$B \to \pi\pi$ form factors from QCD Light-Cone Sum Rules

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Motivation

- importance of the $B \to 2\pi$ form factors: (see also Danny van Dyk's talk)
  - used in $B \to \pi\pi\ell\nu_\ell$ for exclusive $|V_{ub}|$ determination
  - enter the rich set of observables in $B \to \pi\pi\ell\nu_\ell$,

    [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, (2013)]
  - factorizable parts of $B \to 3\pi$ nonleptonic amplitudes

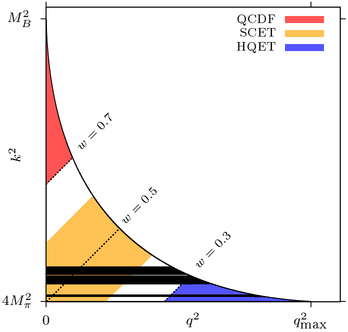
- can we apply Light-Cone Sum Rules in QCD to $B \to 2\pi$ FFs?

- Dalitz plot:

  dipion with a small invariant mass

  and large recoil:

  $k^2 \lesssim 1\ \text{GeV}^2$, $0 \leq q^2 \leq 12\text{-}14\ \text{GeV}^2$.



An exploratory study

[Ch. Hambrock, AK, 1511.02509 [hep-ph]]

- the method: similar to the LCSRs for $B \to \pi$ form factors,
- we consider only $\bar{B}^0 \to \pi^+\pi^0\ell^-\nu_\ell$, isospin 1, $L = 1, 3, , ..$
- only LO, twist-2 approximation for dipion DAs available
- nonperturbative input: dipion distribution amplitudes (DAs)
- challenges revealed...
- problems to be addressed:
  - how important are $L > 1$ partial waves of 2π state in $B \to \pi\pi$?
  - $B \to \rho$ dominance in the P-wave?

The method of LCSRs

- The correlation function: $k = k_1 + k_2, \bar{k} = k_1 - k_2$

$$\Pi_\mu(q, k_1, k_2) =$$
$$= i \int d^4x\, e^{iqx} \langle \pi^+(k_1)\pi^0(k_2)| T\{\bar{u}(x)\gamma_\mu(1-\gamma_5) b(x), im_b \bar{b}(0)\gamma_5 d(0)\}|0\rangle$$
$$= i\epsilon_{\mu\alpha\beta\rho} q^\alpha k_1^\beta k_2^\rho\, \Pi^{(V)} + q_\mu \Pi^{(A,q)} + k_\mu \Pi^{(A,k)} + \bar{k}_\mu \Pi^{(A,\bar{k})} ,$$

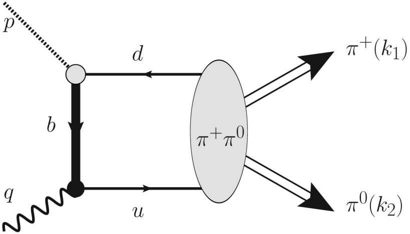
- the invariant amplitudes $\Pi^{(V),(A,q),..)}(p^2, q^2, k^2, q\cdot\bar{k})$, $p = (k+q)$

- OPE valid at $q^2 \ll m_b^2$ (b-quark virtual)
  $k^2 \ll m_b^2$ (2-pion system produced near the LC)

- LO diagram:
  $\langle b(x)\bar{b}(0)\rangle \rightarrow S_b(x, 0)$

- vacuum → on-shell dipion hadronic matrix elements of nonlocal $\bar{u}(x)d(0)$ operators

- with ρ-meson "embedded"



Dipion light-cone DAs

- introduced and developed for $\gamma^*\gamma \to 2\pi$ processes

  [M. Diehl, T. Gousset, B. Pire and O. Teryaev, (1998)
  D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes and J. Horejsi, (1994)
  M. V. Polyakov, (1999)]

- twist-2 DAs:

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int_0^1 du\, e^{iu(k\cdot x)}\Phi_{\parallel}^{I=1}(u,\zeta,k^2)\,,$$

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = 2\sqrt{2}i\frac{k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}}{2\zeta-1}\int_0^1 du\, e^{iu(k\cdot x)}\Phi_{\perp}^{I=1}(u,\zeta,k^2)$$

- the "angular" variable: $\zeta = k_1^+/k^+,\;\; 1-\zeta = k_2^+/k^+\,,\;\; \zeta(1-\zeta) \geq \frac{m_\pi^2}{k^2}\,.$

$q\cdot\bar{k} = \frac{1}{2}(2\zeta-1)\lambda^{1/2}(p^2,q^2,k^2)\,,$ in dipion c.m. $(2\zeta-1) = (1-4m_\pi^2/k^2)^{1/2}cos\theta_\pi\,,$

- normalization conditions → pion timelike form factors ,

$$\int_0^1 du \left\{ \begin{array}{l} \Phi_{\parallel}^{I=1}(u,\zeta,k^2) \\ \Phi_{\perp}^{I=1}(u,\zeta,k^2) \end{array} \right. = (2\zeta-1)\left\{ \begin{array}{ll} F_\pi^{em}(k^2) & \text{pion e.m. form factor} \\ F_\pi^t(k^2) & \text{pion “tensor” form factor} \end{array} \right.$$

- $\Phi_{\perp,\parallel}(u,\zeta,k^2)$ at $k^2 > 4m_\pi^2$ contain Im part
- $F_\pi^{em}(0) = 1$,
- "tensor" charge of the pion $F_\pi^t(0) = 1/f_{2\pi}^{\perp}$

Result for the correlation function in twist-2 approx.

- at LO, twist-2 accuracy:

$$\Pi_\mu(q,k_1,k_2) = i\sqrt{2}m_b \int\limits_0^1 \frac{du}{(q+uk)^2-m_b^2}\Big\{\Big[(q\cdot\bar{k})k_\mu - \Big((q\cdot k)+uk^2\Big)\bar{k}_\mu + i\epsilon_{\mu\alpha\beta\rho}q^\alpha k_1^\beta k_2^\rho\Big]\frac{\Phi_\perp(u,\zeta,k^2)}{2\zeta-1} - m_b k_\mu \Phi_\parallel(u,\zeta,k^2)\Big\}\,.$$

- read off invariant amplitudes: $\Pi^{(V)}$, $\Pi^{(A,k)}$, $\Pi^{(A,\bar{k})}$, $\Pi^{(A,q)}=0$
- transform to a form of dispersion integral in the variable p^2:
  $s(u) = \frac{m_b^2 - q^2\bar{u} + k^2 u\bar{u}}{u}$

$$\Pi^{(r)}(p^2,q^2,k^2,\zeta) = \sum_{i=\parallel,\perp} f_i^{(r)}(p^2,q^2,k^2,\xi)\int\limits_{m_b^2}^{\infty}\frac{ds}{s-p^2}\left(\frac{du}{ds}\right)\Phi_i(u(s),\zeta,k^2)\,.$$

- a problem: nonanalyticity of the Källen function:

$$q\cdot\bar{k} = (1/2)(2\zeta-1)\lambda^{1/2}(p^2,q^2,k^2) \quad \rightarrow \text{kinematical singularity}$$

$$\lambda^{1/2}(p^2,q^2,k^2) = (p^2-(\sqrt{q^2}-\sqrt{k^2})^2)^{1/2}(p^2-(\sqrt{q^2}+\sqrt{k^2})^2)^{1/2}.$$

☹ no consistent sum rule for $\Pi^{(A,k)}$

Hadronic dispersion relation

- the ground *B*-meson state contribution:

$$\Pi_\mu(q,k_1,k_2) = \frac{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle f_B m_B^2}{m_B^2 - p^2} + \dots ,$$

- expansion of $B \to \pi\pi$ matrix element in form factors:

$$\begin{aligned} i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle = & -F_\perp(q^2,k^2,\zeta)\,\frac{4}{\sqrt{k^2\lambda_B}}\, i\epsilon^{\mu\alpha\beta\gamma}\, q_\alpha\, k_{1\beta}\, k_{2\gamma} \\ & + F_t(q^2,k^2,\zeta)\,\frac{q^\mu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k^\mu - \frac{k\cdot q}{q^2}q^\mu\right) \\ & + F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}^\mu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k^\mu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q^\mu\right), \end{aligned}$$

- quark-hadron duality in the *B*-channel, ⇒ effective threshold s_0, Borel transformation , $p^2 \to M^2$

Final results: LCSRs for the form factors at twst-2 LO

- in both sum rules only the chiral-odd twist-2 DA contributes:

$$\frac{F_\perp(q^2,k^2,\zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2}f_B m_B^2(1-2\zeta)} \int\limits_{u_0(s_0)}^{1} \frac{du}{u} \Phi_\perp(u,\zeta,k^2)\, e^{\frac{m_B^2}{M^2} - \frac{m_b^2-q^2\bar{u}+k^2u\bar{u}}{uM^2}} ,$$

$$\frac{F_\parallel(q^2,k^2,\zeta)}{\sqrt{k^2}} = \frac{m_b}{\sqrt{2}f_B m_B^2(1-2\zeta)} \int\limits_{u_0(s_0)}^{1} \frac{du}{u^2} \left(m_b^2 - q^2 + k^2u^2\right) \Phi_\perp(u,\zeta,k^2)\, e^{\frac{m_B^2}{M^2} - \frac{m_b^2-q^2\bar{u}+k^2}{uM^2}}$$

- an additional relation between the axial-current form factors:

$$\frac{1}{\sqrt{\lambda_B}}(m_B^2-q^2-k^2)F_0(q^2,k^2,\zeta) = F_t(q^2,k^2,\zeta)+2\frac{\sqrt{k^2}\sqrt{q^2}(2\zeta-1)}{\sqrt{\lambda_B}}F_\parallel(q^2,k^2,\zeta)\Big] .$$

- can we obtain a sum rule also for F_t ?

What do we know about LCDAs

[M. V. Polyakov, Nucl. Phys. B 555 (1999) 231.]

- double expansion in Legendre and Gegenbauer polynomials:

$$\Phi_\perp(u,\zeta,k^2) = \frac{6u(1-u)}{f_{2\pi}^\perp}\sum_{n=0,2,..}^{\infty}\;\sum_{\ell=1,3,..}^{n+1} B_{n\ell}^\perp(k^2)\,C_n^{3/2}(2u-1)\,\beta_\pi P_\ell^{(0)}\left(\frac{2\zeta-1}{\beta_\pi}\right),$$

- Gegenbauer moments, $B_{n\ell}^\perp(k^2)$ - complex functions at $k^2 > 4m_\pi^2$

- instanton vacuum model for the coefficients,
  $n=0,2,4$, valid at small $k^2 \sim 4m_\pi^2$ *[M. V. Polyakov and C. Weiss, (1999)]*

$$B_{01}^\perp(k^2) = 1+\frac{k^2}{12M_0^2}\,,\; B_{21}^\perp(k^2) = \frac{7}{36}\left(1-\frac{k^2}{30M_0^2}\right)\,,\;\; B_{23}^\perp(k^2) = \frac{7}{36}\left(1+\frac{k^2}{30M_0^2}\right)\,,$$

$$B_{41}^\perp(k^2) = \frac{11}{225}\left(1-\frac{5k^2}{168M_0^2}\right)\,,\; B_{43}^\perp(k^2) = \frac{77}{675}\left(1-\frac{k^2}{630M_0^2}\right)\,,\; B_{45}^\perp(k^2) = \frac{11}{135}\left(1+\frac{k^2}{56M_0^2}\right)\,.$$

$f_{2\pi}^\perp = 4\pi^2 f_\pi^2/3M_0 \simeq 650$ MeV, where $f_\pi = 132$ MeV is the pion decay constant.

- we confined ourselves by $k^2 \sim k^2_{min} \simeq 4m_\pi^2$ for an exploratory numerical analysis

Sum rules for partial waves

- The form factors expanded in partial waves:

$$F_{\perp,\parallel}(q^2,k^2,\zeta) = \sum_{\ell=1}^{\infty} \sqrt{2\ell+1}\, F_{\perp,\parallel}^{(\ell)}(q^2,k^2)\, \frac{P_\ell^{(1)}(\cos\theta_\pi)}{\sin\theta_\pi}\,,$$

$\zeta \sim \cos\theta$, $P_l^{(m)}$ -the (associated) Legendre polynomials

- sum rules for separate partial waves

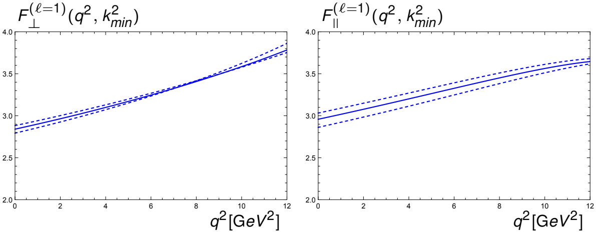
$$F_\perp^{(\ell)}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2} f_{2\pi}^\perp} \frac{\sqrt{\lambda_B}\, m_b}{m_B^2 f_B} e^{m_B^2/M^2} \sum_{n=0,2,..} \sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\, B_{n\ell'}^\perp(k^2) J_n^\perp(q^2,k^2,M^2,s_0^B)\,,$$

$$F_\parallel^{(\ell)}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2} f_{2\pi}^\perp} \frac{m_b^3}{m_B^2 f_B} e^{m_B^2/M^2} \sum_{n=0,2,4,..} \sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\, B_{n\ell'}^\perp(k^2) J_n^\parallel(q^2,k^2,M^2,s_0^B)\,,$$

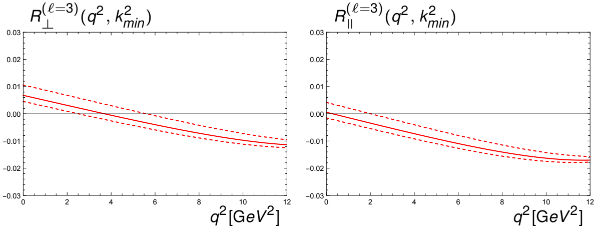
  - $I_{\ell\ell'}$ - integrals over Legendre polynomials,
  - $J_n^{\perp,\parallel}$ - the Borel-weighted integrals over $C_n^{3/2}(2u-1)$
  - in the limit of asymptotic DA, ($B_{01} \neq 0$, $B_{n>0,\ell} = 0$),
    only P-wave form factors are $\neq 0$

Numerical results

- *P*-wave form factors: (only twist-2)



- *P*-wave dominance: ratios of *F*- and *P*-wave form factors



- - - - uncertainties from the variation of M^2.

How much $B \to \rho$ contributes to the $B \to 2\pi$?

- dispersion relations for the $B \to \pi\pi$ *P*-wave ($\ell = 1$) form factors:

$$\frac{\sqrt{3}F_\perp^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - i m_\rho \Gamma_\rho(k^2)} \frac{V^{B\to\rho}(q^2)}{m_B + m_\rho} + ...$$

and

$$\frac{\sqrt{3}F_\parallel^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - i m_\rho \Gamma_\rho(k^2)} (m_B + m_\rho) A_1^{B\to\rho}(q^2) + ...$$

$$\Gamma_\rho(k^2) = \frac{m_\rho^2}{k^2}\left(\frac{k^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2}\right)^{3/2} \theta(k^2 - 4m_\pi^2)\Gamma_\rho^{tot},$$

- using the definition of $B \to \rho$ FFs:

$$\langle \rho^+(k)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle = \epsilon_{\mu\alpha\beta\gamma}\epsilon_\alpha^{*(\rho)} p^\beta k^\gamma \frac{2V^{B\to\rho}(q^2)}{m_B + m_\rho}$$
$$-i\epsilon_\mu^{*(\rho)}(m_B + m_\rho)A_1^{B\to\rho}(q^2) + ...$$

LCSRs for $B \to \rho$ form factors

e.g., [P. Ball and V. M. Braun, Phys. Rev. D ***55*** *(1997) 5561]*

- LCSRs for $B \to \rho$ form factors ($\Gamma_\rho = 0$) in terms of the ρ-meson DAs in the twist-2 approximation:

$$V^{B\to\rho}(q^2) = \frac{(m_B + m_\rho)m_b}{2m_B^2 f_B} f_\rho^\perp e^{\frac{m_B^2}{M^2}} \int\limits_{u_0}^{1} \frac{du}{u}\, \phi_\perp^{(\rho)}(u)\, e^{-\frac{m_b^2 - q^2\bar{u} + m_\rho^2 u\bar{u}}{uM^2}} \,,$$

$$A_1^{B\to\rho}(q^2) = \frac{m_b^3}{2(m_B + m_\rho)m_B^2 f_B} f_\rho^\perp e^{\frac{m_B^2}{M^2}} \int\limits_{u_0}^{1} \frac{du}{u^2}\, \phi_\perp^{(\rho)}(u)\left(1 - \frac{q^2 - m_\rho^2 u^2}{m_b^2}\right) e^{-\frac{m_b^2 - q^2\bar{u} + m_\rho^2 u\bar{u}}{uM^2}} \,.$$

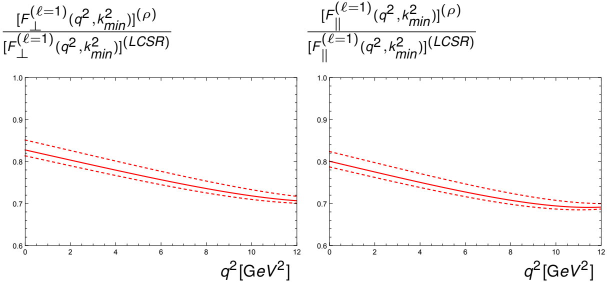
- both sum rules determined by the chiral-odd ρ-meson DA:

$$\langle \rho^+(k)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = -if_\rho^\perp\left(\epsilon_\mu^{*(\rho)} k_\nu - k_\mu \epsilon_\nu^{*(\rho)}\right) \int\limits_0^1 du e^{iuk\cdot x} \phi_\perp^{(\rho)}(u)\,,$$

- the Gegenbauer polynomial expansion:

$$\phi_\perp^{(\rho)}(u) = 6u(1-u)\left(1 + \sum_{n=2,4,,..} a_n^{(\rho)\perp} C_n^{3/2}(2u-1)\right)\,,$$

Numerical estimates



Relative contribution of ρ-meson to the $B \longrightarrow \pi^+\pi^0$ P-wave form factors

$F_{\perp}^{(\ell=1)}(q^2, k^2_{min})$ *(left panel) and* $F_{||}^{(\ell=1)}(q^2, k^2_{min})$ *(right panel) from LCSRs.*

Dashed lines - the uncertainty due to the variation of the Borel parameter.

Is ρ -contribution to $B \to \pi\pi$ “inclusive"?

- as recently argued: *[A.Bharucha, D.Straub and R.Zwicky,1503.05534]*
  “ the ρ -state effectively includes the non-resonant background in the P-wave dipion state in the experimental as well as the LCSR prediction for $B \to \rho$ ...”
- in reality the experimentalists always fit their results on $e^+e^- \to \pi^+\pi^-$ or on $\tau \to \pi^-\pi^0\nu_\tau$ to a combination of ρ and $\rho'(1450)$ etc. resonances to fit f_ρ
- e.g., dispersion relation for the vector FF looks like :

$$\frac{\sqrt{3}F_\perp^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}\frac{V^{B\to\rho}(q^2)}{m_B + m_\rho} + \frac{g_{\rho'\pi\pi}}{m_{\rho'}^2 - k^2 - im_{\rho'}\Gamma_{\rho'}(k^2)}\frac{V^{B\to\rho'}(q^2)}{m_B + m_{\rho'}} + ...$$

The missing sum rule for the form factor F_t

PRELIMINARY !

[AK, work in progress]

- using a different correlation function:

$$i\int d^4x\, e^{iqx}\langle\pi^+(k_1)\pi^0(k_2)|T\{\bar{u}(x)im_b\gamma_5 b(x), im_b\bar{b}(0)\gamma_5 d(0)\}|0\rangle\,.$$
$$=\Pi^{(5)}(p^2,q^2,k^2,q\cdot\bar{k})$$

- $B\to\pi\pi$ matrix element of the pseudoscalar current relating to the divergence of the axial-vector current:

$$\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}(x)im_b\gamma_5 b(x)|\overline{B}^0\rangle = F_t(q^2,k^2,\zeta)\sqrt{q^2}$$

- following the same method,

LCSR for F_t

PRELIMINARY !

- twist-2 accuracy, only the chiral-even DA enters:

$$\frac{F_t(q^2,k^2,\zeta)}{\sqrt{q^2}} = \frac{m_b^2}{\sqrt{2} f_B m_B^2} \int\limits_{u_0}^{1} \frac{du}{u^2} \left(m_b^2 - q^2 + k^2 u^2 \right) \Phi_{\parallel}(u,\zeta,k^2)\, e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2 \bar{u} + k^2 u \bar{u}}{u M^2}} ,$$

- main input $B_{01}(k^2) = F_\pi(k^2)$
- related to the $B \to \rho$ form factor $A_0(q^2)$
- numerical analysis, comparison with LCSR for $B \to \rho$ [in progress]

Further development and applications

- ansatz for Gegenbauer functions $B_{nl}^{\perp,\parallel}(k^2)$ at $k^2 \lesssim 1$ GeV^2
  from LCSRs for pion FFs at $k^2 < 0 \oplus$ dispersion representations at $k^2 > 4m_\pi^2$

- including twist-3,4 and $\bar{q}qG$ components of OPE,
  identifying twist-3,4 DAs and their double expansions,
  the methods used for vector meson DAs [V.Braun et al.]

- NLO gluon radiative corrections

- $B \to \pi^+\pi^-, \pi^0\pi^0$ channels, including dipions in S, D, ..-waves ($\ell = 0, 2, ..$), scalar f_0 and tensor f_2 dominance?,...
  need more inputs for corresponding DAs

- LCSR for S-wave $K\pi$ state in $B \to K\pi$
  [U. G. Mei§ner, W. Wang, (2014)]
  no ζ dependence, asymptotic DA, timelike form factor from ChPT

- $B \to K\pi(K^*)$ form factors,
  SU(3)-violating asymmetry of Gegenbauer moments

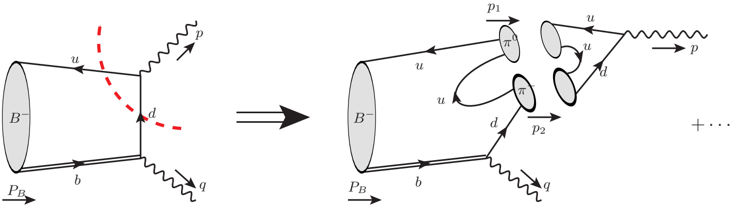
Alternative method to access $B \to \pi\pi$ FFs

[S.Cheng, AK, J.Virto, work in progress]

- LCSRs with B-meson DA and $\bar{u}\gamma_\mu d$ interpolating current
- the method introduced to calculate $B \to P, V$ form factors,

  [A.K., N. Offen, Th. Mannel (2006)] , *"SCET sum rules", [F. De Fazio, Th. Feldmann, T.Hurth (2006)]*

  NLO corrections to B $\to \pi$,[Y-M. Wang, Y-L.Shen (2015)



- insert a dispersion.relation for $B \to 2\pi$ form factors
  and a (dispersion rel. $\oplus$ experiment) parametrization for F_π
- not a direct calculation, given the shape of the $B \to 2\pi$ form factors, these sum rules can provide normalization

Conclusions

- $B \to PP$ ($P = \pi, K$) form factors are calculable from LCSRs with dipion DAs at small dipion mass and large recoil
- first exploratory study: all $B \to \pi^+\pi^0$ form factors at LO, twist-2
- provide quantitative estimates for P-wave dominance, ρ-meson dominance in P-wave, etc.
- more information /dedicated studies on dipion DAs needed
- LCSR can provide "building blocks" for nonleptonic $B \to 3P$
- will help to build viable models for dimeson spectra measured in $B \to \pi\pi\ell\nu_\ell$ and $B \to K\pi\ell\ell$