

Probing CP violation in Non-Leptonic B Decays

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Bad Honnef

General Picture

- CP is a fundamental symmetry. Violation of this symmetry has important implications: matter-antimatter asymmetry.
- CP violation has been observed. It is not a fundamental or approximate symmetry of Nature.
- In the SM, CP is broken by a single complex phase in the CKM matrix.
- Any beyond the SM physics (NP) will likely have new CP violating phases.
- Non-Leptonic Decays offer unique probes of the SM and NP phases.

Outline of Talk- Part I

- The topic is too broad for a single talk. I will focus on a select set of issues.
- Two main probes of CPV are direct CP violation (DCPV) and indirect CP violation in Non-Leptonic decays.
- However, Non-Leptonic decays are difficult to calculate. This makes extraction of information about CP phases challenging.
- However CPV measurements can provide clues to the underlying dynamics (SM or NP) even with hadronic uncertainties.
- Not everything about the decay dynamics is needed to extract CP information. I will demonstrate this with $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ and $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ decays.

Outline of Talk- Part II

- In $B \rightarrow V_1 V_2$ decays there is another probe of CPV- the triple product asymmetries (T.P.A). Many additional CPV signals which are some combinations of these three CP asymmetries also appear in the angular distribution of such decays.
- T.P.A. have been measured. They can provide useful information about the structure of NP. However, they have been studied relatively less theoretically.
- I will talk about CPV/TPA focussing mainly in charmless $B \rightarrow V_1 V_2$ decays and highlight some new observables which are just being explored.

CPV in the SM

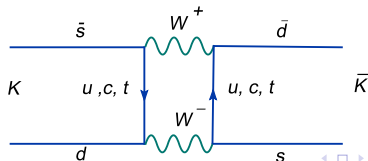
- CPV in the SM comes from charged current interactions:

$$(u \quad c \quad t) V_{CKM} \begin{pmatrix} d \\ s \\ t \end{pmatrix} W$$

- In the SM, CP violation is due to a complex phase in the CKM matrix:

$$V_{CKM} \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1 + iA^2\lambda^4\eta) & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

where $\lambda = 0.22$. CPV first observed in K system.



Why is B Special?

In the SM, CP violation is due to a complex phase in the CKM matrix:

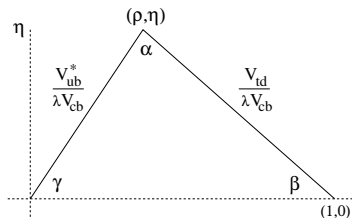
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where $\lambda = 0.22$.

Note: (i) relative sizes of CKM matrix elements, (ii) large phases occur only in corners: V_{ub} and V_{td} .

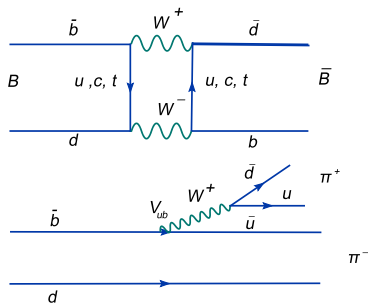
Unitarity Triangle:

$$V_{CKM} \simeq \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & |V_{ts}| & |V_{tb}| \end{pmatrix}$$



- $Arg[V_{td}] \sim \eta \sim O(1)$ - Large
- $Arg[V_{ub}] \sim \eta \sim O(1)$ - Large

These elements can be probed in B decays:



- $B - \bar{B} \sim (V_{td})^2 \sim e^{-2i\beta}$ $\beta \sim 21.9 \pm 0.7$ degrees(HFAG)
- $Arg[V_{ub}] = \gamma \sim 60^\circ - 70^\circ$ - CPV in the B system is large in the SM.

New Phases from New Physics

- CPV in the SM is large.
- All CPV $\propto \eta$
- V_{CKM} is unitary: $V_{CKM}^\dagger V_{CKM} = 1 \Rightarrow 3$ angles and 6 phases.
- Weak Interactions couple only to LH quarks: Can reabsorb 5 phases in quark field definitions
- Only one weak phase η .
- Consider a NP scenario, e.g. Left-Right Symmetric Models:
- New phases associated with the RH mixing matrix, V_R .
- Can no longer absorb the phases of V_R : 6 new phases.

Direct CP Violation

- Direct CP violation is a simple measure of CP violating phases in the theory.
- Consider $B \rightarrow f$ and the CP conjugate process $\bar{B} \rightarrow \bar{f}$.
- Define **direct** CP asymmetry:

$$a_{dir}^{CP} \equiv \frac{\Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\bar{B} \rightarrow \bar{f})} \sim \sin \phi ,$$

where ϕ is the CP violating weak phase difference.

- Non zero direct CP violation requires interference of 2 amplitudes. Consider the decay $B \rightarrow f$. Suppose

$$\begin{aligned} A(B \rightarrow f) &= A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2} , \\ A(\bar{B} \rightarrow \bar{f}) &= A_1 e^{-i\phi_1} e^{i\delta_1} + A_2 e^{-i\phi_2} e^{i\delta_2} . \end{aligned}$$

Hadronic Input

- Hence **direct** CP asymmetry:

$$a_{dir}^{CP} \equiv \frac{\Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\bar{B} \rightarrow \bar{f})} = -\frac{2A_1A_2 \sin \Phi \sin \Delta}{A_1^2 + A_2^2 + 2A_1A_2 \cos \Phi \cos \Delta},$$

where $\Phi \equiv \phi_1 - \phi_2$ and $\Delta \equiv \delta_1 - \delta_2$.

- Note: direct CP asymmetry depends on unknown strong interaction parameters. Cannot extract weak phase information (Φ) without hadronic input.
- We do not always need hadronic input to test for CP.
- If there is no weak phase in the Lagrangian then strong interactions will not grow any new weak phase. So any CP violation measurement will vanish.

Hadronic Input

- Suppose we have a decay $B \rightarrow f$:

$$\begin{aligned}A(B \rightarrow f) &= Ae^{i\phi} \\A(\bar{B} \rightarrow \bar{f}) &= Ae^{-i\phi}.\end{aligned}$$

$$a_{dir}^{CP} \sim |A(B \rightarrow f)|^2 - |A(\bar{B} \rightarrow \bar{f})|^2 = 0$$

- Many decays in SM are dominated by a single amplitude and hence a measurement of non zero a_{dir}^{CP} violation is a clear signal of new physics- to first order. No hadronic uncertainty involved!
- However NP effects have to be large- larger than the SM sub leading amplitudes. Experiments have not found evidence for such large effects.

Mixing Induced CP Violation

- There is another signal of CP violation. Use $B^0-\bar{B}^0$ mixing. Choose final state f accessible to both B^0 and \bar{B}^0 . (Simplest is CP eigenstate.) Then $B^0 \rightarrow f$ and $B^0 \rightarrow \bar{B}^0 \rightarrow f$ interfere. This leads to indirect or mixing induced CP violation.
- Large $B^0-\bar{B}^0$ mixing: get **indirect** CP asymmetry:

$$\Gamma(B^0(t) \rightarrow f) \sim B + a_{dir} \cos(\Delta Mt) + a_{mix} \sin(\Delta Mt)$$

$$B \equiv \frac{1}{2} (|A|^2 + |\bar{A}|^2) , \quad a_{dir} \equiv \frac{1}{2} (|A|^2 - |\bar{A}|^2) , \quad a_{mix} \equiv \text{Im} \left(e^{-2i\beta} A^* \bar{A} \right) .$$

- Can we just measure CP violation phases?
- Point: $\Gamma(B^0(t) \rightarrow f)$ gives 3 measurements.
- Note: if there is only a single decay amplitude in $B^0 \rightarrow f$, i.e. $A_2 = 0$, then $a_{dir} = 0$, but $a_{mix} \neq 0$. This is the most interesting case, since all dependence on hadronic physics cancel.
- Idea: measure α, β, γ in ways independent of strong phases.
- In general we need *one* theory input parameter to measure CP phases.

Example: $B_{d,s} \rightarrow K \bar{K}$

- $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ is a pure $\bar{b} \rightarrow \bar{s}$ penguin decay. That is, its amplitude receives contributions only from gluonic and electroweak penguin (EWP) diagrams.

$$\begin{aligned} \mathcal{A}_s &= \lambda_u^{(s)} P'_u + \lambda_c^{(s)} P'_c + \lambda_t^{(s)} P'_t \\ &= |\lambda_u^{(s)}| e^{i\gamma} P'_{uc} - |\lambda_t^{(s)}| P'_{tc} , \end{aligned}$$

where $\lambda_q^{(q')} \equiv V_{qb}^* V_{qq'}$.

- Also, above we have explicitly written the weak-phase dependence, while P'_{uc} and P'_{tc} contain strong phases.
- The amplitude $\bar{\mathcal{A}}_s$ describing the CP-conjugate decay $\bar{B}_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ can be obtained from the above by changing the signs of the weak phases (in this case, γ).

- There are three measurements which can be made of $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$: the branching ratio, and the direct and indirect CP-violating asymmetries. These yield the three observables

$$\begin{aligned} X' &\equiv \frac{1}{2} (|\mathcal{A}_s|^2 + |\bar{\mathcal{A}}_s|^2) , \\ Y' &\equiv \frac{1}{2} (|\mathcal{A}_s|^2 - |\bar{\mathcal{A}}_s|^2) , \\ Z'_I &\equiv \text{Im} \left(e^{-2i\beta_s} \mathcal{A}_s^* \bar{\mathcal{A}}_s \right) . \end{aligned}$$

- Assuming one takes the values for $|\lambda_u^{(s)}|$, $|\lambda_t^{(s)}|$ from independent measurements.
- \mathcal{A}_s then depends only on the magnitudes of P'_{uc} and P'_{tc} , and their relative strong phase δ' and the two CPV phases γ and β_s .
- If one of these phases is used as an input this makes a total of four unknown parameters. These cannot be determined from only three observables – additional input is needed.

- These pairs of decays have been studied by several authors:
- If we use β_s as input then γ can be extracted with a theory input. If the value of γ disagrees with γ from other independent measurement then we have evidence of NP.
- See for example: [Datta, London archive:0105073](#), ; [Descotes-Genon, Matias, Virto, archive:0603239](#).
- If we use γ as input β_s can be extracted with a theory input. [Ciuchini, Perini, Silvestrini hep-ph/0703137](#).

- Note that, if $\lambda_u^{(s)} P'_{uc}$ were negligible, we would only have two unknowns – $|P'_{tc}|$ and β_s . These could be determined from the measurements of X' and Z'_j (Y' would vanish).
- P'_{uc} in $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ is the subleading amplitude. Extract it from expt. Use its U-spin-conjugate decay $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$. This is the theory input.

- $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ is a pure $\bar{b} \rightarrow \bar{d}$ penguin decay, whose amplitude can be written

$$\mathcal{A}_d = |\lambda_u^{(d)}| e^{i\gamma} P_{uc} + |\lambda_t^{(d)}| e^{-i\beta} P_{tc} .$$

- We proceed as follows. The three experimental measurements of the B_d^0 decay correspond to the three observables

$$X \equiv \frac{1}{2} (|\mathcal{A}_d|^2 + |\bar{\mathcal{A}}_d|^2) = |\lambda_u^{(d)}|^2 |P_{uc}|^2 + |\lambda_t^{(d)}|^2 |P_{tc}|^2 - 2|\lambda_u^{(d)}||\lambda_t^{(d)}||P_{uc}||P_{tc}| \cos \delta \cos \alpha ,$$

$$Y \equiv \frac{1}{2} (|\mathcal{A}_d|^2 - |\bar{\mathcal{A}}_d|^2) = -2|\lambda_u^{(d)}||\lambda_t^{(d)}||P_{uc}||P_{tc}| \sin \delta \sin \alpha ,$$

$$Z_I \equiv \text{Im} \left(e^{-2i\beta} \mathcal{A}_d^* \bar{\mathcal{A}}_d \right) = |\lambda_u^{(d)}|^2 |P_{uc}|^2 \sin 2\alpha - 2|\lambda_u^{(d)}||\lambda_t^{(d)}||P_{uc}||P_{tc}| \cos \delta \sin \alpha .$$

- X , Y and Z_I are related to the branching ratio (\mathcal{B}_d), the direct CP asymmetry (C_d) and the indirect CP asymmetry (S_d) of $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ as follows:

$$X = \kappa_d \mathcal{B}_d \quad , \quad Y = \kappa_d \mathcal{B}_d C_d \quad , \quad Z_I = \kappa_d \mathcal{B}_d S_d \quad ,$$

where

$$\kappa_d = \frac{8\pi m_{B_d}^2}{\tau_d p_c} \quad .$$

- It is useful to define a fourth observable:

$$\begin{aligned} Z_R &\equiv \text{Re} \left(e^{-2i\beta} \mathcal{A}_d^* \bar{\mathcal{A}}_d \right) = |\lambda_u^{(d)}|^2 |P_{uc}|^2 \cos 2\alpha + |\lambda_t^{(d)}|^2 |P_{tc}|^2 \\ &\quad - 2|\lambda_u^{(d)}| |\lambda_t^{(d)}| |P_{uc}| |P_{tc}| \cos \delta \cos \alpha \quad . \\ Z_R^2 &= X^2 - Y^2 - Z_I^2 \quad . \end{aligned}$$

Thus, one can obtain Z_R from measurements of X , Y and Z_I , up to a sign ambiguity.

- The quantity $|P_{uc}|$ can then be written in terms of the observables as

$$|P_{uc}|^2 = \frac{1}{|\lambda_u^{(d)}|^2} \frac{Z_R - X}{\cos 2\alpha - 1} = \frac{\kappa_d \mathcal{B}_d}{|\lambda_u^{(d)}|^2} \frac{\pm \sqrt{1 - C_d^2 - S_d^2} - 1}{\cos 2\alpha - 1}.$$

- In the SU(3) limit $|P'_{uc}| = |P_{uc}|$ and so now using this we can extract β_s from $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$.
- Claim (CPS) is β_s can be obtained with an error of $< 1^\circ$ even for 100 % SU(3) breaking.

- CPS worked with numbers based on QCDF calculations. We use real numbers (Bhattacharya et.al. archive: 1203.3435).
- From BaBar , we have

$$\begin{aligned} \mathcal{B}_d &= (1.08 \pm 0.28 \pm 0.11) \times 10^{-6} \\ \mathcal{S}_d &= -1.28_{-0.73-0.16}^{+0.80+0.11}, \\ \mathcal{C}_d &= -0.40 \pm 0.41 \pm 0.06 \end{aligned}$$

while Belle finds

$$\begin{aligned} \mathcal{B}_d &= (0.87_{-0.20}^{+0.25} \pm 0.09) \times 10^{-6} \\ \mathcal{S}_d &= -0.38_{-0.77}^{+0.69} \pm 0.09 \\ \mathcal{C}_d &= 0.38 \pm 0.38 \pm 0.05 \end{aligned}$$

We see that essentially all values of $\sqrt{\mathcal{C}_d^2 + \mathcal{S}_d^2}$ are still experimentally allowed.

- Thus, we use flavor SU(3) symmetry to relate $|P_{uc}|$ and $|P'_{uc}|$, allowing for a 100% symmetry breaking.

Calculate SU(3) breaking

- Consider again the $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ amplitude. Eliminate the t -quark by CKM unitarity

$$\mathcal{A}_s = T' \lambda_u^{(s)} + P' \lambda_c^{(s)},$$

where $T' \equiv P'_u - P'_t$, $P' \equiv P'_c - P'_t$.

- Now, in QCDf the key observation here is that the difference $T' - P'$ is free of these dangerous IR divergences. Hence this quantity is under good theoretical control ([Descotes-Genon, Matias, Virto, archive:0603239](#))
- Some hadronic factors cancel in the ratio of SU(3) breaking.

$$\begin{aligned}
PP & : \quad \frac{|P'_{uc}|}{|P_{uc}|} = \frac{M_{B_s}^2 F_0^{B_0^0 \rightarrow K} (M_K^2)}{M_{B_d}^2 F_0^{B_0^0 \rightarrow K} (M_K^2)} = 0.86 \pm 0.15 , \\
PV & : \quad \frac{|P'_{uc}|}{|P_{uc}|} = \frac{M_{B_s}^2 F_+^{B_0^0 \rightarrow K} (M_{K^*}^2)}{M_{B_d}^2 F_+^{B_0^0 \rightarrow K} (M_{K^*}^2)} = 0.86 \pm 0.15 , \\
VP, VV_0 & : \quad \frac{|P'_{uc}|}{|P_{uc}|} = \frac{M_{B_s}^2 A_0^{B_0^0 \rightarrow K^*} (M_{K^*}^2)}{M_{B_d}^2 A_0^{B_0^0 \rightarrow K^*} (M_{K^*}^2)} = 0.87 \pm 0.19 , \\
VV_{\parallel}, VV_{\perp} & : \quad \frac{|P'_{uc}|}{|P_{uc}|} = \frac{M_{B_s} (F_-^{B_0^0 \rightarrow K^*} (M_{K^*}^2) \pm F_+^{B_0^0 \rightarrow K^*} (M_{K^*}^2))}{M_{B_d} (F_-^{B_0^0 \rightarrow K^*} (M_{K^*}^2) \pm F_+^{B_0^0 \rightarrow K^*} (M_{K^*}^2))} \\
& = 0.79 \pm 0.16 .
\end{aligned}$$

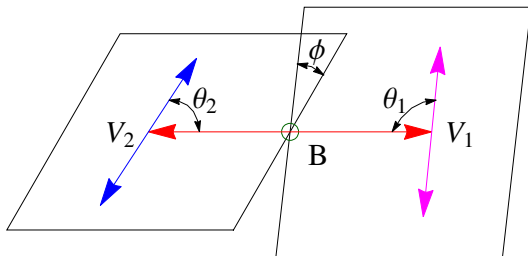
- Challenge here would be to reliably calculate the SU(3). This is the only input needed to extract weak phase information.

CPV with $B \rightarrow V_1 V_2$ Decays

- $B \rightarrow V_1 V_2$ decays can be of several types:
- Both vector mesons are on-shell and observed through their decays to other final state particles.
- One or both the V can be off-shell. Example Semileptonic Decays: $B \rightarrow D^{(*)}(\rho)W^*$ with $W^* \rightarrow l\bar{\nu}_l$.
- The final state particles can be reached by both B_d^0 and \bar{B}_d^0 mesons (B_s^0 or \bar{B}_s^0).E.g. $B_d^0 \rightarrow K^*\bar{K}^*$ and $B_s^0 \rightarrow J/\psi\phi, \phi\phi$. Mixing effects have to be included and this becomes a time dependent problem.
- The final particles can be reached through a scalar background(resonant or non-resonant). Example: $B \rightarrow V_1 V_2 \rightarrow f$ and $B \rightarrow V_1 S \rightarrow f$. One has to include the interference effects.

Background

- Spin 0 meson (B) \rightarrow 2 Spin 1 mesons (Vectors)
- Relative angular momentum : $L_{VV} = 0, 1, 2$
- Vectors identified through their decay modes : Eg. $\phi \rightarrow K\bar{K}$.
- Angular analysis to separate out helicity amplitudes :
 - 1.) Functions of helicity angles θ_1, θ_2 , and ϕ .
 - 2.) Observables can be dependent on time.



BVV Amplitudes

For the process:

- $B(p) \rightarrow V_1(k_1, \varepsilon) V_2(k_2, \eta)$

$$M_{\lambda_1, \lambda_2} = a \varepsilon_{\lambda_1}^* \cdot \eta_{\lambda_2}^* + \frac{b}{m_B^2} (p \cdot \varepsilon_{\lambda_1}^*) (p \cdot \eta_{\lambda_2}^*) + i \frac{c}{m_B^2} \epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu \varepsilon_{\lambda_1}^{*\rho} \eta_{\lambda_2}^{*\sigma},$$

where $q \equiv k_1 - k_2$.

- The amplitude c is $L = 1$ and is parity-odd. The amplitudes a and b are combinations of $L = 0$ and $L = 2$ partial waves.
- Helicity conservations allows $M_{+,+}$, $M_{-,-}$, $M_{0,0}$. Use A_+ , A_- , A_0 .

BVV Transversity Amplitudes

- Another useful parametrization is the transversity basis:

$$M = A_0 \varepsilon_1^{*L} \cdot \varepsilon_2^{*L} - \frac{1}{\sqrt{2}} A_{\parallel} \bar{\varepsilon}_1^{*T} \cdot \bar{\varepsilon}_2^{*T} - \frac{i}{\sqrt{2}} A_{\perp} \bar{\varepsilon}_1^{*T} \times \bar{\varepsilon}_2^{*T} \cdot \hat{p},$$

where \hat{p} is the unit vector along the direction of motion of V_2 in the rest frame of V_1 , $\varepsilon_i^{*L} = \bar{\varepsilon}_i^* \cdot \hat{p}$, and $\bar{\varepsilon}_i^{*T} = \bar{\varepsilon}_i^* - \varepsilon_i^{*L} \hat{p}$.

- A_0 , A_{\parallel} , A_{\perp} are related to a , b and c of via

$$A_{\parallel} = \sqrt{2} a, \quad A_0 = -ax - \frac{m_1 m_2}{m_B^2} b (x^2 - 1),$$

$$A_{\perp} = 2\sqrt{2} \frac{m_1 m_2}{m_B^2} c \sqrt{x^2 - 1},$$

where $x = k_1 \cdot k_2 / (m_1 m_2)$.

- $A_+ = (A_{\parallel} + A_{\perp}) / \sqrt{2}$, $A_- = (A_{\parallel} - A_{\perp}) / \sqrt{2}$ and $M_{0,0} = A_0$.

$B \rightarrow V_1 V_2$: CP phases from $B \rightarrow V_1 V_2$ Decays

- In $B \rightarrow V_1 V_2$ decays an angular analysis is required to extract the different helicity amplitudes.
- Many correlations among the amplitudes appear in the angular distribution from which CPV phases can be extracted.
- These CPV phases can be in mixing or decay amplitude.
- Because there are many observables the CP structure of the SM or NP can be explored.
- As an example we can discuss the B_s mixing from the decay $B_s \rightarrow J/\psi\phi$.

Measuring β_s

- B_s^0 and \bar{B}_s^0 can decay to $f \equiv J/\psi\phi$.
- The indirect CPA measures

$$\text{Im} \left(\frac{q}{p} \frac{\bar{\mathcal{A}}_s^f}{\mathcal{A}_s^f} \right),$$

where \mathcal{A}_s^f and $\bar{\mathcal{A}}_s^f$ are the amplitudes for $B_s^0 \rightarrow f$ and $\bar{B}_s^0 \rightarrow f$, respectively.

- $q/p = (V_{tb}^* V_{ts} / V_{tb} V_{ts}^*) = \exp(2i \arg(V_{tb}^* V_{ts}))$. This is phase-convention dependent.
- Assuming \mathcal{A}_s^f is dominated by a single decay amplitude. $\bar{\mathcal{A}}_s^f / \mathcal{A}_s^f = (V_{cb} V_{cs}^* / V_{cb}^* V_{cs}) = \exp(2i \arg(V_{cb} V_{cs}^*))$, which is also phase-convention dependent.

- However, the product of these two quantities is

$$\frac{q \bar{\mathcal{A}}_s^f}{p \mathcal{A}_s^f} = \frac{V_{tb}^* V_{ts}}{V_{cb}^* V_{cs}} \frac{V_{cb} V_{cs}^*}{V_{tb} V_{ts}^*} = e^{2i\beta_s},$$

where

$$\beta_s \equiv \arg \left[-\frac{V_{tb}^* V_{ts}}{V_{cb}^* V_{cs}} \right].$$

This is phase-convention independent, and hence physical. The indirect CPA measures $\sin 2\beta_s$.

- Including "penguin pollution" or new physics or both([Bhattacharya, Datta, London 1209.1413](#)).

$$\begin{aligned}
 A^h &= \lambda_c^{(s)}(C' + P'_{ct} - \frac{2}{3}P'_{EW}) + \lambda_u^{(s)}(P'_{ut} - \frac{2}{3}P'_{EW}) + A_{NP} \\
 &\equiv e^{i \arg(V_{cb}^* V_{cs})} \left[A_1^h + e^{i\gamma} A_2^h \right] ,
 \end{aligned}$$

where $\lambda_q^{(q')} \equiv V_{qb}^* V_{qq'}$. This holds for the four helicities $h = 0, \perp, \parallel, S$.

- Experimental analysis: Assumes $A_2^h=0$ for all helicities. There are 8 assumptions. Using a convention when the overall phase vanishes then we have $A_h = \bar{A}_h$.
- The tagged angular distribution has enough observables to fit for β_s .
- The point is the 8 assumptions are not needed.

$$\frac{d^4\Gamma(B_s^0 \rightarrow J/\psi\phi)}{dt d\vec{\Omega}} \propto \sum_{k=1}^{10} h_k(t) f_k(\vec{\Omega}) .$$

$$h_k(t) = \frac{1}{2} e^{-\Gamma_s t} [c_k \cos \Delta m_s t + d_k \sin \Delta m_s t + a_k \cosh(\Delta\Gamma_s/2)t + b_k \sinh(\Delta\Gamma_s/2)t] .$$

- By measuring the time-dependent angular distribution and fitting to the four time-dependent functions, Γ_s and $\Delta\Gamma_s$ can be determined, as well as the coefficients a_k - d_k .

- By applying the angular analysis to the full amplitudes A_h and \bar{A}_h , one can still extract $\phi_S^{c\bar{c}s}$, even if there is PP or NP.
- In the general case there are two complex set A_h and \bar{A}_h . The a_k-d_k are expressed in terms of 16 unknown parameters: the magnitudes of the A_h and \bar{A}_h (8), their relative phases (7), and $\phi_S^{c\bar{c}s}$.
- The angular observables can be used to get 15 of these parameters.
- For the phase differences we define

$$\begin{aligned}\delta_{ij} &\equiv \arg(A_i) - \arg(A_j) , \\ \bar{\delta}_{ij} &\equiv \arg(\bar{A}_i) - \arg(\bar{A}_j) , \\ D_{ij} &\equiv \arg(\bar{A}_i) - \arg(A_j) \sim O(A_2/A_1) ,\end{aligned}$$

where i, j are any of the 4 helicities $0, \parallel, \perp, S$.

- In the one amplitude method $A_h = \bar{A}_h \Rightarrow D_{ij} = 0$.
- The point is only theory input is necessary: e.g. an estimate of $D_{0,0}$.
- If there is evidence of deviation from the SM we would like to know if the NP is in the decay or mixing.
- NP in the decay can be explored by the CPV quantities in the angular distribution, $a_k - d_k$. If the second amplitude (PP or NP) is tiny all these quantities are also very small.
- I will focus on charmless penguin/penguin dominated decays - for $B_s \rightarrow J/\psi\phi$ see archive:0910.2929.

$B \rightarrow V_1 V_2$: Naive Amplitude Estimate in the SM

- $B \rightarrow V_1 V_2$ when the vectors are light (charmless decays) there are naive estimates for: $A_L(A_0), A_-, A_+ (A_\perp, A_\parallel)$.
- Consider $b \rightarrow f \bar{q} q$ where $f = s, d$ and $q = u, d, s$. Weak interactions are $(V - A)$ and so the weak transition is

$$b_L \rightarrow f_L \bar{q}_R q_L.$$

- Helicity A_0 no helicity flip $\sim O(1)$.
 A_- one helicity flip $\sim O(\Lambda_{QCD}/m_B)$.
 A_+ two helicity flips $\sim O(\Lambda_{QCD}^2/m_B^2)$.
- For $B \rightarrow V_1 V_2$ where $V_{1,2}$ are light:

$$f_L \gg f_- \gg f_+.$$

$$f_i = \frac{\Gamma_i}{\Gamma_{total}}$$

where $i = 0, -, +$.

Data violates Naive Polarization Pattern

- Large theoretical uncertainties in penguin amplitudes.

Decay	Final State	f_L
$B \rightarrow \phi K^*$	ϕK^{*0}	0.480 ± 0.030
	ϕK^{*+}	0.50 ± 0.05
$B_s \rightarrow \phi\phi$	$\phi\phi$	$0.348 \pm 0.18(stat) \pm 0.82$
$B \rightarrow \rho K^*$	$\rho^0 K^{*0}$	0.57 ± 0.12
	$\rho^+ K^{*0}$	0.48 ± 0.08
$B_d \rightarrow K^* \bar{K}^*$	$K^{*0} \bar{K}^{*0}$	$0.80^{+0.12}_{-0.13}$
	$K^{*+} \bar{K}^{*0}$	$0.75^{+0.16}_{-0.26}$
$B_s \rightarrow K^* \bar{K}^*$	$K^{*0} \bar{K}^{*0}$	$0.31 \pm 0.12 \pm 0.04$
$B \rightarrow \rho\rho$	$\rho^+ \rho^-$	$0.978^{+0.025}_{-0.022}$
	$\rho^0 \rho^0$	$0.75^{+0.12}_{-0.15}$
	$\rho^+ \rho^0$	0.950 ± 0.016

Table: Longitudinal polarization fraction f_L for various $B \rightarrow V_1 V_2$ decays

CPV in $B \rightarrow V_1 V_2$ Decays - Time Independent case

- In the angular distribution, besides the direct CP violation (DCPV) one can have another measurement of CP violation which is called the triple product asymmetry (TPA).
- $\text{DCPV} \sim \sin \Delta\phi \sin \Delta\delta$ while $\text{TPA} \sim \sin \Delta\phi \cos \Delta\delta$. Hence DCPV and TPA complement each other. If the strong phases are small then TPA are maximized.
- There is another measurement which is not CPV. Fake TP which go as $\sim \cos \Delta\phi \sin \Delta\delta$. This observable can constrain NP if the NP has the same weak phase as the SM. In this case DCPV and TPA vanish.

Triple Product Correlations

- In the B rest frame we can construct T.P

$$T.P = \vec{p} \cdot (\vec{\epsilon}_1 \times \vec{\epsilon}_2).$$

- We can define a T-odd asymmetry

$$A_T = \frac{\Gamma[T.P > 0] - \Gamma[T.P < 0]}{\Gamma[T.P > 0] + \Gamma[T.P < 0]}.$$

- For true CP violation, we need to compare A_T and \bar{A}_T

$$A_{T.P}^{true} = A_T + \bar{A}_T \propto \sin \Delta\phi \cos \Delta\delta,$$

$$A_{T.P}^{fake} = A_T - \bar{A}_T \propto \cos \Delta\phi \sin \Delta\delta.$$

Measuring T.P.A.

- The T.P appear in the angular distribution of $B \rightarrow V_1 V_2 \rightarrow (V_1 \rightarrow P_1 P_1')((V_2 \rightarrow P_2 P_2')$.
- We can define two T.P's

$$A_T^{(1)} \equiv \frac{\text{Im}(A_{\perp} A_0^*)}{A_0^2 + A_{\parallel}^2 + A_{\perp}^2} \quad , \quad A_T^{(2)} \equiv \frac{\text{Im}(A_{\perp} A_{\parallel}^*)}{A_0^2 + A_{\parallel}^2 + A_{\perp}^2} \quad .$$

- For the CP conjugate decay one defines two T.P's

$$\bar{A}_T^{(1)} \equiv -\frac{\text{Im}(\bar{A}_{\perp} \bar{A}_0^*)}{\bar{A}_0^2 + \bar{A}_{\parallel}^2 + \bar{A}_{\perp}^2} \quad , \quad \bar{A}_T^{(2)} \equiv -\frac{\text{Im}(\bar{A}_{\perp} \bar{A}_{\parallel}^*)}{\bar{A}_0^2 + \bar{A}_{\parallel}^2 + \bar{A}_{\perp}^2} \quad .$$

- For true CP violation, we need to compare A_T and \bar{A}_T

$$A_{T.P}^{true,1,2} = \frac{1}{2} \left(A_T^{(1,2)} + \bar{A}_T^{(1,2)} \right) \propto \sin \Delta\phi \cos \Delta\delta,$$

$$A_{T.P}^{fake,1,2} = \frac{1}{2} \left(A_T^{(1,2)} - \bar{A}_T^{(1,2)} \right) \propto \cos \Delta\phi \sin \Delta\delta.$$

T.P. General

- If the decay is dominated by a single amplitude (single weak phase):

$$A_h \approx a_h e^{i\phi} e^{i\delta_h}.$$

Then $A_{T.P.}^{true} \approx 0$ but $A_{T.P.}^{fake}$ may be non-zero.

- If the transverse amplitudes $A_T \ll A_0$ then both the true and fake T.P are suppressed by $\frac{|A_{\perp}|}{|A_0|}$ and $\frac{|A_{\perp}A_{\parallel}|}{|A_0|^2}$ even in the presence of new CP violating sources.
- Since T.P. require large transverse amplitudes the interesting decays are penguin decays/penguin dominated decays which have large A_T .

T.P. Estimates

- The transverse amplitudes are written in terms of helicity amplitudes

$$A_{\parallel} = \frac{1}{\sqrt{2}}(A_{+} + A_{-}),$$

$$A_{\perp} = \frac{1}{\sqrt{2}}(A_{+} - A_{-}).$$

- Due to the fact that the weak interactions are left-handed, the helicity amplitudes obey the hierarchy

$$\left| \frac{A_{+}}{A_{-}} \right| = r_T = \frac{\Lambda_{QCD}}{m_b}.$$

- Thus, in the heavy-quark limit, $A_{\parallel} = -A_{\perp}$ which means $A_T^{(2)}$, which is proportional to $\text{Im}(A_{\perp} A_{\parallel}^*)$, vanishes. Hence in the heavy quark limit both $A_{T.P}^{true,2}$ and $A_{T.P}^{fake,2}$ vanish. (Datta, Durisamy, London e-Print: arXiv:1103.2442).

$\bar{b} \rightarrow \bar{s}$ transitions within the SM -pure penguin

- Amplitude within the SM (pure penguin modes) : (Loosely : γ comes from phase of V_{ub}^*)

$$A_h = e^{-i\phi_M/2} \left[P'_{tc,h} e^{i\delta_{tc,h}} + e^{i(\gamma+\phi_M/2)} P'_{uc,h} e^{i\delta_{uc,h}} \right]. \text{ Example Decay } B \rightarrow \phi K^*.$$

- Leading order in Wolfenstein Parameter λ : $P'_{tc,h} \propto |V_{tb}^* V_{ts}| \sim \mathcal{O}(\lambda^2)$.
- Next-to-leading order in λ : $P'_{uc,h} \propto |V_{ub}^* V_{us}| \sim \mathcal{O}(\lambda^4)$.
- If we neglect $P'_{uc,h}$ there is only decay amplitude and so all CPV measurements- direct CP and Triple product asymmetries vanish.
- $A_{T.P}^{true,2} \sim \lambda^2 \frac{\Lambda_{QCD}}{m_b}$ and $A_{T.P}^{fake,2} \sim \frac{\Lambda_{QCD}}{m_b}$.
- $A_{T.P}^{true,1} \sim \lambda^2$ and $A_{T.P}^{fake,1} \sim 0(1)$.

NP in $\bar{b} \rightarrow \bar{s}$ decay

- Assume NP larger than sub-dominant SM term.
- Amplitude with (large) NP in the decay :

$$A_h = P_{tc,h} e^{i\delta_{tc,h}} (1 + R_h^{NP} e^{i\phi^{NP}} e^{i\Delta_h^{NP}}).$$

- Δ_h^{NP} is the difference between NP strong phase and $\delta_{tc,h}$.
- NP strong phases may themselves be helicity dependent.
- $R_h^{NP} = P^{NP,h}/P_{tc,h}$: $R_h^{NP} \gg R_h^{SM} \sim \mathcal{O}(\lambda^2) \Rightarrow$ New Physics.
- CP violation appears due to the interference of two terms.
 \Rightarrow CP-violating observables are proportional to $R_h!$

Look for large CPV (direct, indirect, TP) for signals of NP in $\bar{b} \rightarrow \bar{s}$ decay.

Corrections to the heavy quark limit

- There are corrections to the prediction that $A_T^{(2)} = 0$, in $B \rightarrow \phi K^*$. The estimate for $A_T^{(2)}$ is, based on QCD factorization.

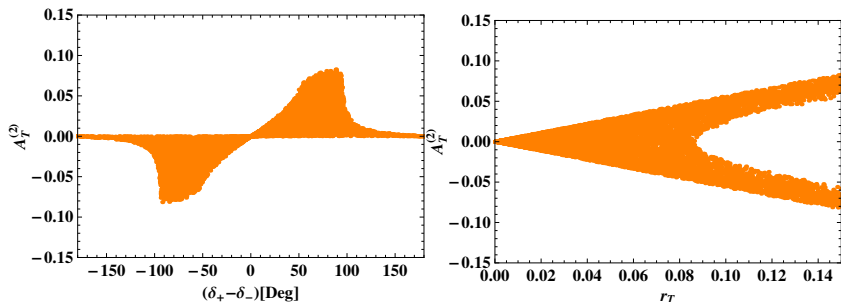


Figure: The left (right) panel of the figure shows $A_T^{(2)}$ for the decay $B_d \rightarrow \phi K^{*0}$ as a function of $(\delta_+ - \delta_-)$ and r_T .

- There we see that $|A_T^{fake,2}| \leq 9\%$ is predicted.

Corrections to the heavy quark limit

- The estimate for $A_{\mathcal{T}}^{(1)}$ is :

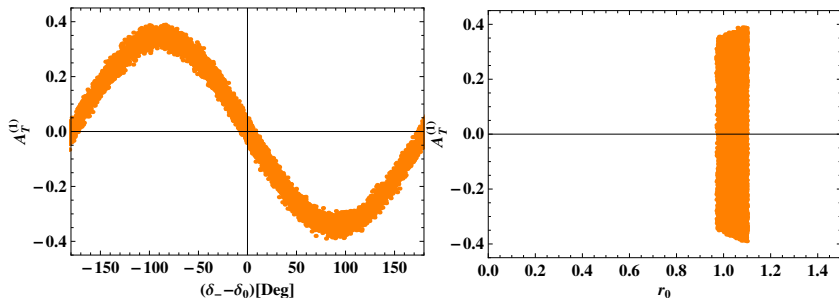


Figure: The left (right) panel of the figure shows $A_{\mathcal{T}}^{(1)}$ for the decay $B_d \rightarrow \phi K^{*0}$ as a function of $(\delta_+ - \delta_-)$ ($r_{\mathcal{T}}$).

- There we see that $|A_{\mathcal{T}}^{fake,1}| \leq 40\%$ is predicted. This prediction is not unexpected given the large size of the transverse amplitudes.

Experiments

- The relevant $B_d \rightarrow \phi K^{*0}$ polarization observables are shown in Table below.

Polarization fractions	
$f_{\parallel} = 0.480 \pm 0.030$	$f_{\perp} = 0.241 \pm 0.029$
Phases	
$\phi_{\parallel}(\text{rad}) = 2.40^{+0.14}_{-0.13}$	$\phi_{\perp}(\text{rad}) = 2.39 \pm 0.13$
$\Delta\phi_{\parallel}(\text{rad}) = 0.11 \pm 0.13$	$\Delta\phi_{\perp}(\text{rad}) = 0.08 \pm 0.13$
CP asymmetries	
$A_{CP}^0 = 0.04 \pm 0.06$	$A_{CP}^{\perp} = -0.11 \pm 0.12$

Table: $B_d \rightarrow \phi K^{*0}$ polarization observables .

- Note the T.P. are directly measurable from the angular distribution.
- $A_{T.P.}^{fake,2} \sim \sin(\phi_{\perp} - \phi_{\parallel})$ $A_{T.P.}^{true,2} \sim \sin(\Delta\phi_{\perp} - \Delta\phi_{\parallel})$.
- $A_{T.P.}^{fake,1} \sim \sin(\phi_{\perp})$ $A_{T.P.}^{true,2} \sim \sin(\Delta\phi_{\perp})$.

Experimental T.P's

- Using the numbers above we can calculate:

$$A_{T.P}^{fake,2} = \frac{1}{2}(A_{T,B}^{(2)} - \bar{A}_{T,\bar{B}}^{(2)}) = 0.002 \pm 0.049 ,$$

$$A_{T.P}^{fake,1} = \frac{1}{2}(A_{T,B}^{(1)} - \bar{A}_{T,\bar{B}}^{(1)}) = -0.23 \pm 0.03 .$$

- The measured value of $A_{T.P}^{fake,2}$ is therefore in agreement with the SM prediction in the heavy quark limit.
- The actual T.P are

$$A_{T.P}^{true,2} = \frac{1}{2}(A_{T,B}^{(2)} + \bar{A}_{T,\bar{B}}^{(2)}) = -0.004 \pm 0.025,$$

$$A_{T.P}^{true,1} = \frac{1}{2}(A_{T,B}^{(1)} + \bar{A}_{T,\bar{B}}^{(1)}) = 0.013 \pm 0.053.$$

Hence consistent with SM or with NP with same weak phase as the SM. No evidence for large NP contribution to the amplitude.

NP in $\bar{b} \rightarrow \bar{s}$ decay- Tree and Penguins

- The decays $B \rightarrow \rho K^*$ are interesting. There are Tree and Penguin contributions.
- They probe $b \rightarrow su\bar{u}$ and $b \rightarrow sd\bar{d}$ transitions. They are the vector counterpart of the $B_d \rightarrow K\pi$ modes.

$$\begin{aligned}
 A(B^+ \rightarrow \rho^+ K^{*0}) &= P'_{ct} , \\
 \sqrt{2}A(B^+ \rightarrow \rho^0 K^{*+}) &= -P'_{ct} - T' e^{i\gamma} - P'_{EW} , \\
 A(B^0 \rightarrow \rho^- K^+) &= -T' e^{i\gamma} - P'_{ct} , \\
 \sqrt{2}A(B^0 \rightarrow \rho^0 K^0) &= P'_{ct} - P'_{EW} .
 \end{aligned}$$

- Large CPV in $B^0 \rightarrow K^- \pi^+$. What are the T.P in $B \rightarrow \rho K^*$ modes. Note these modes also have large transverse polarization. Can these modes shed light on the $K - \pi$ “puzzle”.

Time dependent Angular Distribution: $B \rightarrow V_1 V_2$

- Decays like $B_d \rightarrow \rho^0 K^0$, $B_d \rightarrow K^* \bar{K}^*$, $B_s \rightarrow J/\psi \phi$, $\phi \phi (\bar{b} \rightarrow \bar{s} s \bar{s})$, $K^* \bar{K}^*$ ($\bar{s} d \bar{d}$), the final state can be reached by both B_q and \bar{B}_q decays so mixing effects have to be included.
- Assuming that $V_{1,2}$ both decay into pseudoscalars (i.e. $V_1 \rightarrow P_1 P'_1$, $V_2 \rightarrow P_2 P'_2$), the angular distribution of the decay is then given in terms of the vector $\vec{\omega} = (\cos \theta_1, \cos \theta_2, \Phi)$:

$$\frac{d^3\Gamma(t)}{d\vec{\omega}} = \frac{9}{32\pi} \sum_{i=1}^6 K_i(t) f_i(\vec{\omega}) .$$

- Functions $K_i(t)$ are expressed in terms of ϕ_q , Γ_q , $\Delta\Gamma_q$, the B_q^0 oscillation frequency Δm_q and transversity amplitudes $A_{i(=0,\parallel,\perp)}$.

Time-integrated untagged distribution

The time-integrated untagged angular distribution can be obtained by integrating the $K_i(t) + \bar{K}_i(t)$ observables over time:

$$\frac{d^3 \langle \Gamma(B_s^0 \rightarrow f) \rangle}{d\vec{\omega}} = \frac{9}{32\pi} \sum_{i=1}^6 \langle K_i \rangle f_i(\vec{\omega}),$$

$$\langle \Gamma(B_s^0 \rightarrow f) \rangle = \frac{1}{2} \int_0^\infty dt (\Gamma^{B_s} + \Gamma^{\bar{B}_s}), \quad \langle K_i \rangle = \frac{1}{2} \int_0^\infty dt (K_i(t) + \bar{K}_i(t)).$$

The general structure is

$$\langle K_i \rangle \propto \mathcal{A}_i^{ch} + \mathcal{A}_i^{sh} y_q,$$

where $y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}$. The \mathcal{A}_i^{ch} are used to extract the polarization fractions and triple products.

Time-integrated untagged distribution

- If y_q is small (e.g. y_d) then

$$\langle K_i \rangle \propto \mathcal{A}_i^{ch}$$

The polarization fractions and triple products can be extracted from $\langle K_i \rangle$ which appear in the angular distribution.

- If y_q cannot be neglected (e.g. y_s) then we need the input $\mathcal{A}_{\Delta\Gamma}^i \equiv \mathcal{A}_i^{sh}/\mathcal{A}_i^{ch}$ (known in SM, Fleischer et.al.).
- Use:

$$\tau_{B_s}^{eff,i} = \frac{\int_0^\infty t(K_i(t) + \bar{K}_i(t))dt}{\int_0^\infty (K_i(t) + \bar{K}_i(t))dt} = \frac{\tau_{B_s}}{(1 - y_s^2)} \frac{(1 + 2\mathcal{A}_{\Delta\Gamma}^i y_s + y_s^2)}{(1 + \mathcal{A}_{\Delta\Gamma}^i y_s)}.$$

Polarization Fractions

In the SM (one amp): We have $\mathcal{A}_i^{sh} = \mp \mathcal{A}_i^{ch}$, where the minus sign is for $i = 1, 2, 5$, the plus sign for $i = 3$, and both quantities vanish when $i = 4, 6$. With NP these relations are no longer true.

The polarization fractions can be extracted from $\langle K_i \rangle$, $i = 1, 2, 3$

$$\langle K_h \rangle = \frac{\tau_{B_s}}{2(1-y_s^2)} \left[(|A_h|^2 + |\bar{A}_h|^2) - \eta_h 2 \left(\text{Re}(A_h^* \bar{A}_h) \cos \phi_s + \text{Im}(A_h^* \bar{A}_h) \sin \phi_s \right) y_s \right].$$

and $\eta_h = \eta_{0,\parallel,\perp} = (1, 1, -1)$.

$$f_h = \frac{|A_h|^2 + |\bar{A}_h|^2}{|A_0|^2 + |\bar{A}_0|^2 + |A_{\parallel}|^2 + |\bar{A}_{\parallel}|^2 + |A_{\perp}|^2 + |\bar{A}_{\perp}|^2} = \frac{\mathcal{A}_h^{ch}}{\sum_{i=1,2,3} \mathcal{A}_i^{ch}}.$$

Triple Products

We now turn to the measurement of TP's

$$\begin{aligned}
 \langle K_4 \rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)} \left[\left(\text{Im}(A_{\perp} A_{\parallel}^*) - \text{Im}(\bar{A}_{\perp} \bar{A}_{\parallel}^*) \right) \right. \\
 &\quad - \left(\left(\text{Im}(A_{\perp} \bar{A}_{\parallel}^*) - \text{Im}(\bar{A}_{\perp} A_{\parallel}^*) \right) \cos \phi_s \right. \\
 &\quad \quad \left. \left. + \left(\text{Re}(A_{\perp} \bar{A}_{\parallel}^*) + \text{Re}(\bar{A}_{\perp} A_{\parallel}^*) \right) \sin \phi_s \right) y_s \right], \\
 \langle K_6 \rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)} \left[\left(\text{Im}(A_{\perp} A_0^*) - \text{Im}(\bar{A}_{\perp} \bar{A}_0^*) \right) \right. \\
 &\quad - \left(\left(\text{Im}(A_{\perp} \bar{A}_0^*) - \text{Im}(\bar{A}_{\perp} A_0^*) \right) \cos \phi_s \right. \\
 &\quad \quad \left. \left. + \left(\text{Re}(A_{\perp} \bar{A}_0^*) + \text{Re}(\bar{A}_{\perp} A_0^*) \right) \sin \phi_s \right) y_s \right].
 \end{aligned}$$

The TP's in the untagged distribution can be from the decay as well as mixing and can be measured by constructing asymmetries involving the angular variables (Rosner, Gronau).

Triple Products- Untagged Decays

We begin with $i = 4$, for which $f_4(\vec{\omega}) = -2 \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\Phi$. We define $u \equiv \sin 2\Phi$. Construct the T.P as an asymmetry in u .

$$\begin{aligned} \mathcal{A}_u &= \frac{1}{2} \left[\frac{\langle \Gamma(B_s^0 \rightarrow \phi\phi), u > 0 \rangle - \langle \Gamma(B_s^0 \rightarrow \phi\phi), u < 0 \rangle}{\langle \Gamma(B_s^0 \rightarrow \phi\phi), u > 0 \rangle + \langle \Gamma(B_s^0 \rightarrow \phi\phi), u < 0 \rangle} \right] \\ &= -\frac{2}{\pi} [\mathcal{A}_T^{(2,true)}]_{\text{exp}} \quad , \quad [\mathcal{A}_T^{(2,true)}]_{\text{exp}} = \frac{\langle K_4 \rangle}{\langle \Gamma(B_s^0 \rightarrow \phi\phi) \rangle} . \end{aligned}$$

For $i = 6$ with $f_6(\vec{\omega}) = -\sqrt{2} \sin 2\theta_1 \sin 2\theta_2 \sin \Phi$. We define $v \equiv \text{sign}(\cos \theta_1 \cos \theta_2) \sin \Phi$, which has the following associated TP asymmetry :

$$\begin{aligned} \mathcal{A}_v &= \frac{1}{2} \left[\frac{\langle \Gamma(B_s^0 \rightarrow \phi\phi), v > 0 \rangle - \langle \Gamma(B_s^0 \rightarrow \phi\phi), v < 0 \rangle}{\langle \Gamma(B_s^0 \rightarrow \phi\phi), v > 0 \rangle + \langle \Gamma(B_s^0 \rightarrow \phi\phi), v < 0 \rangle} \right] \\ &= -\frac{\sqrt{2}}{\pi} [\mathcal{A}_T^{(1,true)}]_{\text{exp}} \quad , \quad [\mathcal{A}_T^{(1,true)}]_{\text{exp}} = \frac{\langle K_6 \rangle}{\langle \Gamma(B_s^0 \rightarrow \phi\phi) \rangle} . \end{aligned}$$

Triple Products- Untagged Decays

The relation between $[\mathcal{A}_T^{(2,1)}]_{\text{exp}}$ and the theoretical expression for the TP in the decay are.

$$[\mathcal{A}_T^{(2,true)}]_{\text{exp}} = \frac{1}{2} \left(\text{Im}(A_{\perp} A_{\parallel}^*) - \text{Im}(\bar{A}_{\perp} \bar{A}_{\parallel}^*) \right) \left[\frac{(1 + A_{\Delta\Gamma}^{(4)} y_s)}{(1 - y_s^2)} \frac{\tau_{B_s}}{\langle \Gamma(B_s^0 \rightarrow \phi\phi) \rangle} \right].$$

$$[\mathcal{A}_T^{(1,true)}]_{\text{exp}} = \frac{1}{2} \left(\text{Im}(A_{\perp} A_0^*) - \text{Im}(\bar{A}_{\perp} \bar{A}_0^*) \right) \left[\frac{(1 + A_{\Delta\Gamma}^{(6)} y_s)}{(1 - y_s^2)} \frac{\tau_{B_s}}{\langle \Gamma(B_s^0 \rightarrow \phi\phi) \rangle} \right].$$

$$A_{\Delta\Gamma}^{(4,6)} = \mathcal{A}_{4,6}^{sh} / \mathcal{A}_{4,6}^{ch}.$$

$B \rightarrow V_1 V_2$ with scalar background

- Penguin-dominated decays : Eg. $B_s \rightarrow \phi\phi, K^* \bar{K}^*$. Scalar background contribute to the final state.([arXiv:1306.1911](#), [Bhattacharya, Datta, Duraisamy, London](#)).
- Vectors detected via hadronic decay come with scalar backgrounds
Eg. $\phi \rightarrow K^+ K^-$, Background Scalar : $K^+ K^-$ s-wave.

- Additional contributions to Amplitude :

$$A(B \rightarrow V_1 V_2) + A(B \rightarrow V_1 S_2) + A(B \rightarrow S_1 V_2) + A(B \rightarrow S_1 S_2).$$

- 3 helicity amplitudes in $B \rightarrow VV$: 1 Longitudinal and 2 transverse.
- Scalar background adds additional helicities : (SV, VS, SS).

Identical final-state vector mesons : 2 additional helicities (VS = -SV).

Distinguishable final-state vector mesons : 3 additional helicities.

The differential decay rate

- Most general amplitude has the following terms :

$$A_{VV} : A_0 \cos \theta_1 \cos \theta_2 + \frac{A_{\parallel}}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \cos \phi + i \frac{A_{\perp}}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \sin \phi$$

$$A_{VS} : -\frac{A_+^{(VS)}}{\sqrt{6}} (\cos \theta_1 - \cos \theta_2) - \frac{A_-^{(VS)}}{\sqrt{6}} (\cos \theta_1 + \cos \theta_2)$$

$$A_{SS} : -\frac{A_s}{3};$$

- $A_{\pm}^{(VS)} = (A_{VS} \pm A_{SV})/\sqrt{2}$. $A_-^{(VS)}$ and $A_+^{(VS)}$ are CP even and CP odd.
- If $V_1 = V_2$ then $A_+^{(VS)} \equiv 0$.

The differential decay rate

- The differential decay rate is then :

$$\frac{d^4\Gamma}{dt d\vec{\Omega}} \propto |A_{VV} + A_{VS} + A_{SS}|^2$$

- Triple product and DCPV can be constructed from (A_{\perp}) and $A_{+}^{(VS)}$ which are CP-odd amplitudes.
- CP-violating terms are the result of interference between CP-odd and CP-even amplitudes.

- Angular distribution with six helicities : (${}^6C_2 + 6 = 21$)

$$\frac{d^4\Gamma}{dt d\vec{\Omega}} = \frac{9}{8\pi} \sum_{i=1}^{21} K_i(t) X_i(\theta_1, \theta_2, \phi)$$

$$\text{where } K_i(t) = \frac{1}{2} e^{-\Gamma t} \left[a_i \cosh\left(\frac{\Delta\Gamma}{2}t\right) + c_i \cos(\Delta mt) \right. \\ \left. + b_i \sinh\left(\frac{\Delta\Gamma}{2}t\right) + d_i \sin(\Delta mt) \right]$$

- Appropriately integrate over phase space to extract K_i 's using :

$$\int X_i(\vec{\Omega}) f_j(\vec{\Omega}) d\vec{\omega} = \delta_{ij}$$

- Note: It is not possible to distinguish between $\text{Re}[A_5 A_0^*]$ and $|A_+^{(VS)}|^2 - |A_-^{(VS)}|^2$ since the angular function is the same : $X \propto \cos\theta_1 \cos\theta_2$.
- Time-dependent fit to K_i 's give the observables : a_i, b_i, c_i, d_i .

$$\underline{B_s \rightarrow K^{*0} \bar{K}^{*0}}$$

- CP conjugate K_i 's can be obtained from :

$$\begin{aligned} \bar{K}_i(t) = \frac{1}{2} e^{-\Gamma t} & \left[\bar{a}_i \cosh\left(\frac{\Delta\Gamma}{2}t\right) + \bar{c}_i \cos(\Delta mt) \right. \\ & \left. + \bar{b}_i \sinh\left(\frac{\Delta\Gamma}{2}t\right) + \bar{d}_i \sin(\Delta mt) \right] \end{aligned}$$

where $\bar{a}_i = a_i$, $\bar{b}_i = b_i$, $\bar{c}_i = -c_i$, $\bar{d}_i = -d_i$

- Untagged analysis angular distribution
- Asymmetric integration over helicity angles obtain :

$$K_i^{\text{untagged}} = K_i + \bar{K}_i = e^{-\Gamma t} \left[a_i \cosh\left(\frac{\Delta\Gamma}{2}t\right) + b_i \sinh\left(\frac{\Delta\Gamma}{2}t\right) \right].$$

- Observables a_i and b_i from time-dependent fit to K_i^{untagged}

Physics from the angular analysis

- a_i, b_i, c_i, d_i are functions of 12 magnitudes $|A_h|, |\bar{A}_h|$, 11 relative phases and the $B_q - \bar{B}_q$ mixing phase (ϕ_q).
- Construction of observables is model-independent. However, several CP-violating observables are expected to be small in the SM. Large values of these can indicate new physics in B_q decay.

- Examples of CP-violating observables : ($\phi_s = -2\text{Arg}(V_{tb}^* V_{ts})$)

Direct CP Asymmetry (c_i, a_i) : $\text{Re}[A_i A_i^* - \bar{A}_i \bar{A}_i^*] \quad i = j \Rightarrow |A|^2 - |\bar{A}|^2$

Indirect CP Asymmetry (d_i, b_i) : $\text{Im} \left[(A_i^* \bar{A}_j - \bar{A}_i A_j^*) e^{-i\phi_M} \right]$

True triple product (a_i, c_{12}) : $\text{Im} [A_{\perp} A_i^* - \bar{A}_{\perp} \bar{A}_i^*]$

Mixing-induced triple product (b_i, d_{12}) : $\text{Im} [(\bar{A}_{\perp} A_i^* + A_{\perp}^* \bar{A}_i) e^{-i\phi_M}]$

$\bar{b} \rightarrow \bar{s}$ transitions within the SM- $B_s \rightarrow \phi\phi$

- Amplitude within the SM (pure penguin modes) : (Loosely : γ comes from phase of V_{ub}^*)

$$A_h = e^{-i\phi_M/2} \left[P'_{tc,h} e^{i\delta_{tc,h}} + e^{i(\gamma+\phi_M/2)} P'_{uc,h} e^{i\delta_{uc,h}} \right]. \text{ Example Decay } B \rightarrow \phi K^*.$$

- Leading order in Wolfenstein Parameter λ : $P'_{tc,h} \propto |V_{tb}^* V_{ts}| \sim \mathcal{O}(\lambda^2)$.
- Next-to-leading order in λ : $P'_{uc,h} \propto |V_{ub}^* V_{us}| \sim \mathcal{O}(\lambda^4)$.
- If we neglect $P'_{uc,h}$ there is only decay amplitude and so all CPV measurements- direct CP and Triple product asymmetries vanish.
- $A_{T.P}^{true,2} \sim \mathcal{A}_u \sim \lambda^2 \frac{\Lambda_{QCD}}{m_b}$ and $A_{T.P}^{fake,2} \sim \frac{\Lambda_{QCD}}{m_b}$.
- $A_{T.P}^{true,1} \sim \mathcal{A}_v \sim \lambda^2$ and $A_{T.P}^{fake,1} \sim 0(1)$.

$B_s \rightarrow \phi\phi$

- Two identical vectors in the final state :
5 helicity amplitudes (3VV, SS, VS₋)
- Studied by LHCb in detail : arXiv:1407.2222 (published in PRD)
- LHCb(new) results for $B_s^0 \rightarrow \phi\phi$.

Observable	Measurement
$ A_0 _{exp}^2$	0.365 ± 0.022 (stat) ± 0.012 (syst)
$ A_{\perp} _{exp}^2$	0.291 ± 0.024 (stat) ± 0.010 (syst)
$ A_{\parallel} _{exp}^2$	0.344 ± 0.024 (stat) ± 0.014 (syst)
$\cos(\delta_{\parallel} - \delta_0)$	-0.844 ± 0.068 (stat) ± 0.029 (syst)
\mathcal{A}_u	-0.003 ± 0.017 (stat) ± 0.006 (syst)
\mathcal{A}_v	-0.017 ± 0.017 (stat) ± 0.006 (syst)

$B_s \rightarrow \phi\phi$

- $\phi_s = -0.17 \pm 0.15 \pm 0.03$.
- Scalar background consistent with zero.
- No evidence of new CP violation in $b \rightarrow s\bar{s}s$ transitions as in $B_d \rightarrow \phi K^*$.
- As more experimental precision is achieved the discarded subleading SM amplitude will have to be included.
- We can move on to $b \rightarrow s\bar{d}d$. Motivated by the $K-\pi$ puzzle. Examples are $B_d \rightarrow \rho K^*$, $B_s \rightarrow K^* \bar{K}^*$.

$$\underline{B_s \rightarrow K^{*0} \bar{K}^{*0}}$$

- Final state has distinguishable vectors : 6 helicity amplitudes
- The same final state is accessible to both B_s and \bar{B}_s
- $K^{*0}(890)$ is identified through its decay to $K^+ \pi^-$
- Scalar background : $K^{*0}(1430)$ (Large width) is noticed.
- Time-dependent tagged analysis could be difficult
- Interesting physics even in untagged time-dependent analysis

$B_s \rightarrow K^{*0} \bar{K}^{*0}$ LHCb

- LHCb (1503.05362) measured 8 CP violating observables in the time integrated untagged decay.
- Triple product constructed with (A_{\perp}) and DCPV with $A_{+}^{(VS)}$
- $A_T^i \sim \text{Im} [A_{\perp} A_i^* - \bar{A}_{\perp} \bar{A}_i^*]$ where $A_i = A_0, A_{\parallel}, A_{-}^{(VS)}, A_S$.
- $A_D^i \sim \text{Re} [A_{+}^{(VS)} A_i^* - \bar{A}_{+}^{(VS)} \bar{A}_i^*]$ where $A_i = A_0, A_{\parallel}, A_{-}^{(VS)}, A_S$.
- Found large scalar background from $K_0^*(1430)$ and $K_0^*(800)$.

New-Physics Scenarios

- Typical effective NP operator :

$$H_{AB}^{NP} \sim (\bar{b} \gamma_A s)(\bar{q} \gamma_B q) \text{ where } A, B \text{ stands for left(L) or right(R)}$$

- Expansion parameters : Λ_{QCD}/m_B and R_h^{NP}
- RR and LL operators only contribute to A_{\parallel} , A_{\perp} , and A_{SS}
 - \Rightarrow Direct CPV involving $A^{(VS)+}$ are suppressed
 - Reasonable triple products
- RL and LR operators don't contribute to VS helicities
 - \Rightarrow Triple products involving A_{\parallel} and A_{\perp} are small
 - Other CP violating observables are reasonable, including direct CPV
- a_i, b_i, c_i, d_i can help distinguish between different NP scenarios

Table: Triple product and direct CP asymmetries measured in this analysis. The first uncertainties are statistical and the second systematic.

Asymmetry	Value
A_T^1	$0.003 \pm 0.041 \pm 0.009$
A_T^2	$0.009 \pm 0.041 \pm 0.009$
A_T^3	$0.019 \pm 0.041 \pm 0.008$
A_T^4	$-0.040 \pm 0.041 \pm 0.008$
A_D^1	$-0.061 \pm 0.041 \pm 0.012$
A_D^2	$0.081 \pm 0.041 \pm 0.008$
A_D^3	$-0.079 \pm 0.041 \pm 0.023$
A_D^4	$-0.081 \pm 0.041 \pm 0.010$

Conclusions

- Non- Leptonic Decays offer unique probes of CP violation.
- Extraction of CPV phases require control of hadronic quantities.
- Challenge is to identify the most reliable theory input.
- $B \rightarrow V_1 V_2$ decays offer many probes of CP.
- Experiments have measured these observables and will continue to do so with more precision. Challenge is to study carefully the implications of these measurements.