Lattice Computations of $K \to \pi\pi$ Decays & Lessons for B-Decays

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Future Challenges in Non-Leptonic B-Decays
Theory and Experiment

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Outline of talk

1. Introduction
2. Status of RBC-UKQCD Collaboration's calculations of $K \to \pi\pi$ decay amplitudes. *
3. Electromagnetic corrections to decay amplitudes.

* RBC=Riken Research Center, Brookhaven National Laboratory, Columbia University; UKQCD = Edinburgh + Southampton.

- Items from the current RBC-UKQCD Kaon Physics Programme not discussed here:
  Long-distance contributions to flavour changing processes

$$\int\!\!\int d^4x\, d^4y\, \langle f \,|\, T[Q_1(x)\, Q_2(y)]\, |\, i\rangle\, .$$

(i) K_L-K_S mass difference (and ϵ_K)

(ii) Rare kaon decays $K \to \pi\ell^+\ell^-$ and $K^+ \to \pi^+\nu\bar{\nu}$

2. Status of RBC-UKQCD calculations of $K \to \pi\pi$ decays

- In May 2015 RBC-UKQCD* published our first result for ϵ'/ϵ computed at physical quark masses and kinematics, albeit still with large errors:

$$\frac{\epsilon'}{\epsilon}\bigg|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$$

to be compared with

$$\frac{\epsilon'}{\epsilon}\bigg|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}\,.$$

RBC-UKQCD, arXiv:1505.07863

- This is by far the most complicated project that I have ever been involved with.
- This single result hides much important (and much more precise) information which we have determined along the way.
- I will review the main obstacles to computing $K \to \pi\pi$ decay amplitudes, the techniques used to overcome them, present our main results and discuss the implications for non-leptonic B-decays.

* RBC=Riken Research Center, Brookhaven National Laboratory, Columbia University; UKQCD = Edinburgh + Southampton.

1. A_0 and A_2 amplitudes with unphysical quark masses and with the pions at rest.

   "K to $\pi\pi$ decay amplitudes from lattice QCD,"

   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Lehner, Q.Liu, R.D. Mawhinney, C.T.S, A.Soni, C.Sturm, H.Yin and R. Zhou, Phys. Rev. D **84** (2011) 114503 [arXiv:1106.2714 [hep-lat]].

   "Kaon to two pions decay from lattice QCD, $\Delta I = 1/2$ rule and CP violation"

   Q.Liu, Ph.D. thesis, Columbia University (2010)

2. A_2 at physical kinematics and a single coarse lattice spacing.

   "The $K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD,"

   T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.S., A.Soni, and C.Sturm

   Phys. Rev. Lett. **108** (2012) 141601 [arXiv:1111.1699 [hep-lat]],

   "Lattice determination of the $K \to (\pi\pi)_{I=2}$ Decay Amplitude A_2"

   Phys. Rev. D **86** (2012) 074513 [arXiv:1206.5142 [hep-lat]]

   "Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD,"

   P.A. Boyle, N.H. Christ, N. Garron, E.J. Goode, T. Janowski, C. Lehner, Q. Liu, A.T. Lytle, C.T. Sachrajda, A. Soni, and D.Zhang, Phys. Rev. Lett. **110** (2013) 15, 152001 [arXiv:1212.1474 [hep-lat]].

3. A_2 at physical kinematics on two finer lattices $\Rightarrow$ continuum limit taken.
   "$K \to \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit,"
   T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Janowski, C.Jung, C.Kelly, C.Lehner, A.Lytle, R.D.Mawhinney, C.T.S., A.Soni, H.Yin, and D.Zhang
   Phys. Rev. D **91** (2015) 7, 074502 [arXiv:1502.00263 [hep-lat]].
4. A_0 at physical kinematics and a single coarse lattice spacing.
   "Standard-model prediction for direct CP violation in $K \to \pi\pi$ decay,"
   Z.Bai, T.Blum, P.A.Boyle, N.H.Christ, J.Frison, N.Garron, T.Izubuchi, C.Jung, C.Kelly, C.Lehner, R.D.Mawhinney, C.T.S, A. Soni, and D. Zhang,
   Phys. Rev. Lett. **115** (2015) 21, 212001 [arXiv:1505.07863 [hep-lat]].

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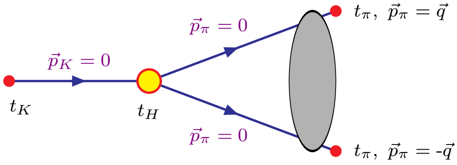


- $K \to (\pi\pi)_{I=2}$ correlation function is dominated by lightest state, i.e. the state with two-pions at rest. Maiani and Testa, PL B245 (1990) 585

$$C_{I=2}(t_\pi) = B_1 e^{-2m_\pi t_\pi} + B_2 e^{-2E_\pi t_\pi} + \cdots$$

  - Solution 1: Study an excited state. Lellouch and Lüscher, hep-lat/0003023
  - Solution 2: Introduce suitable boundary conditions such that the $\pi\pi$ ground state is $|\pi(\vec q)\pi(-\vec q)\rangle$. RBC-UKQCD, C.h.Kim hep-lat/0311003

- For $K \to (\pi\pi)_{I=0}$ decays the correlation function is dominated by the vacuum intermediate state:

$$C_{I=0}(t_\pi) = A + B_1 e^{-2m_\pi t_\pi} + B_2 e^{-2E_\pi t_\pi} + \cdots$$

For B-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.

Boundary conditions for A_2

- For A_2, there is no vacuum subtraction and we can use the Wigner-Eckart theorem to write

$$\underbrace{\langle(\pi\pi)^{I=2}_{I_3=1}\,|}_{\frac{1}{\sqrt{2}}(\langle\pi^+\pi^0|+\langle\pi^0\pi^+|)}\; Q^{\Delta I=3/2}_{\Delta I_3=1/2,i}\,|\,K^+\rangle = \frac{3}{2}\,\underbrace{\langle(\pi\pi)^{I=2}_{I_3=2}\,|}_{\langle\pi^+\pi^+|}\,Q^{\Delta I=3/2}_{\Delta I_3=3/2,i}\,|\,K^+\rangle\,,$$

  and impose anti-periodic conditions on the d-quark in one or more directions.
- If we impose the anti-periodic boundary conditions in all 3 directions then the ground state is

$$\Big|\pi\left(\frac{\pi}{L},\frac{\pi}{L},\frac{\pi}{L}\right)\,\pi\left(\frac{-\pi}{L},\frac{-\pi}{L},\frac{-\pi}{L}\right)\Big\rangle\,.$$

- With an appropriate choice of L and the number of directions, we can arrange that $E_{\pi\pi}=m_K$.
- Isospin breaking by the boundary conditions is harmless here.

CTS & G.Villadoro, hep-lat/0411033

- These are based on the Poisson summation formula:

$$\frac{1}{L}\sum_{n=-\infty}^{\infty} f(p_n^2) = \int_{-\infty}^{\infty}\frac{dp}{2\pi} f(p^2) + \sum_{n\neq 0}\int_{-\infty}^{\infty}\frac{dp}{2\pi} f(p^2)\, e^{inpL}\,,$$

- For single-hadron states the finite-volume corrections decrease exponentially with the volume $\propto e^{-m_\pi L}$. For multi-hadron states, the finite-volume corrections generally fall as powers of the volume.
- For two-hadron states, there is a huge literature following the seminal work by Lüscher and the effects are generally understood.
  - The spectrum of two-pion states in a finite volume is given by the scattering phase-shifts. M. Luscher, Commun. Math. Phys. 105 (1986) 153, Nucl. Phys. B354 (1991) 531.
  - The $K \to \pi\pi$ amplitudes are obtained from the finite-volume matrix elements by the Lellouch-Lüscher factor which contains the derivative of the phase-shift. L.Lellouch & M.Lüscher, hep-lat/:0003023, C.h.Kim, CTS & S.R.Sharpe, hep-lat/0507006 $\cdots$
  - Recently we have also determined the finite-volume corrections for $\Delta m_K = m_{K_L} - m_{K_S}$. N.H.Christ, X.Feng, G.Martinelli & CTS, arXiv:1504.01170
- For three-hadron states, there has been a major effort by Hansen and Sharpe leading to much theoretical clarification.

M.Hansen & S.Sharpe, arXiv:1408.4933, 1409.7012, 1504.04248

One more thing!

- Since we cannot perform simulations with lattice spacings $< 1/M_W$ or $1/m_t$ we exploit the standard technique of the Operator Product Expansion and write schematically:

$$\text{Physics} = \sum_i C_i(\mu) \times \langle f|O_i(\mu)|i\rangle\,.$$

- Until recently, the (perturbative) Wilson coefficients $C_i(\mu)$ were typically calculated with much greater precision than our knowledge of the matrix elements.
  - The C_i are typically calculated in schemes based on dimensional regularisation (such as $\overline{\text{MS}}$) which are intrinsically perturbative.
  - We can compute the matrix elements non-perturbatively, with the operators renormalised in schemes which have a non-perturbative definition (such as RI-MOM schemes) but not in purely perturbative schemes based on dim.reg.

G.Martinelli, C.Pittori, CTS, M.Testa and A.Vladikas, hep-lat/9411010

- Thus the determination of the C_i in $\overline{\text{MS}}$-like schemes is not the complete perturbative calculation. Matching between $\overline{\text{MS}}$ and non-perturbatively defined schemes must also be performed.
  - This is beginning to be done.
  - We are now careful to present tables of matrix elements of operators renormalized in RI-MOM schemes, which can be used to gain better precision once improved perturbative calculations are performed.

Error budgets in our calculation of A_2

RBC-UKQCD, T.Blum et al., arXiv:1502:00263

Source	ReA_2	ImA_2
NPR (nonperturbative)	0.1%	0.1%
NPR (perturbative)	2.9%	7.0%
Finite volume corrections	2.4%	2.6%
Unphysical kinematics	4.5%	1.1%
Wilson coefficients	6.8%	10%
Derivative of the phase shift	1.1%	1.1%
Total	9%	12%

- *Wilson Coefficients* and *NPR(perturbative*) errors are not from our lattice calculation.
- Step-scaling can be used to increase the scale at which the matching is performed.

Results for A_2

- Our first results for A_2 at physical kinematics were obtained at a single, rather coarse, value of the lattice spacing ($a \simeq 0.14$ fm). Estimated discretization errors at 15%. arXiv:1111.1699, arXiv:1206.5142
- Our recent results were obtained on two new ensembles, 48^3 with $a \simeq 0.11$ fm and 64^3 with $a \simeq 0.084$ fm so that we can make a continuum extrapolation:

$$\begin{aligned} \mathrm{Re}(A_2) &= 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}} \times 10^{-8}\ \mathrm{GeV}. \\ \mathrm{Im}(A_2) &= -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} \times 10^{-13}\ \mathrm{GeV}\,. \end{aligned}$$

arXiv:1502.00263

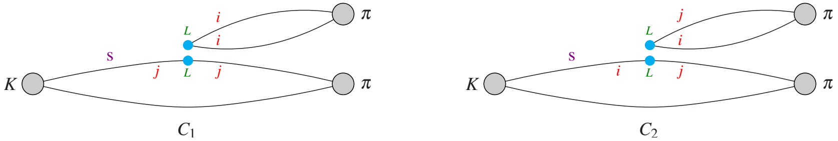
- Although the precision can still be significantly improved (partly by perturbative calculations), the calculation of A_2 at physical kinematics can now be considered as standard.

RBC-UKQCD Collaboration, arXiv:1212.1474

- Re A_2 is dominated by a simple operator:

$$O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \left\{ (\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L \right\} + (\bar{s}^i u^i)_L\,(\bar{u}^j d^j)_L$$

and two diagrams:



- Re A_2 is proportional to $C_1 + C_2$.
- The contribution to Re A_0 from Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same overall sign.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3} C_1$.
- We find instead that $C_2 \approx -C_1$ so that A_2 is significantly suppressed!
- **We believe that the strong suppression of Re A_2 and the (less-strong) enhancement of Re A_0 is a major factor in the $\Delta I = 1/2$ rule.**

Evidence for the Suppression of Re A_2

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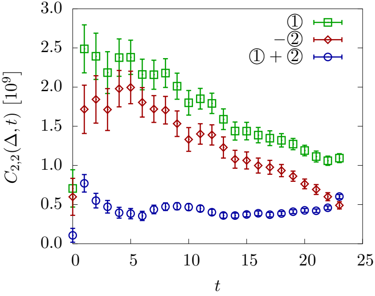
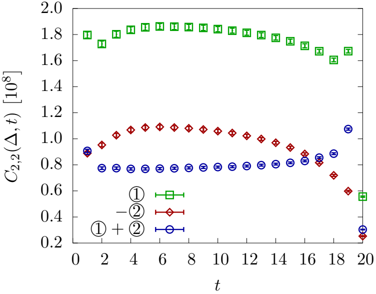


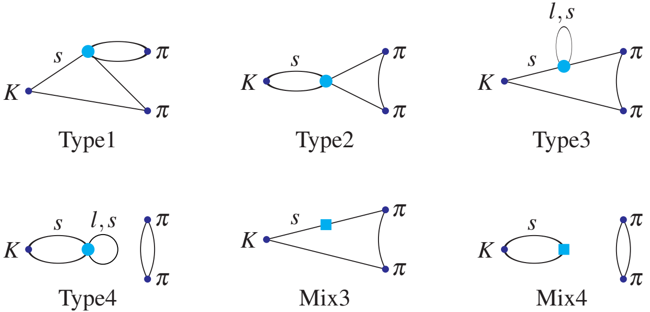
Physical Kinematics



$m_\pi \simeq 330$ MeV at threshold.

- Notation ⓘ $\equiv C_i,\ i = 1, 2$.
- Of course before claiming a quantitative understanding of the $\Delta I = 1/2$ rule we needed to compute Re A_0 at physical kinematics and reproduce the experimental value of 22.5.
- Much early phenomenology was based on the vacuum insertion approach. although the qualitative picture we find had been suggested by Bardeen, Buras and Gerard in 1987.

- The calculation is much more difficult for the $K \to (\pi\pi)_{I=0}$ amplitude A_0:
  - The presence of disconnected diagrams, vacuum subtraction, ultra-violet power divergences, $\cdots$



  - $|\pi^+(\pi/L)\pi^-(\text{-}\pi/L)\rangle$ has a different energy from $|\pi^0(\vec{0})\pi^0(\vec{0})\rangle$.

    CTS & G.Villadoro, hep-lat/0411033

  - We have developed the implementation of G-parity boundary conditions in which $(u,d) \to (\bar{d}, -\bar{u})$ at the boundary .

    U. Wiese, Nucl.Phys. B375 (1992) 45 , RBC-UKQCD, C.h.Kim hep-lat/0311003

- RBC-UKQCD have computed A_0 with the two pions at rest and with unphysical masses, finding e.g. arXiv:1106.2714, Qi Liu Columbia Un.Thesis

$$\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} = 9.1 \pm 2.1 \qquad 877\,\mathrm{MeV\ kaon\ decaying\ into\ two\ }422\,\mathrm{MeV\ pions}$$
$$\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} = 12.0 \pm 1.7 \qquad 662\,\mathrm{MeV\ kaon\ decaying\ into\ two\ }329\,\mathrm{MeV\ pions}$$

- Whilst both these results are obtained at unphysical kinematics and are different from the physical value of 22.5, it is nevertheless interesting to understand the origin of these enhancements.
- 99% of the contribution to the real part of A_0 and A_2 come from the matrix elements of the current-current operators.
- For a calculation of ϵ'/ϵ at physical kinematics, RBC-UKQCD are developing G-parity boundary conditions (estimate timescale $\sim$ 2 years).

- Computations were performed on a $32^3 \times 64$ lattice with the Iwasaki and DSDR gauge action and $N_f = 2+1$ flavours of Möbius Domain Wall Fermions)

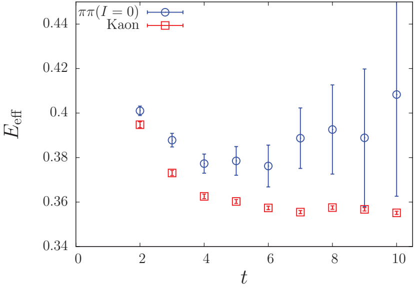
$$a^{-1} = 1.379(7)\,\mathrm{GeV},\, m_\pi = 143.2(2.0)\,\mathrm{MeV},\, (E_\pi = 274.8(1.4)\,\mathrm{MeV})$$

- The $\pi\pi$ energies are

$$E_{\pi\pi}^{I=0} = (498 \pm 11)\,\mathrm{MeV} \quad E_{\pi\pi}^{I=2} = (565.7 \pm 1.0)\,\mathrm{MeV}$$

  to be compared with $m_K = (490.6 \pm 2.4)$ MeV.

- Lüscher's quantisation condition $\Rightarrow E_{\pi\pi}^{I=0}$ corresponds to $\delta_0 = (23.8 \pm 4.9 \pm 1.2)^\circ$, which is somewhat smaller than phenomenological expectations.



$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu)\right] Q_i(\mu). \qquad \left(\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}\right)$$

Wilson coefficients from Buchalla, Buras, Lautenbacher, hep-ph/9512380

i	Re(A_0)(GeV)	Im(A_0)(GeV)
1	$1.02(0.20)(0.07) \times 10^{-7}$	0
2	$3.60(0.90)(0.28) \times 10^{-7}$	0
3	$-1.28(1.69)(1.20) \times 10^{-10}$	$1.53(2.03)(1.44) \times 10^{-12}$
4	$-2.01(0.69)(0.36) \times 10^{-9}$	$1.80(0.61)(0.32) \times 10^{-11}$
5	$-8.93(2.23)(1.84) \times 10^{-10}$	$1.54(0.38)(0.32) \times 10^{-12}$
6	$3.51(0.89)(0.23) \times 10^{-9}$	$-3.56(0.90)(0.24) \times 10^{-11}$
7	$2.38(0.40)(0.00) \times 10^{-11}$	$8.49(1.44)(0.00) \times 10^{-14}$
8	$-1.28(0.04)(0.00) \times 10^{-10}$	$-1.71(0.05)(0.00) \times 10^{-12}$
9	$-7.38(1.97)(0.48) \times 10^{-12}$	$-2.41(0.64)(0.16) \times 10^{-12}$
10	$7.29(2.62)(0.68) \times 10^{-12}$	$-4.72(1.69)(0.44) \times 10^{-13}$
Total (stat only)	$4.66(0.96)(0.27) \times 10^{-7}$	$-1.90(1.19)(0.32) \times 10^{-11}$
Final (incl. syst)	$4.66(1.00)(1.21) \times 10^{-7}$	$-1.90(1.23)(1.04) \times 10^{-11}$

- Representative Errors

Description	Error	Description	Error
Finite lattice spacing	8%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			26%

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- As a results of our work, the computation of A_2 is now “standard".
- It appears that the explanation of the $\Delta I = 1/2$ rule has a number of components, of which the significant cancelation between the two dominant contributions to ReA_2 is a major one.
- We have completed the first calculation of ϵ'/ϵ with controlled errors $\Rightarrow$ motivation for further refinement (systematic improvement by collecting more statistics, working on larger volumes, $\geq$2 lattice spacings etc.)
- ϵ'/ϵ is now a quantity which is amenable to lattice computations.
- So far we are not able to apply these techniques to B-decays in a practicable way.
  - The bottleneck is our inability to isolate the required M_1M_2 state, whose contribution to the correlation functions is highly suppressed.
  - We need smart new ideas.

3. Isospin Breaking

- Until recently all lattice simulations were performed in the isospin limit with $m_u = m_d$.
  - This is gradually being overcome, with several studies of isospin breaking in the spectrum.
  - A highlight has been the *Ab initio calculation of the neutron-proton mass difference* by the BMW collaboration. S.Borsanyi et al., arXiv:1406.4088
- An important simplification in the study of the spectrum with electromagnetic corrections is the absence of infrared divergences.
- In order to include electromagnetic effects in hadronic processes on the other hand, it is necessary to be able to deal with the presence of infrared divergences.
  - This is necessary e.g. to determine CKM matrix elements at $O(1\%)$ or better and other tests of the standard model at this level of precision.
- Our recent paper *QED Corrections to Hadronic Processes in Lattice QCD* is the first (and only) one to propose a method for how this might be done.

N.Carrasco, V.Lubicz, G.Martinelli, CTS, N.Tantalo, C.Tarantino & M.Testa, arXiv:1502.00257.

- For illustration, I consider consider leptonic decays of the pion but the discussion is general; I do not use ChPT.
  - The discussion applies to the leptonic and semileptonic decays of all pseudoscalar mesons and can be readily adapted to other processes.
  - For a ChPT based discussion of f_π, see J.Gasser & G.R.S.Zarnauskas, arXiv:1008.3479)
- At $O(\alpha^0)$

$$\Gamma(\pi^+ \to \ell^+ \nu_\ell) = \frac{G_F^2\,|V_{ud}|^2 f_\pi^2}{8\pi}\, m_\pi\, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2 .$$

  - All the hadronic effects are contained in the leptonic decay constant f_π.

$$\langle 0\,|\,\bar{d}\gamma^\mu\gamma^5\,u\,|\,\pi^+(p)\rangle = ip^\mu f_\pi\,.$$

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- At $O(\alpha)$ infrared divergences are present and we have to consider

$$\begin{aligned}\Gamma(\pi^+ \to \ell^+\nu_\ell(\gamma)) &= \Gamma(\pi^+ \to \ell^+\nu_\ell) + \Gamma(\pi^+ \to \ell^+\nu_\ell\gamma) \\ &\equiv \Gamma_0 + \Gamma_1\,,\end{aligned}$$

where the suffix denotes the number of photons in the final state.

- Each of the two terms on the rhs is infrared divergent, the divergences cancel in the sum.
- The cancelation of infrared divergences between contributions with virtual and real photons is an old and well understood issue.

F.Bloch and A.Nordsieck, PR 52 (1937) 54

- The question for our community is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.
- This is a generic problem if em corrections are to be included in the evaluation of a decay process.

- In principle, particularly as techniques and resources improve in the future, it may be better to compute Γ_1 nonperturbatively over a larger range of photon energies.
- At present we do not propose to compute Γ_1 nonperturbatively. Rather we consider only photons which are sufficiently soft for the point-like (pt) approximation to be valid.
  - A cut-off ΔE of $O(10\,\text{-}\,20\,\mathrm{MeV})$ appears to be appropriate both experimentally and theoretically.

    KLOE collaboration, hep-ex/0509045; arXiv:0907.3594, NA62
  - Question: What is the likely photon energy resolution at LHCb and Belle II in the rest frame of the decaying mesons?
- We now write

$$\boxed{\Gamma_0+\Gamma_1(\Delta E)=\lim_{V\to\infty}(\Gamma_0-\Gamma_0^{\mathrm{pt}})+\lim_{V\to\infty}(\Gamma_0^{\mathrm{pt}}+\Gamma_1(\Delta E))\,.}$$

  - The second term on the rhs can be calculated in perturbation theory. It is infrared convergent, but does contain a term proportional to $\log\Delta E$.
  - The first term is also free of infrared divergences.
  - Γ_0 is calculated nonperturbatively and Γ_0^{pt} in perturbation theory.

$$\boxed{\Gamma_0+\Gamma_1(\Delta E)=\lim_{V\to\infty}(\Gamma_0-\Gamma_0^{\rm pt})+\lim_{V\to\infty}(\Gamma_0^{\rm pt}+\Gamma_1(\Delta E))\,.}$$

1. Introduction ✓
2. What is G_F at $O(\alpha)$? ✗
3. Proposed calculation of $\Gamma_0-\Gamma_0^{\rm pt}$ ✗
4. Calculation of $\Gamma_0^{\rm pt}+\Gamma_1(\Delta E)$ ✗
5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ ✓
6. Summary and Conclusions ✓

Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$

- For sufficiently small ΔE the structure dependent contributions to Γ_1 can be neglected.
- How big might they be for experimentally accessible values of ΔE?
  To estimate this for f_π and f_K we use Chiral Perturbation Theory.

  J.Bijnens, G.Ecker and J.Gasser, hep-ph/9209261,
  J.Bijnens, G.Colangelo, G.Ecker and J.Gasser, hep-ph/9411311.
  V. Cirigliano and I. Rosell, arXiv:0707.3439 [hep-ph]],
  L. Ametller, J. Bijnens, A. Bramon and F. Cornet, hep-ph/9302219.
- We define

$$R_1^A(\Delta E) = \frac{\Gamma_1^A(\Delta E)}{\Gamma_0^{\alpha,\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)}\,, \qquad A = \{\mathrm{SD,INT}\}\,,$$

  where SD and INT refer to the structure dependent and interference (between SD and pt) contributions respectively.
  - Note that the notation I am using here differs from that in the paper.

- Start with a decomposition in terms of Lorenz invariant form factors of the hadronic matrix element

$$H^{\mu\nu}(k,p_\pi) \quad = \quad \int d^4x\, e^{ikx}\, T\langle 0|j^\mu(x)J_W^\nu(0)|\pi(p_\pi)\rangle$$

and separate the contribution corresponding to the approximation of a point-like pion $H_{\rm pt}^{\mu\nu}$, from the structure dependent part $H_{\rm SD}^{\mu\nu}$,

$$H^{\mu\nu} = H_{\rm SD}^{\mu\nu} + H_{\rm pt}^{\mu\nu}\ .$$

- $H_{\rm pt}^{\mu\nu}$ is simply given by

$$H_{\rm pt}^{\mu\nu} = f_\pi\left[g^{\mu\nu} - \frac{(2p_\pi-k)^\mu(p_\pi-k)^\nu}{(p_\pi-k)^2-m_\pi^2}\right]\ .$$

- The structure dependent component can be parametrised by four independent invariant form factors which we define as

$$H_{\rm SD}^{\mu\nu} \quad = \quad H_1\left[k^2g^{\mu\nu}-k^\mu k^\nu\right] + H_2\left\{\left[(k\cdot p_\pi-k^2)k^\mu - k^2(p_\pi-k)^\mu\right](p_\pi-k)^\nu\right\}$$

$$-i\frac{F_V}{m_\pi}\,\epsilon^{\mu\nu\alpha\beta}k_\alpha p_{\pi\,\beta} + \frac{F_A}{m_\pi}\left[(k\cdot p_\pi-k^2)g^{\mu\nu}-(p_\pi-k)^\mu k^\nu\right]\ .$$

Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ (cont)

- For the decay into a real photon, only F_V and F_A contribute.

- At $O(p^4)$ in chiral perturbation theory,

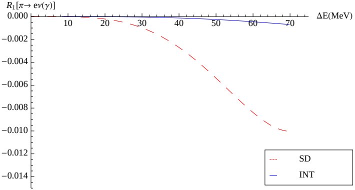
$$F_V = \frac{m_P}{4\pi^2 f_\pi} \quad \text{and} \quad F_A = \frac{8m_P}{f_\pi}\left(L_9^r + L_{10}^r\right),$$

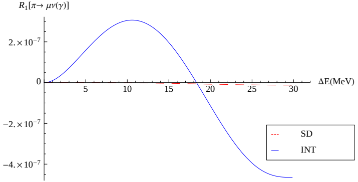
  where $P = \pi$ or K and L_9^r, L_{10}^r are Gasser-Leutwyler coefficients.

- The numerical values of these constants have been taken from the review by M.Bychkov and G.D'Ambrosio in the PDG. F_V and F_A are 0.0254 and 0.0119 for the pion and 0.096 and 0.042 for the Kaon (for the pion these values of the form factors, obtained from direct measurements, can be found in the supplement to the PDG.)

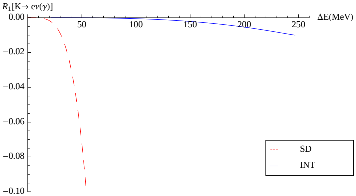
Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ (cont)

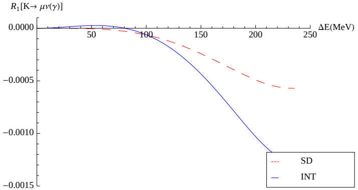
Pion





Kaon





Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ (cont)

- For heavy-light mesons we don't have such ChPT calculations.
- For the B-meson in particular we have another small scale $< \Lambda_{\rm QCD}$, $m_{B^*} - m_B \simeq 45$ MeV so that we may expect that we will have to go to smaller ΔE in order to be able to neglect SD effects.
- Calculations based on the extreme approximation of single pole dominance suggest that this is likely to be the case.

  D. Becirevic, B. Haas and E. Kou, arXiv:0907.1845 [hep-ph]
- To be investigated further!

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- Lattice calculations of some physical quantities are approaching $O(1\%)$ precision $\Rightarrow$ we need to include isospin-breaking effects, including electromagnetic effects, to make the tests of the SM even more stringent.
- For decay widths and scattering cross sections including em effects introduces infrared divergences.
- We propose a method for dealing with these divergences, illustrating the procedure by a detailed study of the leptonic (and semileptonic) decays of pseudoscalar mesons.
- Although challenging, the method is within reach of present simulations and we are now implementing the procedure in an actual numerical computation.
  - Power-like FV corrections, $O(1/(L\Lambda_{\mathrm{QCD}})^n)$, to be evaluated.
  - $O(\alpha\alpha_s)$ matching factors to be studied.

- We need guidance from LHCb and Belle-II experimenters on the treatment of soft photons.
- One can certainly envisage relaxing the condition $\Delta E \ll \Lambda_{\rm QCD}$, including the emission of real photons with energies which do resolve the structure of the initial hadron. Such calculations can be performed in Euclidean space under the same conditions as above, i.e. providing that there is a mass gap.
  - In that case we generalise the master formula to

$$\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\rm pt}) + \lim_{V\to\infty}(\Gamma_1(\Delta E) - \Gamma_1^{\rm pt}(\Delta E)) + \lim_{V\to\infty}(\Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}(\Delta E)).$$

  - The important point is to organise the calculation into terms, each of which is infrared convergent.
  - At present we are exploring how best to calculate

$$\lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\rm pt})\,.$$