Normalisation in Coulex experiments

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Outline:

1. Normalisation constants in GOSIA:
   - independent normalisation,
   - user-given normalisation constants.

2. Possible techniques

   (elastic scattering, known lifetimes, target excitation …),

3. Selected applications.

Intro

- Low-energy Coulex with heavy ions → sensitive tool to probe electromagnetic structure of nuclei.

- In conjuction with complementary spectroscopic data → understanding of low-lying collective modes and nuclear deformations .

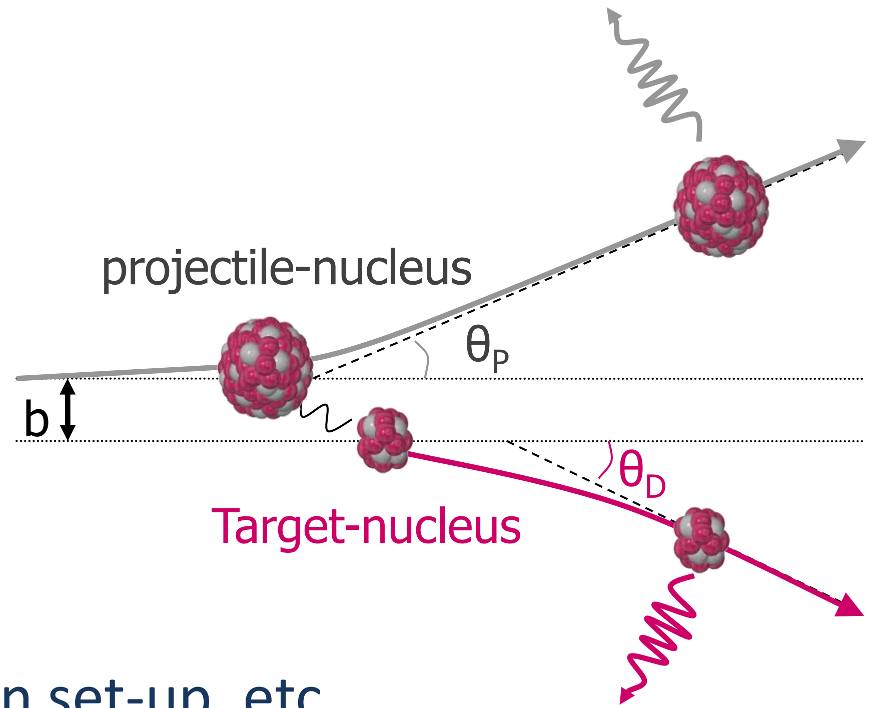
- New challenges when studying exotic nuclei with Coulex →
  low statistics, often lack of complementary data
  i.e. precise lifetime values especially for short-lived, neutron-rich nuclei.

- New solutions for the normalisation for the measured Coulex cross sections.

Normalisation of measured Coulex cross sections

Why needed ?

- To convert **measured** γ-ray intensities to **absolute** excitation cross sections.



- Possible complications:
  deadtime, beam intensity,
  efficiency of the particle detection set-up, etc…

- GOSIA uses normalisation constants to relate experimental and calculated intensities.

- GOSIA does not require the absolute normalization.

Normalisation constants used in GOSIA

- to relate calculated and experimental γ-ray intensities

$$\sum_i (\boxed{C} I_i^c - I_i^e)^2 / \sigma_i^2$$

experimental uncertainty

calculated
γ-ray intensity

experimental γ-ray intensity for
the *i*-th measured transition

normalisation constant
for a given experiment

the product of the:
→ Rutherford cross section,
→ absolute efficiency of particle detection set-up,
→ solid angle covered by the particle detection setup

Normalisation constants used in GOSIA

- to relate calculated and experimental γ-ray intensities
- relative normalisation constants C_m to link *m* experimental data sets (→ different scattering angle, target, etc...)

$$\sum_m \sum_i (C_{\mathrm{gl}} \sum_i (C I_i^c - I_i^e)^2 / \sigma_i^2$$

experimental uncertainty

calculated γ-ray intensity

experimental γ-ray intensity for the *i*-th measured transition

relative normalisation constant for each *m* data sets

- C_m can be **specified by user** or **fitted** by GOSIA.
- C_{global} extracted in the minimisation proccess.

χ^2 in GOSIA

weights ascribed to the various subsets of data

normalisation constants

$$\chi^2 = \sum_{i=1}^{N\,\mathrm{exp}} \sum_{j=1}^{N\,\mathrm{det}} w_{ij} \sum_{k(ij)}^{N\,\gamma\mathrm{exp}} \frac{1}{\sigma_k^2} \left(C_{ij} I_k^c - I_k^e\right)^2$$

$$+ \sum_d w_d \sum_{n_d} \frac{1}{\sigma_{n_d}^2} \left(D_{n_d}^c - D_{n_d}^e\right)^2$$

spectroscopic data points
(lifetimes, BR, mixing coefficients …)

$$+ \sum_{m\,(i\,j)}^{N\,\gamma\mathrm{calc}} \left(\frac{I_j^c(i,j)}{I_n^c(i,j)} - u(i,j) \right)^2 \cdot \frac{1}{u^2(i,j)}$$

„observation upper limit"
of γ-ray intensities

Possible techniques of normalization (1/2)

Normalisation constants can be either **specified by user** or **fitted independently.**

Several techniques possible, the choice depends on the specific of the experiment

→ normalisation constants defined by the user based on:

elastic scattering, target excitation

(*will be discussed later …*)

GOSIA calculates the best normalization factors (i.e. the ones providing the minimum value of $\chi 2$ for a given set of ME's) for each single γ detector independently.

Option **INR** (**I**ndependent **N**o**R**malization) in the GOSIA input file

(more details → see page 122 in the GOSIA manual:

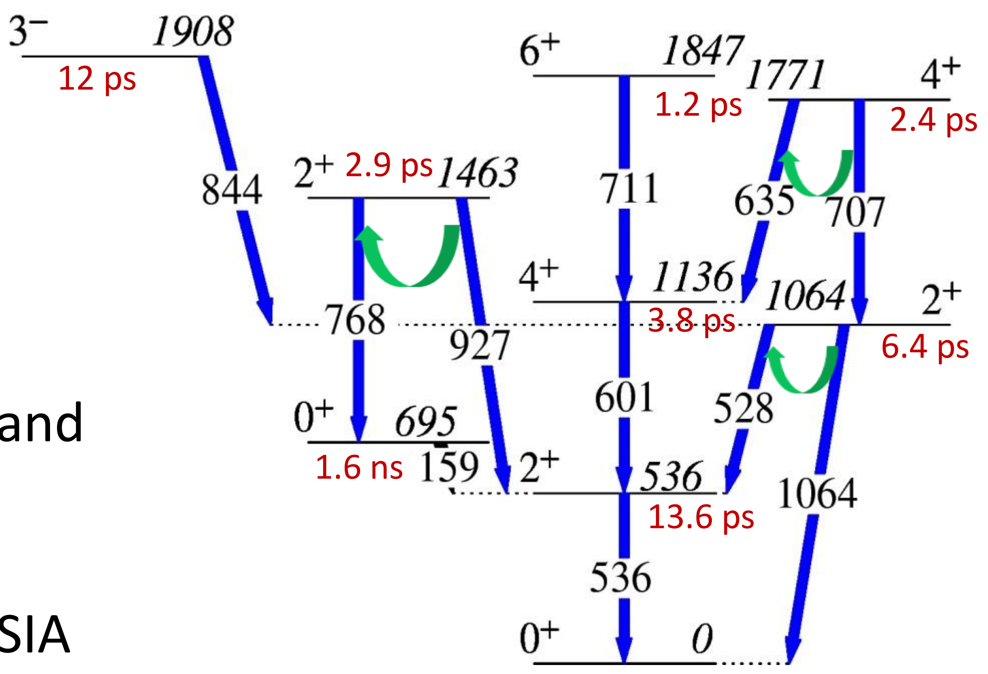
http://www.pas.rochester.edu/~cline/Gosia/index.html)

Independent normalization

- When should be used ?
- Always if possible !

⇩

Number of fitted matrix elements and experimental data <u>large enough</u> to neglect the impact of introducing few more parameters fitted by GOSIA



^{100}Mo

Coulex, HIL, Warsaw, 2007

K. Wrzosek-Lipska et al., PRC 86, 064305 (2012)

Multistep Coulex (with stable beams):

- lifetimes (+ other spectroscopic data) known
  → no need for other kind of normalization
- beam intensities ~ 10^9pps -> high statistic

Knowledge of spectroscopic data

- In **even-even nuclei** normalisation is usually fulfilled by an **independent lifetime measurements** → examples for exotic nuclei are the cases of:
  - **^{74,76}Kr** E. Clement *et al.*, PRC **75**, 054313 (2007),
  - **$^{182-188}$Hg** N. Bree *et al.*, PRL **112**, 162701 (2014).

- For **odd-mass** or **odd-odd nuclei multiple mixing ratios** of the γ-ray transitions become important .

- Low-energy transitions in **heavy nuclei** can also be **strongly converted**

  ➔ strongest excitation path may not necessary result an intense γ-ray decay.

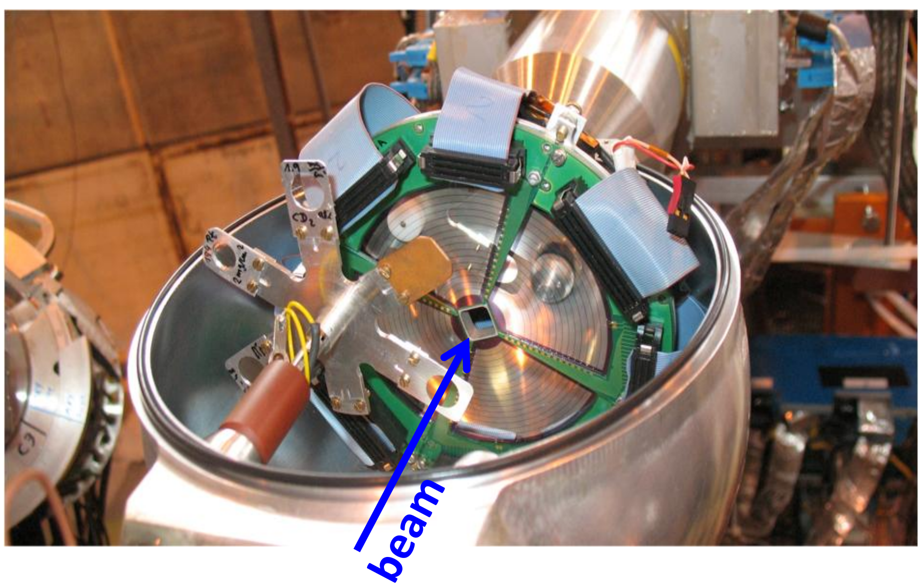
  → normalisation to the next higher-lying transition usually possible:

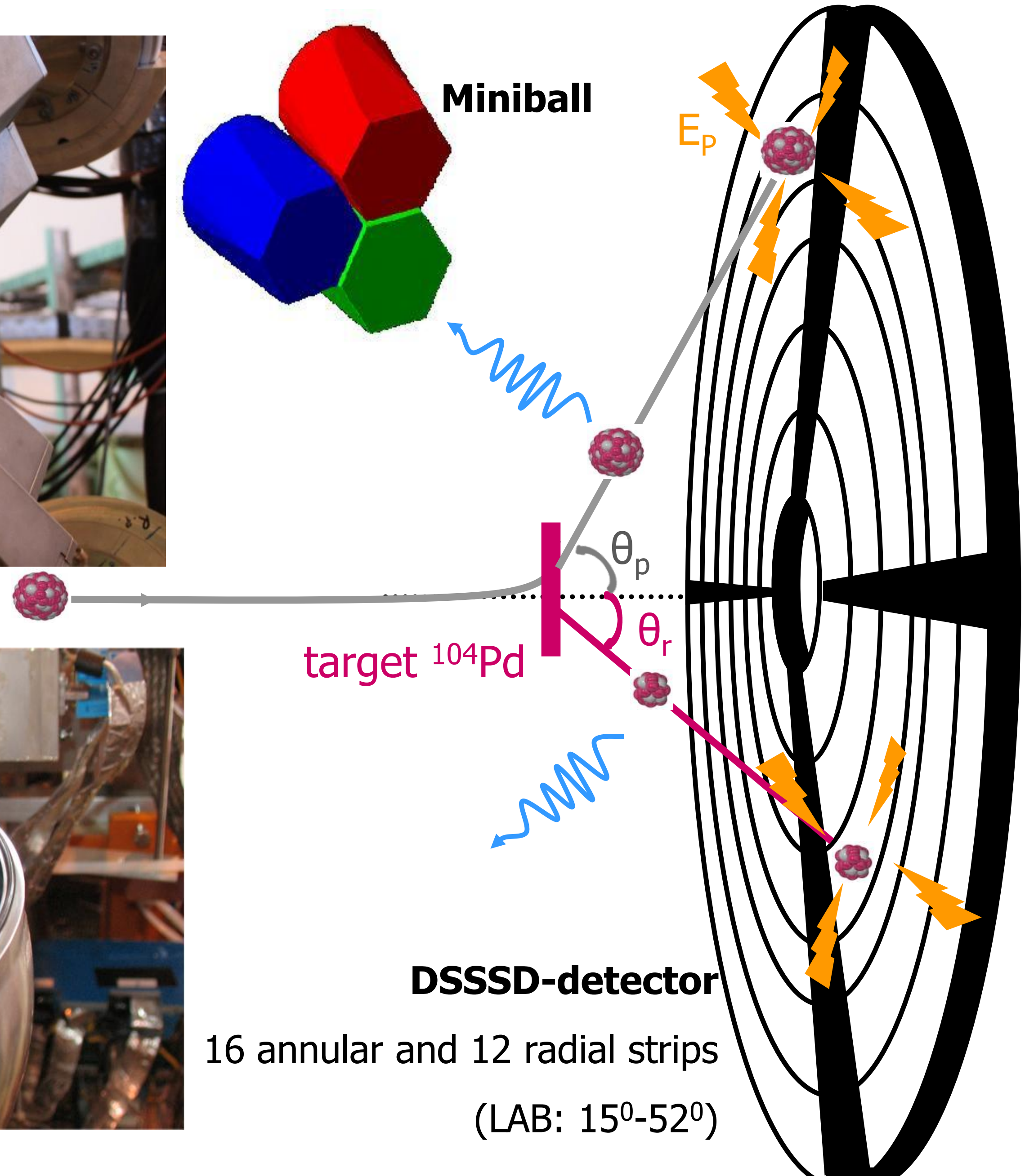
  ^{224}Ra L. P. Gaffney *at el.*, Nature **497**, 199 (2013).

How to proceed when there is a lack of the spectroscopic information ?

Coulex of RIB's @ ISOLDE

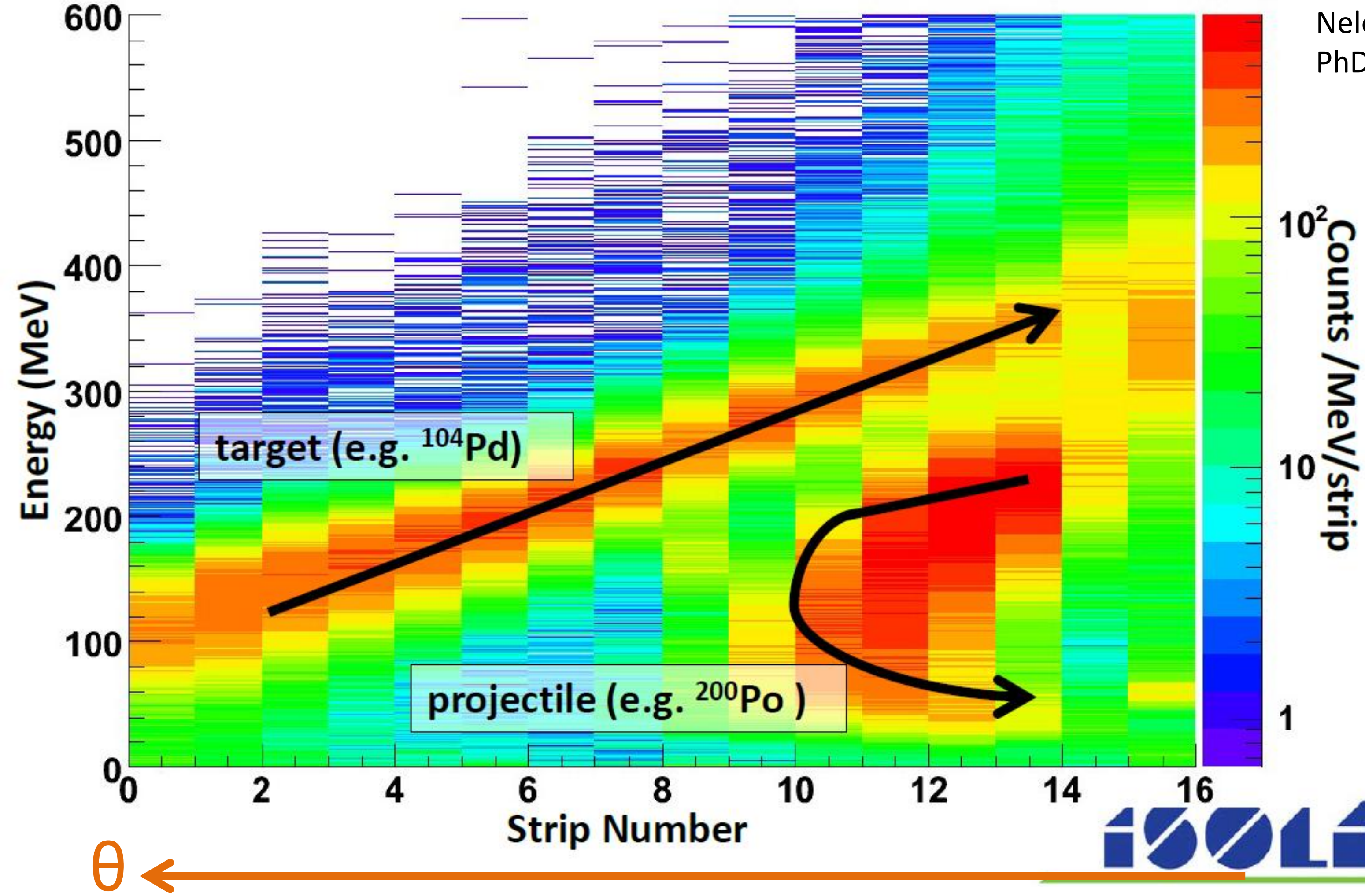
Fig.: N. Bree, PhD thesis, KU Leuven





Coulex experiments @ REX-ISOLDE

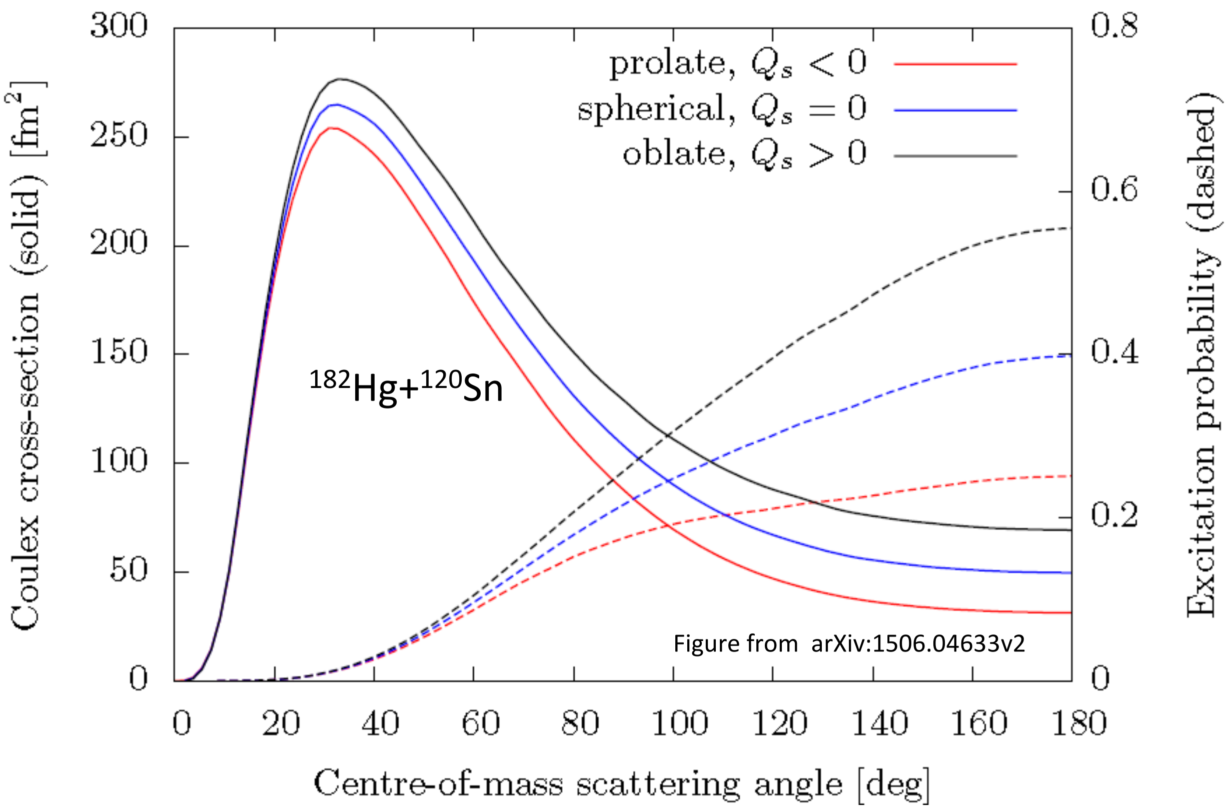
highest possible Z for the target-nucleus vs well **kinematic separation**

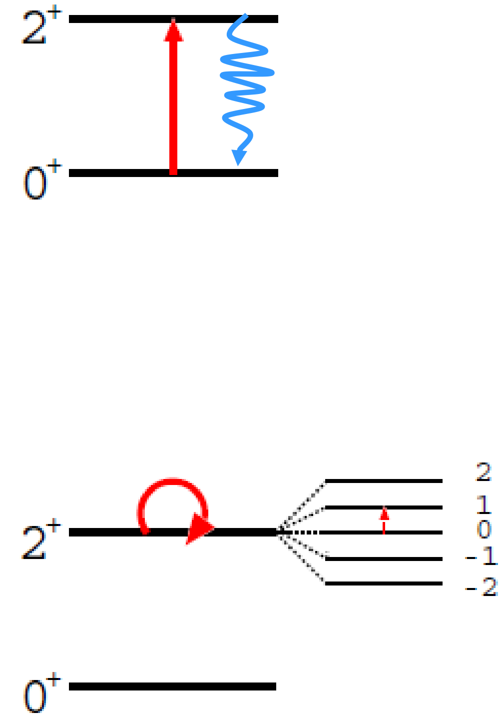


Nele Kesteloot,
PhD thesis, KU Leuven

Coulex of exotic nuclei

- lack of complementary experimental data: τ, BR, δ(E2/M1)
- beam intensities rather low: particle detectors at forward angles to maximise the statistics
- low statistics, usually one- step or two-step excitation and only one gamma line observed





$\langle 2^+||\mathrm{E2}||2^+\rangle \sim Q_S$

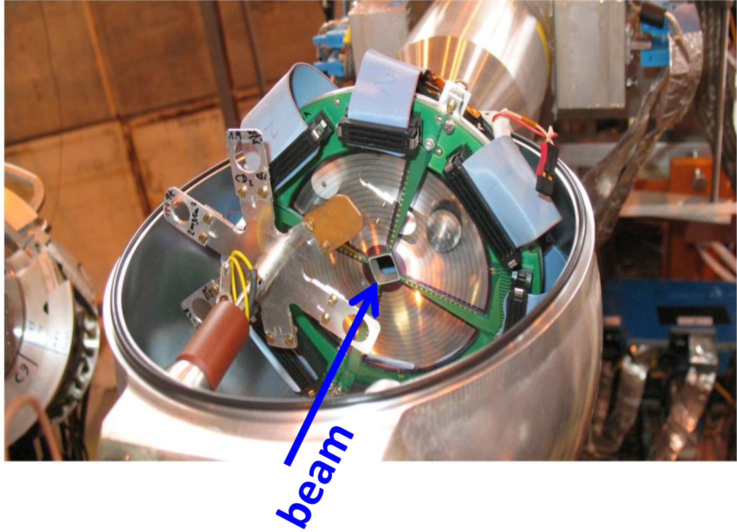
$$\frac{d\sigma_{clx}}{d\Omega} = \frac{d\sigma_{Ruth}}{d\Omega} \cdot P(i \to f) = \left(\frac{a}{2}\right)^2 \frac{1}{\sin^4(\frac{\vartheta_p}{2})} \cdot P(i \to f)$$

$$P_{2_1^+} \propto |\langle 2_1^+||\mathrm{E2}||0_1^+\rangle|^2 \cdot (1 + \langle 2_1^+||E2||2_1^+\rangle \cdot K(\theta, \mathrm{E}_p))$$

Relative normalization (1/2)

Coulex with RIB's:

- very limited number of experimental data
- better to avoid introducing additional free parameters (normalization constants)



Relative normalisation of a number of data sets corresponding to different angular ranges:

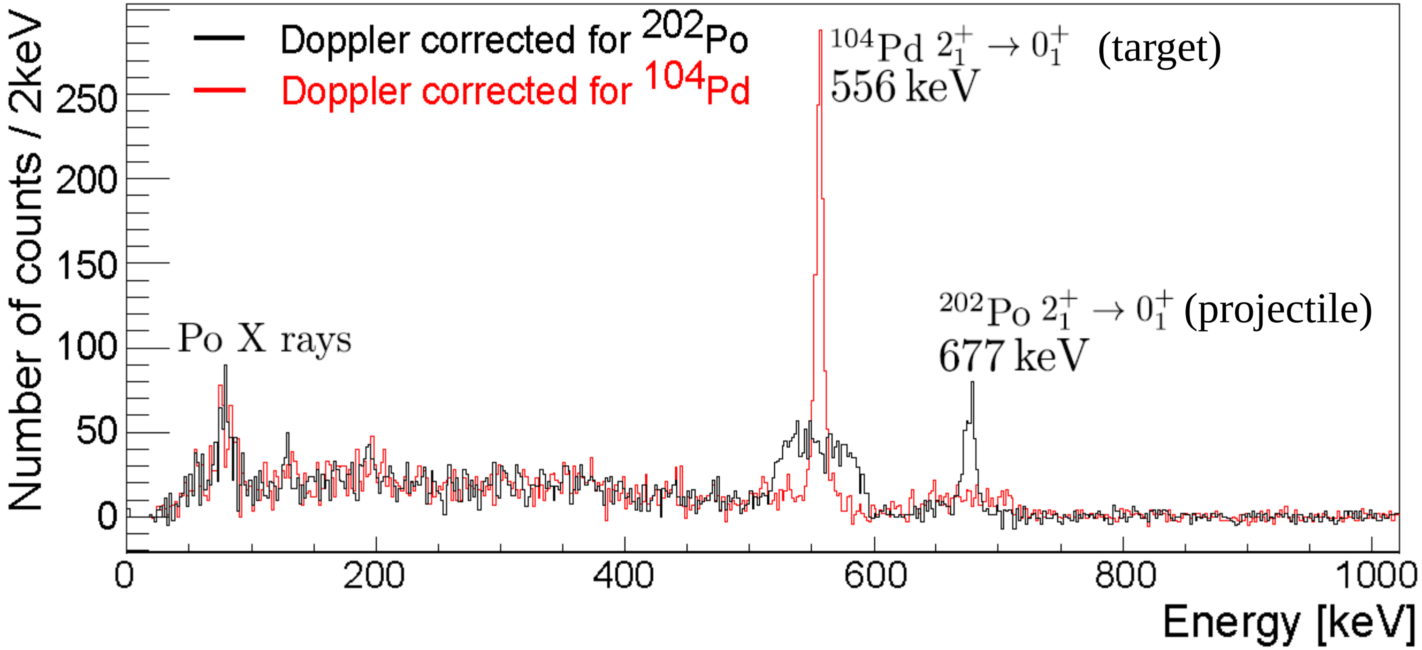
1. **Elastic scattering**
2. **Normalisation to the target excitation**

These normalisation constants are specified by user.

Elastic-scattering (Rutherford) cross section – historically the simplest and most direct method

however,…
precise knowledge of the scattering angular range, well understood deat time, beam current is required…

Relative normalization (2/2)

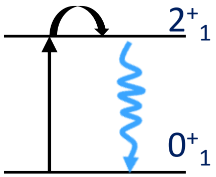


Nele Kesteloot, PhD thesis, KU Leuven, PRC 92, 054301 (2015)

- one-step Coulex of exotic beams
- spectroscopic data not known

w/o other kind of normalization
  – impossible to obtain solution

any combination of BE2 ↔ Q (2^+_1) will reproduce the γ yield



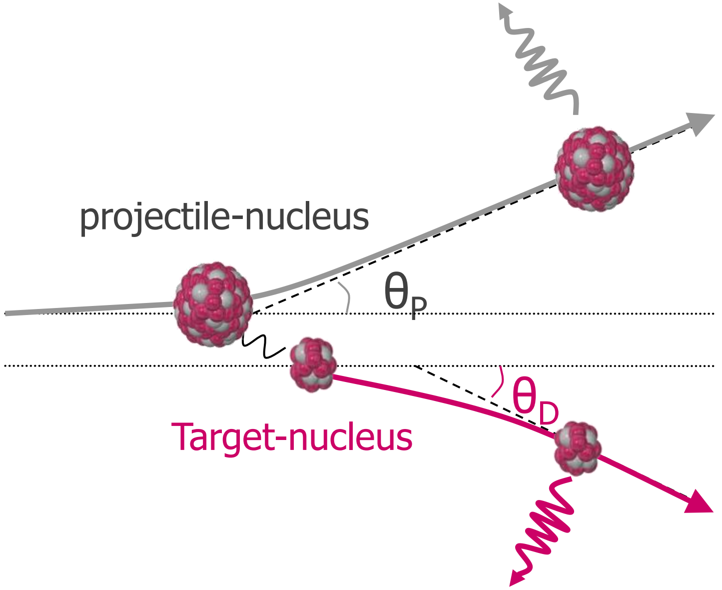
Normalization to the target excitation

Target excitation

1. Several conditions when choosing a target for the RIB's Coulex (e.g. kinematic separation, gammas overlaping).

2. One of them → electromagnetic structure (*B(E2)*'s, *Qs*) of the target nucleus is known.

3. The observed **excitation of the target** can be described with the literature values of *ME*s and used to **normalise** the **excitation cross sections** for the **projectile**



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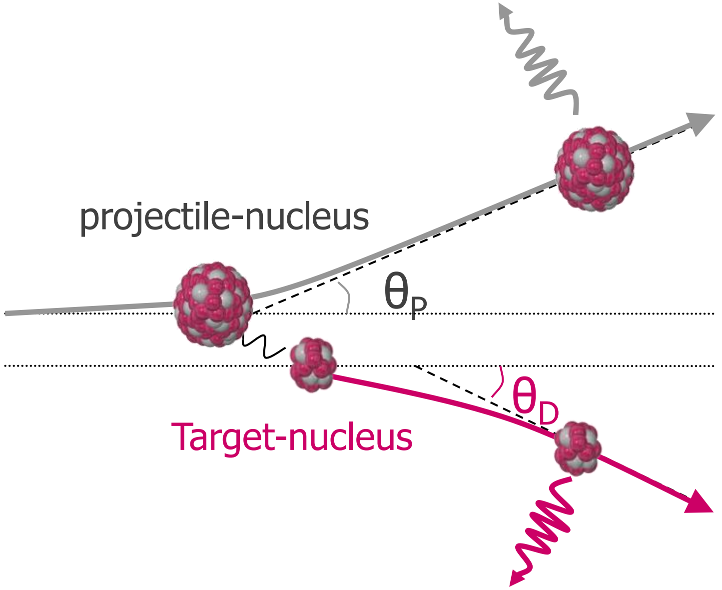


The observed number of γ rays in the transition de-exciting an excited state in the **target nucleus** →

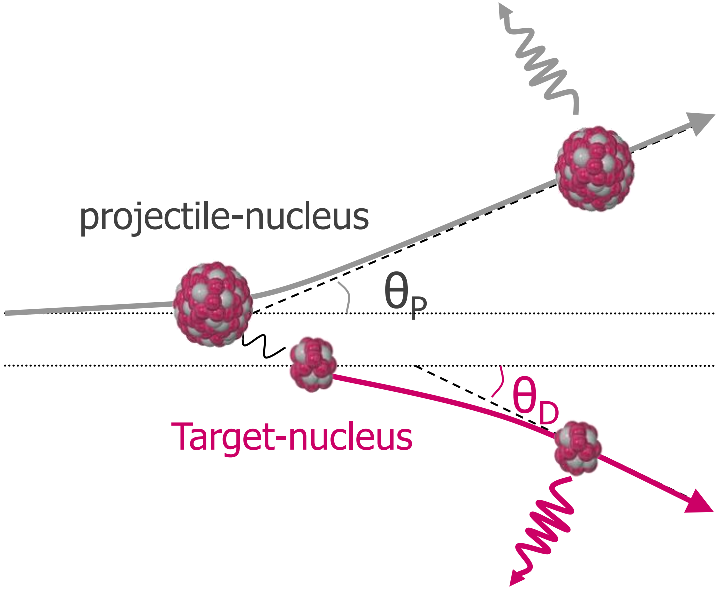
$$N_t = L \cdot \frac{\rho d N_A}{A_t} \cdot b_t \epsilon_\gamma(E_t)\epsilon_{\text{part}}\sigma_t$$

time-integrated luminosity of the beam

total g-ray branching ratio for the transition

integrated cross-section of exciting given state in the target

Target excitation



1. Several conditions when choosing a target for the RIB's Coulex (e.g. kinematic separation, gammas overlaping).

2. One of them → electromagnetic structure (*B(E2)*'s, *Qs*) of the target nucleus is known.

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The observed number of γ rays in the transition de-exciting an excited state in the **target nucleus** →

$$N_t = L \cdot \frac{\rho d N_A}{A_t} \cdot b_t \epsilon_\gamma(E_t) \epsilon_{\text{part}} \sigma_t$$

time-integrated luminosity of the beam

total g-ray branching ratio for the transition

integrated cross-section of exciting given state in the target

$$\boxed{\frac{N_p}{N_t} = \frac{b_p \epsilon_\gamma(E_p) \sigma_p}{b_t \epsilon_\gamma(E_t) \sigma_t}}$$

The observed number of γ rays in the transition de-exciting an excited state in the **projectile nucleus** →

$$N_p = L \cdot \frac{\rho d N_A}{A_t} \cdot b_p \epsilon_\gamma(E_p) \epsilon_{\text{part}} \sigma_p$$

Normalisation to the target excitation – GOSIA2

- developed to handle the simultaneous analysis of both target and projectile excitation;
- limited to one combination of beam and target (available at www.slcj.uw.edu.pl/gosia);
- two input files have to be prepared: one for target, one for beam;
- GOSIA2 minimises χ^2 function for the target (this includes calculation of normalisation factors) and then uses the same normalisation factors as a starting point when it starts minimising χ^2 for the beam;
- normalisation factors are shared as parameters across both χ^2 functions and after several iterations best set of normalisation factors found;
- for high CM angles - diagonal matrix element for the target important

$$eQ_{sp} = \sqrt{\frac{16\pi}{5}}\frac{1}{\sqrt{2I+1}}(I,I,2,0|I,I)\langle I\|\hat{M}(E2)\|I\rangle$$

Limitation of GOSIA2

- data collected on more than one target
- error calculation not incorporated – „by hand”
- if one-step excitation for both target and projectile, one can use standard error progression (contributions from:
    - uncertainty of target yield
    - uncertainty of projectile yield
    - uncertainty of the B(E2) of the target)

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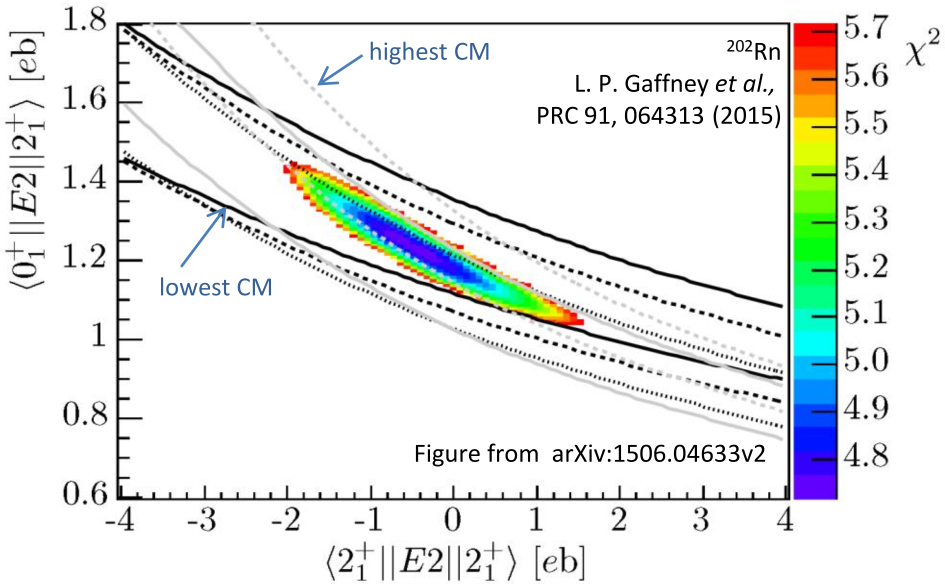
- if several angular ranges and quadrupole moment important – χ^2 surface



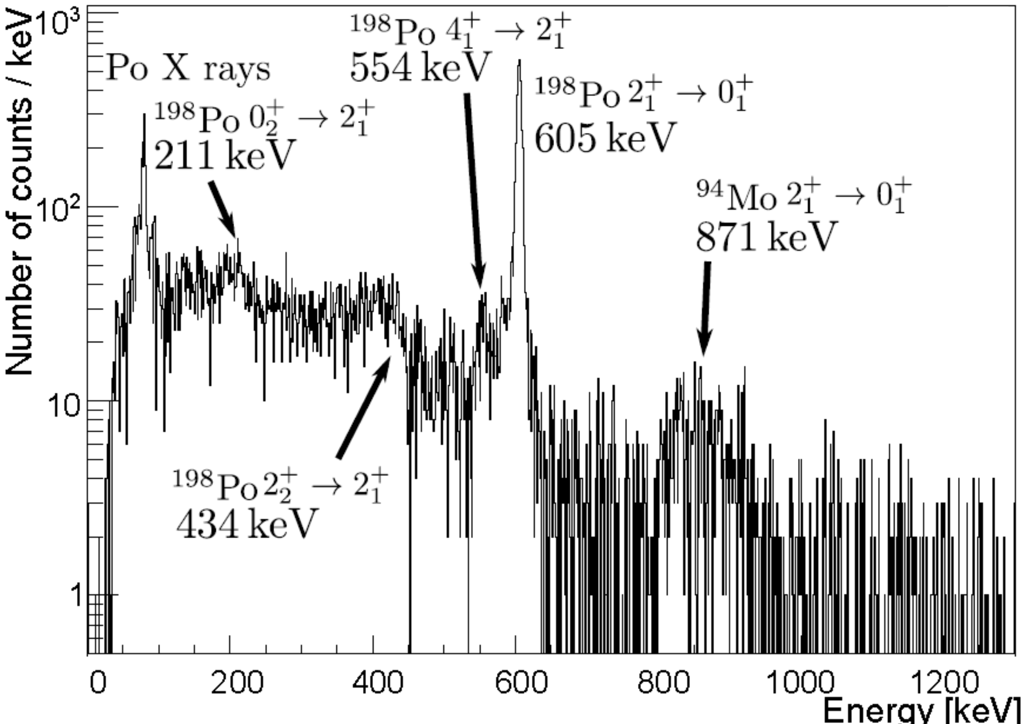
- χ^2 calculated for various combinations of Q and B(E2) for the beam.
- solution corresponds to the minimum of the total χ^2_{total} for both beam and target nuclei.
- The 1σ uncertainty countour defined as the region of the surface for which $\chi^2 < \chi^2_{total,min} + 1$

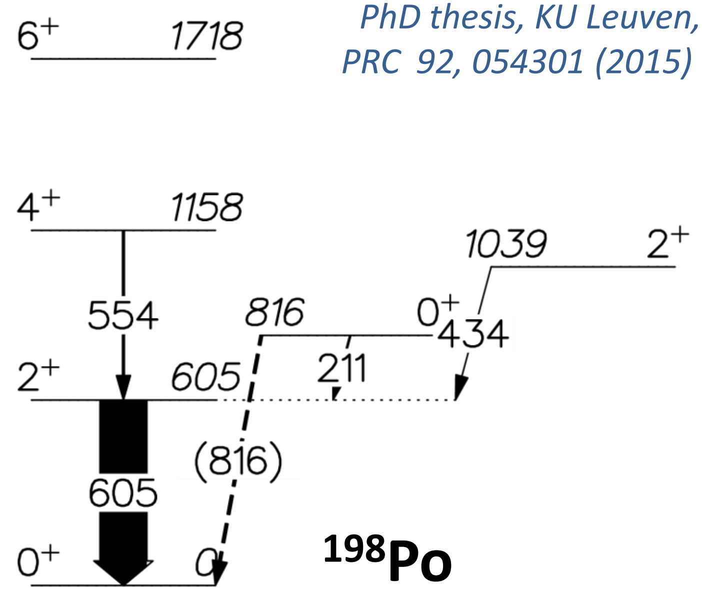
- if more than two matrix elements involved – almost impossible !

Multistep Coulex of exotic even-even nuclei

^{198}Po @ 2.85 MeV/A on ^{94}Mo target, REX-ISOLDE

Nele Kesteloot, PhD thesis, KU Leuven, PRC 92, 054301 (2015)





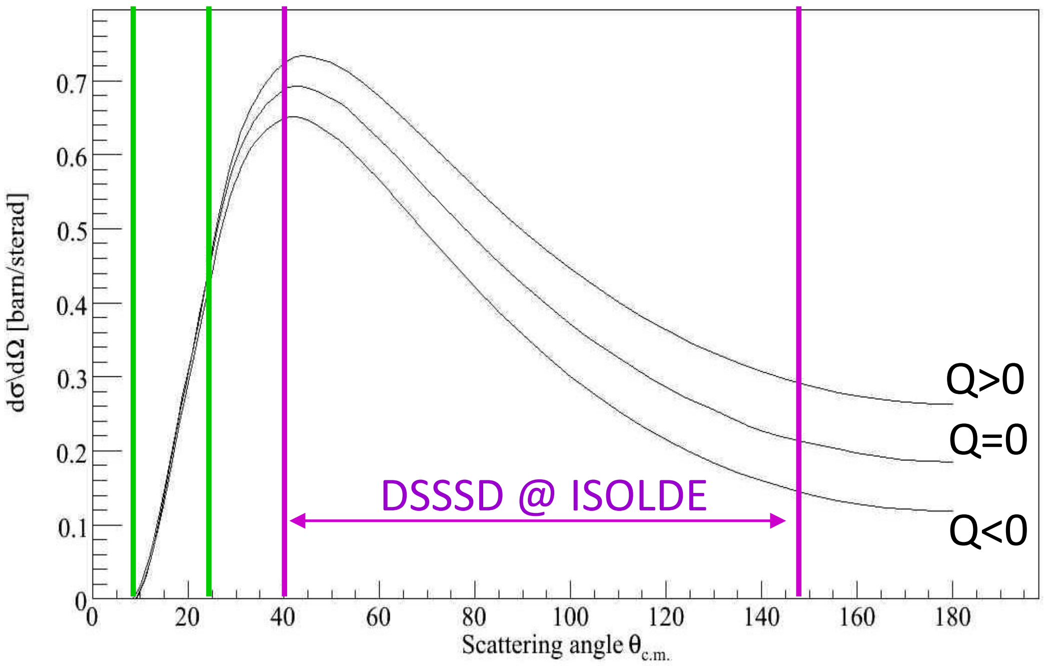
Case:

1. No complementary spectroscopic data.
2. More than two matrix elements involved.
3. Normalisation to the target excitation

Problems:

1. Error calculation !
2. How to include contribution from uncertainty originating from the target excitation ?

Possible solutions



1. Normalisation to the *B(E2)* extracted from data sets where **no correlations** are observed

   lowest angular range → influence of quadrupole moment negligible → determination of the $B(E2; 2^+_1 \rightarrow 0^+_1)$

   ^{44}Ar: M. Zielińska et al., PRC 80, 014317 (2009)

2. Multistep Coulex and normalization to target excitation.

GOSIA 2 → *B(E2)* → contain information on uncertainty originating from the target excitation

+

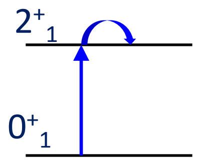
standard GOSIA → error calculation (including correlations) of ME2's coupling higher - lying states

^{196,198}Po: N. Kesteloot, PhD thesis, KU Leuven, PRC 92, 054301 (2015)

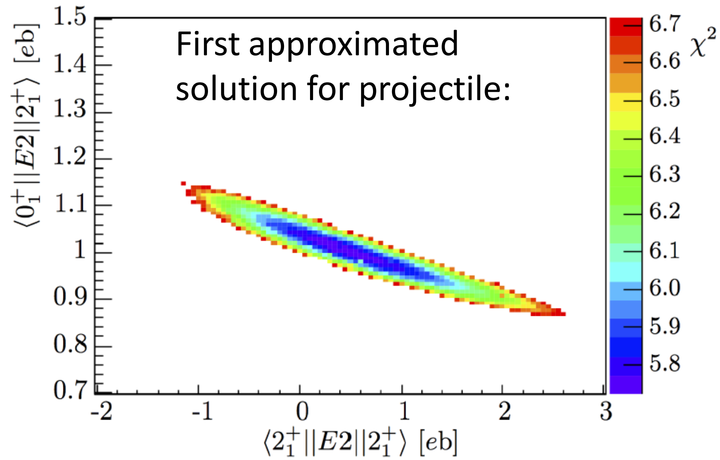
$^{182-188}$Hg: N. Bree, PhD thesis, KU Leuven

GOSIA2 ⟺ standard GOSIA analysis

GOSIA2; first approximation for projectile

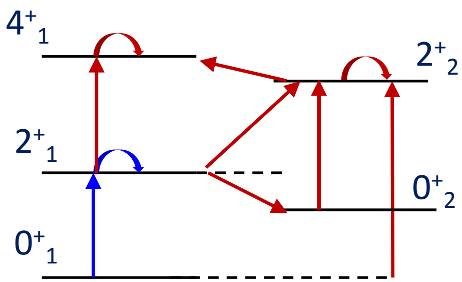


projectile simplified level scheme

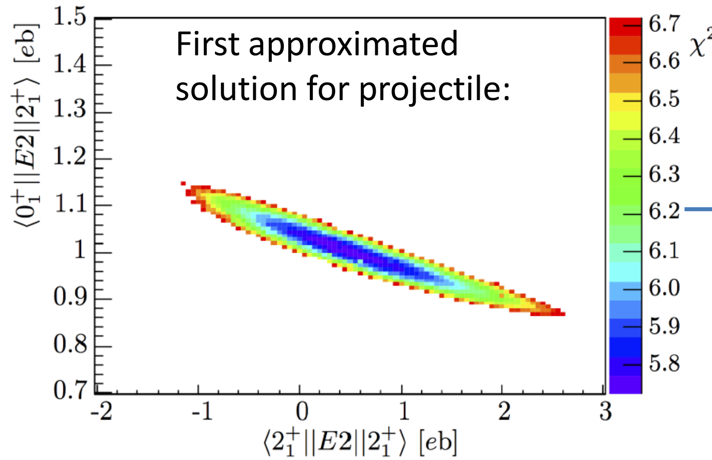


GOSIA2 ⟺ standard GOSIA analysis

GOSIA2; first approximation for projectile



projectile full level scheme



$\langle 0_1^+ \| E2 \| 2_1^+ \rangle$

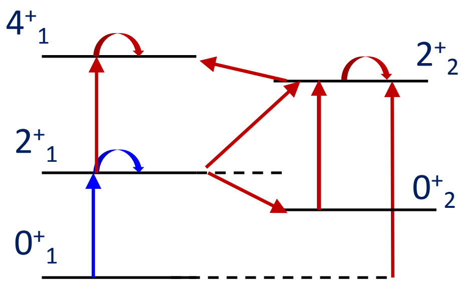
standard GOSIA; **target** excitation

normalisation constants C_{ij}

standard GOSIA; full minimisation for the projectile

GOSIA2 ⟺ standard GOSIA analysis

GOSIA2; first approximation for projectile



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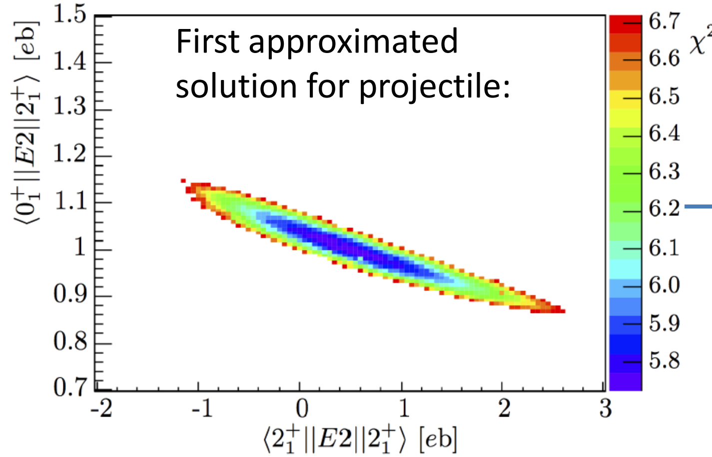

$\langle 0^+_1 \| E2 \| 2^+_1 \rangle$

standard GOSIA; target excitation

normalisation constants C_{ij}

standard GOSIA; full minimisation for the projectile

$\langle 0^+_1 \| E2 \| 2^+_1 \rangle$

best fit matrix elements

NO

GOSIA2; all matrix elements fixed, only **$<0^+_1||E2||2^+_1>$** and **$<2^+_1||E2||2^+_1>$ free**

$\langle 0^+_1 \| E2 \| 2^+_1 \rangle$

$\langle 2^+_1 \| E2 \| 2^+_1 \rangle$

converged ?

YES

final solution !

Summary:

1. Normalisation constants required to convert **measured** γ-ray intensities

   to **absolute** Coulomb-excitation cross sections.

2. Gosia does not require an absolute normalisation for a given experiment.

3. Normalization constants in Gosia analysis can be either **specified by user** or **fitted independently.**

4. Independent normalization (INR option in GOSIA)
   a) known τ or B(E2, $2^+_1 \rightarrow 0^+_1$) determined from the lowest ϑ^{CM}
   b) number of experimental data large enough

   standard Gosia

5. only one state populated + no life time information
   → normalization to the target excitation required

   Gosia2

6. Multistep Coulex (no additional data available) – combined standard Gosia ↔ Gosia2 analysis:

   → final error bars of fitted matrix elements include correlations between matrix elements and contain uncertainty originating from the target excitation.

For more details and additional information…

Analysis methods of safe Coulomb-excitation experiments with radioactive ion beams using the GOSIA code

M. Zielińska[1,a], L. P. Gaffney[2,b,c], K. Wrzosek-Lipska[2,3], E. Clément[4], T. Grahn[5,6], N. Kesteloot[2,7], P. Napiorkowski[3], J. Pakarinen[5,6], P. Van Duppen[2], and N. Warr[8]

arXiv:1506.04633v2