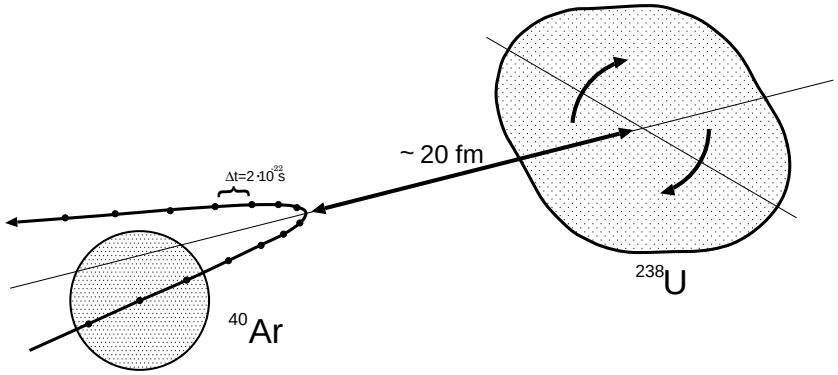
Theoretical description of low-energy Coulomb excitation

Magda Zielińska
IRFU/SPhN, CEA Saclay

- Why Coulomb excitation?
- Basic introduction to Coulex theory
- Approximations and limitations

Coulomb excitation: what's so great about it?

- population of excited states via purely electromagnetic interaction between the collision partners



- renaissance of the technique as ideally suited for state-of-the-art RIB facilities:
  - beam energies available perfect for Coulomb excitation (2-5 MeV/A)
  - high cross sections (excitation of 2_1^+: barns)
  - practical at the neutron-rich side
  - easy way to access non-yrast states and study their properties

Basic facts about Coulex

• Due to the purely electromagnetic interaction the nucleus undergoes a transition from state $|i\rangle$ to $|f\rangle$.

• Then it decays to the lower state, emitting a γ-ray (or a conversion electron).

• The matrix elements $\langle f||M(E2)||i\rangle$ describe the excitation and decay pattern $\rightarrow$ they are directly connected with γ-ray intensities observed in the experiment.

• In the intrinsic frame of the nucleus they are related to the deformation parameters.

Basic facts about Coulex experiments

• Due to the purely electromagnetic interaction the nucleus undergoes a transition from state $|i\rangle$ to $|f\rangle$.

$\rightarrow$ Cline's "safe energy" criterion – if the distance between nuclear surfaces is greater than 5 fm, the nuclear interaction is negligible.

• Then it decays to the lower state, emitting a γ-ray (or a conversion electron).

• The matrix elements $\langle f||M(E2)||i\rangle$ describe the excitation and decay pattern $\rightarrow$ they are directly connected with γ-ray intensities observed in the experiment.

• In the intrinsic frame of the nucleus they are related to the deformation parameters.

Basic facts about Coulex experiments

• Due to the purely electromagnetic interaction the nucleus undergoes a transition from state $|i\rangle$ to $|f\rangle$.

$\rightarrow$ Cline's "safe energy" criterion – if the distance between nuclear surfaces is greater than 5 fm, the nuclear interaction is negligible.

$\rightarrow$ to properly describe the excitation process - particle detectors needed

• Then it decays to the lower state, emitting a γ-ray (or a conversion electron).

• The matrix elements $\langle f||M(E2)||i\rangle$ describe the excitation and decay pattern $\rightarrow$ they are directly connected with γ-ray intensities observed in the experiment.

• In the intrinsic frame of the nucleus they are related to the deformation parameters.

Basic facts about Coulex experiments

• Due to the purely electromagnetic interaction the nucleus undergoes a transition from state $|i\rangle$ to $|f\rangle$.

$\rightarrow$ Cline's "safe energy" criterion – if the distance between nuclear surfaces is greater than 5 fm, the nuclear interaction is negligible.

$\rightarrow$ to properly describe the excitation process – particle detectors needed

• Then it decays to the lower state, emitting a γ-ray (or a conversion electron).
$\rightarrow$ gamma detectors needed

• The matrix elements $\langle f||M(E2)||i\rangle$ describe the excitation and decay pattern $\rightarrow$ they are directly connected with γ-ray intensities observed in the experiment.

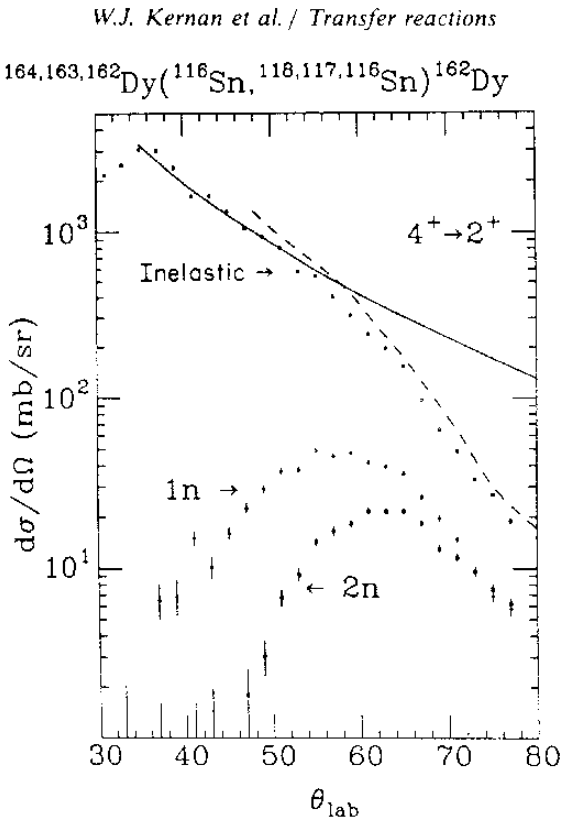
• In the intrinsic frame of the nucleus they are related to the deformation parameters.

Safe energy

- Cline's "safe energy" criterion: purely electromagnetic interaction if the distance between nuclear surfaces is greater than 5 fm

$$D_{min} = 1.25 \cdot (A_p^{1/3} + A_t^{1/3}) + 5.0 \quad [\mathrm{fm}]$$

- empirical criterion based on systematic studies of inelastic and transfer cross-sections at beam energies of few MeV/A

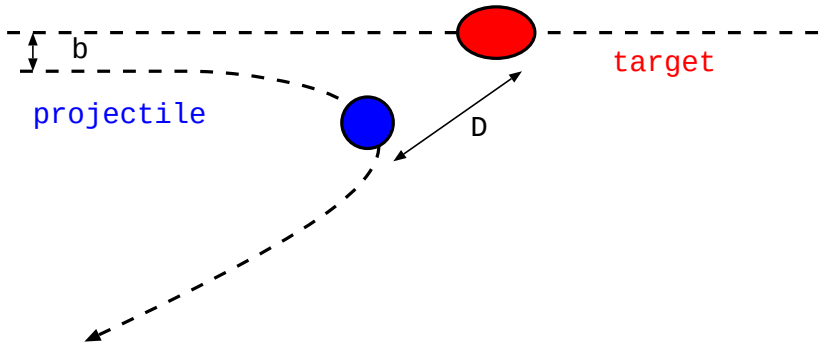


- other criteria established for high-energy Coulex

- one-neutron sub-barrier transfer recently observed in Coulex of ^{42}Ca on ^{208}Pb

”Safe energy” requirement

...is due to the fact that D_{min} has to be sufficiently large.
But it’s not the only way to ensure it!



$$E(\theta_{CM}) = 0.72 \frac{Z_P Z_T}{D_{min}} \cdot \frac{A_P + A_T}{A_T} \left(1 + \frac{1}{sin\left(\frac{\theta_{CM}}{2}\right)}\right) \qquad [MeV]$$

Two possibilities to prepare an experiment:

- choose adequate beam energy (D > D_{min} for all θ)
  low-energy Coulomb excitation
- limit scattering angle, i.e. select impact parameter b (E_b, θ) > D_{min}
  high-energy Coulomb excitation

Semi-clasiccal approximation vs full quantum treatment

- Electromagnetic interaction well-known $\rightarrow$ one can easily calculate Coulomb excitation cross section for any states of the investigated nucleus when its internal structure is known (i.e. matrix elements of electromagnetic transitions)

  - Straightforward method – quantum mechanical treatment: high number of partial waves, coupled channel equations... not very practical :(

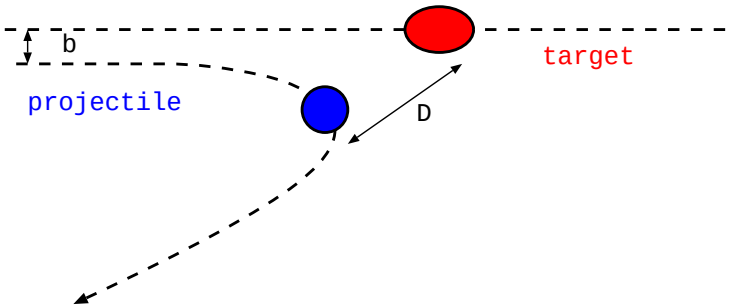
  - Simplified and replaced by a semiclassical approach without any significant loss of accuracy

Simplified description of Coulomb excitation

- Projectile is moving along a hyperbolic orbit and excitation of nuclear states is caused by the time-dependent electromagnetic field between the collision partners
- The trajectories are described by the classical equations of motion, quantum mechanics is used to describe the effect of the electromagnetic field on the nucleus
- Other simplifying assumptions:
  - small energy transfer
  - interaction expanded in a multipole series and only monopole-multipole terms taken into account
  - time separation of the collision (10^{-19}s) and deexcitation process (>10^{-13}s)

Validity of classical trajectories

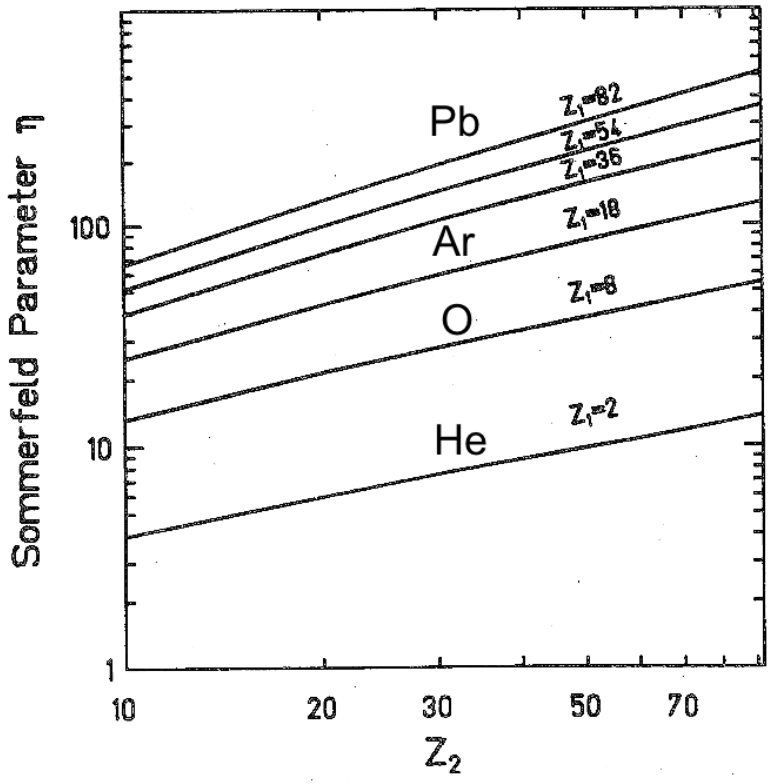
- trajectories can be described by the classical equations of motion, excitation process is described using quantum mechanics.



$\lambda_{\text{projectile}} \ll D$
$\Rightarrow$ Sommerfeld parameter η

$$\eta = \frac{D}{2\lambda\!\!\!^{-}} = \frac{Z_p Z_t e^2}{\hbar v} \gg 1$$

- condition well fulfilled in heavy-ion induced Coulomb excitation



- semiclassical treatment is expected to deviate from the exact calculation by terms of the order $\sim 1/\eta$: most important systematic error in Coulex analysis

Limitation to monopole-multipole terms

The excitation process can be described by the time-dependent Hamiltionan H:

$$H = H_P + H_T + V(r(t))$$

$H_{P/T}$: free Hamiltonian of the projectile/target nucleus
V(t) : time-dependent electromagnetic interaction

If the wave function is expressed by eigenfunctions of the free $H_{P/T}$:

$$\psi(t) = \sum_n a_n(t)\phi_n$$

one gets a set of coupled equations for time-dependent excitation amplitudes $a_n(t)$

$$i\hbar\frac{da_n(t)}{dt} = \sum_m \langle\phi_n|V(t)|\phi_m\rangle \exp(i(E_n - E_m)/\hbar)a_m(t)$$

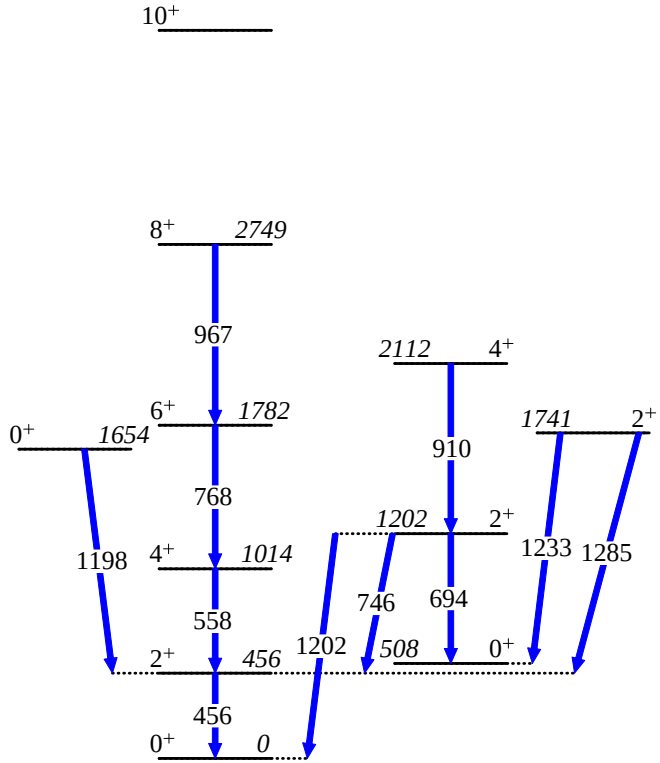
Then V(t) can be expanded in multipoles:

$V(r(t)) = Z_T Z_P e^2/r$ monopole-monopole (Rutherford) term
$+\sum_{\lambda\mu} V_P(E\lambda,\mu) + \sum_{\lambda\mu} V_T(E\lambda,\mu)$ electric multipole-monopole excitation,
$+\sum_{\lambda\mu} V_P(M\lambda,\mu) + \sum_{\lambda\mu} V_T(M\lambda,\mu)$ magnetic excitation (small at low v/c)
+ higher order multipole-multipole terms (neglected – estimated at $\sim$ 0.5 %)

Multi-step excitation and coupled equations

$$i\hbar\frac{da_n(t)}{dt} = \sum_{m,\lambda}\langle\phi_n\|M\lambda\|\phi_m\rangle \exp(i(E_n - E_m)/\hbar)a_m(t)$$

- in heavy-ion induced Coulomb excitation the interaction strength gives rise to multiple excitation
- a nuclear state can be populated directly, via several intermediate states
- excitation probability of an individual state may depend on many matrix elements involved in different excitation paths
- high number of coupled equations for excitation amplitudes $a_n(t)$
- dedicated data analysis codes needed



Deexcitation process

For a given set of matrix elements $\langle \phi_n \| M\lambda \| \phi_m \rangle$ the set of coupled equations

$$i\hbar \frac{da_n(t)}{dt} = \sum_{m,\lambda} \langle \phi_n \| M\lambda \| \phi_m \rangle \exp(i(E_n - E_m)/\hbar) a_m(t)$$

is solved in order to determine level populations.
The same set of matrix elements describes the deexcitation process:

$$P(M\lambda; I_i \to I_f) = \frac{8\pi(\lambda+1)}{\lambda((2\lambda+1)!!)^2} \cdot \frac{1}{\hbar} \cdot \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} \cdot B(M\lambda; I_i \to I_f)$$

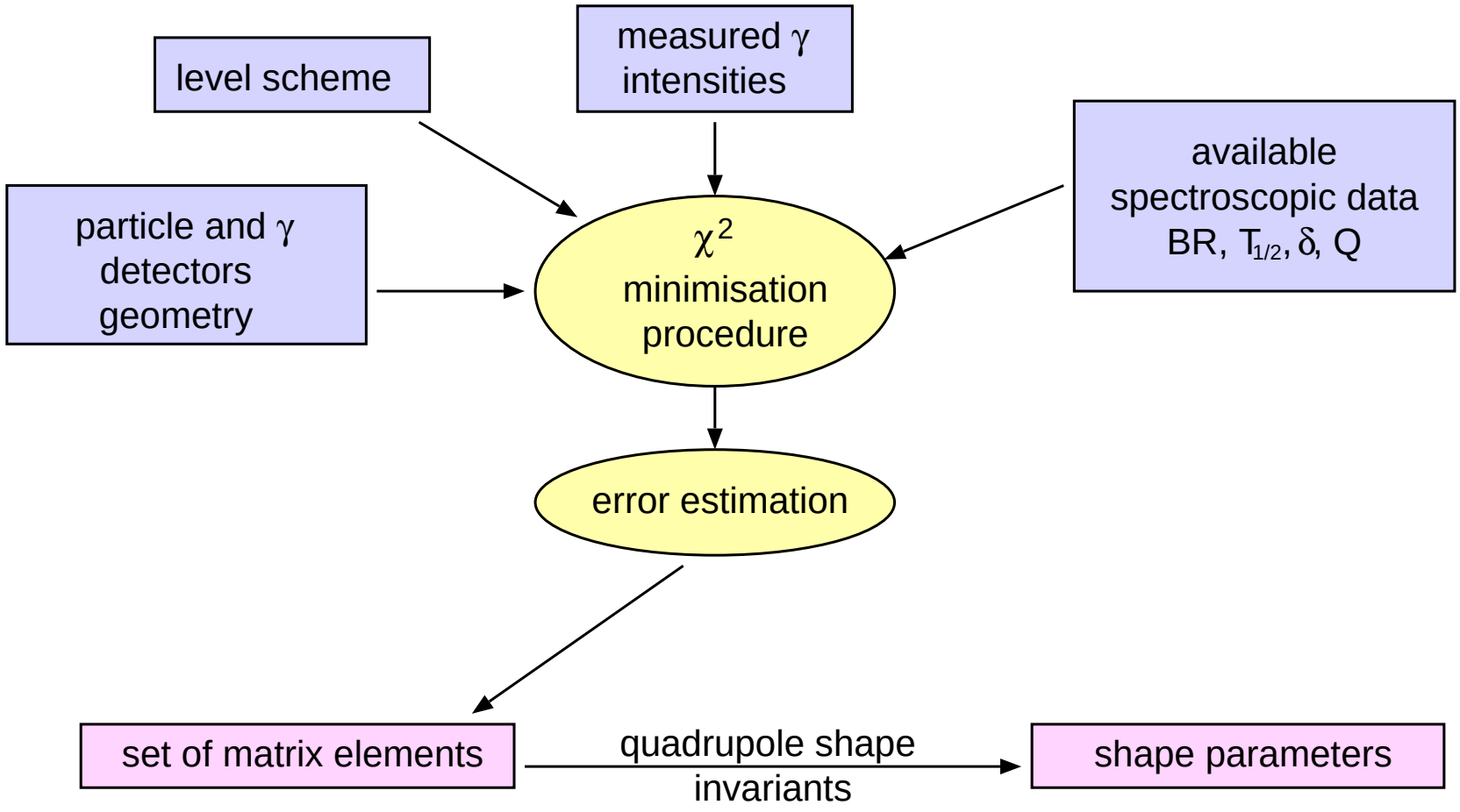
$$B(M\lambda; I_i \to I_f) = \frac{1}{2I_i+1} \langle \phi_n \| M\lambda \| \phi_m \rangle^2$$

The calculation includes effects influencing γ-ray intensities: internal conversion, γ-ray angular distribution, its attenuation due to finite size of Ge detectors, deorientation

GOSIA code

GOSIA: Rochester - Warsaw semiclassical Coulomb excitation least-squares search code

Developed in early eighties by T. Czosnyka, D. Cline, C.Y. Wu (Bull. Am. Phys. Soc. 28 (1983) 745.) and continuously upgraded



Approximations used in GOSIA

1. semi-classical approximation
   - symmetrisation of the trajectory to take into account the energy transfer
2. limitation to the monopole-multipole term
3. other effects taken into account in the description of the excitation process:

- correction for the dipole polarisation effect: quadrupole interaction V(E2) multiplied by a factor

$$1 - d \cdot \frac{E_p A_t}{Z_t^2 (1 + A_p/A_t)} \frac{a}{r}$$

  where d = 0.005 (empirical E1 polarisation strength, from photo-nuclear absorption cross section or GDR energy + dipole sum rule)

  Alder and Winther, Coulomb excitation, appendix J

  important for high-lying levels, high CM angles, heavy beams: ^{104}Ru - 10% change of population of 10^+_γ if effect increased 2 times

- integration over scattering angles covered by particle detectors and incident energy (beam stopping in the target) - changing meshpoints may give an effect of few %, especially for multi-step excitation

Effects taken into account when describing decay

- start from statistical tensors calculated in the excitation stage
  - information on excitation probability and initial sub-state population
- cascade feeding from higher-lying states
- deorientation of the angular distribution (due to recoil in vacuum): Brenn and Spehl two-state model:
  ^{104}Ru - 2% change of matrix elements if effect increased by 20%
- relativistic transformation of solid angles
- attenuation due to finite size of gamma-ray detectors
- simplified (cylindrical) detector geometry

- all approximations have usually an effect $\sim 5\%$ on gamma-ray intensities (often similar to statistical uncertainties, increasing with number of steps needed)
- uncertainties lower than this are rather suspicious (unless they reflect the precision of a lifetime measurement, but the quality of such measurement should also be verified)

Number of parameters versus number of data points

- number of matrix elements coupling low-lying states is higher than number of transitions observed in a Coulex experiment
- some of them have much smaller influence on gamma-ray intensities than others
- even if angular dependence of cross sections is measured, often problem remains underdetermined
  - especially if E1, E3 matrix elements are declared, or for odd nuclei – M1
- additional spectroscopic data needed
  - these data are not used to fix any of the parameters, but enter the χ^2 function exactly like gamma-ray intensities
- in rare very undetermined cases theoretical relations between the ME's may be used (which couplings are negligible, similar, etc...)

Global vs local minimum

Standard question: is this a unique solution, or maybe a different combination of matrix elements can reproduce the experimental data equally well?

Genetic Algorithm in GOSIA: JACOB (P. Napiorkowski, HIL Warsaw)

GOSIA:

- often trapped in a local minimum
- various starting points have to be carefully checked (combinations of signs and magnitudes)
- only for very simple cases ”plug and play”

JACOB:

- scan of the χ^2 surface, ”promising” minima localised
- integration procdure repeated for each of them, real solutions identified
- alternative method for error estimation (in development)