{YM}(T) - \frac{16}{\pi^2} T \left[B^{(2)}_{YM}(T) - B^{(3)}_{YM}(T) \right],$$

$$B^{(1)}_{YM}(T) = \int_0^{\omega_{eff}} d\omega \frac{\omega}{e^{\beta\omega} - 1}, \quad \beta^{-1} = T,$$

$$B^{(2)}_{YM}(T) = \int_0^{\omega_{eff}} d\omega \; \omega^2 \ln\left(1 - e^{-\beta\omega}\right),$$

$$B^{(3)}_{YM}(T) = \int_0^{\omega_{eff}} d\omega \; \omega^2 \ln\left(1 - e^{-\beta\omega'}\right), \; \omega' = \sqrt{\omega^2 + 3\Delta^2}$$

The derivation of $P'_{YM}(T)$

$$P'_{YM}(T) = \frac{6}{\pi^2}\Delta^2 P^{(1)}_{YM}(T) + \frac{16}{\pi^2}T\left[P^{(2)}_{YM}(T) - P^{(3)}_{YM}(T)\right],$$

$$P^{(1)}_{YM}(T) = \int_0^\infty d\omega \frac{\omega}{e^{\beta\omega}-1},$$

$$P^{(2)}_{YM}(T) = \int_0^\infty d\omega\ \omega^2 \ln\left(1 - e^{-\beta\omega}\right),$$

$$P^{(3)}_{YM}(T) = \int_0^\infty d\omega\ \omega^2 \ln\left(1 - e^{-\beta\bar{\omega}}\right),\ \bar{\omega} = \sqrt{\omega^2 + (3/4)\Delta^2}$$

THE GLUON MATTER EoS

$$P_{GM}(T) = B_{YM}(T) + P'_{YM}(T) + P_{PT}(T) = P_{NP}(T) + P_{PT}(T),$$

$$P_{NP}(T) = \frac{6}{\pi^2}\Delta^2 P_1(T) + \frac{16}{\pi^2}T[P_2(T) + P_3(T) - P_4(T)],$$

$$P_1(T) = \int_{\omega_{eff}}^{\infty} d\omega \frac{\omega}{e^{\beta\omega} - 1},$$

$$P_2(T) = \int_{\omega_{eff}}^{\infty} d\omega\ \omega^2 \ln\left(1 - e^{-\beta\omega}\right),$$

$$P_3(T) = \int_0^{\omega_{eff}} d\omega \; \omega^2 \ln\left(1 - e^{-\beta\omega'}\right),$$

$$P_4(T) = \int_0^{\infty} d\omega \; \omega^2 \ln\left(1 - e^{-\beta\bar{\omega}}\right).$$

$$\omega' = \sqrt{\omega^2 + 3\Delta^2} = \sqrt{\omega^2 + m'^2_{eff}}$$

$$\bar{\omega} = \sqrt{\omega^2 + \frac{3}{4}\Delta^2} = \sqrt{\omega^2 + \bar{m}^2_{eff}}.$$

In the formal PT limit ($\Delta^2 = 0$) from these relations it follows that $\bar{\omega} = \omega' = \omega$. And always $\beta = T^{-1}$.

THERMODYNAMICAL QUANTITIES

The energy density

$$\epsilon(T) = T\left(\frac{\partial P_{GM}(T)}{\partial T}\right) - P_{GM}(T) = Ts(T) - P_{GM}(T)$$

The entropy

$$s(T) = \frac{\partial P_{GM}(T)}{\partial T}$$

The scale-setting scheme

$$q_{eff}^2 = \mathbf{q}_{eff}^2 + \omega_c^2 = \omega_{eff}^2 + \omega_c^2, \quad \omega_c = 2\pi n_c T_c$$

$$\omega_{eff} = \sqrt{q_{eff}^2 - \omega_c^2}, \quad \omega_{eff} \leq q_{eff}$$

$$\omega_{eff} = \sqrt{q_{eff}^2} = 1\ GeV$$

$$\Delta^2 = 0.4564\ GeV^2$$

$$\bar{\omega} = \sqrt{\omega^2 + \bar{m}_{eff}^2}, \quad \bar{m}_{eff} = (\sqrt{3}/2)\Delta = 0.585\ GeV,$$

$$\omega' = \sqrt{\omega^2 + m'^2_{eff}}, \quad m'_{eff} = \sqrt{3}\Delta = 1.17\ GeV.$$

$$\bar{m} = \frac{1}{2}m'$$

The confinement dynamics is nontrivially taken into account directly through the mass gap, and through the thermalization of the Bag constant itself.

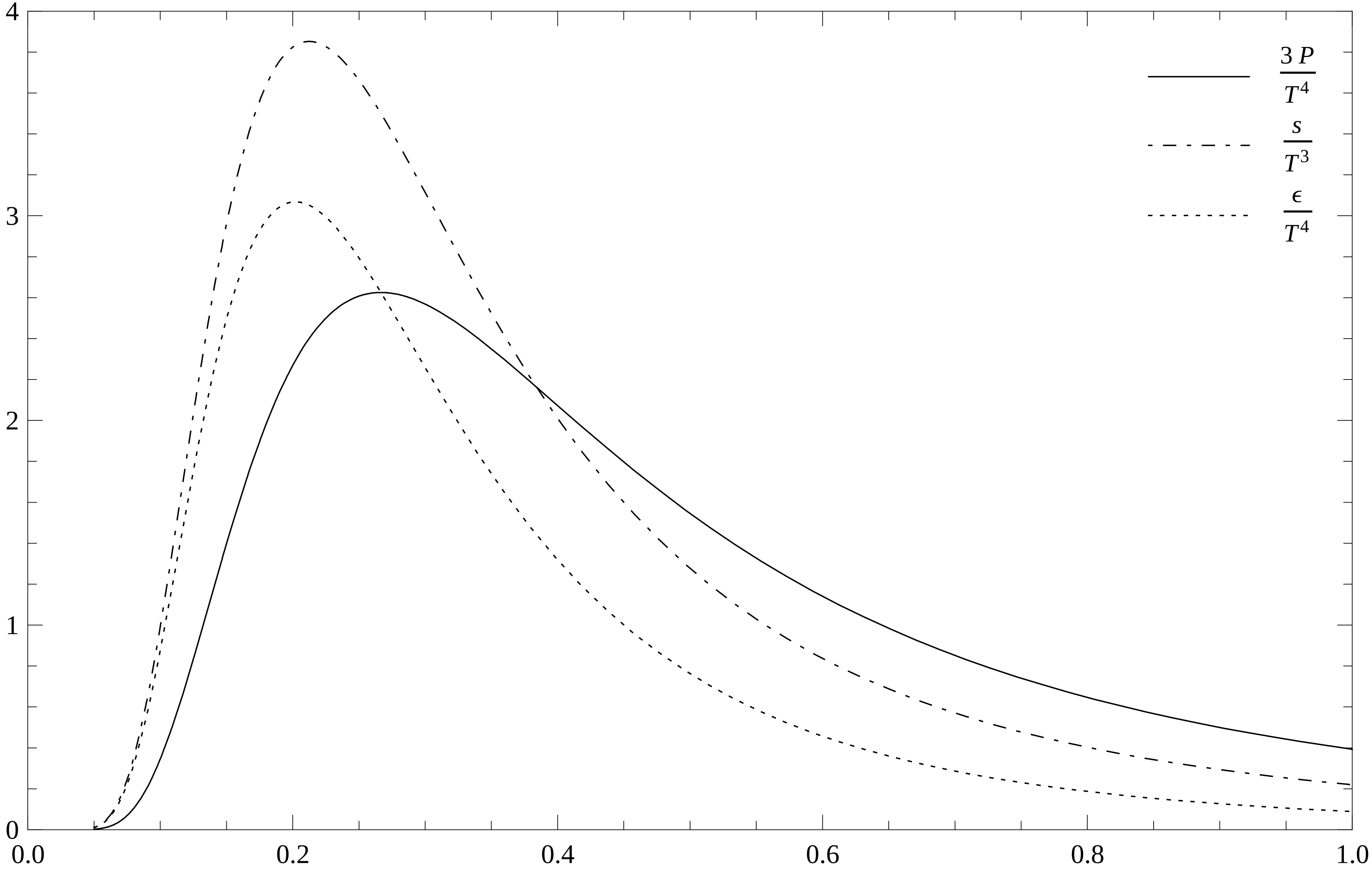


Figure 1: $T^* = 266.5\ MeV$.

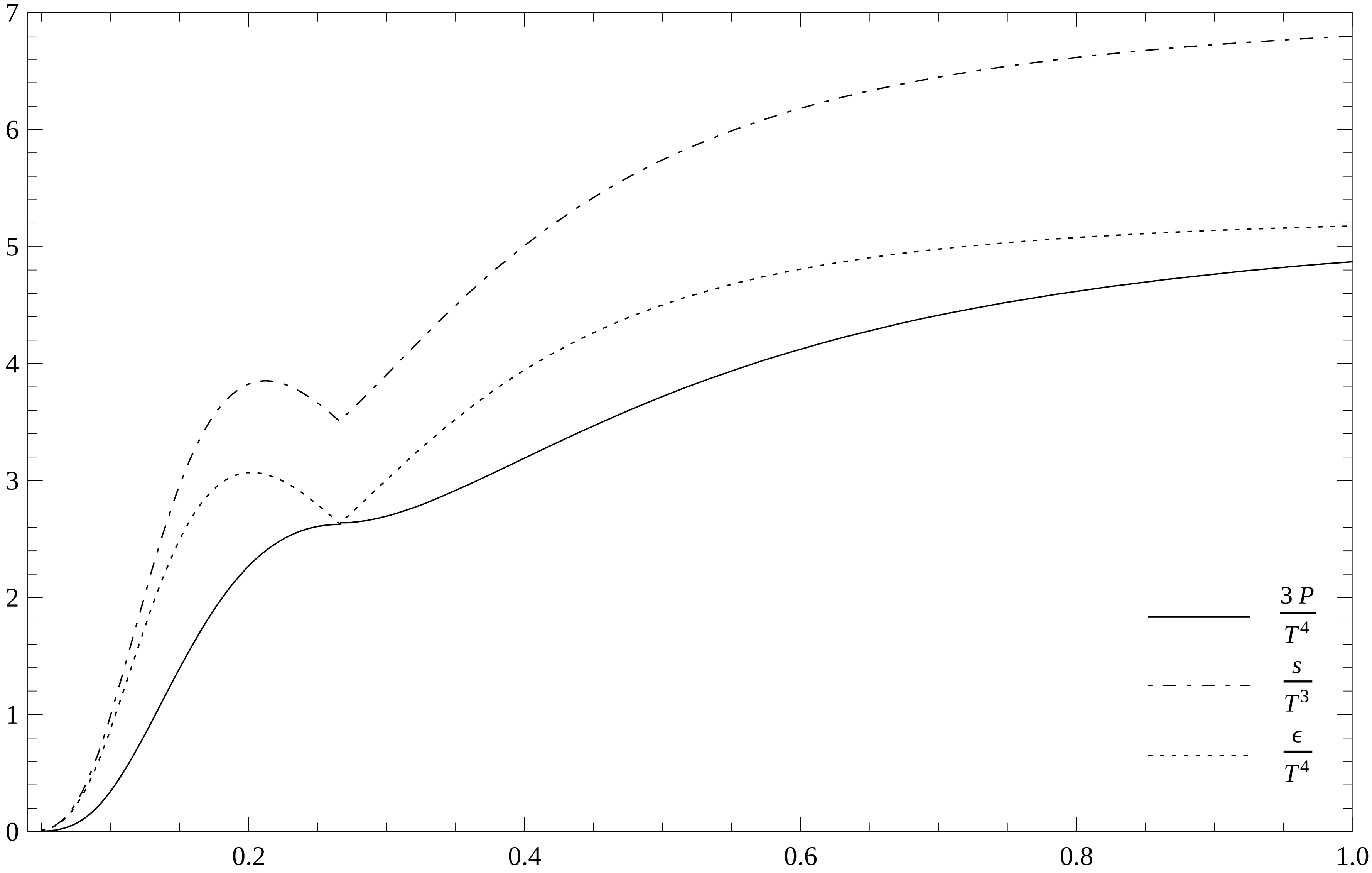
$$P_{GM}(T) = P_{NP}(T) + P_{PT}(T),$$

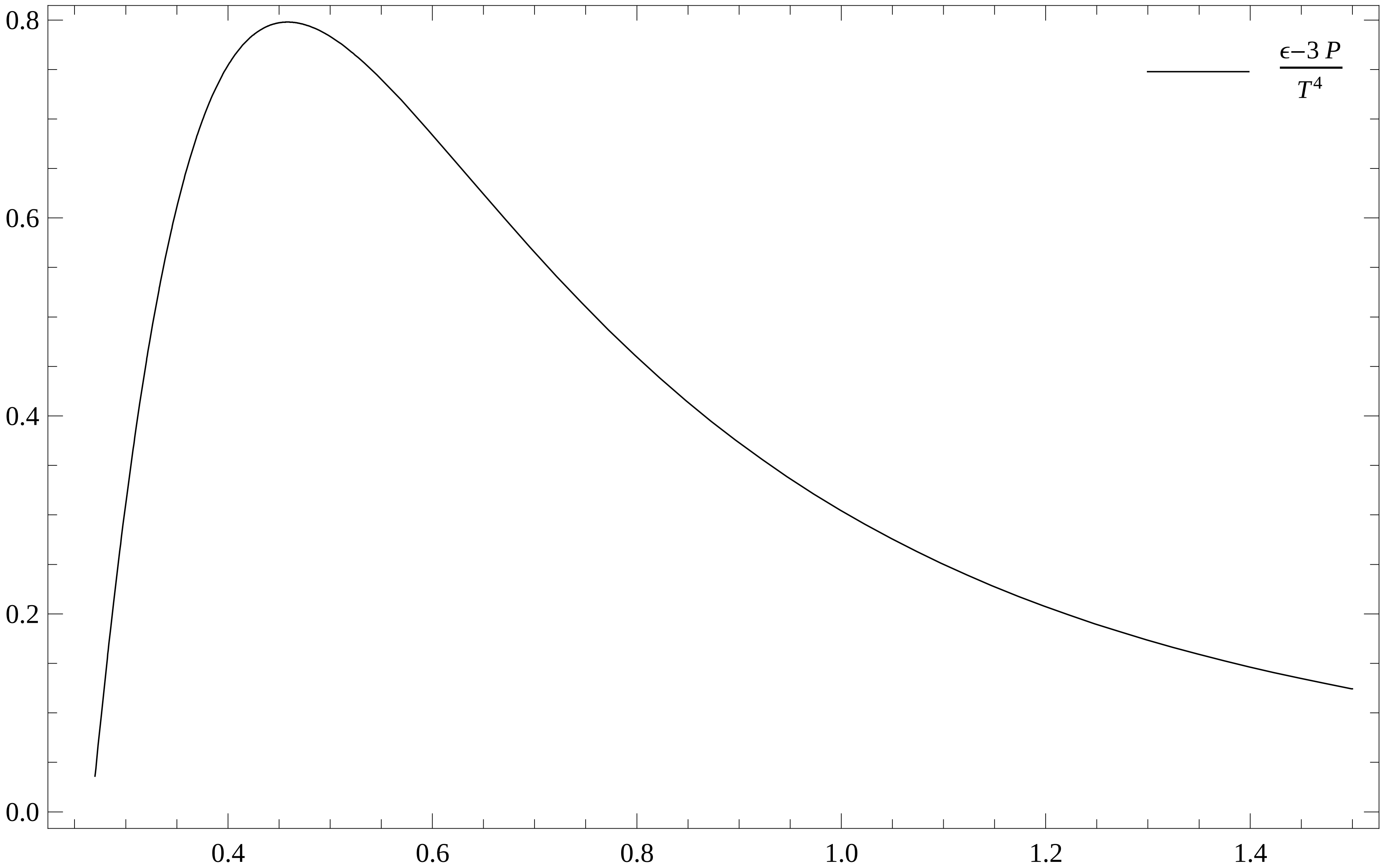
$$P_{PT}(T) = -16 \int \frac{d^4 q}{(2\pi)^4} \left\{ \ln \left[1 + \frac{3\alpha_s^{PT}(q^2)}{4 + 3\alpha_s^{TNP}(q^2)} \right] - \frac{3}{4} \alpha_s^{PT}(q^2) \right\},$$

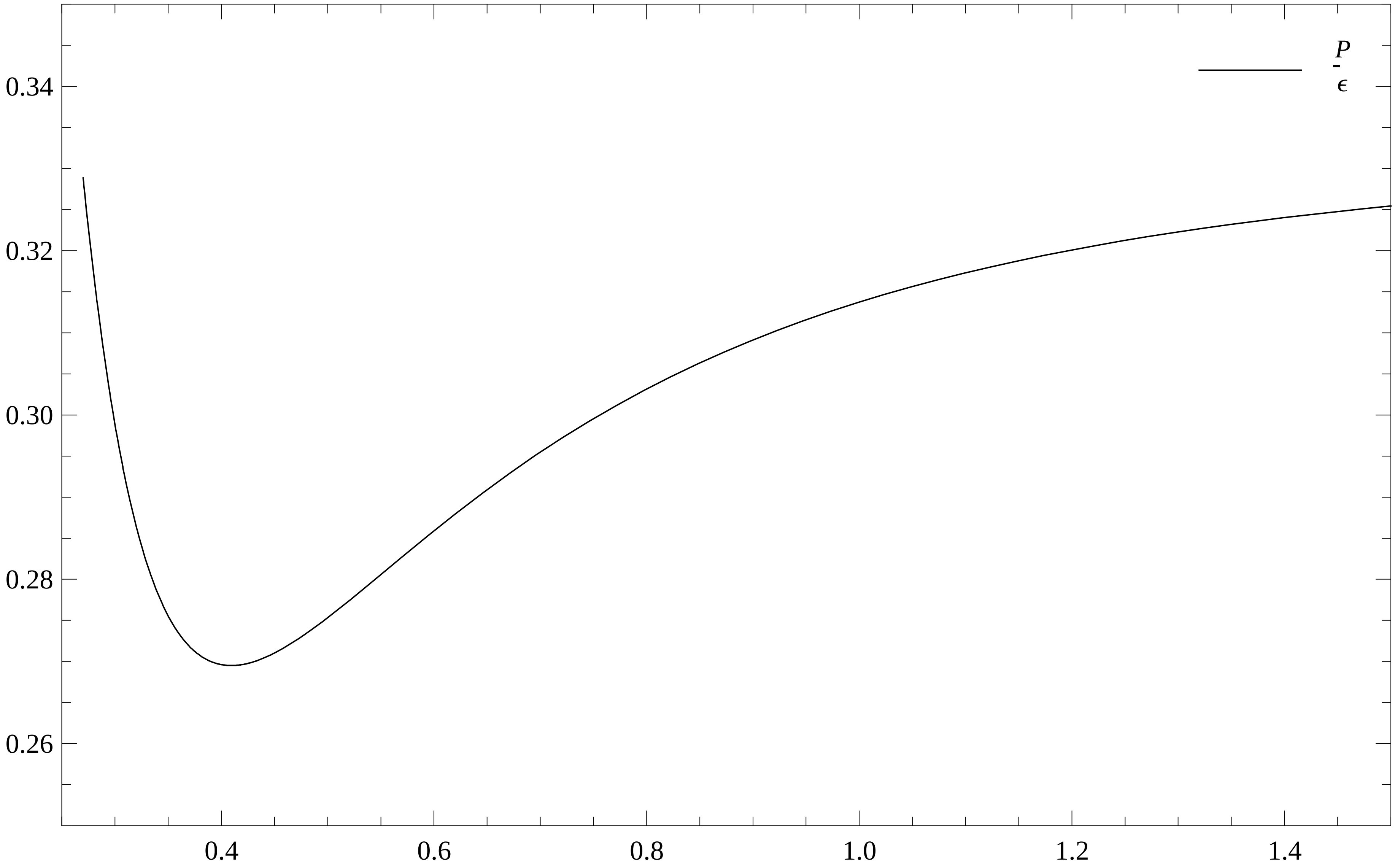
$$P_{SB}(T) = \frac{8}{45} \pi^2 T^4, \quad T \to \infty \quad (\beta \to 0).$$

$$P_{GM}(T) = \theta \left(1 - \frac{T}{T^*} \right) P_{NP}(T) + \theta \left(\frac{T}{T^*} - 1 \right) \left[P_{SB}(T) - P_{NP}(T) \right],$$

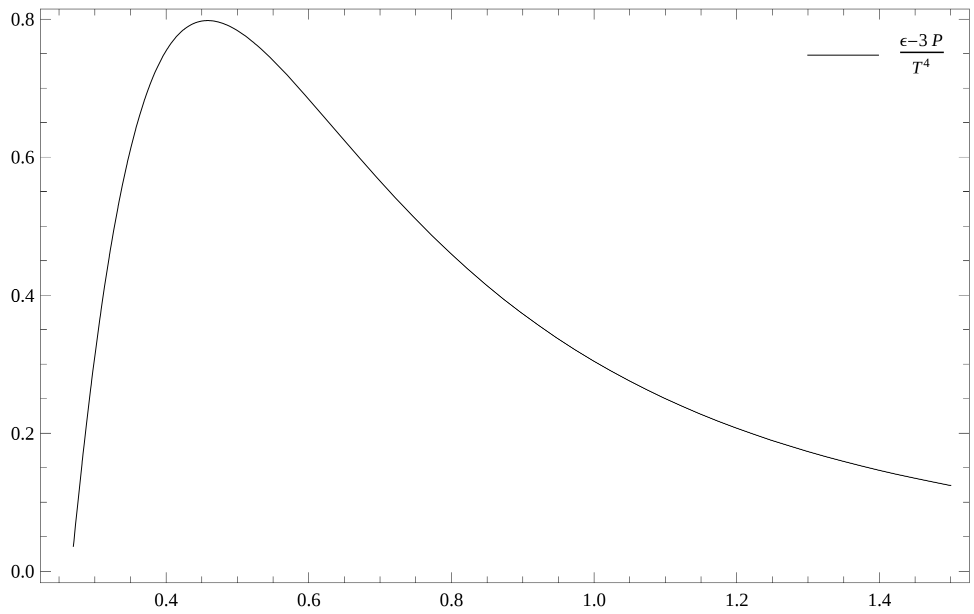
$$P_{GM}(T) = P_{NP}(T) + \theta \left(\frac{T}{T^*} - 1 \right) \left[P_{SB}(T) - 2P_{NP}(T) \right],$$

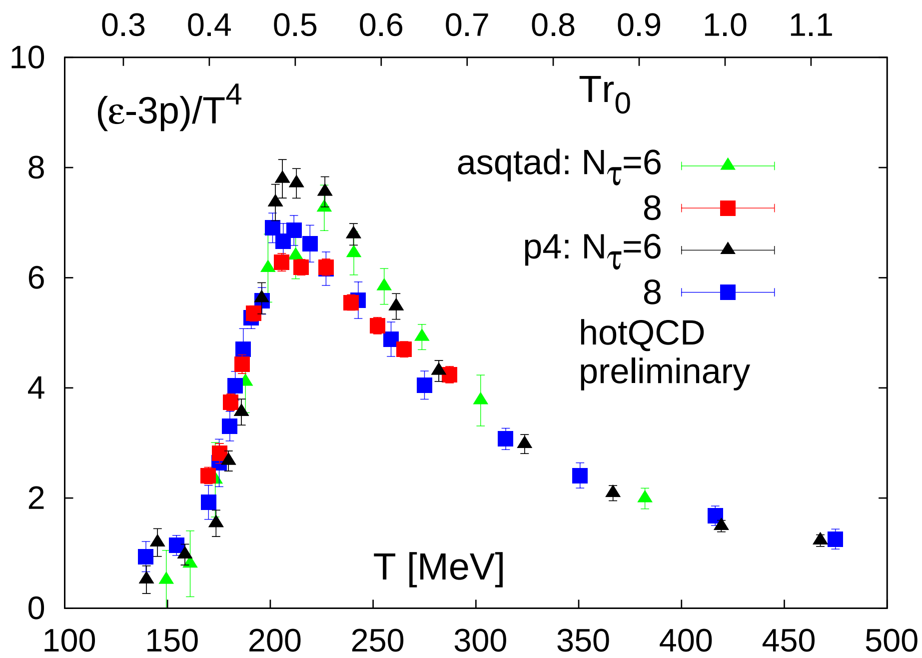

7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1.0
$\frac{3P}{T^4}$
$\frac{s}{T^3}$
$\frac{\epsilon}{T^4}$

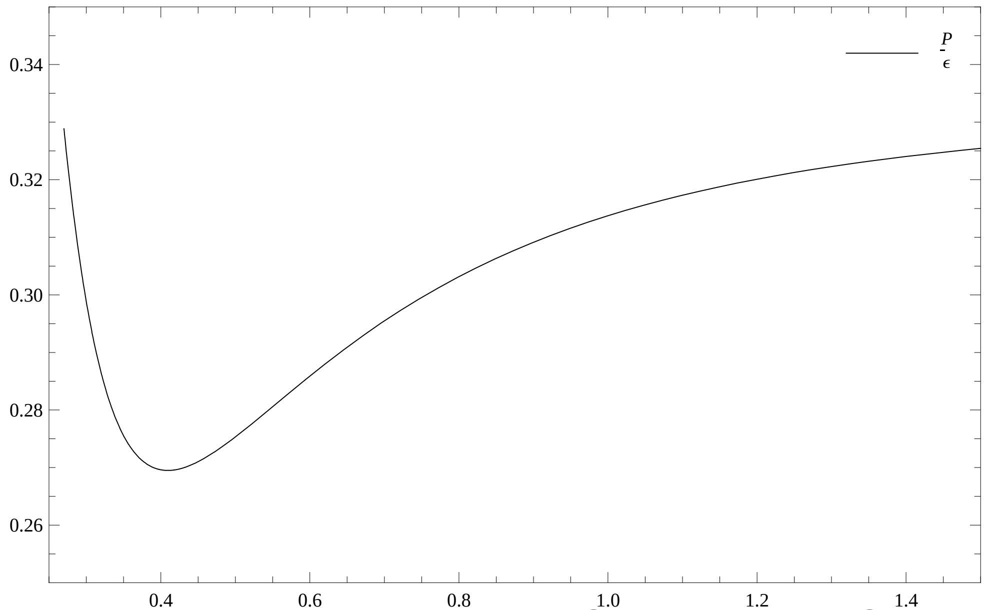

0.8
0.6
0.4
0.2
0.0
0.4
0.6
0.8
1.0
1.2
1.4
ϵ−3 P / T4



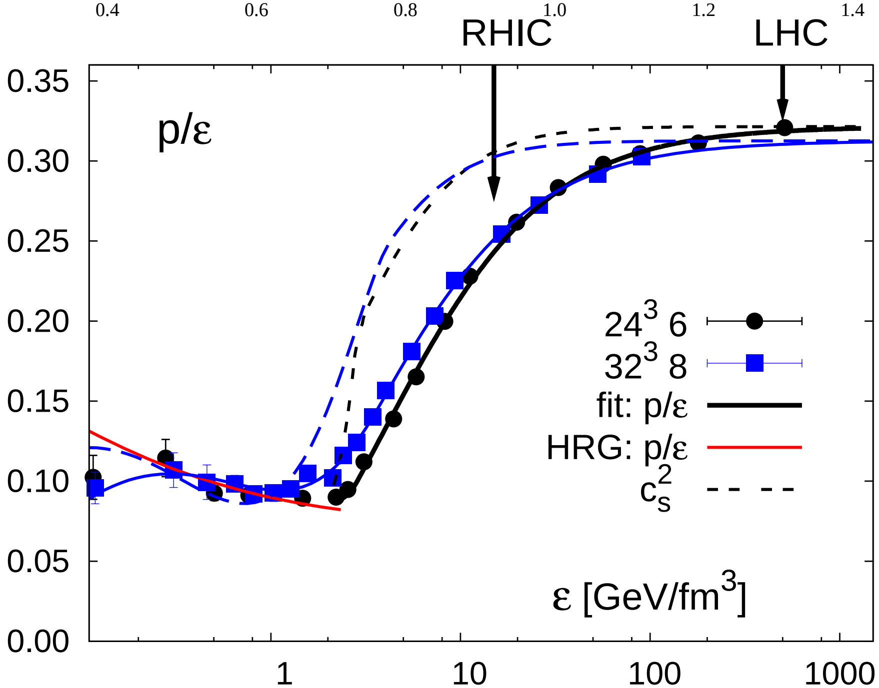
P
ϵ
0.34
0.32
0.30
0.28
0.26
0.4
0.6
0.8
1.0
1.2
1.4


0.8
0.6
0.4
0.2
0.0
0.4
0.6
0.8
1.0
1.2
1.4
$\frac{\epsilon-3P}{T^4}$

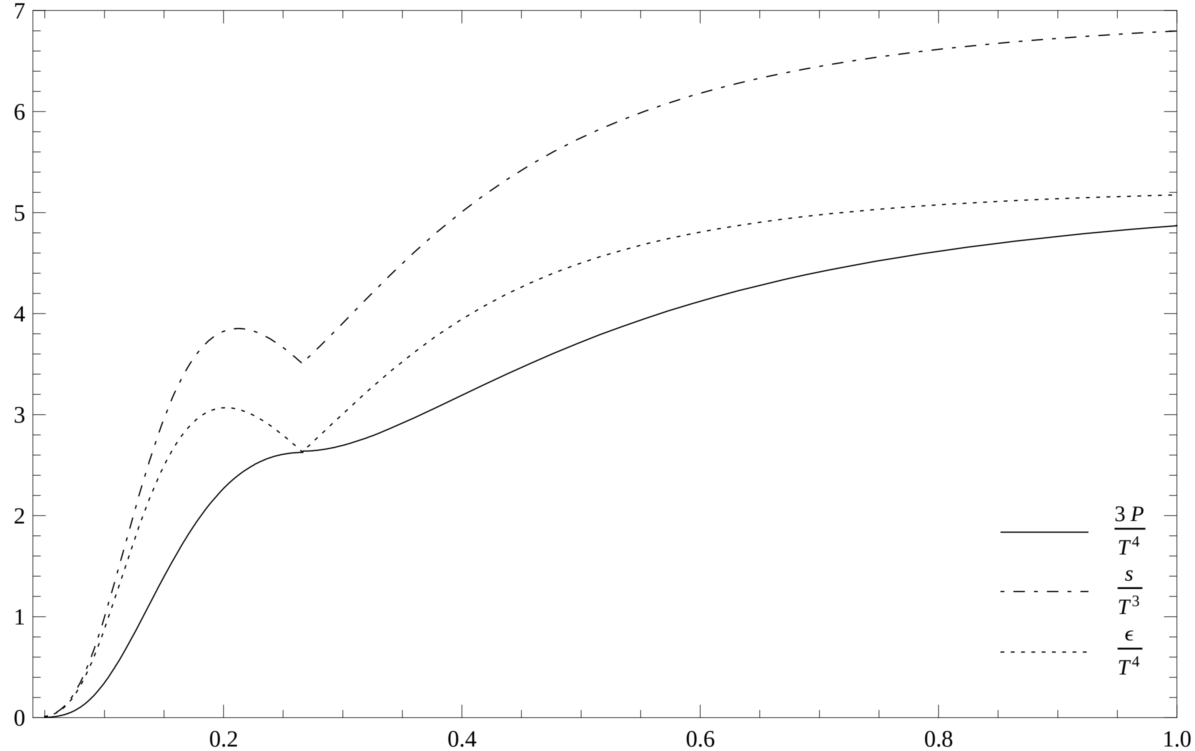

0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1
(ε-3p)/T^4
Tr_0
asqtad: N_τ=6
8
p4: N_τ=6
8
hotQCD
preliminary
T [MeV]
0 2 4 6 8 10
100 150 200 250 300 350 400 450 500



P/ϵ
0.34
0.32
0.30
0.28
0.26
0.4
0.6
0.8
1.0
1.2
1.4



RHIC
LHC
p/ε
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
24^3 6
32^3 8
fit: p/ε
HRG: p/ε
c_s^2
ε [GeV/fm^3]
1
10
100
1000


7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1.0
$\frac{3P}{T^4}$
$\frac{s}{T^3}$
$\frac{\epsilon}{T^4}$