$B \to \pi\pi$ form factors

Overview and results at large dipion masses

Danny van Dyk

based on Phys. Rev. D89 (2014) 1,014015 (with Faller,Feldmann,Khodjamirian,Mannel)

and QFET-2016-02 (with Böer, Feldmann)

Universität Zürich

Future Challenges in Non-Leptonic B Decays
Bad Honnef, 11.02.2016

Overview

Introduction to $B \to \pi\pi$ form factors

– $B \to \pi\pi$ form factors (FFs) parametrize hadronic matrix elements

$$\langle \pi^+(k_1)\pi^-(k_2)|\,\overline{\psi}_u\;\Gamma\;\psi_b|B^-(p)\rangle$$

  – relevant for semileptonic decays $B \to \pi\pi\ell\overline{\nu}$
    and nonleptonic three-body decays (e.g. $B \to \pi\pi\pi$) [previous talk by Th. Mannel]

– SM: one vector FF, three axialvector FFs:

$$\langle \pi^+(k_1)\pi^-(k_2)|\,\overline{\psi}_u\;\gamma^\mu(1-\gamma_5)\;\psi_b|B^-(p)\rangle$$
$$= F_V\varepsilon^{\mu p k_1 k_2} + F_{A_1}k_1^\mu + F_{A_2}k_2^\mu + F_{A_3}p^\mu$$

  – dependend on three Lorentz invariants:

$$F_x \equiv F_x(q^2, k^2, (k_1-k_2)\cdot q)$$

where $q \equiv p - k_1 - k_2$ [compare Lee, Ming and Wise PRD 46 (5040-5048) 1992]

Partial wave decomposition

q^2 invariant mass of the lepton pair

$k^2 \equiv (k_1 + k_2)^2$ invariant mass two-pion system

$\cos\theta_\pi$ helicity angle of the positively charged pion in the two-pion rest frame

$$q \cdot (k_1 - k_2) = \frac{\beta_\pi}{2}\sqrt{\lambda}\cos\theta_\pi$$

where $\beta_\pi^2 \equiv 1 - 4M_\pi^2/k^2$, and $\lambda \equiv \lambda(M_B^2, q^2, k^2)$

– decompose form factors in partial waves:

$$F_x(q^2, k^2, q \cdot (k_1 - k_2)) = \sum_{\ell}^{\infty} F_x^{(\ell)}(q^2, k^2) P_\ell(\cos\theta_\pi)$$

– ℓ: angular momentum of two-pion system
– partial-wave form factors $F_x^{(\ell)}$ now only depend on two variables: q^2 and k^2

Helicity form factors

– project onto polarization associated with $b \to u$ current:

$$\varepsilon(q,\sigma)^*_\mu \, \langle \pi\pi | \, \overline{u} \, \gamma^\mu (1-\gamma_5) b \, |\overline{B}\rangle \; \sim \; F_\sigma$$

for each polarization $\sigma = t, 0, \pm 1$ [Bharucha, Feldmann, Wick 1004.3249]

- straight-forward for $\sigma = 0$ or $\sigma = t$

$$\varepsilon(q,0(t))^*_\mu \, \langle \pi\pi | \, \overline{u} \, \gamma^\mu (1-\gamma_5) b \, |\overline{B}\rangle \; = \; F_{0(t)}$$

- for $\sigma = \pm 1$ rather use

$$\varepsilon(q,\pm 1)^*_\mu \, \langle \pi\pi | \, \overline{u} \, \gamma^\mu (1-\gamma_5) b \, |\overline{B}\rangle \; = \; (F_\perp \pm F_\parallel) \frac{\beta_\pi}{\sqrt{2}} \sin\theta_\pi e^{\pm i\phi}$$

where ϕ: angle between decay planes

Final parametrization

[Faller, Feldmann, Khodjamirian, Mannel, DvD 1310.6660]

- apply partial wave decomposition to helicity FFs
- decompose longitudinal and time-like FFs in $P_\ell^{(m=0)}(\cos\theta_\pi)$
  - F_0 and F_t start with S-wave $\pi\pi$ systems

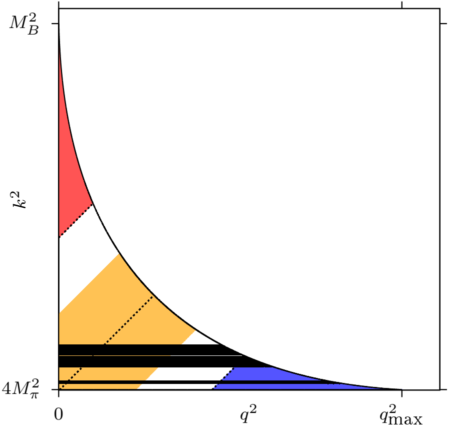
$$F_{0(t)} = F_{0(t)}^S + \sqrt{3} F_{0(t)}^P \cos\theta_\pi + \sqrt{5} F_{0(t)}^D \frac{3\cos^2\theta_\pi - 1}{2} + \ldots$$

- decompose transverse FFs in $P_\ell^{(m=1)}(\cos\theta_\pi)/\sin(\theta_\pi)$
  - $F_\perp$ and $F_\|$ start with P-wave only, decompose in associated Legendre polynomials

$$F_{\perp(\|)} = \frac{\sqrt{3}}{\sqrt{2}} F_{\perp(\|)}^P + \frac{\sqrt{15}}{\sqrt{2}} F_{\perp(\|)}^D \cos\theta_\pi + \ldots$$

- each partial-wave FF: $F_\sigma^\ell \equiv F_\sigma^\ell(q^2, k^2)$

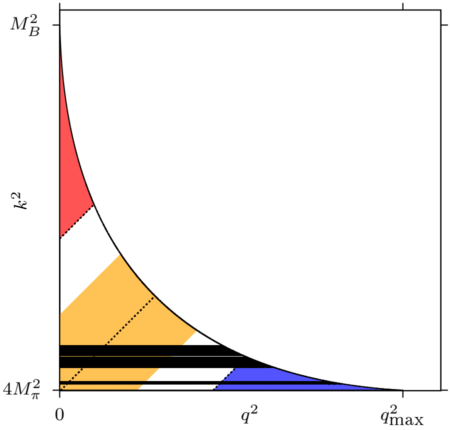
Phase Space & Theoretical Methods



HHχPT region

- both pions soft in B rest frame
- approximate HQET symmetry relations [Faller,Feldmann,Khodjamirian,Mannel,DvD 1310.6660]
- Heavy-Hadron Chiral Perturbation Theory (HHχPT) applicable [Burdman, Donoghue, Phys. Lett. B280 287 (1992)]
- Dispersive representation, using $\pi\pi$ phase as input, and matching onto HHχPT [Kang, Kubis, Hanhart, Meißner 1312.1193]

Phase Space & Theoretical Methods

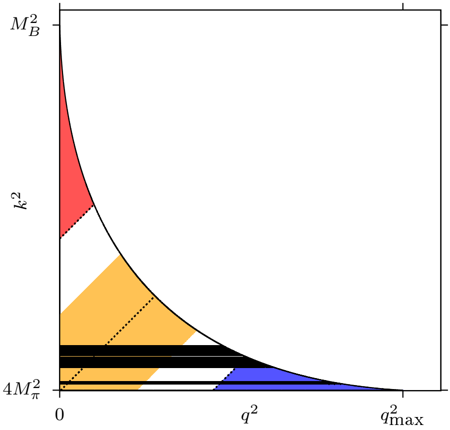


LCSR region

- both pions collinear in B rest frame
- approximate SCET symmetry relations [Faller,Feldmann,Khodjamirian,Mannel,DvD 1310.6660]
- Light-Cone Sum Rules with two-pion LCDAs
- all partial waves accessible for $F_\perp$, and $F_\|$ [Hambrock, Khodjamirian 1511.02509]
- additional sum rule for F_0, F_t w.i.p.

dedicated talk on this by Alexander Khodjamirian

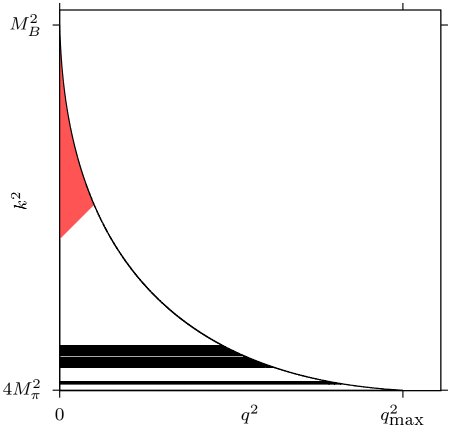
Phase Space & Theoretical Methods



QCDF region

- both pions energetic in B rest frame
  - near back-to-back kinematics
- QCD-improved factorization approach applicable(?)
  - to be shown through explicit calculation!
  - remainder of this talk

Phase Space & Theoretical Methods



QCDF region

- both pions energetic in B rest frame
  - near back-to-back kinematics
- QCD-improved factorization approach applicable(?)
  - to be shown through explicit calculation!
  - remainder of this talk

Results at large dipion masses

Hadronic Kinematics

– for convenience express results in

$$k^2 \qquad\qquad E_{1(2)} \equiv \frac{p \cdot k_{1(2)}}{M_B}$$

– required power counting

$$k^2 \simeq (E_1 + E_2)^2 \sim M_B^2 \qquad E_1 - E_2 = \frac{\sqrt{\lambda}}{2M_B} \cos\theta_\pi \sim \Lambda_{\text{had.}} \ll M_B$$

implying $q^2 \sim (M_B - E_1 - E_2)^2$

- for generic helicity angle $\cos\theta_\pi$ require $\sqrt{\lambda} \ll M_B^2$
- extrapolation to "mercedes-star" configuration in $B \to \pi\pi\pi$? [Kränkl, Mannel, Virto 1505.04111]

Light-Cone Coordinates and Projectors

$$B^-(p) \to \pi^+(k_1)\pi^-(k_2)\overline{\nu}_\ell(q_1)\ell^-(q_2)$$

- pion momenta are light-like: $k_1^2 = k_2^2 \simeq 0$
- write partonic momenta in light-like directions
  - for quarks that enter $\pi^+(k_1)$:

    $$\simeq u k_1^\mu \,, \qquad \simeq \overline{u} k_1^\mu \,, \qquad \text{with } \overline{u} = 1 - u$$
  - for quarks that enter $\pi^-(k_2)$:

    $$\simeq v k_2^\mu \,, \qquad \simeq \overline{v} k_2^\mu \,, \qquad \text{with } \overline{v} = 1 - v$$
  - B meson:

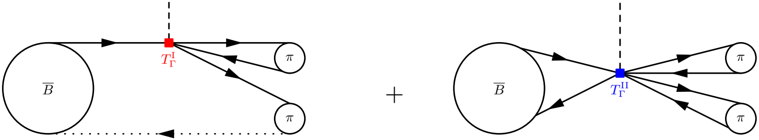
    $$p_b^\mu \simeq p^\mu = M_B v_b^\mu \qquad l_\mu \simeq \omega \frac{k_2^\mu}{(v_b \cdot k_2)}$$
- define projectors for later use:

$$P_{v_b} \equiv \frac{1 + \not{v}_b}{2} \,, \qquad P_{12} \equiv \frac{\not{k}_1 \not{k}_2}{k^2} \,, \qquad P_{21} \equiv \frac{\not{k}_2 \not{k}_1}{k^2} \,,$$

Factorization Theorem

we want to show that

$$
\begin{aligned}
&\langle \pi^+(k_1)\pi^-(k_2)|\overline{\psi}_u\ \Gamma\ \psi_b|B^-(p)\rangle \\
&= \frac{2\pi f_\pi \xi_\pi(E_2;\mu)}{k^2}\int_0^1 du\,\phi_\pi(u,\mu)\,T_\Gamma^{\mathrm{I}}(u,\ldots;\mu) \\
&\quad + \int_0^1 du \int_0^1 dv \int_0^\infty \frac{d\omega}{\omega}\,\phi_\pi(u;\mu)\,\phi_\pi(v;\mu)\,\phi_B^+(\omega;\mu)\,T_\Gamma^{\mathrm{II}}(u,v,\omega,\ldots;\mu) \\
&\quad + \text{power corrections}\,.
\end{aligned}
$$



ξ_π denotes the universal non-factorizable $B \to \pi$ form factor in SCET

[Beneke, Feldmann hep-ph/0008255] [Charles et al. hep-ph/9812358]

Tasks

To proof the factorization theorem we need to show:

#1 the term to leading power in $1/m_b$ arises only from the leading twist (i.e. twist-2) pion distribution amplitude of π^+

#2 spectator interactions that would formally lead to endpoint divergencies in T_Γ^{II} are *universal*, i.e.: they can be absorbed into the soft form factor ξ_π

University of Zurich[UZH]

Physik Institut

Task # 1 - LO Kernel

LO - Kernel T_Γ^{I}

gluon propagator

– large virtuality $\sim \overline{u}k^2 = O\left(m_b^2\right)$

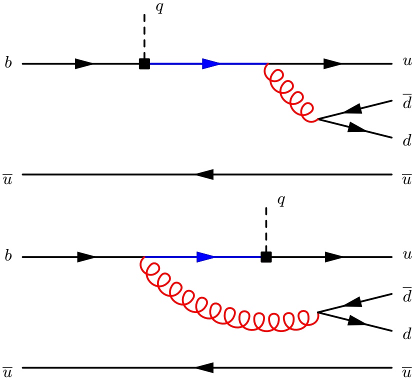
quark propagators

A $\sim k^2 = O\left(m_b^2\right)$

B $\sim (k^2 - 2M_B E_1)\overline{u} - 2M_B E_2 = O\left(m_b^2\right)$

soft spectator:

assume: $\left[\bar{d}\Gamma_X b\right] \to \xi_\pi(E_2)\,\mathrm{tr}\left[\not{k}_2\Gamma_X P_{v_b}\right]$



LO - Kernel T_Γ^{I} Result at Twist 2

$$T_\Gamma^{\mathrm{I}}(u, k^2, E_1, E_2)\Big|_{\text{LO,twist 2}} = i\,\frac{\alpha_s C_F}{N_C}\,\frac{S_A + S_B^{(i)}(u) + S_B^{(ii)}(u)}{\overline{u}}$$

– from diagram A:

$$S_A = \mathrm{tr}\left[\not{k}_2 \gamma_5\, P_{12}\, \Gamma\, P_v\right] \;,$$

– from diagram B:

$$S_B^{(i)}(u) = \frac{k^2}{2}\,\frac{2M_B\,\mathrm{tr}\left[\gamma_5\, P_{21}\, \Gamma\, P_v\right] - \mathrm{tr}\left[\overline{u}\not{k}_1\, \gamma_5\, P_{21}\, \Gamma\right]}{(k^2 - 2M_B E_1)\,\overline{u} - 2M_B E_2}\;,$$

$$S_B^{(ii)}(u) = E_2\,\frac{k^2\,\mathrm{tr}\left[\gamma_5\, P_{21}\, \Gamma\right] - 2M_B\,\mathrm{tr}\left[\not{k}_1\, \gamma_5\, P_{21}\, \Gamma\, P_v\right]}{(k^2 - 2M_B E_1)\,\overline{u} - 2M_B E_2}\;.$$

– does not vanish for $\Gamma = \{$any of the helicity projectors$\}$ ✓

LO - Kernel T_Γ^{I} Comments

- twist-3 contributions from π^+ projector are indeed $1/m_b$ suppressed
  - $\Rightarrow$ task #1: ✓
  - however: numerically sizable due to $\sim \mu_\pi = m_\pi^2/(m_u + m_d)$
- at LO, only two independent terms appear in the convolution integral

$$\int_0^1 \mathrm{d}u \, \frac{\phi(u)}{\overline{u}} \, , \qquad \int_0^1 \mathrm{d}u \, \frac{\phi(u)}{\overline{u}(k^2 - 2E_1 M_B) - 2E_2 M_B}$$

  - implies FF relations
  - more complicated than for $B \to \pi$ due to partial wave decomposition
  - reminder: $E_1 - E_2 \propto \cos\theta_\pi$
  - FF relations near endpoint $\lambda \simeq 0$ from Lorentz symmetry
    $F_0 \simeq \cos\theta_\pi F_{||} \left(1 + O\left(\sqrt{\lambda}/M_B^2\right)\right)$

LO Results

Preliminary

F_0 as example, after partial wave decomposition: $\mathcal{N}$: normalization $\sim \alpha_s f_\pi \xi_\pi$

$$F_0^{(S)} = \mathcal{N}\,\frac{\sqrt{\lambda}}{M_B\,\sqrt{q^2}}\left\{\left(\frac{1}{2}+\frac{\sqrt{q^2}}{3M_B}\right)i_0+\frac{1}{3}\left(1-\frac{\sqrt{q^2}}{M_B}-\frac{q^2}{M_B^2}\right)j_1+\frac{1}{6}\left(1-\frac{q^2}{M_B^2}\right)j_2\right\}$$

$$F_0^{(P)} = \mathcal{N}\,\frac{\sqrt{k^2}}{\sqrt{3}\,M_B}\left\{-i_0+\left(1-\frac{\sqrt{q^2}}{M_B}\right)j_1\right\}$$

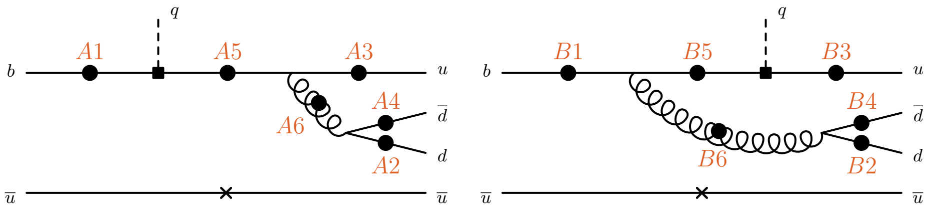
$$F_0^{(D)} = \mathcal{N}\,\frac{\sqrt{\lambda}}{3\sqrt{5}\,M_B^2}\left\{-i_0-\left(\frac{M_B}{\sqrt{q^2}}-1-\frac{\sqrt{q^2}}{M_B}\right)j_1+\left(\frac{M_B}{\sqrt{q^2}}-\frac{\sqrt{q^2}}{M_B}\right)j_2\right\}$$

– i_0 and $j_{1,2}$: moments of the twist-2 LCDA ϕ

$$i_0 \equiv \int_0^1 \frac{\mathrm{d}u\,\phi(u)}{\bar{u}} \qquad\qquad j_n \equiv \int_0^1 \frac{\mathrm{d}u\,\phi(u) M_B^n}{(M_B+\sqrt{q^2}\bar{u})^n}$$

Task # 2 - NLO Spectator Terms

NLO (Spectator Scattering) - Kernel T_Γ^{II}



gluon exchange between $\times$ and any of the named labels

Steps

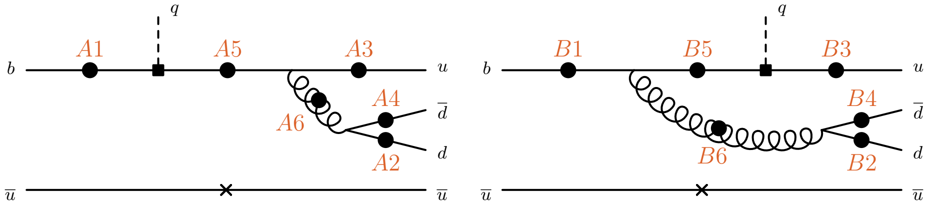
– identify endpoint-divergent contributions ✓

– subtract corresponding terms from $B \to \pi$ form factor [Beneke/Feldmann hep-ph/0008255] ✓

– extract finite contributions to T_Γ^{II} (w.i.p.)

$\Rightarrow$ corrections to LO FF relations

NLO (Spectator Scattering) - Kernel T_{Γ}^{II}



gluon exchange between $\times$ and any of the named labels

Findings

– non-trivial cancellation of $\overline{v} \to 0$ / $\omega \to 0$ endpoint divergencies between: diagrams $A1$ – $A6$, diagrams $B1$ – $B6$, and $B \to \pi$ FF

– non-trivial cancellation of $\overline{u} \to 0$ endpoint divergencies between: diagrams $A1$ – $A6$, and diagrams $B1$ – $B6$

Endpoint divergent terms ($A1 - A6$) (in Feynman gauge)

structure	A1	A2	A3 + A4	A5	A6	A1-A6
$\frac{S_C}{\overline{u}^2}\,\frac{\phi_B^+(\omega)}{\omega}\,\frac{\phi_\pi(v)}{2v\overline{v}}$	0	0	$-C_{FA}\,2v$	0	$C_A\,\frac{v-\overline{v}}{2}$	$2vC_F - \frac{C_A}{2}$
$\frac{S_A}{\overline{u}}\,\frac{\phi_B^-(\omega)}{\omega}\,\frac{\phi_\pi(v)}{\overline{v}^2}$	$C_F\,\frac{1}{v}$	$C_F\,\overline{v}$	$C_{FA}\,\frac{\overline{v}}{v}$	0	$-\frac{C_A}{2}\,\frac{\overline{v}}{v}$	$C_F\,(1+\overline{v})$
$\frac{S_A}{\overline{u}}\,\frac{\phi_B^+(\omega)}{\omega}\,\frac{\Phi_-(v,E_2)}{\overline{v}^2}$	C_F	0	0	0	0	C_F
$2\mu_\pi\,\frac{S_A}{\overline{u}}\,\frac{\phi_B^+(\omega)}{\omega^2}\,\frac{\phi_P(v)}{\overline{v}}$	0	C_F	0	0	0	C_F

$$S_C \equiv 2E_2\,\mathrm{tr}[\gamma_5 P_{12}\Gamma P_v]$$

$$C_{FA} \equiv \frac{C_A}{2} - C_F = \frac{1}{2N_C}$$

$$\Phi_-(v,E_2) \equiv \frac{\mu_\pi}{2E_2}\left(\phi_P(v) - \frac{\phi'_\sigma(v)}{6}\right)$$

$$\beta_{12} \equiv \frac{4E_1E_2}{k^2} - 1$$

Endpoint divergent terms ($B1$ – $B6$) (in Feynman gauge)

structure	B1	B2	B3+B5	B4	B6	B1-B6
$\frac{S_C}{\bar{u}^2}\,\frac{\phi_B^+(\omega)}{\omega}\,\frac{\phi_\pi(v)}{v\bar{v}}$	0	0	0	$C_{FA}\,2v$	$C_A\,\frac{\bar{v}-v}{2}$	$\frac{C_A}{2}-2vC_F$
$\frac{S_B^{(\mathrm{i})}}{\bar{u}}\,\frac{\phi_B^+(\omega)}{\omega}\,\frac{\phi_\pi(v)}{\bar{v}^2}$	0	0	$C_{FA}\,\beta_{12}$	$-C_{FA}\,\beta_{12}$	0	0
$\frac{S_B^{(\mathrm{i})}+S_B^{(\mathrm{ii})}}{\bar{u}}\,\frac{\phi_B^-(\omega)}{\omega}\,\frac{\phi_\pi(v)}{\bar{v}^2}$	$C_F\,\frac{1}{v}$	$C_F\,\bar{v}$	$C_{FA}\,\frac{1}{v}$	$-C_{FA}$	$-\frac{C_A}{2}\,\frac{\bar{v}}{v}$	$C_F\,(1+\bar{v})$
$\frac{S_B^{(\mathrm{i})}+S_B^{(\mathrm{ii})}}{\bar{u}}\,\frac{\phi_B^+(\omega)}{\omega}\,\frac{\Phi_-(v,E_2)}{\bar{v}^2}$	C_F	0	$C_{FA}\,\beta_{12}$	$-C_{FA}\,\beta_{12}$	0	C_F
$\frac{S_B^{(\mathrm{i})}}{\bar{u}}\,\frac{\phi_B^-(\omega)}{\omega}\,\frac{\Phi_-(v,E_2)}{\bar{v}^2}$	0	0	$-C_{FA}\,\beta_{12}$	$C_{FA}\,\beta_{12}$	0	0
$2\mu_\pi\,\frac{S_B^{(\mathrm{i})}+S_B^{(\mathrm{ii})}}{\bar{u}}\,\frac{\phi_B^+(\omega)}{\omega^2}\,\frac{\phi_P(v)}{\bar{v}}$	0	C_F	0	0	0	C_F

$$S_C \equiv 2E_2\,\mathrm{tr}[\gamma_5 P_{12}\Gamma P_v]$$

$$C_{FA} \equiv \frac{C_A}{2} - C_F = \frac{1}{2N_C}$$

$$\Phi_-(v,E_2) \equiv \frac{\mu_\pi}{2E_2}\left(\phi_P(v) - \frac{\phi'_\sigma(v)}{6}\right)$$

$$\beta_{12} \equiv \frac{4E_1E_2}{k^2} - 1$$

Summary and Outlook

Summary

- factorization theorem holds (at first non-trivial order)
  - leading power result emerges from twist-2 pion LCDAs ✓
  - NLO endpoint-divergent (spectator scattering) terms can be reabsorbed into universal soft form factor ξ_π ✓
- explicit results[1]
  - LO expression for T_Γ^{I} (including twist-3 corrections)
  - partial wave FFs up to $\ell = 2$ (D wave)
  - several FF relations near edge of phase space $\lambda(q^2, k^2, M_B^2) = 0$

[1] we did *not* use $\phi(v) = \phi(\overline{v})$

Outlook

[Bell/Böer/Feldmann w.i.p.]

- α_s corrections (at leading power) calculable in QCDF
  - SCET matching and running
- generalization to $\overline{B} \to \overline{K} K \ell \overline{\nu}$, and $\overline{B}_s \to K\pi\ell\overline{\nu}$ trivial
  replace $\phi_\pi(\cdot) \to \phi_K(\cdot)$, etc.
- phenomenologically interesting: $B \to K\pi\ell^+\ell^-$, interplay between current-current and penguin operators

Combining Results

How to extrapolate the FFs across regions of applicability?

- aim: use correlated FF inputs, e.g. in description of $B \to 3\pi$
- suggestion: z series in q^2, as already used for $B \to \pi$, $B \to \rho$ etc.

$$F_\sigma^\ell(q^2, k^2) = P_\sigma(q^2) \sum_n a_n^{\sigma,\ell}(k^2) z^n(q^2, t_-)\,.$$

  - P_σ: low-lying pole
  - z-expansion coefficients $a_{\sigma,i}(k^2)$ still depend on k^2
  - use spectral representation for a_{σ_i}, e.g. Omnès representation
- question: how to implement non-trivial end-point relations?
  - is it feasible to let $t_- \equiv t_-(k^2) = (M_B - \sqrt{k^2})^2$?
  - then: $z(t_-; t_0 = t_-) = 0$, and $F_\sigma^\ell(q^2 = t_-) \propto a_0^{\sigma,\ell}$

Appendix

$B \to \pi$ form factors in QCDF (I)

- three form factors: f_+, f_0, f_T
- renormalize end-point divergencies by absorption into ξ_π
- choice of renormalization scheme: $\xi_\pi \equiv f_+$ to all orders in α_s

$$f_0 = \frac{2E_\pi}{M_B}\xi_\pi\left(1 + O\left(\alpha_s\right)\right) + \frac{\alpha_s C_F}{4\pi}\Delta F_0$$

$$f_T = \frac{M_B + M_\pi}{M_B}\xi_\pi\left(1 + O\left(\alpha_s\right)\right) + \frac{\alpha_s C_F}{4\pi}\Delta F_T$$

- finite spectator scattering contribution *universal*

$$\Delta F_0 = \frac{M_B - 2E_\pi}{2E_\pi}\Delta F_\pi \qquad \Delta F_T = -\frac{M_B + M_\pi}{2E_\pi}\Delta F_\pi$$

where

$$\Delta F_\pi \equiv \frac{8\pi^2 f_B f_\pi}{N_C M_B}\langle\frac{\phi_\pi(v)}{\bar{v}}\rangle\langle\frac{\phi_B^+(\omega)}{\omega}\rangle$$

$B \to \pi$ form factors in QCDF (II)

end-point divergences in NLO spectator scattering

$$
\begin{aligned}
\langle \pi | \overline{u}\, \Gamma\, b | \overline{B} \rangle \Big|_{\text{spect.}} &= C_F \left\langle \frac{(1+\overline{v})\phi(v)}{\overline{v}^2} \right\rangle \left\langle \frac{\phi_B^-(\omega)}{\omega} \right\rangle \\
&+ 2\mu_\pi C_F \left\langle \frac{\phi_P(v)}{\overline{v}} \right\rangle \left\langle \frac{\phi_B^+(\omega)}{\omega^2} \right\rangle \\
&+ \frac{\mu_\pi}{2E_2} C_F \left\langle \frac{\phi_P(v) - \phi'_\sigma(v)/6}{\overline{v}^2} \right\rangle \left\langle \frac{\phi_B^+(\omega)}{\omega} \right\rangle
\end{aligned}
$$

exactly cancels $\overline{v} \to 0$ / $\omega \to 0$ divergences in $A1$ – $B6$!

Endpoint contributions ($A1$ – $A6$)

$$\langle \pi^+(k_1)\pi^-(k_2)|\overline{\psi}_u\,\Gamma\,\psi_b|B^-(p)\rangle\Big|_{(\mathrm{A1-A6})}$$

$$= \frac{2\pi f_\pi\,\xi_\pi(E_2)}{k^2}\,\frac{i\alpha_s C_F}{N_C}\int_0^1 \frac{du}{\overline{u}}\,\phi_\pi(u)\,S_A$$

$$+\,\frac{2\pi f_\pi}{k^2}\,\frac{i\alpha_s^2 C_F\,(C_F-\frac{C_A}{2})}{4\pi N_C}\,\frac{M_B^2}{4E_2}\,\Delta F_\pi\left\langle\frac{\phi_\pi(u)}{\overline{u}^2}\right\rangle \mathrm{tr}\left[\gamma_5\,P_{21}\,\Gamma\,P_v\right]$$

$+$ finite terms.

- S_A the same as the diag. A term in T_Γ^{I}
- highlighted term is endpoint-divergent due to anti-quark momentum in π^+
  - ΔF_π is the endpoint-finite hard-scattering correction to the $B \to \pi$ form factors
  - expect the term to be cancelled in $B1$ – $B6$ based on color transparency and gauge invariance arguments