NON-LEPTONIC B DECAYS IN QCD FACTORISATION

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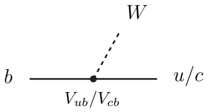
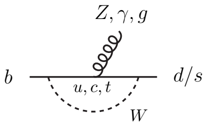
Non-leptonic B decays

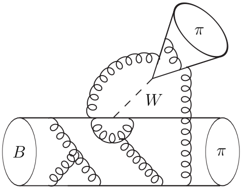
Rich playground for theory and experiment

- many different final states
- CKM studies
- FCNCs
- CP violation
- hadron structure





QCD dynamics



$$\Lambda_{NP} > M_W \gg m_b \gg \Lambda_{QCD}$$

- Soft-Collinear Effective Theory [Bauer, Fleming, Pirjol, Stewart 00; Beneke, Chapovsky, Diehl, Feldmann 02]
- QCD Light-Cone Sum Rules [Khodjamirian 00]
- Lattice QCD?
- Models: naive factorisation, final-state rescattering, . . .

OUTLINE

QCD FACTORISATION

- OPERATOR ANALYSIS
- FACTORISATION FORMULA

PERTURBATIVE CALCULATION

HADRONIC INPUT PARAMETERS

POWER CORRECTIONS

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Charmless B decays

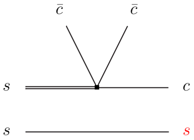
Integrate out UV physics ($\sim M_W, \Lambda_{NP}$)

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i C_i\, Q_i$$

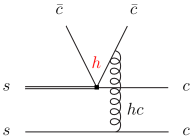
- Wilson coefficients C_i known to NNLL accuracy

[Bobeth, Misiak, Urban 99; Misiak, Steinhauser 04; Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05; Czakon, Haisch, Misiak 06]

Two competing mechanisms



M_1 in asymmetric configuration



additional hard propagators

- power counting?
- short-distance dominated?
- higher Fock states?

Soft-Collinear Effective Theory

Effective field theory for energetic massless particles in a soft background

Momentum modes

	$n_+p \quad n_-p \quad p_\perp$		
hard:	$p_h \sim (m_b, m_b, m_b)$	$p_h^2 \sim m_b^2$	QCD $\to$ SCET-1
hard-collinear:	$p_{hc} \sim (m_b, \Lambda, \sqrt{m_b\Lambda})$	$p_{hc}^2 \sim m_b\Lambda$	SCET-1 $\to$ SCET-2
collinear:	$p_c \sim (m_b, \frac{\Lambda^2}{m_b}, \Lambda)$	$p_c^2 \sim \Lambda^2$	
soft:	$p_s \sim (\Lambda, \Lambda, \Lambda)$	$p_s^2 \sim \Lambda^2$	

+ anti-collinear modes for second energetic direction

QCD $\rightarrow$ SCET-1

Integrate out hard modes

$$Q = \int dt \; \tilde{T}^{I}(t)\, O^{I}(t) \;+\; \int dt\, ds \; \tilde{H}^{II}(t,s)\, O^{II}(t,s)$$

- SCET-1 operators are non-local on the light-cone

$$O^{I}(t) = \big[(\bar{\chi} W_{\bar{c}})(t n_{-}) ... (W_{\bar{c}}^{\dagger} \chi)(0)\big] \; \big[(\bar{\xi} W_{c})(0) ... h_{v}(0)\big]$$

$$O^{II}(t,s) = \big[(\bar{\chi} W_{\bar{c}})(t n_{-}) ... (W_{\bar{c}}^{\dagger} \chi)(0)\big] \; \big[(\bar{\xi} W_{c})(0) ... (W_{c}^{\dagger}\, i\not{D}_{\perp c} W_{c})(s n_{+}) ... h_{v}(0)\big]$$

QCD $\to$ SCET-1

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- collinear and anti-collinear modes decouple: $(p_c + p_{\bar{c}})^2 \sim m_b^2$

  $\Rightarrow$ strong phases are generated at the scale m_b only!

QCD $\to$ SCET-1

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- SCET-1 operators are non-local on the light-cone

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- collinear and anti-collinear modes decouple: $(p_c + p_{\bar{c}})^2 \sim m_b^2$

  $\Rightarrow$ strong phases are generated at the scale m_b only!

- soft and anti-collinear modes decouple: $\chi = S\chi^{(0)}, A_c = SA_c^{(0)} S^\dagger$ [Bauer, Pirjol, Stewart 01]

$$\int dt\; e^{-iut} \,\langle M_2 \,|\, (\bar{\chi} W_{\bar{c}})(tn_-) \ldots (W_{\bar{c}}^\dagger \chi)(0) \,|\, 0 \rangle \;\sim\; \phi_{M_2}(u)$$

SCET-1 $\to$ SCET-2

Integrate out hard-collinear modes

$$O^{II}(t,s): \quad \int ds\, e^{-i\tau s}\, \langle M_1 \,|\, (\bar{\xi} W_c)(0) \ldots (W_c^\dagger i\not{D}_{\perp c} W_c)(sn_+) \ldots h_v(0) \,|\, B\rangle$$

$$\sim \int d\omega\, dv\, J(\tau,\omega,v)\, \phi_B(\omega)\, \phi_{M_1}(v)$$

- hard-collinear jet function J (known to NLO) [Becher, Hill 04; Beneke, Yang 05]
- leading-twist LCDA ϕ_{M_1} and ϕ_B

SCET-1 $\rightarrow$ SCET-2

Integrate out hard-collinear modes

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$$\sim \int d\omega\, dv \; J(\tau,\omega,v)\; \phi_B(\omega)\; \phi_{M_1}(v)$$

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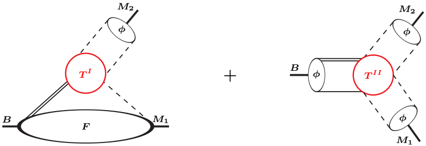
Hard-collinear factorisation fails for $O^{I}(t)$

- endpoint-divergent convolutions $\Rightarrow$ contribution from asymmetric configuration
- can be absorbed into form factor $F^{BM_1}(q^2 = 0)$

Factorisation formula

To leading power in the heavy quark expansion [Beneke, Buchalla, Neubert, Sachrajda 99]

$$\langle M_1 M_2 |Q| B\rangle = F^{BM_1}(0) \int du\, T^I(u)\, \phi_{M_2}(u) + \int d\omega\, du\, dv\, T^{II}(\omega,u,v)\, \phi_B(\omega)\, \phi_{M_1}(v)\, \phi_{M_2}(u)$$



- vertex corrections: $T^I(u) = 1 + \mathcal{O}(\alpha_s)$
- spectator scattering: $T^{II}(\omega,u,v) = \int dz\, H^{II}(u,z)\, J(1-z,\omega,v) = \mathcal{O}(\alpha_s)$

  (power suppressed when M_1 is a heavy meson)

Comparison

"SCET approach" [Bauer, Pirjol, Rothstein, Stewart 04]

- hard-collinear effects are not factorised

  $\langle M_1 \,|\, (\bar{\xi} W_c)(0)...(W_c^\dagger i\not{D}_{\perp c} W_c)(sn_+)...h_v(0) \,|\, B\rangle \;\Rightarrow$ non-local SCET-1 form factor

- charm loops are not factorised

$\Rightarrow$ less computations, complicated hadronic input, stronger fitting component

pQCD [Keum, Li, Sanda 00]

- not based on a systematic heavy quark expansion
- Glauber effects invalidate factorisation? [Li, Mishima 09]

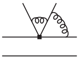
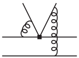
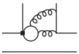
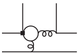
OUTLINE

NNLO calculation

Two hard-scattering kernels for each operator insertion

$$\langle M_1 M_2 | Q_i | B \rangle \simeq F^{BM_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes

Status	2-loop vertex corrections (T_i^I)		1-loop spectator scattering (T_i^{II})	
Trees		[GB 07, 09] [Beneke, Huber, Li 09]		[Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins		[GB, Beneke, Huber, Li 15 + in progress]		[Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

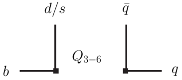
- calculable power correction a_6^p known to NLO [BBNS 01]
- NNLO corrections for $B \to D\pi$ [Huber, Kränkl 15; Kränkl, talk at 9th terascale meeting]

Missing NNLO ingredient

Various contributions to up/charm QCD penguin amplitudes

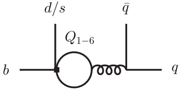
- tree insertions of penguin operators

  2-loop, similar to tree calculation



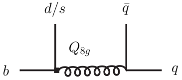
- penguin insertions of current-current and penguin operators

  2-loop, charm propagator introduces additional scale



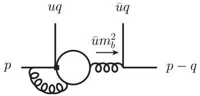
- insertions of magnetic dipole operator

  1-loop, much simpler [Kim, Yoon 11]



$\mathcal{O}(70)$ diagrams at NNLO

- 2 loops, 3 scales (m_b, um_b, m_c), 4 legs
- charm contribution has non-trivial threshold at $\bar{u}m_b^2 \gtrsim 4m_c^2$



2-loop penguin calculation

Automated reduction to scalar master integrals (IBP, Laporta)

⇒ $\mathcal{O}(20)$ additional master integrals compared to 2-loop tree calculation

New proposal to choose ”optimal” basis of master integrals [Henn 13]

- simple iterated integrations in each order in ε-expansion
- no systematic construction of such a basis exists so far

⇒ found optimal basis and calculated all master integrals analytically [GB, Huber 14]

(same technique has been applied for $B \to D\pi$ calculation) [Huber, Kränkl 15]

Status of calculation:

- $Q_{1\text{-}2}$ complete, $Q_{3\text{-}6}$ in progress [GB, Beneke, Huber, Li 15]

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Input parameters

Decay constants: $\hat{f}_B(\mu)$, f_M ($f_M^{\perp}$)

- precise determinations from Lattice + QCD sum rules

Form factors: $F_{BM_1}(q^2 = 0)$

- from QCD LC sum rules (+ extrapolation of Lattice results)

Light meson LCDAs: $\phi_M(u)$

- $\phi_M(u,\mu) = 6u\bar{u} \sum_n a_n^M(\mu)\, C_n^{(3/2)}(2u-1)$

$\Rightarrow$ first few Gegenbauer moments from Lattice + QCD sum rules

B-meson LCDA

Definition

$$\langle 0 \,|\, \bar{q}(tn)\,[tn,0]\,\not{n}\gamma_5\, h_v(0)\,|\,\bar{B}(v)\rangle \;=\; im_B\,\hat{f}_B(\mu)\,\tilde{\phi}_B(t,\mu)$$

- gauge link $[tn,0] = W(tn)W^\dagger(0)$ with $n^2 = 0$
- $\phi_B(\omega,\mu) = \int \frac{dt}{2\pi}\, e^{i\omega t}\, \tilde{\phi}_B(t,\mu)$

  $\omega = n \cdot k$ is a light-cone projection of the spectator quark momentum

Phenomenological applications involve inverse moments

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\, \phi_B(\omega,\mu) \qquad\qquad \frac{\sigma_n(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\, \ln^n\left(\frac{\mu}{\omega}\right)\, \phi_B(\omega,\mu)$$

- λ_B determines normalisation of spectator scattering contribution

Inverse moments

Dominant parametric uncertainty in QCDF

- QCD sum rule estimate $\lambda_B(1\text{GeV}) \simeq (460 \pm 110)$ MeV [Braun, Ivanov, Korchemsky 03]
- $\pi\pi/\pi\rho/\rho\rho$ data seems to prefer $\sim$ 200 MeV ?

λ_B can be measured in $B \to \gamma \ell \nu$ decays

- state-of-the-art analysis (NLL, tree-level $1/m_b$) [Beneke, Rohrwild 11; Braun, Khodjamirian 12]
- Babar 09 data ($E_\gamma > 1$GeV) $\Rightarrow$ $\lambda_B(1\text{GeV}) > 115$ MeV
- Belle 15 data ($E_\gamma > 1$GeV) $\Rightarrow$ $\lambda_B(1\text{GeV}) > 238$ MeV
- good prospects to measure λ_B at Belle-II

RG evolution

Scale dependence known to one-loop order [Lange, Neubert 03]

$$\frac{d\phi_B(\omega,\mu)}{d\ln\mu} = -\left(\Gamma_{\rm cusp}\,\ln\frac{\mu}{\omega}+\gamma\right)\phi_B(\omega,\mu) + \Gamma_{\rm cusp}\int_0^\infty d\eta\,\left[\frac{\omega\,\theta(\eta-\omega)}{\eta(\eta-\omega)}+\frac{\theta(\omega-\eta)}{(\omega-\eta)}\right]_+\phi_B(\eta,\mu)$$

- eigenfunctions of RG kernel [GB, Feldmann, Wang, Yip 13]

$$\phi_B(\omega,\mu) = \int_0^\infty \frac{d\omega'}{\omega'}\,\sqrt{\frac{\omega}{\omega'}}\,J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)\,\rho_B(\omega',\mu)$$

- simple RG evolution in dual space

$$\frac{d\rho_B(\omega',\mu)}{d\ln\mu} = -\left(\Gamma_{\rm cusp}\,\ln\frac{\mu}{\hat\omega'}+\gamma\right)\rho_B(\omega',\mu)$$

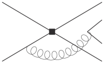
- systematic separation of perturbative + non-pert. effects [Feldmann, Lange, Wang 14]

- conformal symmetry properties [Braun, Manashov 14]

OUTLINE

Power corrections

Main limitation of QCDF approach, e.g. weak annihilation



$$\sim \quad \int d\omega\, du\, dv\;\; T(\omega, u, v)\; \phi_B(\omega)\; \phi_{M_1}(v)\; \phi_{M_2}(u) \quad ?$$

- convolutions diverge at endpoints $\Rightarrow$ non-factorisation in SCET-2
- currently modelled with arbitrary soft rescattering phase

Pure annihilation decays

$10^6\, \text{Br}(B_d \to K^+K^-) = 0.13 \pm 0.05$ ($\Delta D = 1$, exchange topology)

$10^6\, \text{Br}(B_s \to \pi^+\pi^-) = 0.76 \pm 0.13$ ($\Delta S = 1$, penguin annihilation)

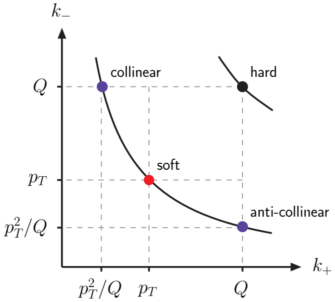
$\Rightarrow$ extract weak annihilation amplitudes from data

[Wang, Zhu 13; Bobeth, Gorbahn, Vickers 14; Chang, Sun, Yang, Li 14]

Factorisation in SCET-2

Similar problem has now been solved in collider physics

- SCET-2 configuration for transverse-momentum-dependent observables
- jet broadening, Higgs p_T spectrum, jet vetos, ...



- collinear and soft modes of same virtuality
- jet and soft functions are endpoint divergent

$$\Rightarrow \int_0^Q \frac{dk_+}{k_+} = \infty$$

Collinear anomaly

Need additional regulator that distinguishes modes by $k_\pm$ [Becher, GB 11]

$$\int d^4k\ \delta(k^2)\,\theta(k^0) \quad \Rightarrow \quad \int d^dk\ \left(\frac{\nu}{k_+}\right)^\alpha \delta(k^2)\,\theta(k^0)$$

$\Rightarrow$ induces rapidity logarithms that cannot be resummed with RG techniques

$$\underbrace{\frac{1}{\alpha} + \ln\left(\frac{\nu}{p_T}\right)}_{\text{soft function}} - \underbrace{\frac{1}{\alpha} - \ln\left(\frac{\nu}{Q}\right)}_{\text{jet function}} = \ln\left(\frac{Q}{p_T}\right)$$

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Rapidity logarithms exponentiate [Becher, Neubert 10; Chiu, Jain, Neill, Rothstein 12]

$$J(p_T)\ S(p_T) = \left(\frac{Q}{p_T}\right)^{-F_B(p_T)} W(p_T)$$

- anomaly exponent $F_B(p_T)$, remainder function $W(p_T)$
- can this be used to calculate weak annihilation amplitudes?

Conclusions

Factorisation at leading power is in mature state

- non-factorisation in SCET-2 hinders progress on power corrections
- new insights from collider physics applications?

NNLO calculation almost complete

- direct CP asymmetries to NLO accuracy
- major update of $B \to PP/PV/VV$ observables

Pheno status will be covered in Martin Beneke's talk