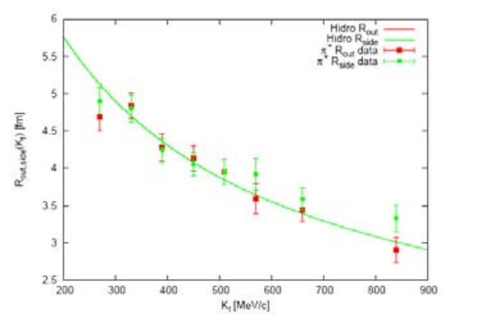
Observables from Relativistic Hydrodynamics



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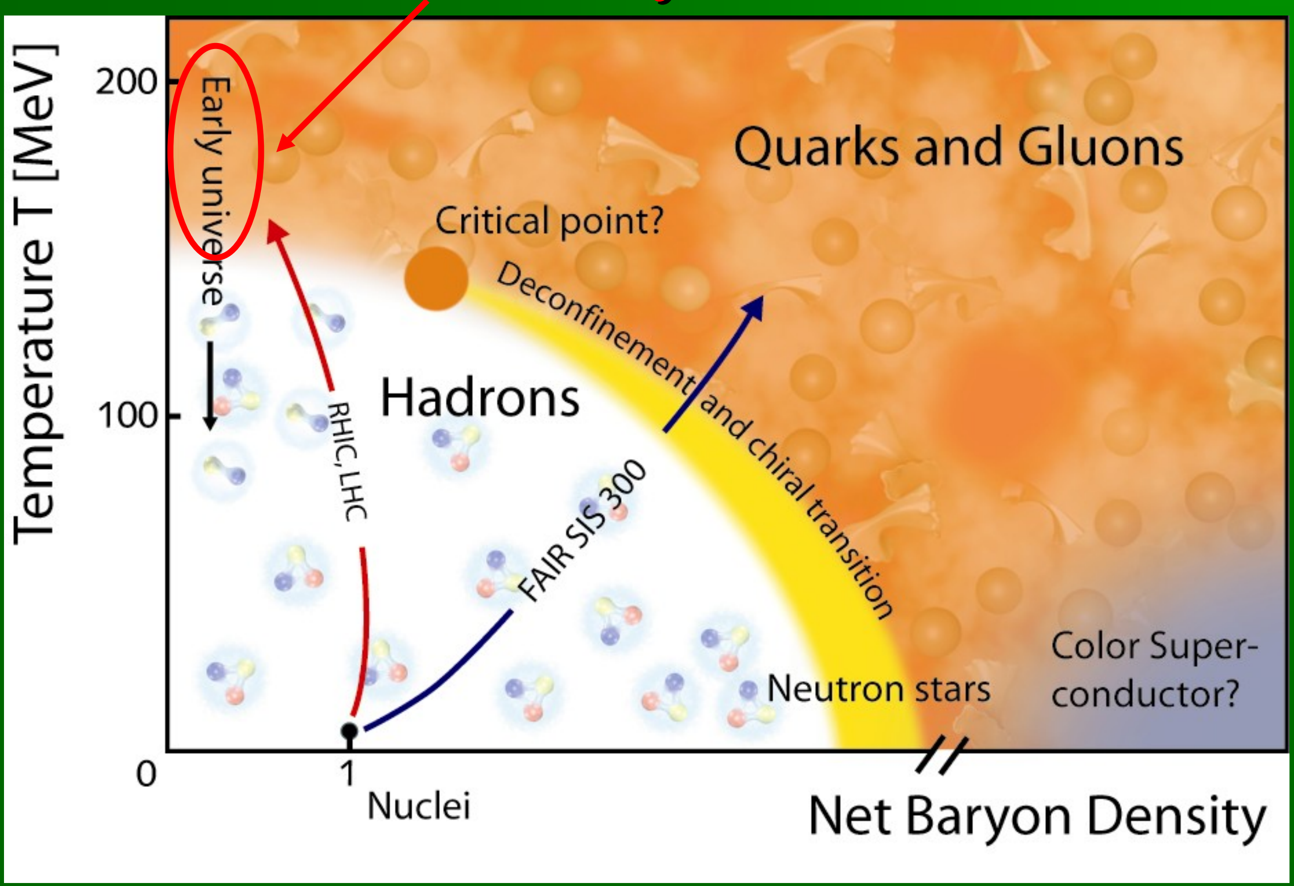
- To compare with data of RHIC PHENIX
- Summary

A well-known diagram

The phase diagram of the quarks

Perfect hydro seen at RHIC

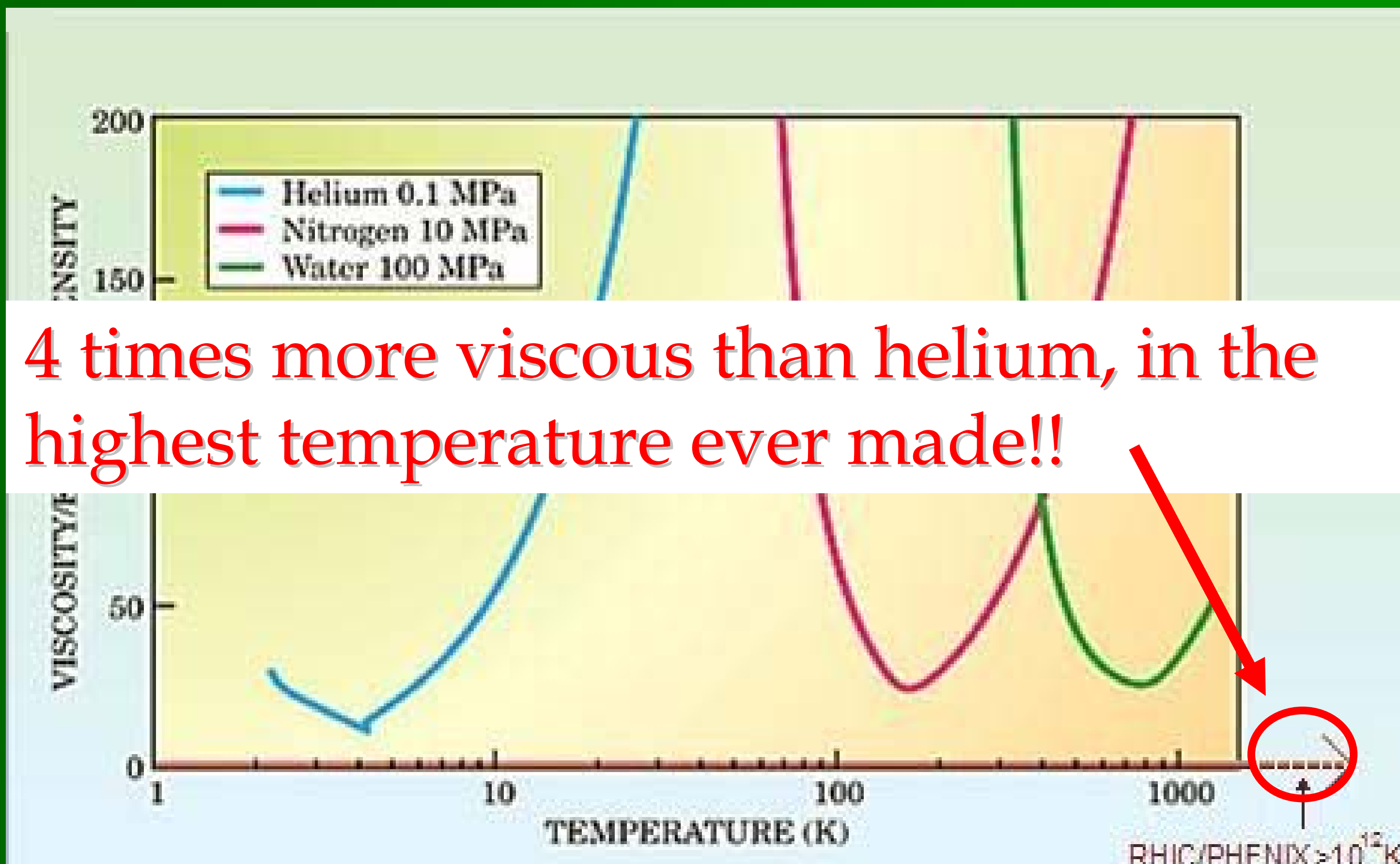


Deconfined state of matter

This is avaible at RHIC

Why we use perfect hidro

The created matter is a (almost) perfect fluid



4 times more viscous than helium, in the highest temperature ever made!!

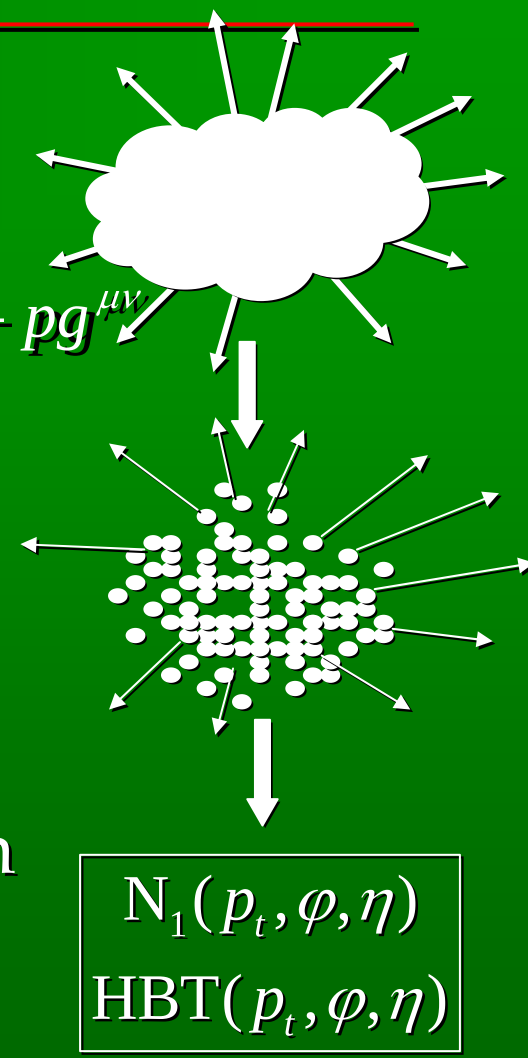
How we use hdrodynamics

- Take hydro equations and EoS

$$\partial_\mu (n u^\mu) = 0 \qquad \epsilon = \kappa p$$

$$\partial_\mu T^{\mu\nu} = 0 \qquad p = nT \qquad T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

- Find a solution (T. Csörgő *et al.)*
- Use a freeze-out condition
  - Eg fixed proper time or fixed temperature
  - Generally a hyper-surface
- Calculate the hadron source function
- Calculate observables
  - E.g. spectra, flow, correlations



Some well-known solution

- Landau's solution (1D, developed for p+p):
  - Accelerating, implicit, complicated, 1D
  - L.D. Landau, Izv. Acad. Nauk SSSR 81 (1953) 51
  - I.M. Khalatnikov, Zhur. Eksp.Teor.Fiz. 27 (1954) 529
  - L.D.Landau and S.Z.Belenkij, Usp. Fiz. Nauk 56 (1955) 309
- Hwa-Bjorken solution:
  - Non-accelerating, explicit, simple, 1D, boost-invariant
  - R.C. Hwa, Phys. Rev. D10, 2260 (1974)
  - J.D. Bjorken, Phys. Rev. D27, 40(1983)
- Others
  - Chiu, Sudarshan and Wang
  - Baym, Friman, Blaizot, Soyeur and Czyz
  - Srivastava, Alam, Chakrabarty, Raha and Sinha

The analized solution

- The solution of T. Csörgő *et al.* (T. Csörgő, L. P. Csernai, Y. Hama és T. Kodama, Heavy Ion Phys. A **21**, 73 (2004))
- They assumed:
  - ellipsoidal symmetry, which is expressed by a scale parameter *s:*

$$s = \frac{x^2}{X(t)^2} + \frac{y^2}{Y(t)^2} + \frac{z^2}{Z(t)^2}$$

  - Scale parameter: the termodynamical quantitees depend only on the scale variable, and the proper time
  - and the expansion 3D anisotropic Hubble:

$$u^\mu = \gamma\left(1, \frac{\dot{X}}{X}x, \frac{\dot{Y}}{Y}y, \frac{\dot{Z}}{Z}z\right)$$

The analized solution

- The mentioned solution of T. Csörgő *et al*. for the thermodynamical quantities (where $p_0 = n_0 T_0$):

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \nu(s)$$

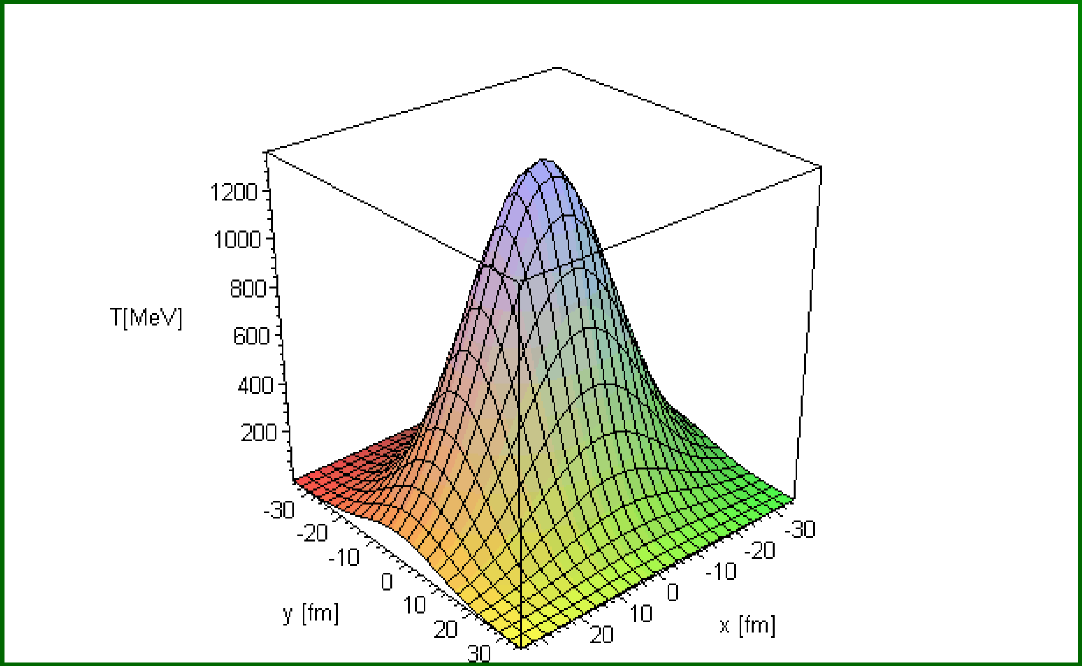
$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{(3/\kappa)} \frac{1}{\nu(s)}$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\left(3+\frac{3}{\kappa}\right)}$$

- Subsituting the velocity field, and ***n*** to the continuity equation, it works if the solution is accelerationless
  - The velocities from the Hubble expansion: $\dot{X}, \dot{Y}, \dot{Z} = const.$
  - It's equal to $$u^\mu = \frac{x^\mu}{\tau}$$

The analized solution

- The ν(s) function is arbitrary, but it's useful to choose ν(s) to an exploding fireball: $\nu(s) = e^{-bs/2}$
- where *b* is a temperature gradient $b = \left.\frac{\Delta T}{T}\right|_r$
- Temperature as a function of proper time:



The source distribution

- Our task is to create a source distribution from the Maxwell-Boltzman statistics

$$S(x,p)d^4x = \mathcal{N} n \exp\left[-\frac{p_\mu u^\mu(x)}{T(x)}\right] H(\tau) d\tau \, p_\mu d^3\Sigma_\mu(x)$$

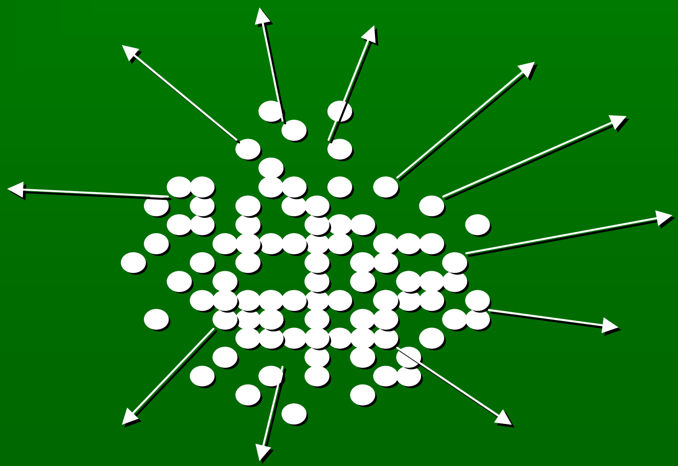
- Extra quantities: N: normalization, n: number density, H(τ): freeze-out density in proper time, $p^\mu d^3\Sigma_\mu(x)$: Cooper-Fry prefactor
- We assume the freeze-out of the hadrons depend only on τ, and it's immediate $H(\tau) = \delta(\tau - \tau_0)$
  - τ_0 is the moment of the freeze-out, each quantity with a 0 index are defined at this very moment

The source distribution

- The Cooper-Fry prefactor describes the hypersurface of the freeze-out
- It's normal to τ_0=*const*. surface

$$p_\mu d^3\Sigma_\mu(x) = p_\mu \frac{u^\mu d^3x}{u^0}$$

$$S(x,p)d^4x = \mathcal{N} n \exp\left[-\frac{p_\mu u^\mu(x)}{T(x)}\right] H(\tau) \frac{p_\mu u^\mu}{u^0} d^4x$$



Invariant Momentum Distribution

- S(x,p) isn't adequate for measurement, because it includes the place where the particles were created
- Integrate respecting to the spatial coordinatas

→ the invariant momentum spectrum

$$N_1(p) = \int_{\mathbb{R}^4} S(x,p) d^4x$$

- Gaussian approximation around maximum
- After integration:

$$N_1(p) = \bar{N} \cdot \bar{E} \cdot \bar{V} \cdot \exp\left[\frac{p^2}{2ET_0} - \frac{E}{T_0} - \frac{p_x^2}{2ET_x} - \frac{p_y^2}{2ET_y} - \frac{p_z^2}{2ET_z} \right]$$

Used quantities

Key of the defined quantities

$$\bar{N} = \mathcal{N} n_0 \left(\frac{2T_0 \tau_0^2 \pi}{E} \right)^{3/2}$$

$$\bar{E} = E - \frac{p_x^2 \left(1 - \frac{T_0}{T_x}\right)}{E} - \frac{p_y^2 \left(1 - \frac{T_0}{T_y}\right)}{E} - \frac{p_z^2 \left(1 - \frac{T_0}{T_z}\right)}{E}$$

$$\bar{V} = \sqrt{\left(1 - \frac{T_0}{T_x}\right)\left(1 - \frac{T_0}{T_y}\right)\left(1 - \frac{T_0}{T_z}\right)}$$

$$T_x = T_0 + \frac{E T_0 \dot{X}_0^2}{b(T_0 - E)}$$

$$T_y = T_0 + \frac{E T_0 \dot{Y}_0^2}{b(T_0 - E)}$$

$$T_z = T_0 + \frac{E T_0 \dot{Z}_0^2}{b(T_0 - E)}$$

$T_{x,y,z}$: anisotrope effective temperatures (slopes) in the momentum spectra:

$$\exp\left[-\frac{p_x^2}{2ET_x} - \frac{p_y^2}{2ET_y} - \frac{p_z^2}{2ET_z} \right]$$

Observables to calculate

And now with $S(x,p)$, and $N_1(p)$ we can calculate the observables:

- the transverse momentum spectrum
- the elliptic flow
- the Bose-Einstein correlation radii

Transverse momentum spectrum

- The detectors of RHIC PHENIX measure at low longitudinal momentum (p_z=0)
- Other transformations are needed

$$p_t = \sqrt{p_x^2 + p_y^2}$$

$$p_x = p_t \cos\phi$$

$$p_y = p_t \sin\phi$$

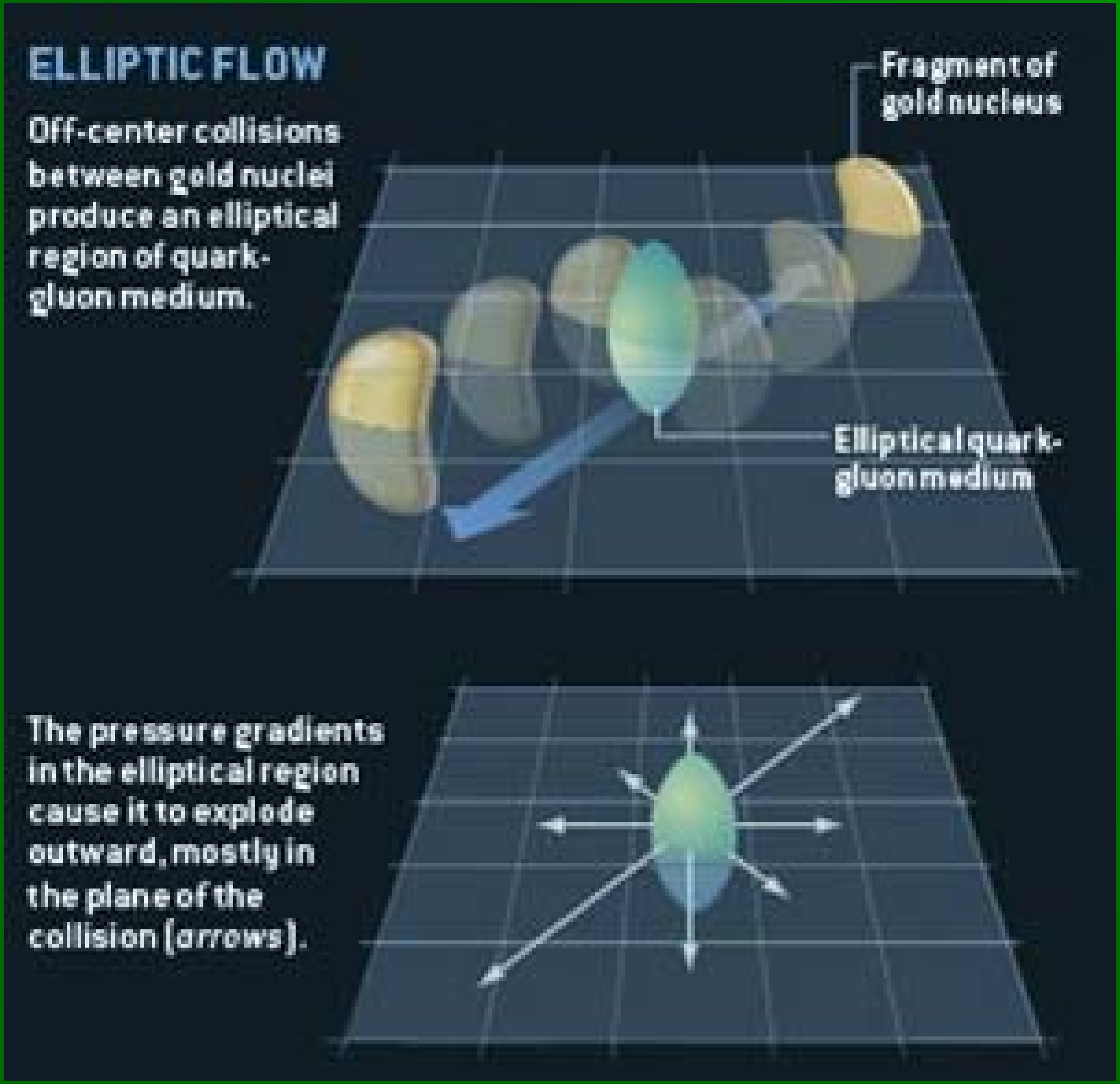
$$\frac{1}{T_{\mathrm{eff}}} = \frac{1}{2}\left(\frac{1}{T_x} + \frac{1}{T_y}\right)$$

$$T_x = T_0 + \frac{E T_0 \dot{X}_0^2}{b(T_0 - E)}$$

- and we have to integrate (on angle phi) in the transverse plane

$$N_1(p_t) = \bar{N}\bar{V}\left(E - \frac{p_t^2 (T_{\mathrm{eff}} - T_0)}{E T_{\mathrm{eff}}}\right)\exp\left[-\frac{p_t^2}{2ET_{\mathrm{eff}}} + \frac{p_t^2}{2ET_0} - \frac{E}{T_0}\right]$$

The elliptic flow I.



How it arises

0 for ideal gas

Here $v_2 > 0$, because of the collective dynamics

The elliptic flow II.

- Matematically it can be defined with the Fourier series

$$N_1(p) = N_1(p_t)\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi)\right]$$

- Each v_n are very small, except v_2

$$v_2 = \frac{\int_0^{2\pi} d\phi N_1(p_t,\phi)\cos(2\phi)}{\int_0^{2\pi} d\phi N_1(p_t,\phi)}$$

- I'll use the modified Bessel functions, defined as follows:

$$I_n(w) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{e}^{w\cos(2\phi)} \cos(2n\phi)d\phi$$

The elliptic flow III.

In the argument of that Bessel function

$$w = \frac{p_t^2}{4E}\left(\frac{1}{T_y} - \frac{1}{T_x}\right)$$

Recall: $$\frac{1}{T_{\text{eff}}} = \frac{1}{2}\left(\frac{1}{T_x} + \frac{1}{T_y}\right)$$

$$T_x = T_0 + \frac{E T_0 \dot{X}_0^2}{b(T_0 - E)}$$

Using an approximation for the modified Bessel functions (if $w \ll 1$):

$$v_2(p_t) = \frac{I_1(w)}{I_0(w)}\left(1 + \frac{2T_0}{E - \frac{p_t^2 (T_{\text{eff}} - T_0)}{E T_{\text{eff}}}}\right)$$

The Bose-Einstein correlation I.

- Measuring two particles correlations
- The only way to determine the size of the source
- R. H. Brown, R. Q. Twiss → HBT radii
- G. Goldhaber, S. Goldhaber, W. Y. Lee, A. Pais
  (Phys. Rev. **120**, 300 (1960))
- By definition:

$$C_2(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1)N_1(p_2)}$$

- We have to consider the quantum-correlations, so the two particle wave function has to be symmetrised

The Bose-Einstein correlation II.

- Here we use an approximation, because $p_1 \approx p_2$ in the regime of measurements

$$K = \frac{p_1 + p_2}{2}$$

$$q = p_1 - p_2$$

- Because of the Bose-Einstein symmetrization, the result is:

$$C_2(q,K) = 1 + \left| \frac{\tilde{S}(q,K)}{\tilde{S}(0,K)} \right|^2$$

- Finally, the result from the used model:

$$C_2(q,K) = 1 + \exp\left[-R_x^2 q_x^2 - R_y^2 q_y^2 - R_z^2 q_z^2 \right]$$

The Bose-Einstein correlation III.

The apparent source radii are:

$$R^2_{x,y,z} \sim \frac{1}{m_t}$$

$$R_x^2 = \frac{T_0 \tau_0^2 (T_x - T_0)}{E_K T_x}$$

$$R_y^2 = \frac{T_0 \tau_0^2 (T_y - T_0)}{E_K T_y}$$

$$R_z^2 = \frac{T_0 \tau_0^2 (T_z - T_0)}{E_K T_z}$$

- Bertsch-Pratt coordinates
  - *out*: the projection of K to the x-y plate
  - *longitudinal*: equal to z
  - *side:* normal to both of them
  - The duration of the freeze-out $\Delta\tau=0$ → in Bertsch-Pratt $R_o=R_s$, and $R_o^2=0.5(R_x^2+R_y^2)$

Comparing to experimental data

- To compare with data from RHIC PHENIX, it's useful to introduce
  - **ε:** momentum space asymmetry (useful for elliptic flow)
  - **u_t^2:** transverse velocity (useful for p_t spectra)

$$\varepsilon = \frac{\dot{X}_0^2 - \dot{Y}_0^2}{\dot{X}_0^2 + \dot{Y}_0^2}$$

$$\frac{1}{u_t^2} = \frac{1}{2}\left(\frac{1}{\dot{X}_0^2} + \frac{1}{\dot{Y}_0^2}\right)$$

- No fit done, just rough comparison
- Most parameters taken from my tutor's article
  - M. Csanád, T. Csörgő, B. Lörstad és A. Ster, J. Phys. G **30**, S1079 (2004)
- Other parameters chosen to achieve good description

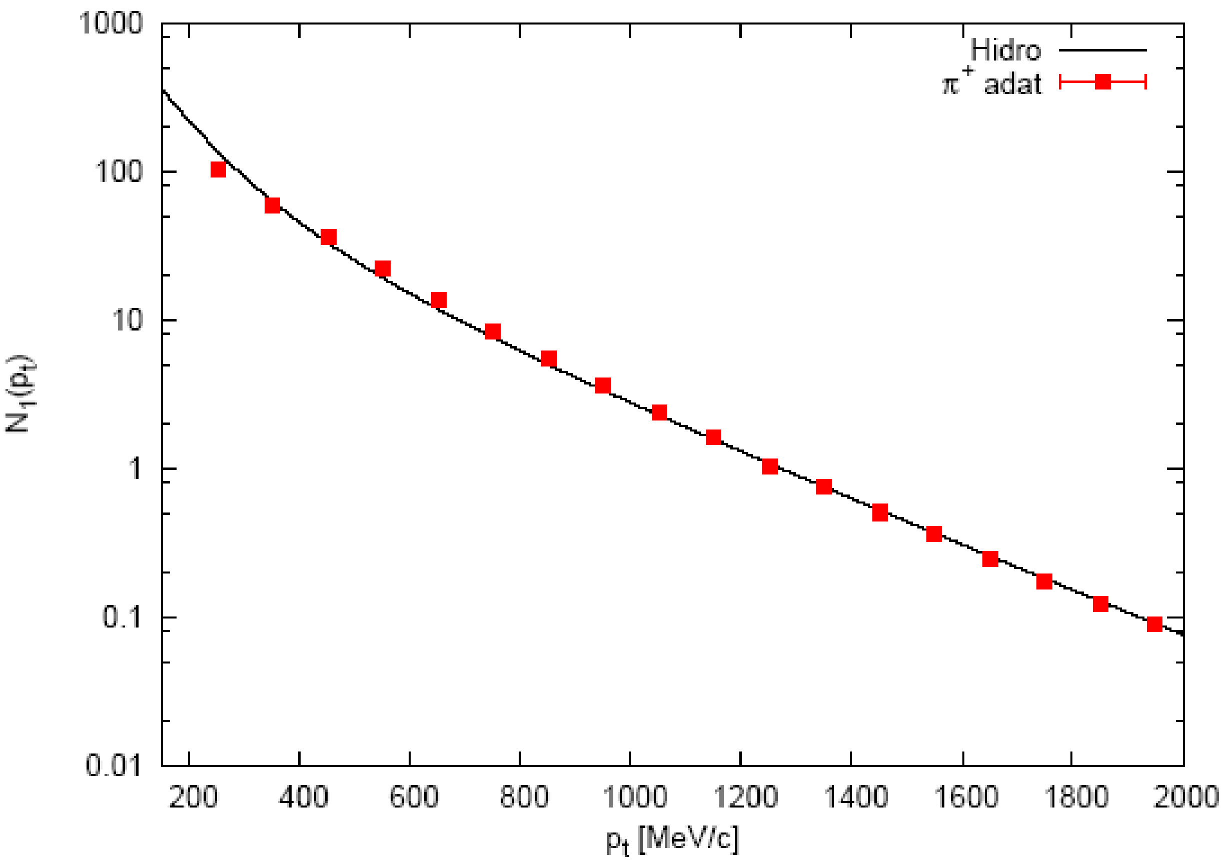
Parameters of the plot

Parameter	value	description
b	-0.1	temperature gradient
N	0.0022	normalization
m	139 MeV	pion mass
T_0	170 MeV	central freeze-out temperature
τ_0	7 fm/c	the freeze-out proper-time
u_t^2	1	transverse velocity at freeze-out
ε	0.2	asymmetry of momentum field
$\dot{Z}_0^2$	2	longitudinal velocity at the freeze-out

Compare with RHIC PHENIX

$$N_1(p_t) = \bar{N}\bar{V}\left(E - \frac{p_t^2(T_{\text{eff}} - T_0)}{ET_{\text{eff}}}\right)\exp\left[-\frac{p_t^2}{2ET_{\text{eff}}} + \frac{p_t^2}{2ET_0} - \frac{E}{T_0}\right]$$

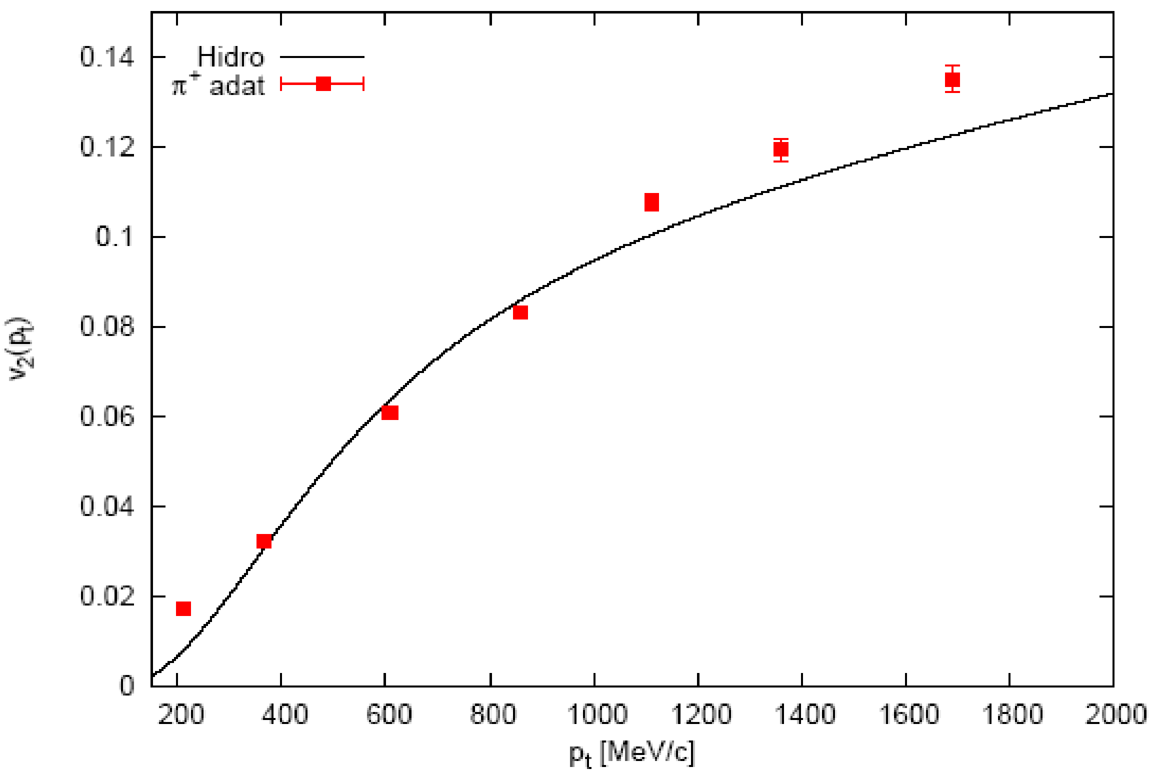
S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. C 69, 034909 (2004)



Compare with RHIC PHENIX

$$v_2(p_t) = \frac{I_1(w)}{I_0(w)}\left(1 + \frac{2T_0}{E - \frac{p_t^2(T_{\text{eff}} - T_0)}{ET_{\text{eff}}}}\right)$$

S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. 91, 182301 (2003)



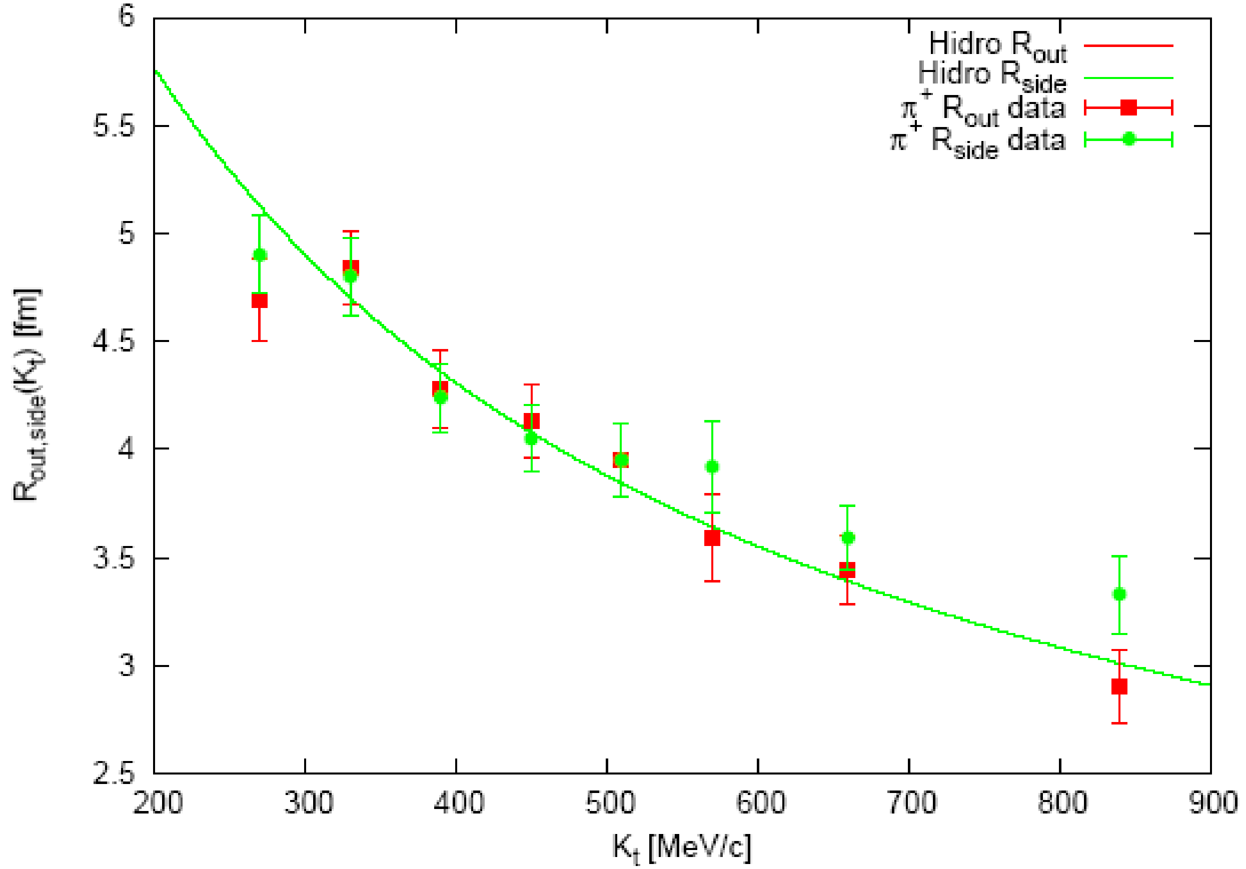
Compare with RHIC PHENIX

$$R_x^2 = \frac{T_0 \tau_0^2 (T_x - T_0)}{E_K T_x}$$

$$R_y^2 = \frac{T_0 \tau_0^2 (T_y - T_0)}{E_K T_y}$$

$$R_z^2 = \frac{T_0 \tau_0^2 (T_z - T_0)}{E_K T_z}$$

S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. 93, 152302 (2004)



Conclusions

- First calculation of these observables from a 1+3D relativistic hydro solution
- The result are in accordance with quantities from other models
- Comparision to experimental data
- Rough agreement
- Future: more observables (dn/dη), real fit

Thank you for your attention!