Padé Theory: a toolkit for hadronic form factors

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Preliminary work done in collaboration with
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SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Cluster of Excellence Precision Physics, Fundamental Interactions and Structure of Matter PRISMA

Future Challenges in Non-leptonic B decays, Bad Honnef, Feb 10, 2016

JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Outline

- Motivation
- Form factors in Semi-leptonic B decays (*warm up*)
- Form factors in Non-leptonic B decays
- Conclusions

Motivation

(very personal)

- Accurate determination of hadronic form factors is relevant for CKM, CPV, NP
- Hadronic form factors appear also (and are very important) in other hot topics: $(g-2)_\mu$, proton radius puzzle, P $\to$ll, ... (where NP are potential)
  - effort on param. + *synergy* between experiment and theory (<u>data driven</u>)

- Can all this knowledge be transported to Non-leptonic B decays?

- Yes, with pleasure!

- Very personal: two requests:
  - first sorry if I misquote
  - I'm open to suggestions: numbers here have no relevance, but the method

Motivation II

(a bit more technical)

• Accurate determination of hadronic form factors is relevant for CKM, CP, NP

• We are at the level (specially with lattice) where systematic errors on the parameterizations are important

• The environment of FF in B decays is theoretically a challenge
(see for example the talks this morning: Edward, Mannel, van Dyk, Khodjamirian)

Semi-leptonic B decays
warm up

$B \to \pi$ FF

Overview of FF parameterizations

The relevant form factor for the decay $B \to \pi \ell \nu_\ell$ is defined ($m_\ell \to 0$):

$$\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = F_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right)$$

The spectrum in q^2 is given by: (as a Ref. Minireview from PDG)

$$\frac{d\Gamma(B \to \pi \ell \nu_\ell)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192 \pi^3 m_B^3} \lambda^{3/2} |F_+(q^2)|^2$$

A dispersion relation for FF:

$$F(q^2) = \frac{\mathrm{Res} F(q^2 = s_p)}{q^2 - s_p} + \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im} F(s')}{s' - q^2 - i\varepsilon} \qquad s_{th} = (m_B + m_\pi)^2$$

$B \to \pi \ell \nu_\ell$ with $0 < q^2 < (m_B - m_\pi)^2$

$$F_+(q^2) = \frac{F_+(0)}{1 - q^2/m_{B^*}^2} \qquad \mathrm{Res} F_+(q^2 = m_{B^*}^2) \propto m_{B^*} f_{B^*} g_{B^* B \pi}$$

$B \to \pi$ FF

Overview of FF parameterizations

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VMD (1 parameter):

$$F_+(q^2) = \frac{F_+(0)}{1 - q^2/m_{B^*}^2}$$

Becirevic, Kaidalov '99 (2 param):

$$F_+(q^2) = \frac{r_1}{1 - q^2/m_{B^*}^2} + \frac{r_2}{1 - q^2/m_{B^{*\prime}}^2}$$ or $$F_+(q^2) = \frac{F_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)}$$

Ball, Zwicky '04 (2 param):

$$F_+(q^2) = F_+(0) \left(\frac{1}{1 - q^2/m_{B^*}^2} + \frac{r q^2/m_{B^*}^2}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)} \right)$$

B→π FF

Overview of FF parameterizations

$$\frac{d\Gamma(B \to \pi \ell \nu_\ell)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2} |F_+(q^2)|^2$$

Boyd, Grinstein, Lebed '95,'97 (z-parameterization, many param):

$$F_+(q^2) = \frac{1}{P(q^2)\phi(q^2, q_0^2)} \sum_{n=0}^{\infty} a_k(q_0^2)[z(q^2, q_0^2)]^k$$

$$z(q^2, q_0^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - q_0^2}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - q^2}}$$

$$P(q^2) = z(q^2, m_{B*}^2)$$

$$t_+ = (m_B + m_\pi)^2$$

Bourrely, Caprini, Lellouch '09 (alternative z-parameterization, many param):

$$F_+(q^2) = \frac{1}{1 - q^2/m_{B*}^2} \sum_{n=0}^{K} b_k(t_0)[z(q^2, q_0^2)]^k$$

B→π FF

Overview of FF parameterizations

$$\frac{d\Gamma(B\to\pi\ell\nu_\ell)}{dq^2}=\frac{G_F^2|V_{ub}|^2}{192\pi^3 m_B^3}\lambda^{3/2}|F_+(q^2)|^2$$

All of them have something in common

I) $$F_+(q^2)=\frac{F_+(0)}{(1-q^2/m_{B^*}^2)(1-\alpha q^2/m_{B^*}^2)}$$

II) $$F_+(q^2)=F_+(0)\left(\frac{1}{1-q^2/m_{B^*}^2}+\frac{rq^2/m_{B^*}^2}{(1-q^2/m_{B^*}^2)(1-\alpha q^2/m_{B^*}^2)}\right)$$

III) $$F_+(q^2)=\frac{1}{P(q^2)\phi(q^2,q_0^2)}\sum_{n=0}^{\infty}a_k(q_0^2)[z(q^2,q_0^2)]^k$$

IV) $$F_+(q^2)=\frac{1}{1-q^2/m_{B*}^2}\sum_{n=0}^{K}b_k(t_0)[z(q^2,q_0^2)]^k$$

B→π FF

Overview of FF parameterizations

$$\frac{d\Gamma(B\to\pi\ell\nu_\ell)}{dq^2}=\frac{G_F^2|V_{ub}|^2}{192\pi^3 m_B^3}\lambda^{3/2}|F_+(q^2)|^2$$

All of them are Padé approximants

I) $F_+(q^2)=\frac{F_+(0)}{(1-q^2/m_{B^*}^2)(1-\alpha q^2/m_{B^*}^2)}$ Partial Padé [Baker,'95]

II) $F_+(q^2)=F_+(0)\left(\frac{1}{1-q^2/m_{B^*}^2}+\frac{rq^2/m_{B^*}^2}{(1-q^2/m_{B^*}^2)(1-\alpha q^2/m_{B^*}^2)}\right)$ Partial Padé

III) $F_+(q^2)=\frac{1}{P(q^2)\phi(q^2,q_0^2)}\sum_{n=0}^{\infty}a_k(q_0^2)[z(q^2,q_0^2)]^k$ Padé Type

IV) $F_+(q^2)=\frac{1}{1-q^2/m_{B*}^2}\sum_{n=0}^{K}b_k(t_0)[z(q^2,q_0^2)]^k$ Padé Type

B→π FF

Overview of FF parameterizations

$$\frac{d\Gamma(B \to \pi \ell \nu_\ell)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2} |F_+(q^2)|^2$$

All of them are Padé approximants

I) $F_+(q^2) = \frac{F_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)}$ Partial Padé [Baker,'95]

II) $F_+(q^2) = F_+(0) \left(\frac{1}{1 - q^2/m_{B^*}^2} + \frac{r q^2/m_{B^*}^2}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)} \right)$ Partial Padé

III) $F_+(q^2) = \frac{1}{P(q^2)\phi(q^2, q_0^2)} \sum_{n=0}^{\infty} a_k(q_0^2)[z(q^2, q_0^2)]^k$ Padé Type

IV) $F_+(q^2) = \frac{1}{1 - q^2/m_{B*}^2} \sum_{n=0}^{K} b_k(t_0)[z(q^2, q_0^2)]^k$ Padé Type

Non of them use Padé Theory

Padé Approximants

Padé approx: $Q(z)f(z) + R(z) = \mathcal{O}\left(z^{q+r+1}\right)$

$R(z), Q(z)$ are polynomials

Let f(z)

$$f(z) = \sum_{k=0} a_k z^k$$

then its PA $P^N_M(z) = \dfrac{\sum_{n=0}^{N} r_n z^n}{\sum_{m=0}^{M} q_n z^n}$

and the PA has a contact with f(z) or order N+M+1

$$\begin{cases} P^N_M(z) = r_0 + (r_1 - r_0 q_1) z + (r_2 - r_1 q_1 + r_0 q_1^2 - r_0 q_2) z^2 + \mathcal{O}(z^3) \\ f(z) = a_0 + a_1 z + a_2 z^2 + \mathcal{O}(z^3) \end{cases}$$

Examples: $P^0_1(z) = \dfrac{a_0}{1 - \frac{a_1}{a_0} z}$ $P^1_1(z) = \dfrac{a_0 + \frac{a_1^2 - a_0 a_2}{a_1} z}{1 - \frac{a_2}{a_1} z}$

Padé Approximants

Padé approx: $Q(z)f(z) + R(z) = \mathcal{O}\left(z^{q+r+1}\right)$

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Stieltjes theorem:

$$\lim_{N\to\infty} P^N_{N+1}(z) \leq f(z) \leq \lim_{N\to\infty} P^N_N(z)$$

(others: Montessus, Pommerenke, Nutall, Baker, Chisholm...)

Example: $f(z) = \frac{1}{z}\log(1-z)$

$$f(z) = -\sum_{k=0} \frac{z^k}{k+1} = -1 - \frac{z}{2} - \frac{z^2}{3} - \frac{z^3}{4} - \frac{z^4}{5} + \mathcal{O}(z^6)$$

Padé Approximants

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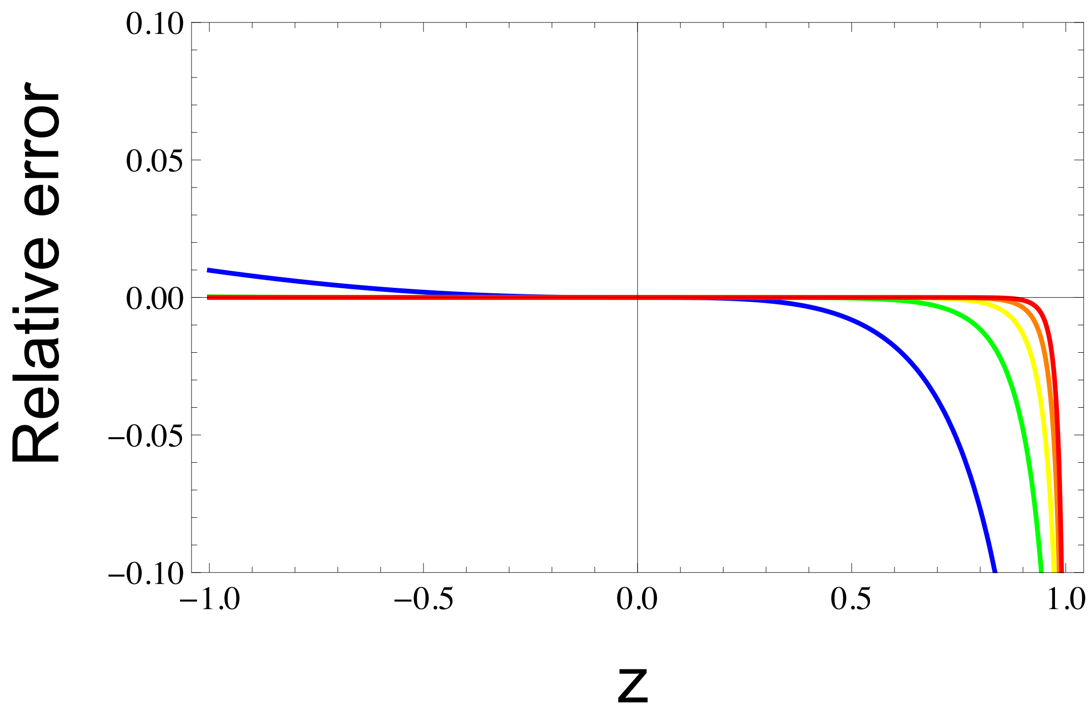
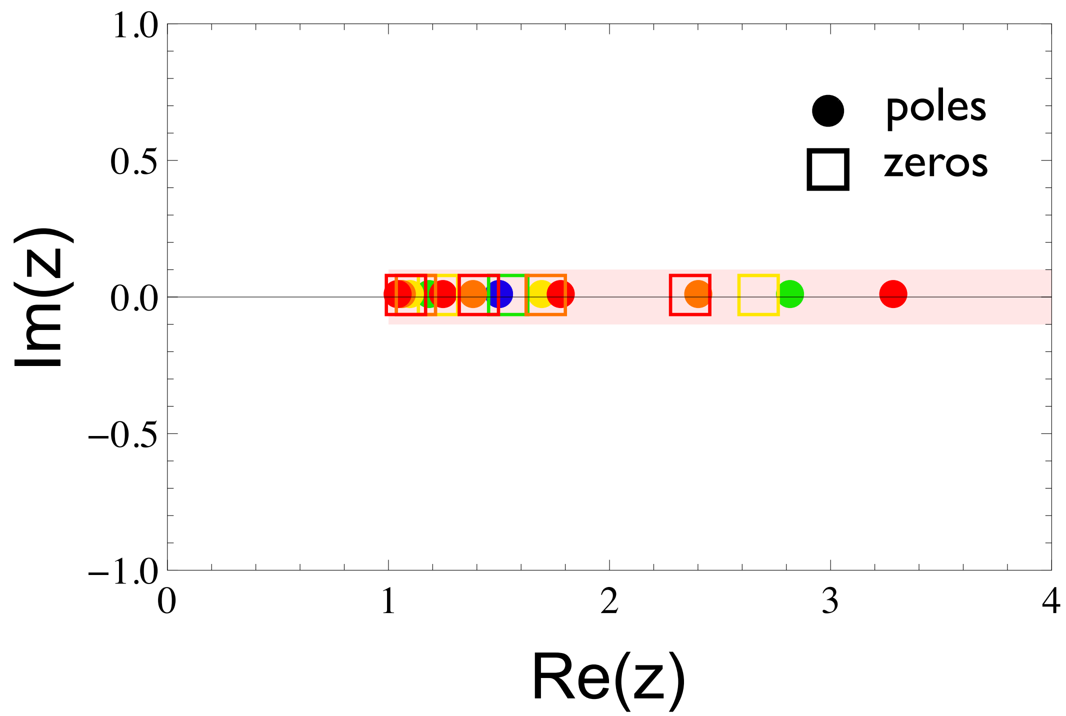
$P_N^N(z)$ for N=1,2,3,4,5





Padé Approximants

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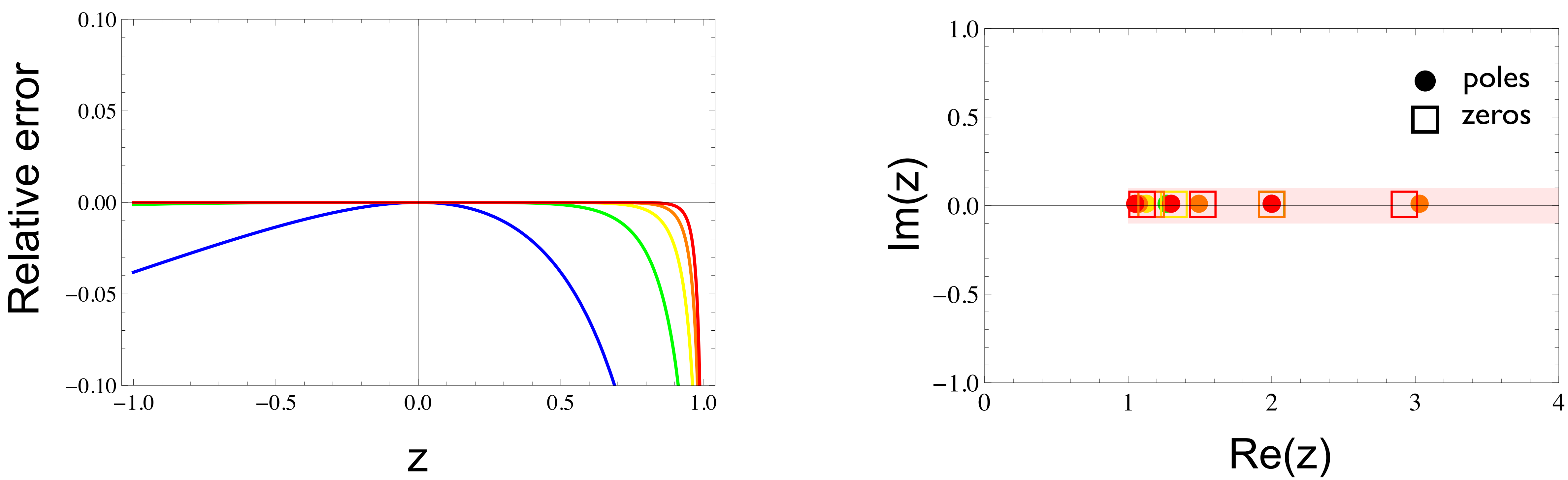
$P^N_{N+1}(z)$ for N=0,1,2,3,4



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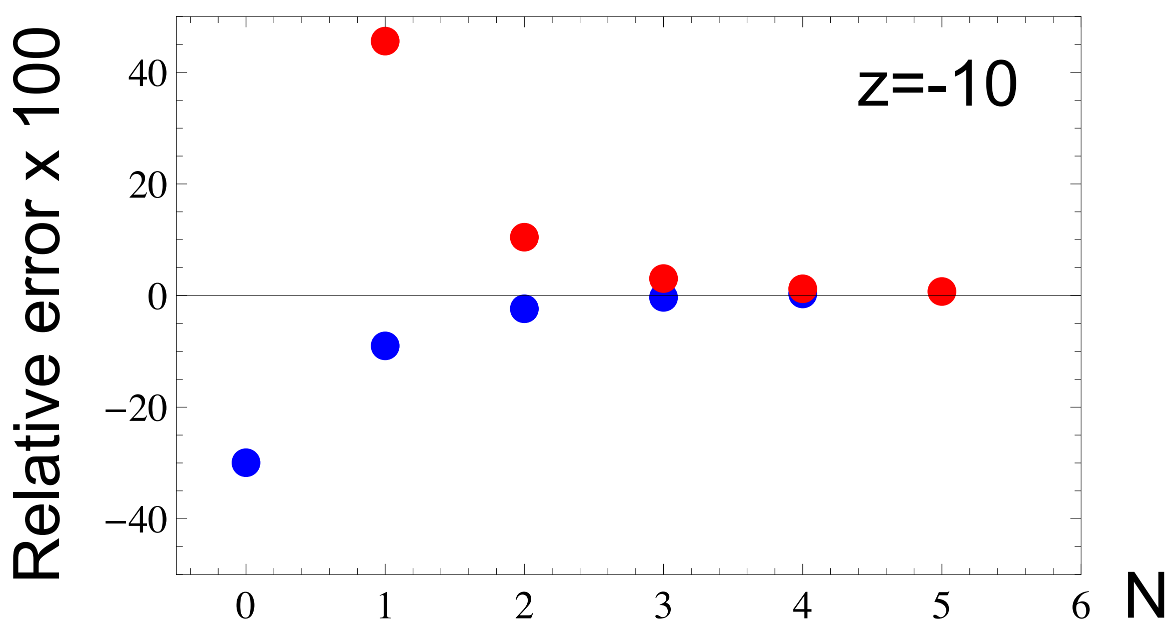

Message: convergence pattern gives systematic error

Padé Approximants

Padé approx: $Q(z)f(z) + R(z) = \mathcal{O}\left(z^{q+r+1}\right)$

$R(z), Q(z)$ are polynomials

Example: vacuum polarization function

$$\Pi(q^2) = \Pi^{(0)}(q^2) + \left(\frac{\alpha_s}{\pi}\right)\Pi^{(1)}(q^2) + \mathcal{O}(\alpha_s^2)$$

let me define $z = \dfrac{q^2}{4m^2}$

$$\Pi^{(0)}(z) = \frac{3}{16\pi^2}\left(\frac{4}{3z} + \frac{20}{9} - \frac{4(1-z)(2z+1)G(z)}{3z}\right)$$

$$G(z) = 2\frac{u\log(u)}{u^2-1} \qquad \text{where} \qquad u \to \frac{\sqrt{1-z^{-1}}-1}{\sqrt{1-z^{-1}}+1}$$

Padé Approximants

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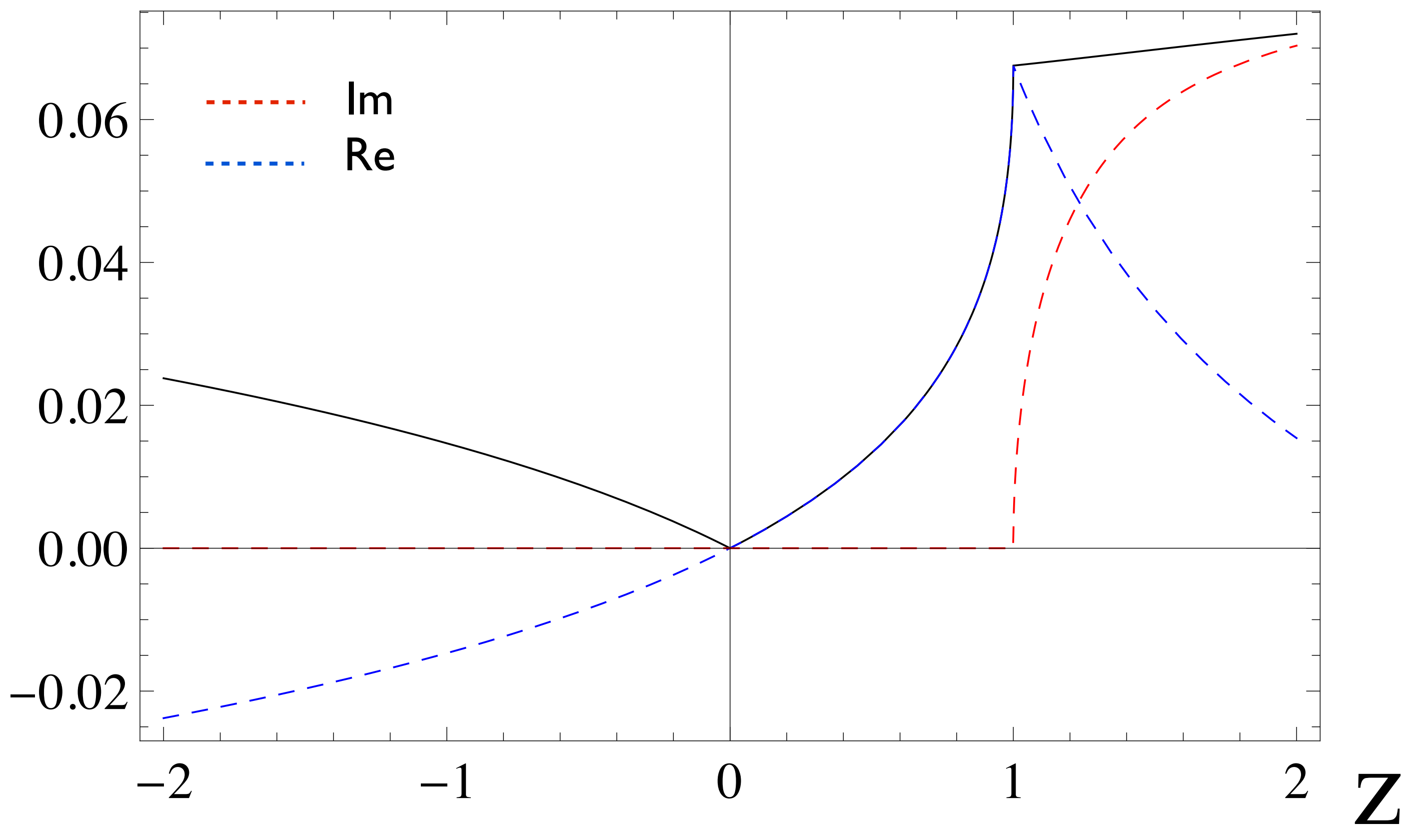

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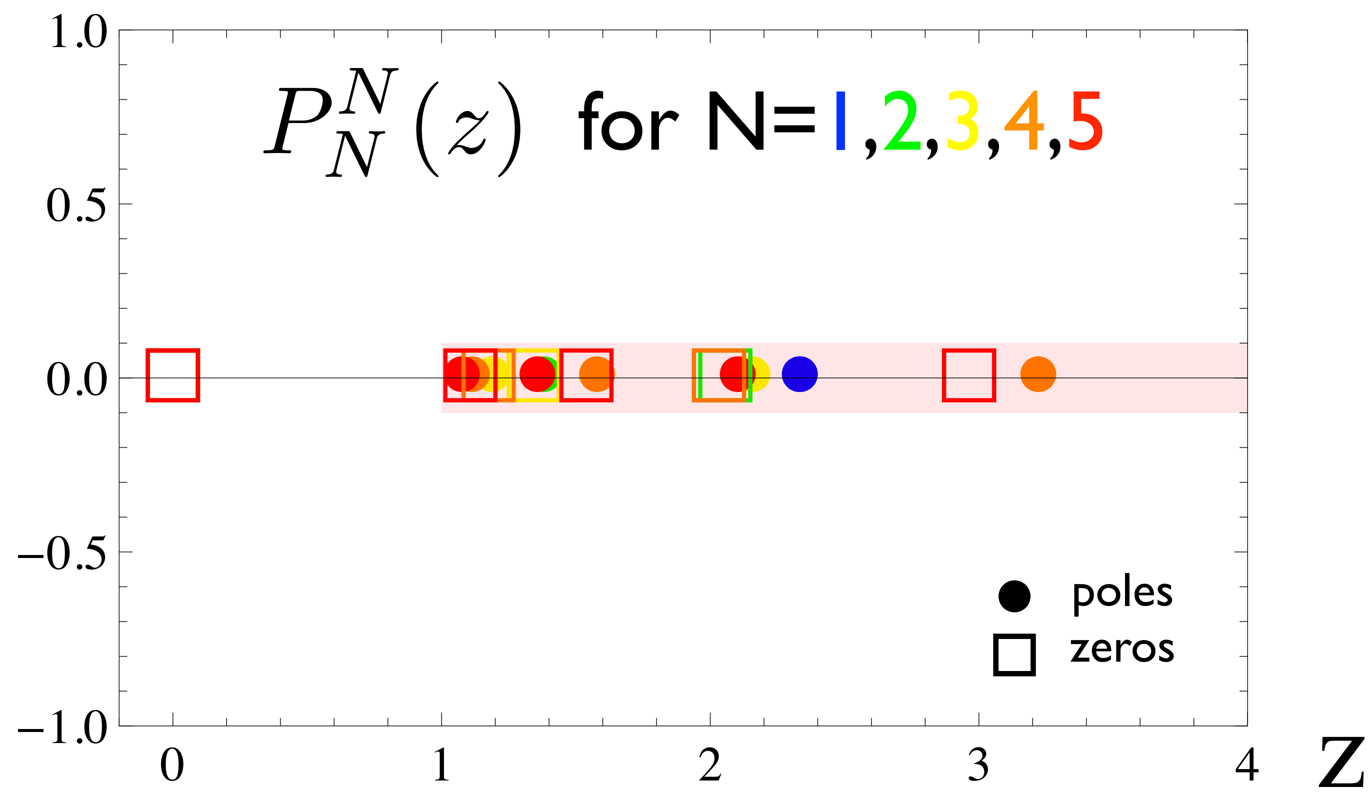

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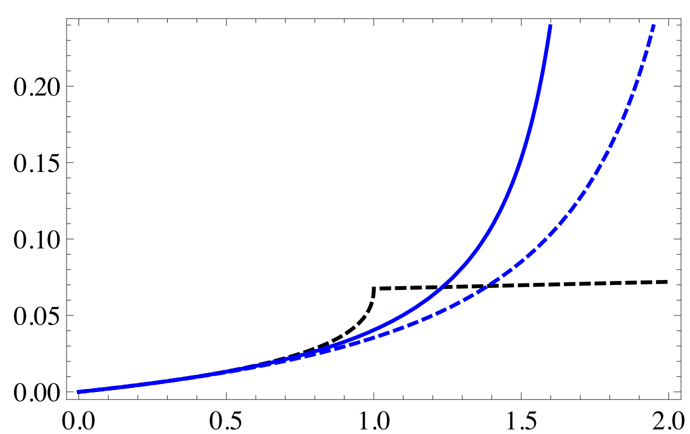
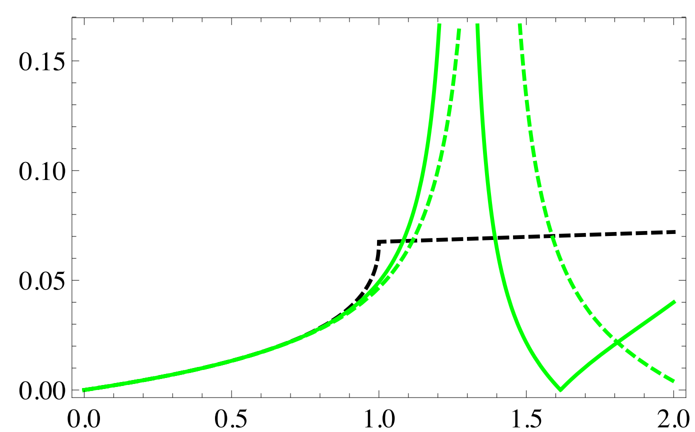
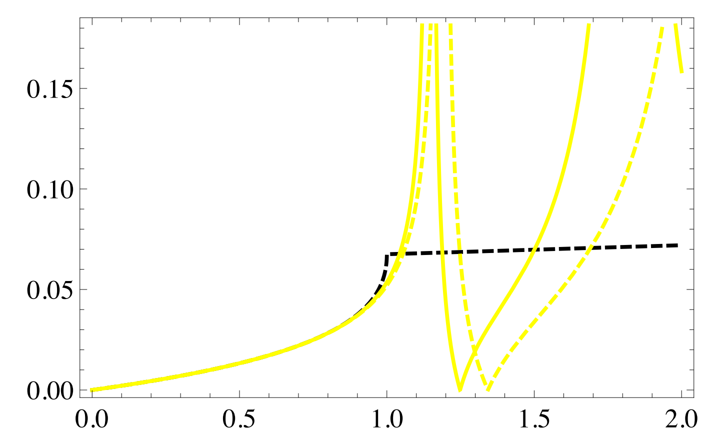
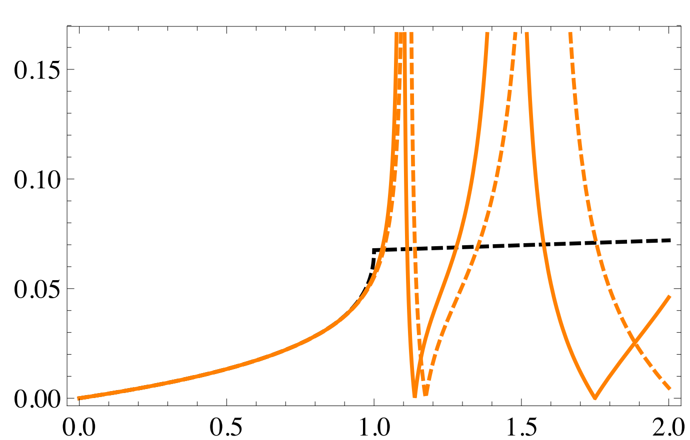
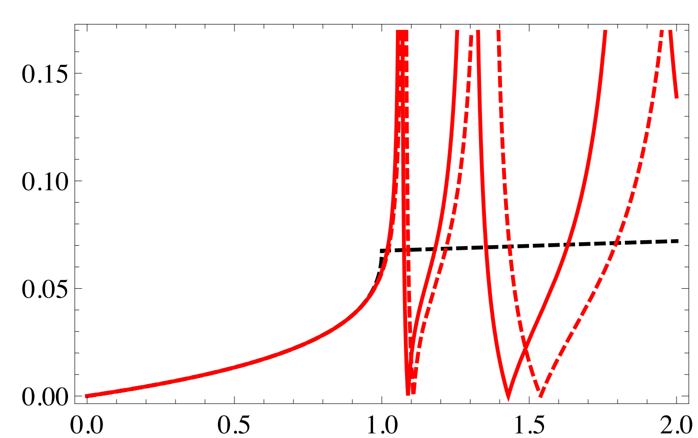










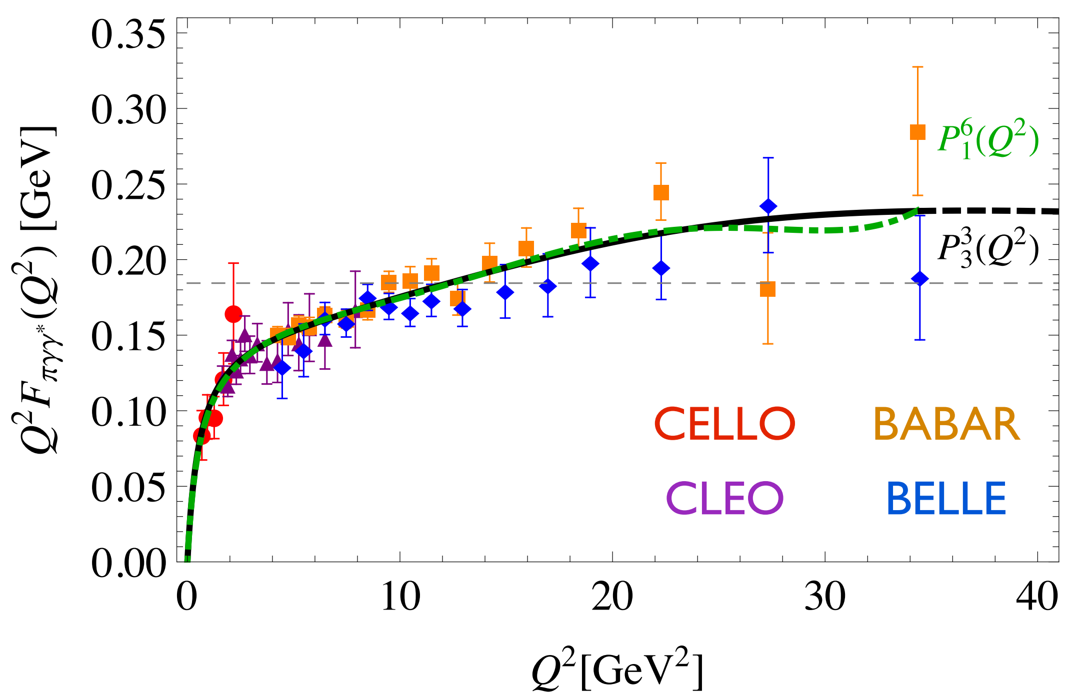
Realistic examples: context of (g-2)$_\mu$

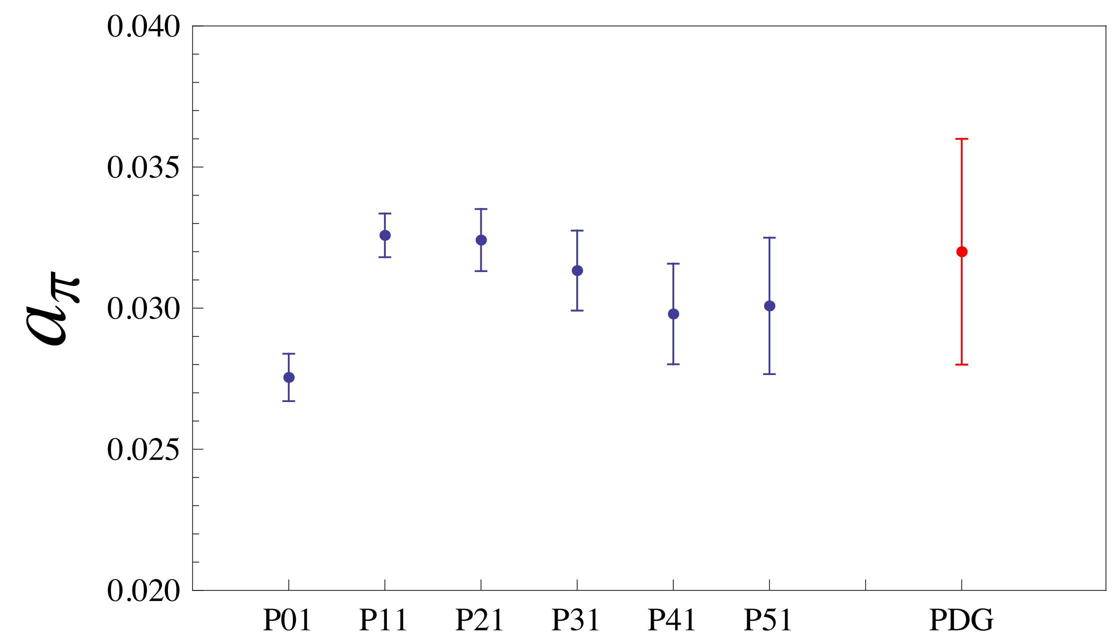
[P.M.'12; P.M., M.Vanderhaeghen'12; R. Escribano, P.M., P. Sanchez-Puertas, '13, '15]

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'09 and Belle'12

$P_1^N(Q^2)$ up to N=5 [P.M, '12]

π^0-TFF





(imposing high-energy QCD)

$P_N^N(Q^2)$ up to N=3

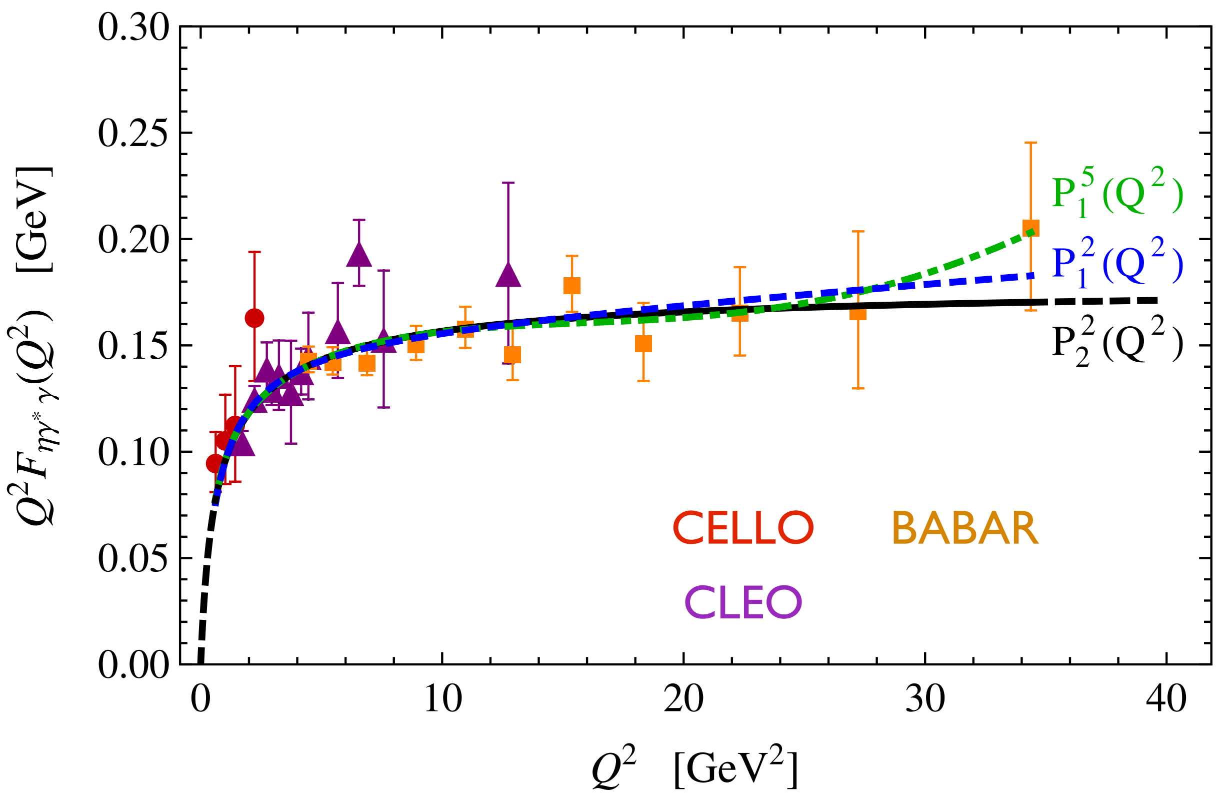
Accurate description of the low-energy region making full use of available experimental data

η-TFF & η'-TFF

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'11+ $\Gamma_{\eta\to\gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, '13]

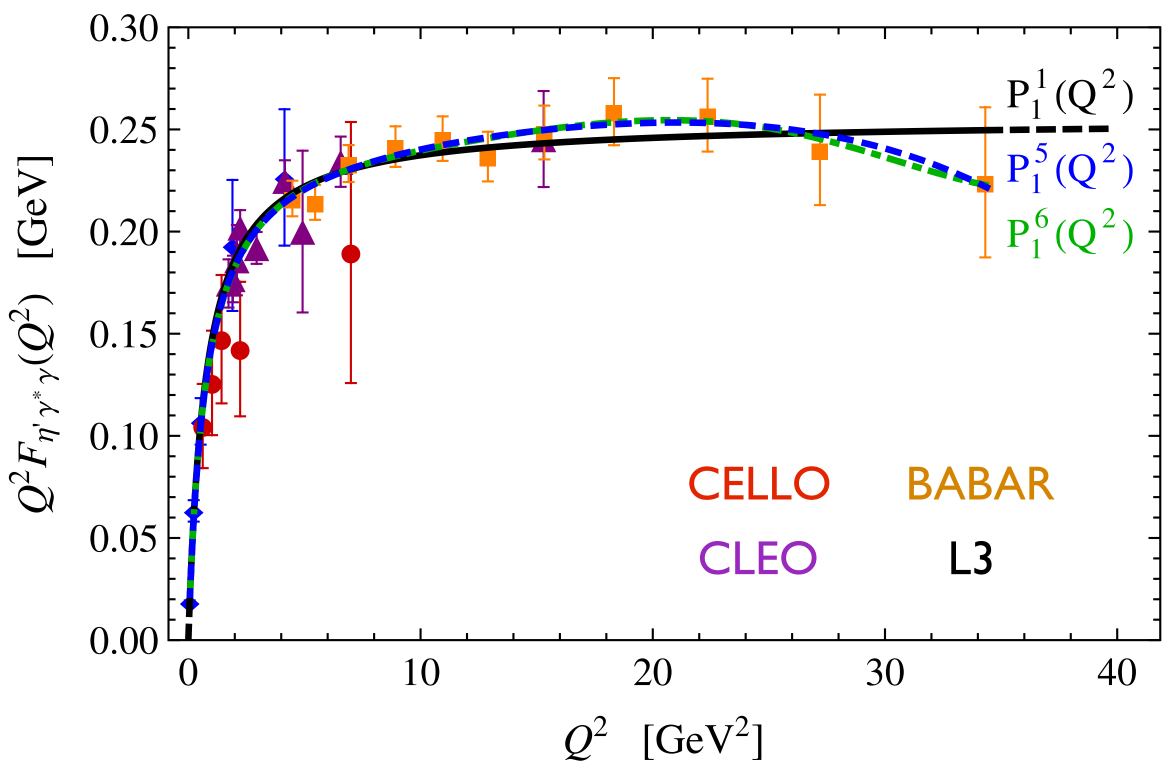
$P_1^N(Q^2)$ up to N=4



$P_N^N(Q^2)$ up to N=2

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2,0) = 0.160(24)\mathrm{GeV}$$

up to N=5



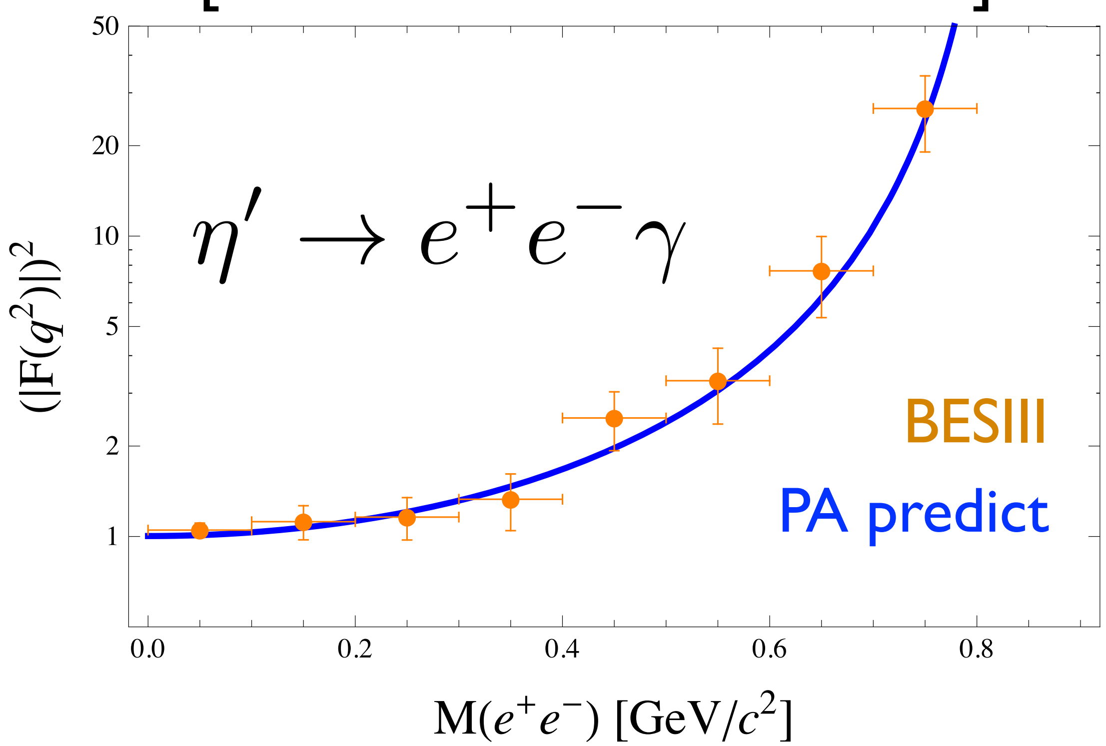
$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma*\gamma}(Q^2,0) = 0.254(4)GeV$$

time-like TFF

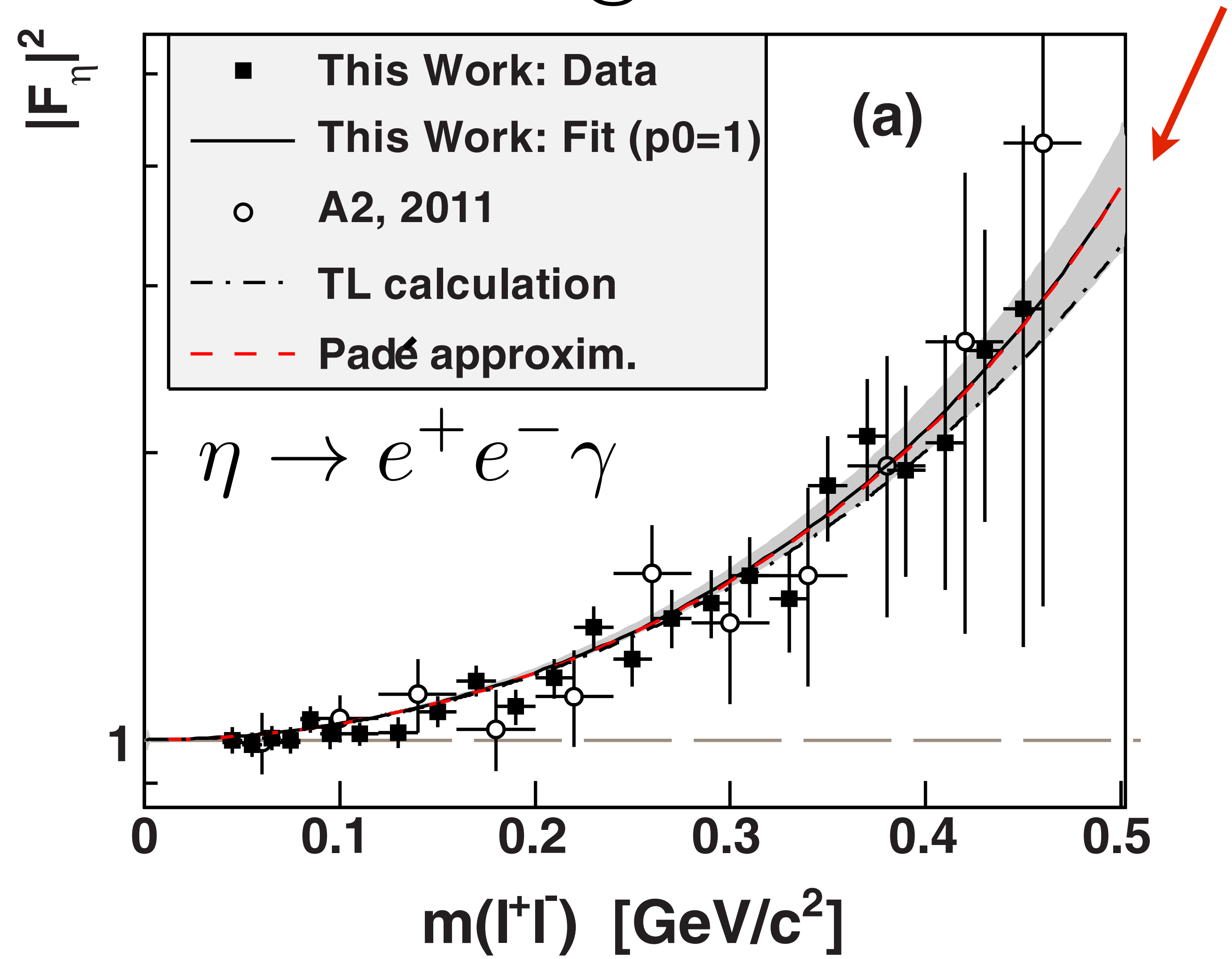
Predictive method!

- Study Dalitz decays $\eta(')\rightarrow\gamma^*\gamma\rightarrow e^+e^-\gamma$
- Prediction of the time-like from space-like data

[BESIII Coll. arXiv:1504.06016]



A2@MAMI

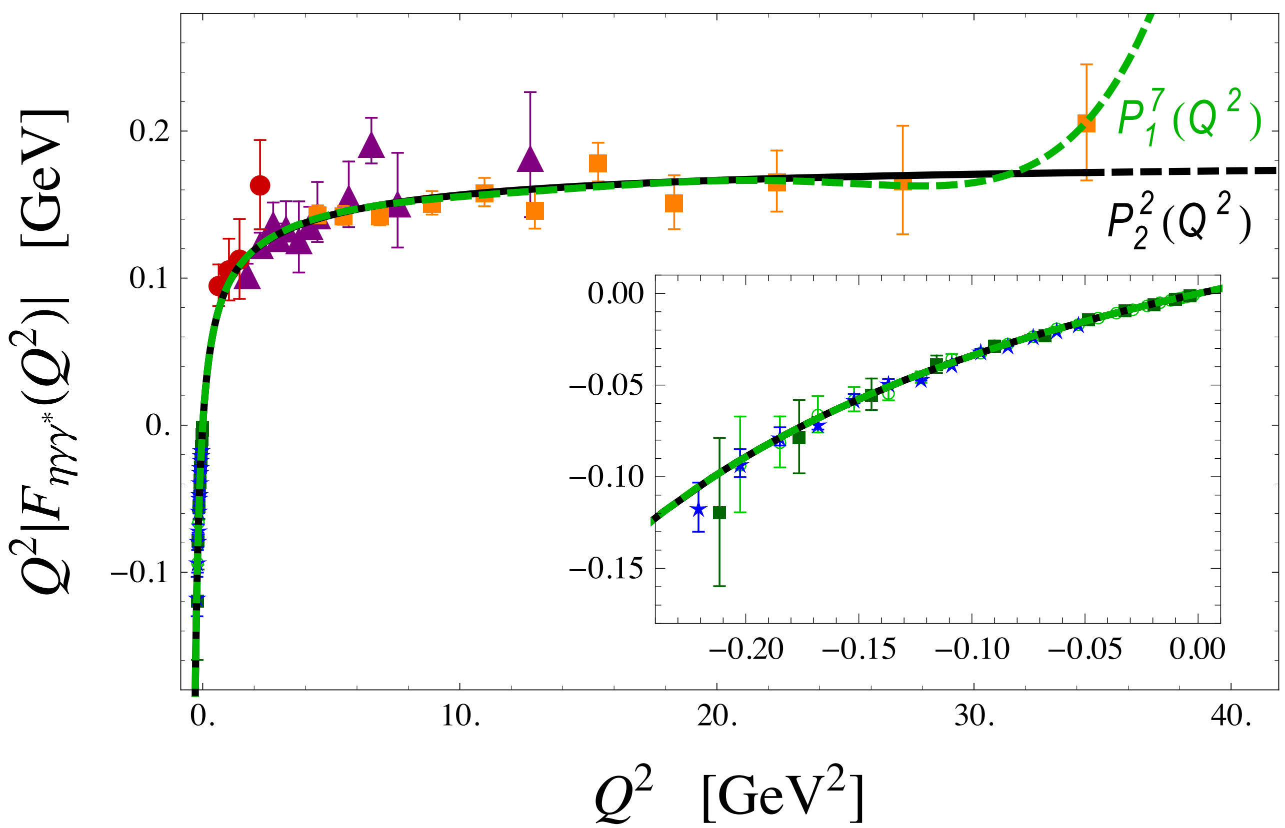


[A2 Coll. PRC89 2014]

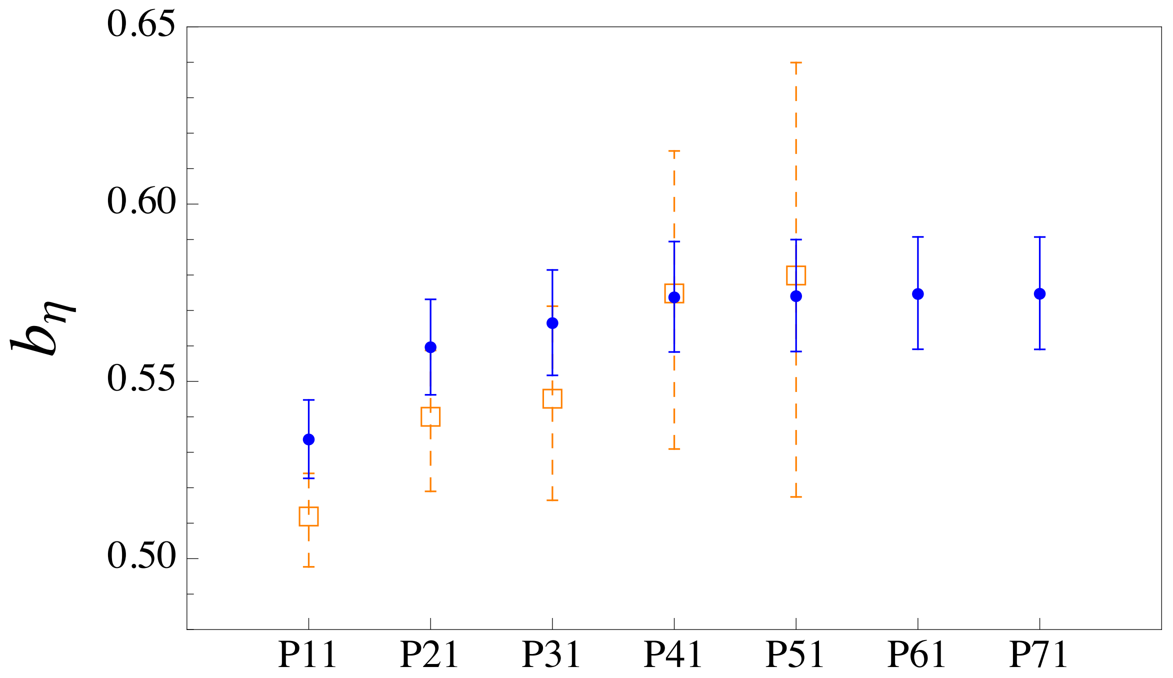
η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+ $\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

[R.Escribano, P.M., P. Sanchez-Puertas, '15]



$P_1^N(Q^2)$ up to N=7



$P_N^N(Q^2)$ up to N=2

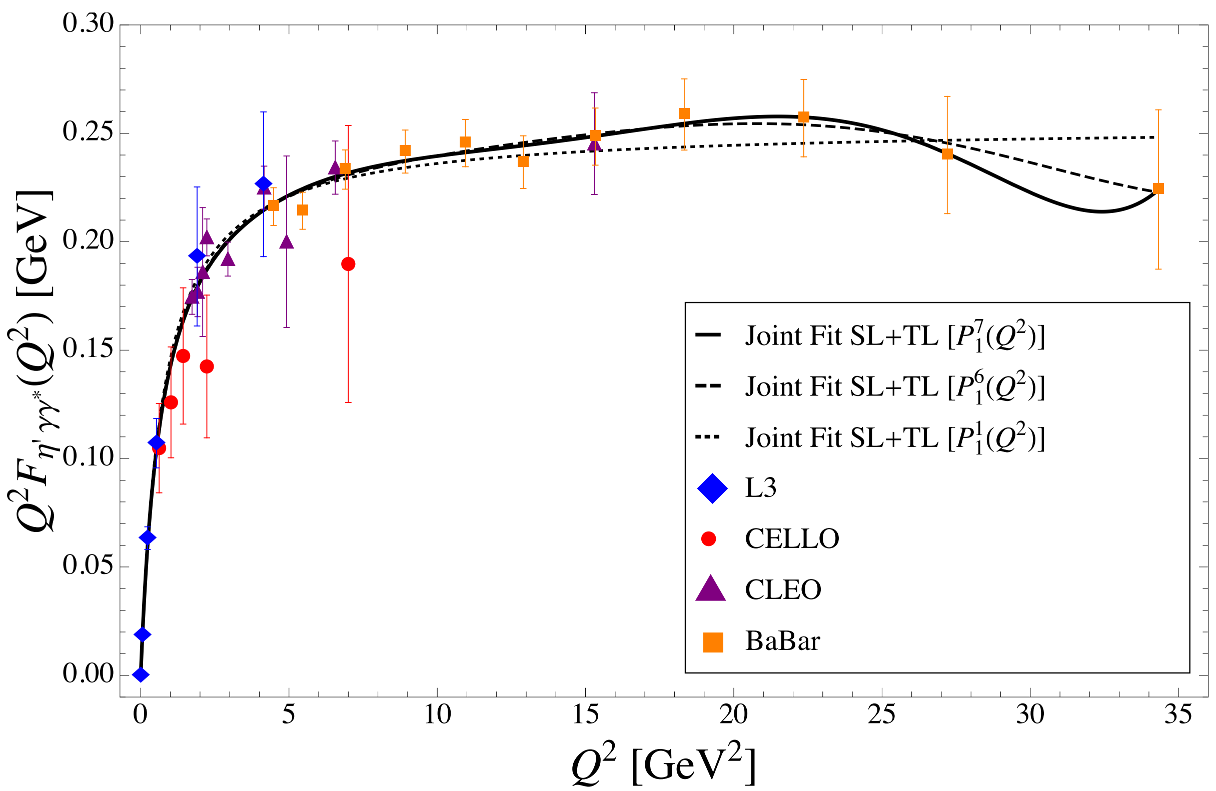
$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2,0) = 0.177(15)GeV$$

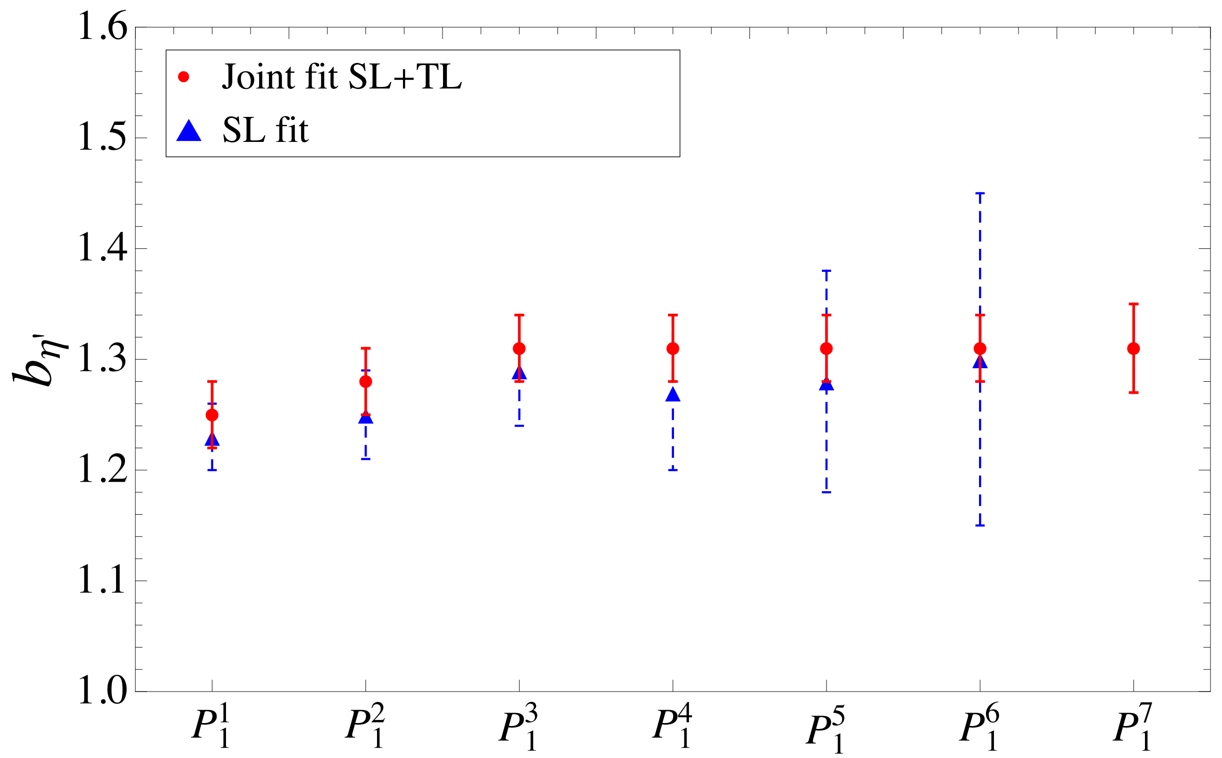
η’-TFF

Fit to Space-like data [CELLO’91, CLEO’98, BABAR’11]+ $\Gamma_{\eta'\to\gamma\gamma}$
+ Time-like data [BESIII]

[R.Escribano, S. Gonzalez-Solis, P.M., P. Sanchez-Puertas, ’15]

$P_1^N(Q^2)$ up to N=7



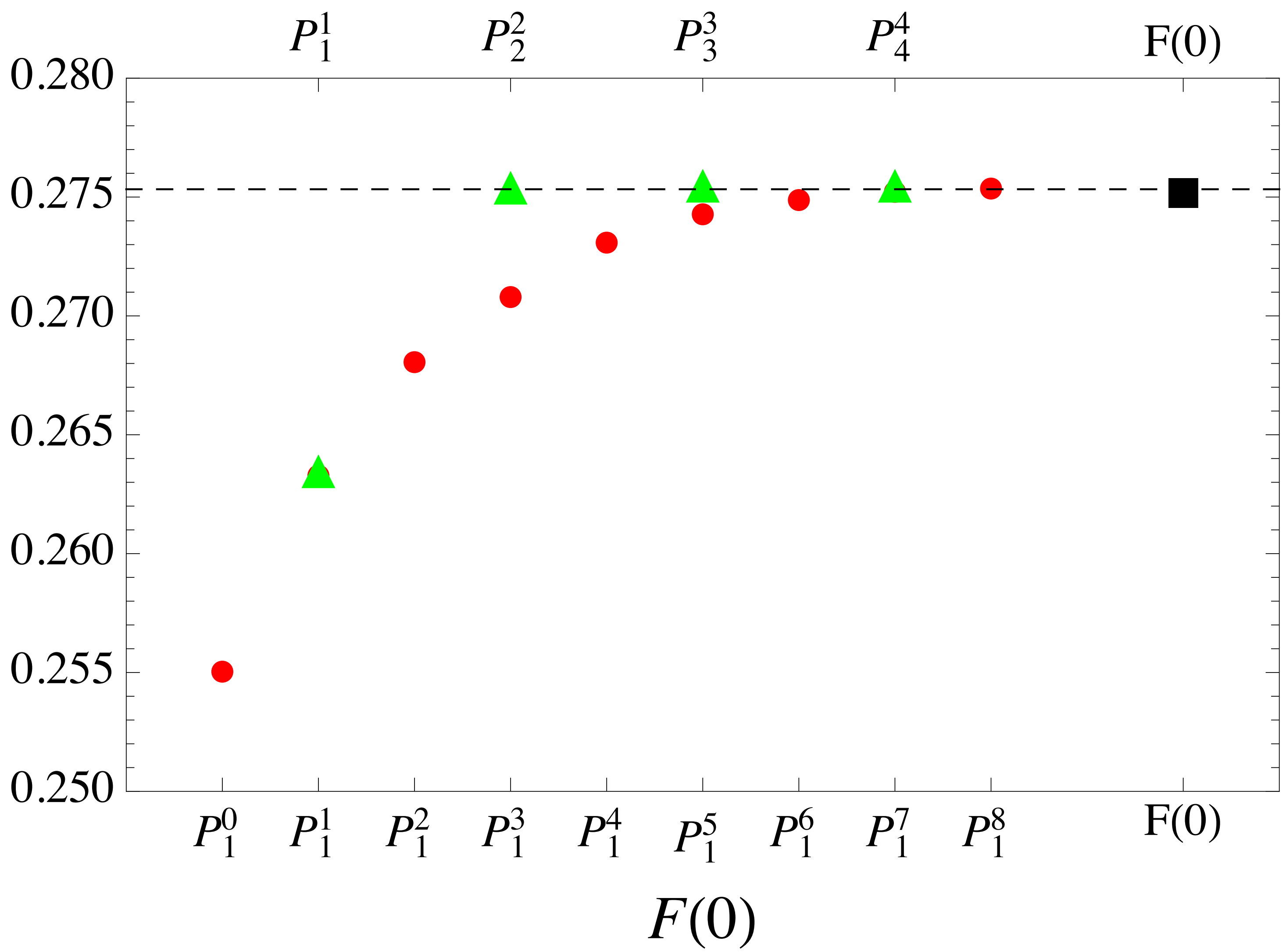


Crucial to extract the most precise η-η’ mixing

[R.Escribano, S. Gonzalez-Solis, P.M., P. Sanchez-Puertas, ’15]

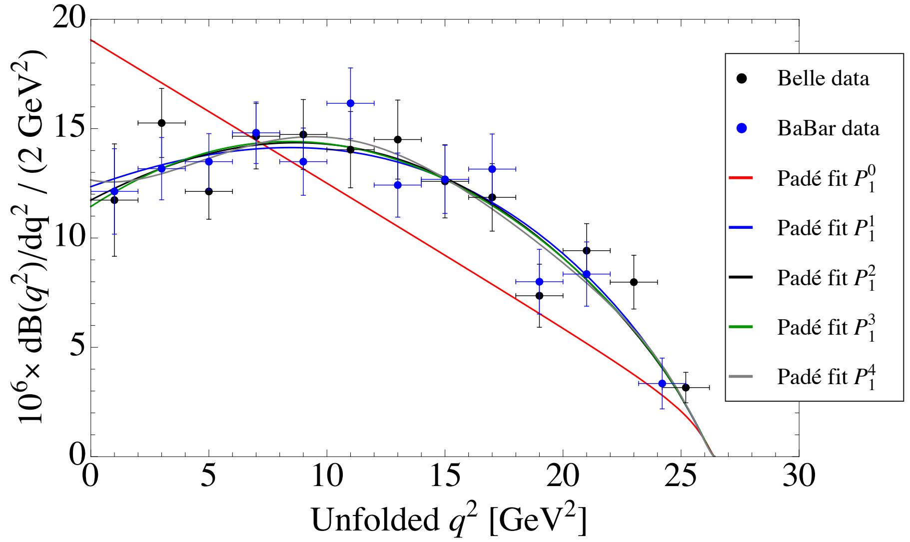
A word on systematics

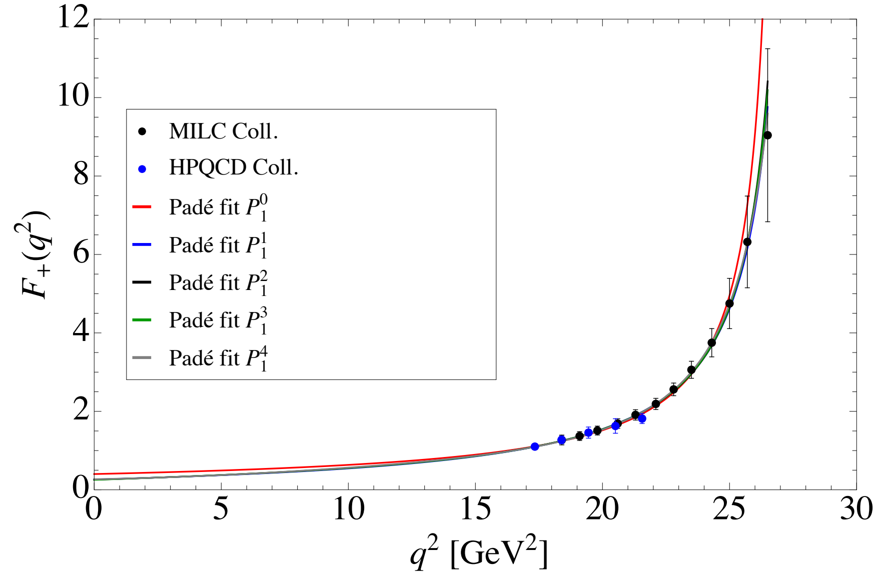
- Consider a model for FF
- Generate a pseudodata set emulating the physical situation
- Build up your PA sequence
- Fit and compare



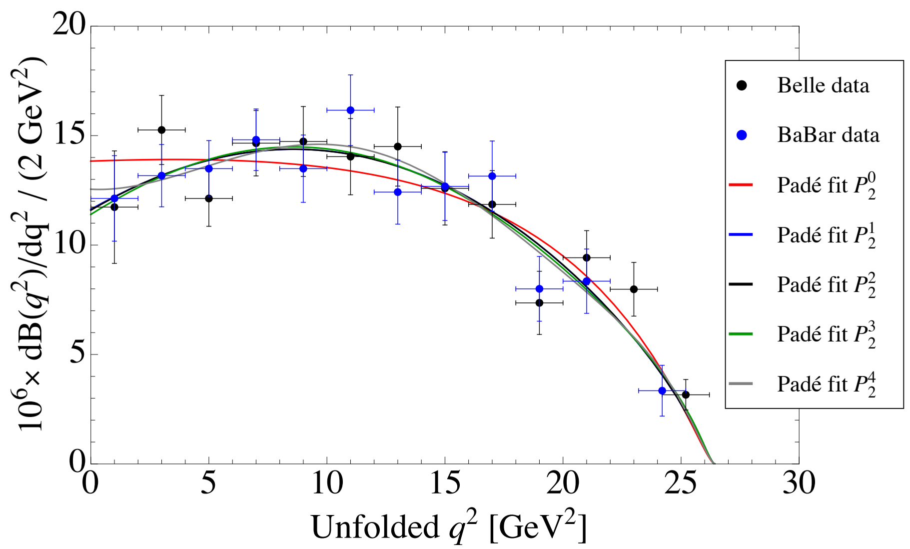
$B \to \pi \ell \nu_\ell$ FF from lattice+experiment

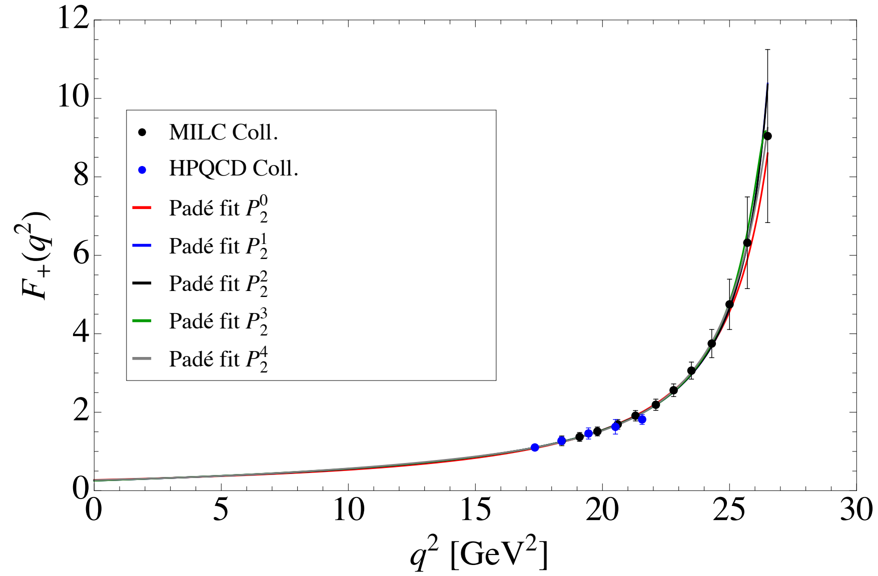
$P_1^N(q^2)$



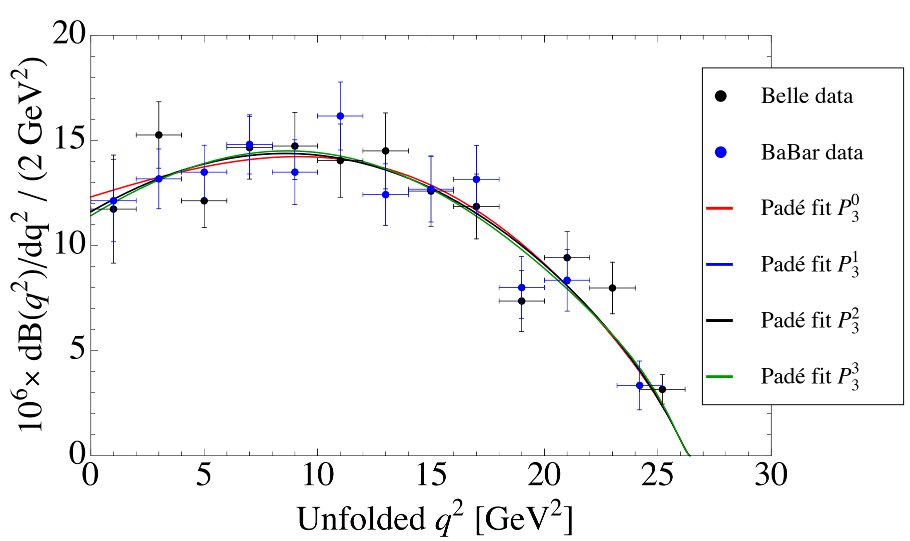


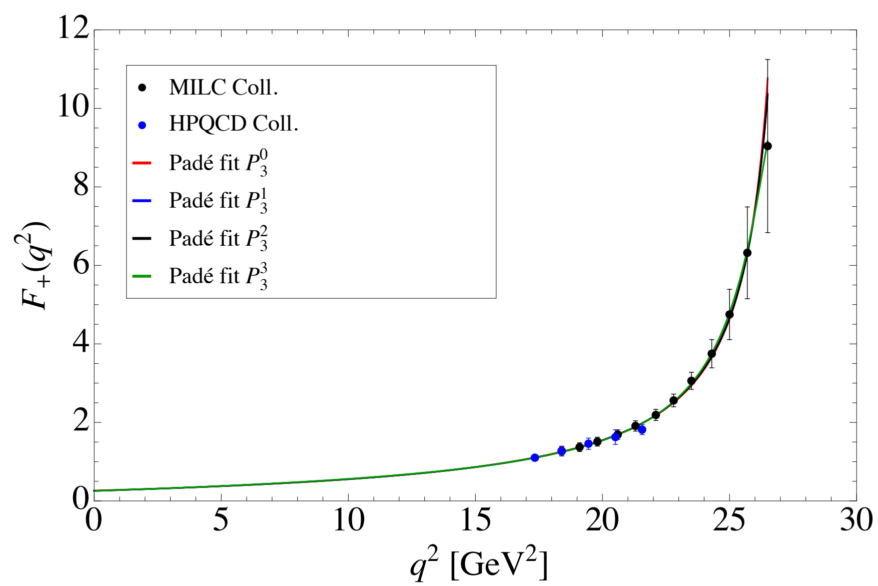
$P_2^N(q^2)$



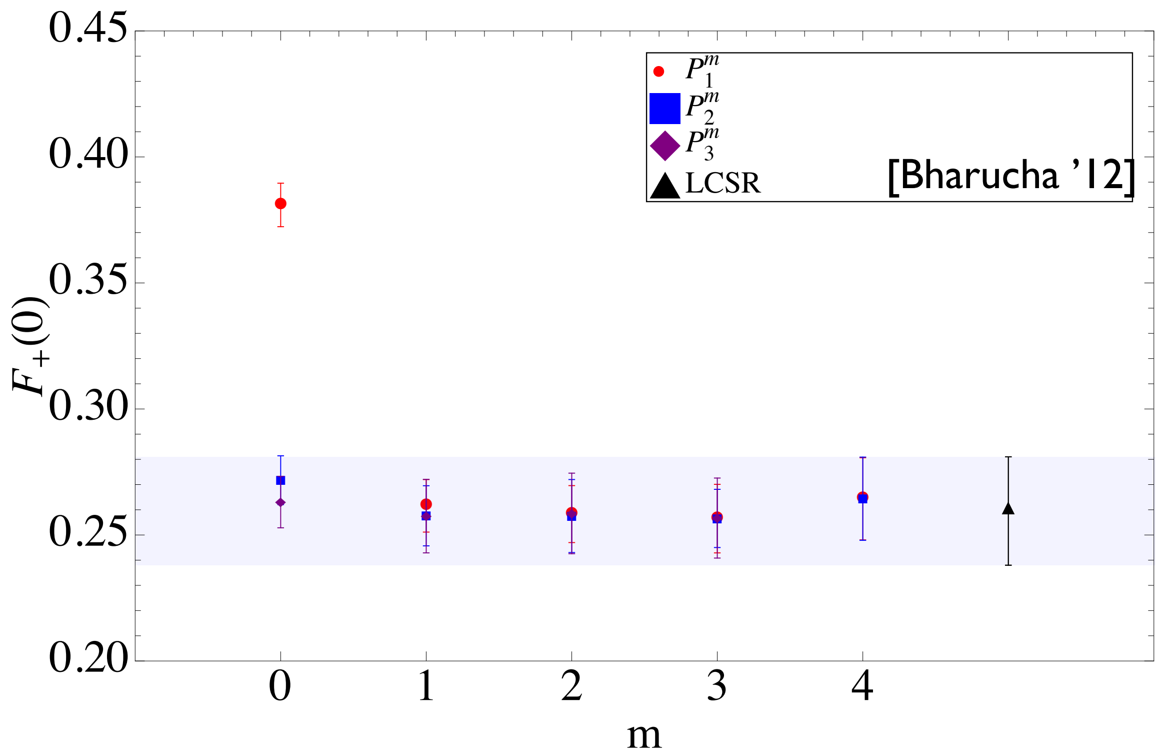


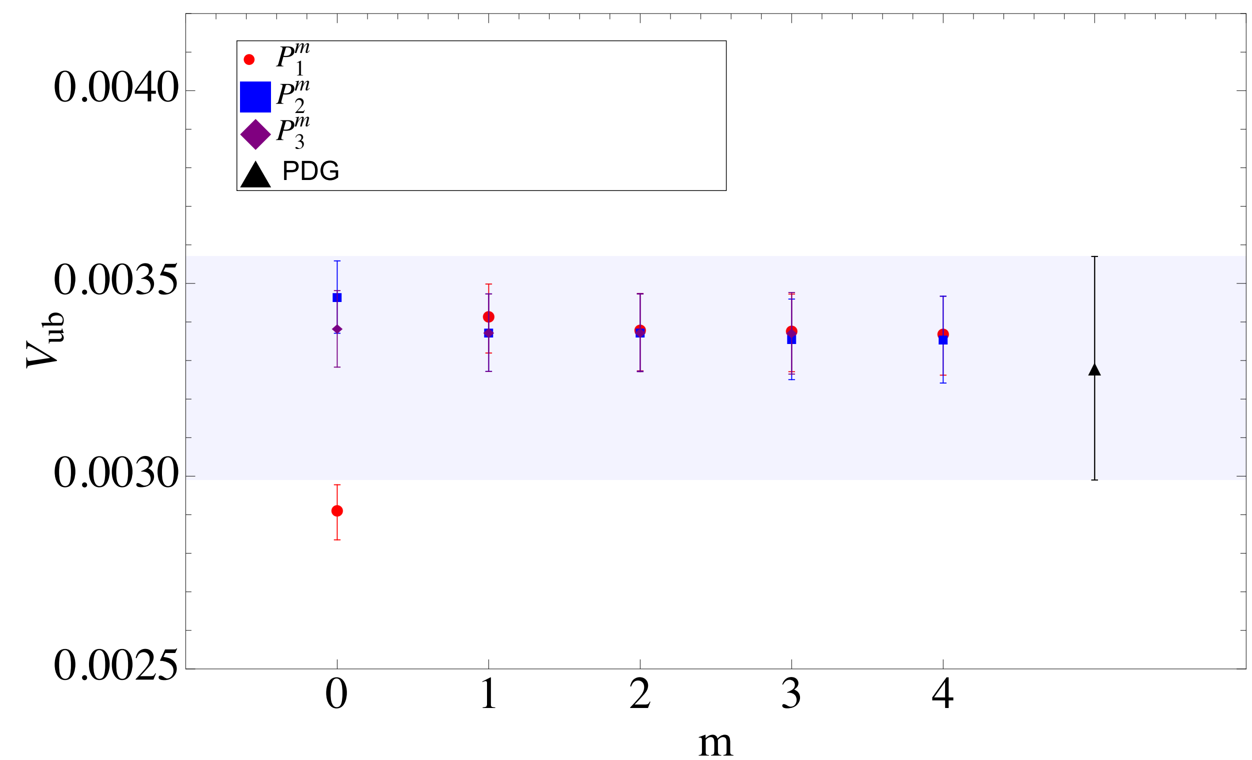
$P_3^N(q^2)$



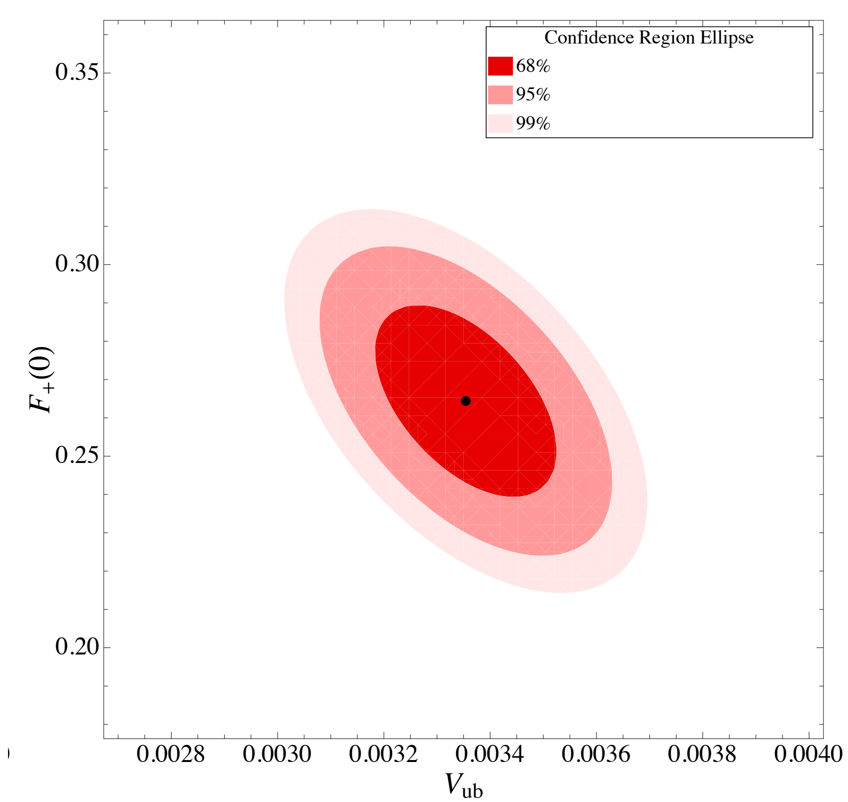


A realistic (preliminary) example





(as a Ref. Minireview from PDG)



$$F_+(0) = 0.264(16)(7)$$
$$|V_{ub}| = 3.37(11)(2)\times 10^{-3}$$

Improvements:

- Update of inputs (new lattice)
- Correlations
- Include Padé-Type approach
- Provide derivatives at $q^2 = 0$

Non-leptonic B decays using Quadratic Approximants

Role of FF in Non-leptonic B decays:

Edward, Mannel, van Dyk, Khodjamirian, Roig, Magalhães...

Quadratic Approximants

Padé approx: $Q(z)f(z) + R(z) = \mathcal{O}\left(z^{q+r+1}\right)$

Quadratic approx: $Q(z)f^2(z) + 2R(z)f(z) + S(z) = \mathcal{O}\left(z^{q+r+s+2}\right)$

$R(z), S(z), Q(z)$ are polynomials

$$\mathbb{Q}_q^{r,s}(z) = \frac{-R(z) \pm \sqrt{[R(z)]^2 - Q(z)S(z)}}{Q(z)}$$
$$= \frac{-S(z)}{R(z) \pm \sqrt{[R(z)]^2 - Q(z)S(z)}}$$

- When S(z)=0, $\mathbb{Q}_q^{r,s}(s) \rightarrow P_q^r(z)$
- Lowest order ~ Breit-Wigner param.
- If info about poles, threshold, LEPs is known, easy to implement
- Satisfy Disp. Rel.
- Relation with z-param.

Quadratic Approximants

[S. González-Solís, PM, P. Sanchez-Puertas, in prep]

Quadratic approx: $Q(z)f^2(z) + 2R(z)f(z) + S(z) = \mathcal{O}\left(z^{q+r+s+2}\right)$

$R(z), S(z), Q(z)$ are polynomials

General form for the Q[1,1,1]:

$$\mathbb{Q}_1^{1,1}(z) = \frac{-(R_0 + R_1 z) \pm \sqrt{[R_0 + R_1 z]^2 - (Q_0 + Q_1 z)(S_0 + S_1 z)}}{(Q_0 + Q_1 z)}$$

We need to solve the equation:

$$(Q_0 + Q_1 z)[f(z)]^2 + 2(R_0 + R_1 z)f(z) + (S_0 + S_1 z) = \mathcal{O}(z^5)$$

z-param $\Leftrightarrow$ $z = \mathbb{Q}_1^{1,1}$ with $R_0 = -1, S_0 = Q_0 = 0, R_1 = S_1 = Q_1 = 1/2$

z-param can be generalized! (correct high-energy, above-threshold poles...)

Quadratic Approximants

[S. González-Solís, PM, P. Sanchez-Puertas, in prep]

Quadratic approx: $Q(z)f^2(z) + 2R(z)f(z) + S(z) = \mathcal{O}\left(z^{q+r+s+2}\right)$

$R(z), S(z), Q(z)$ are polynomials

Example: $f(z) = \frac{1}{z}\log(1-z)$

We need to solve the equation:

$$(Q_0 + Q_1 z)[f(z)]^2 + 2(R_0 + R_1 z)f(z) + (S_0 + S_1 z) = \mathcal{O}(z^5)$$

Two solutions:

$$\mathbb{Q}_1^{1,1}(z) = \frac{(\frac{7}{8} - \frac{5}{12}z) \pm \sqrt{[-\frac{7}{8} + \frac{5}{12}z]^2 - (1 - \frac{13}{12}z)(-\frac{11}{4} + \frac{1}{24}z)}}{(1 - \frac{13}{12}z)}$$

Quadratic Approximants

[S. González-Solís, PM, P. Sanchez-Puertas, in prep]

Quadratic approx: $Q(z) f^2(z) + 2R(z) f(z) + S(z) = \mathcal{O}\left(z^{q+r+s+2}\right)$

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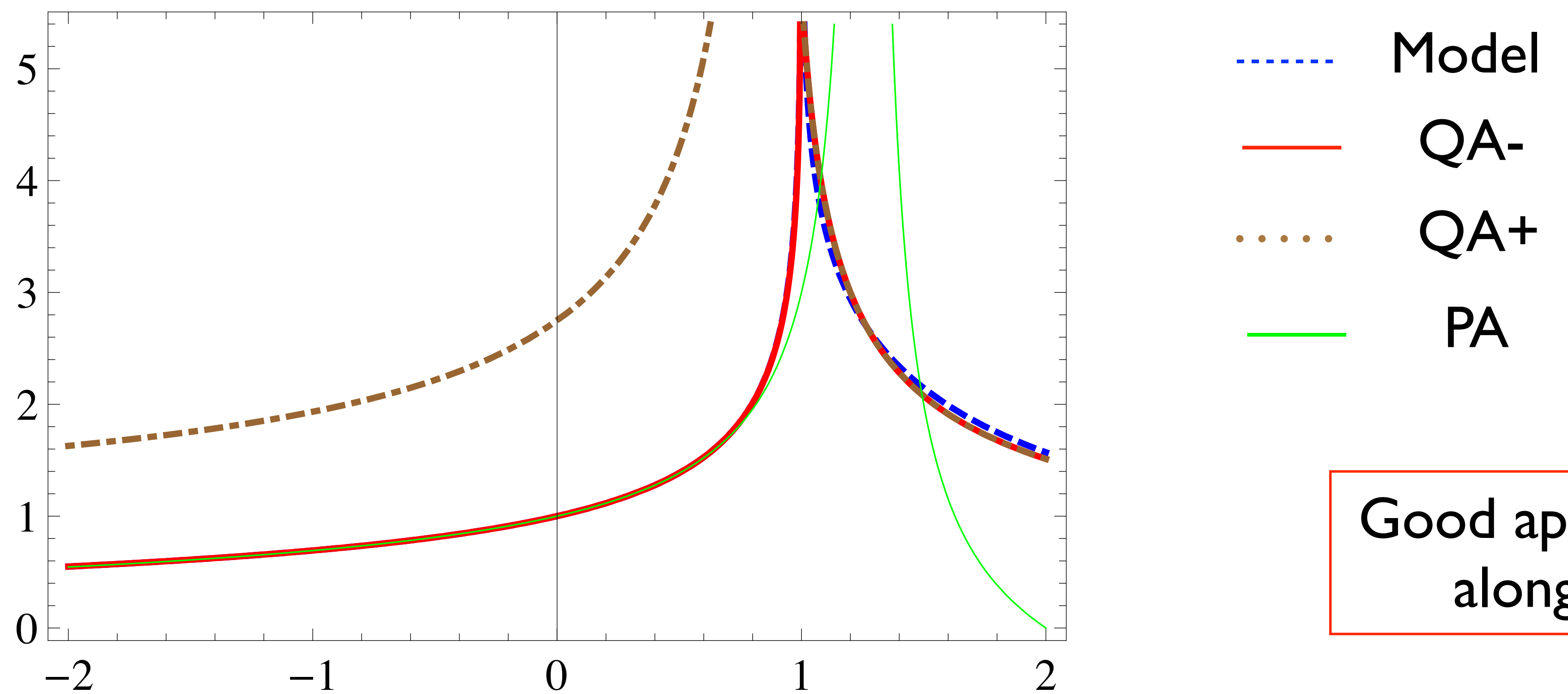


Good approach even along the cut

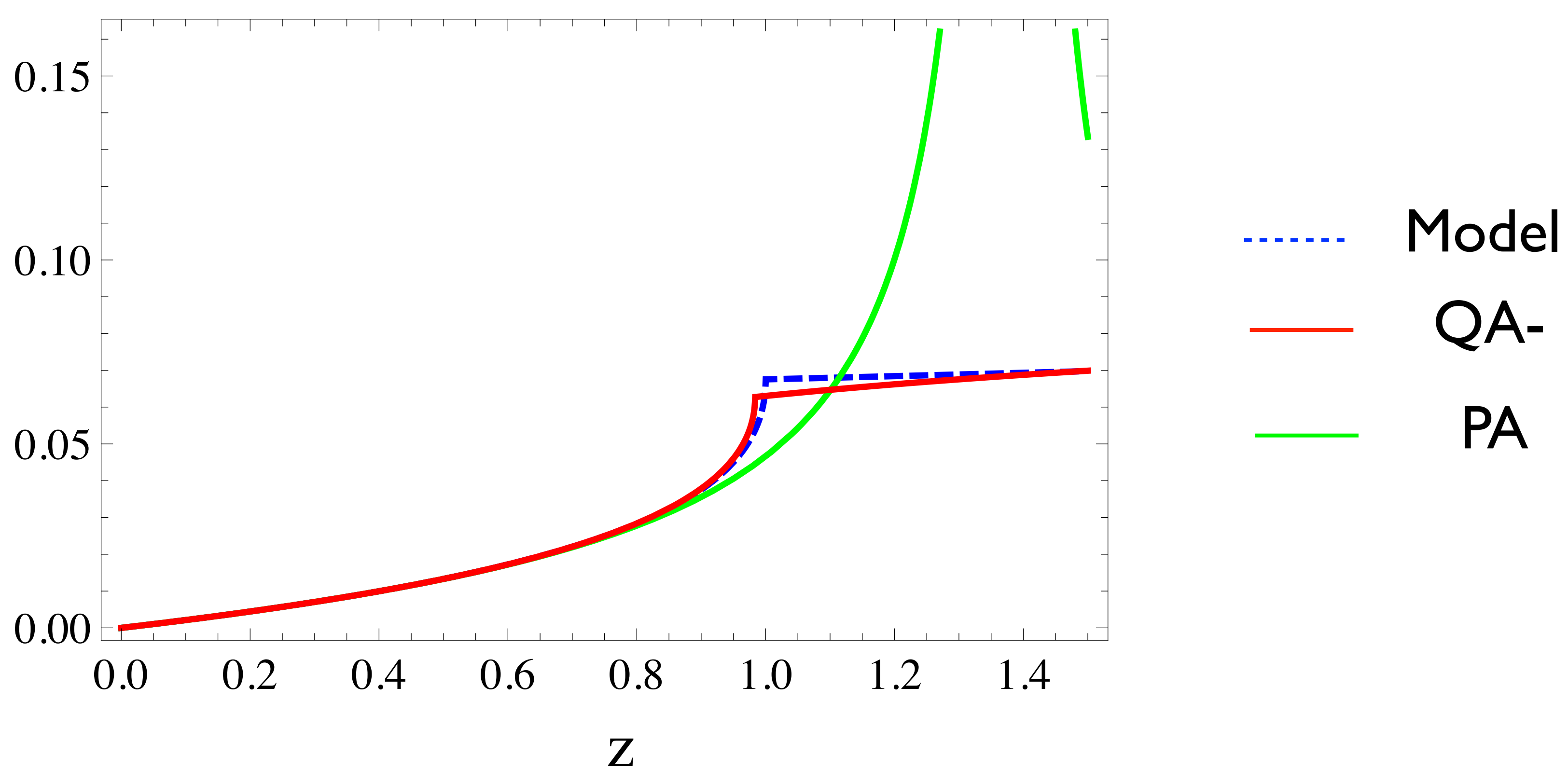
Quadratic Approximants

Quadratic approx: $$Q(z)f^2(z)+2R(z)f(z)+S(z)=\mathcal{O}\left(z^{q+r+s+2}\right)$$

$R(z), S(z), Q(z)$ are polynomials

Example: vacuum polarization function

$$\Pi^{(0)}(z)=\frac{3}{16\pi^2}\left(\frac{4}{3z}+\frac{20}{9}-\frac{4(1-z)(2z+1)G(z)}{3z}\right)$$



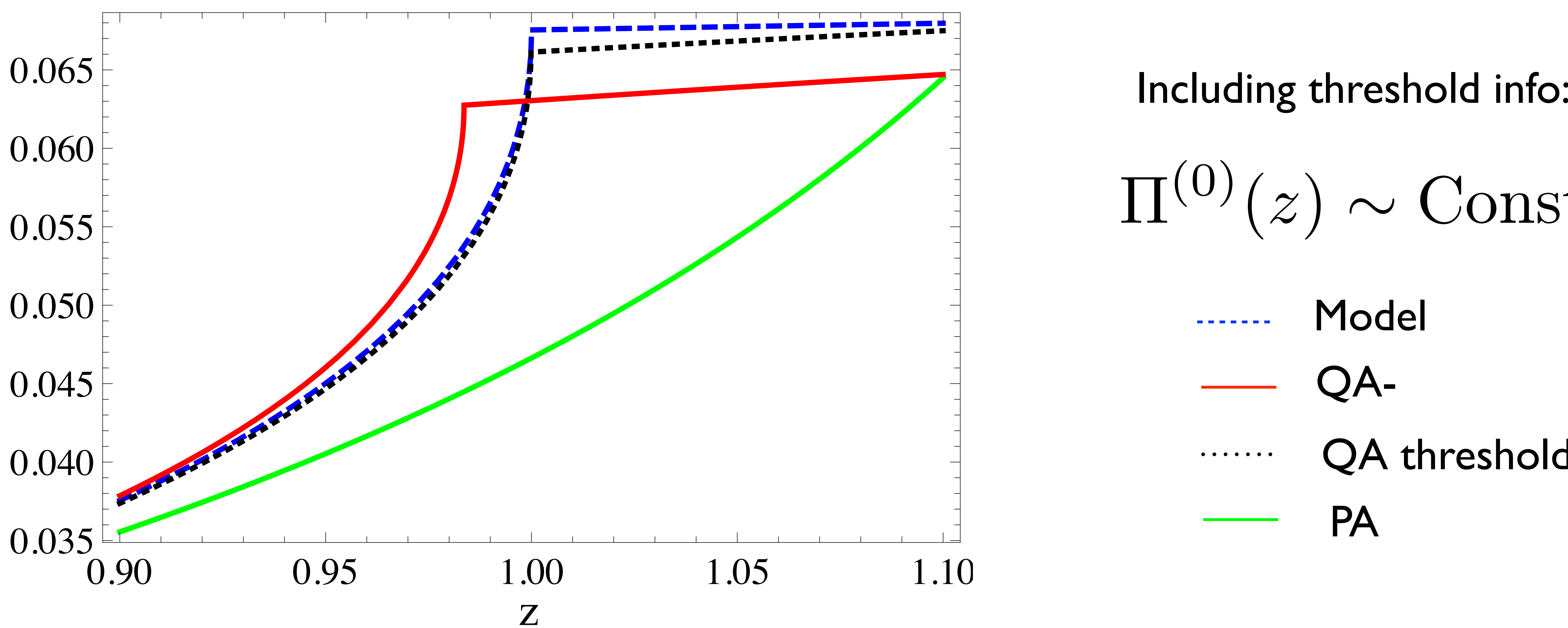
Quadratic Approximants

Quadratic approx: $Q(z)f^2(z) + 2R(z)f(z) + S(z) = \mathcal{O}\left(z^{q+r+s+2}\right)$

$R(z), S(z), Q(z)$ are polynomials

Example: vacuum polarization function

$$\Pi^{(0)}(z) = \frac{3}{16\pi^2}\left(\frac{4}{3z} + \frac{20}{9} - \frac{4(1-z)(2z+1)G(z)}{3z}\right)$$



Including threshold info:

$$\Pi^{(0)}(z) \sim \text{Const}$$

Quadratic Approximants

Examples

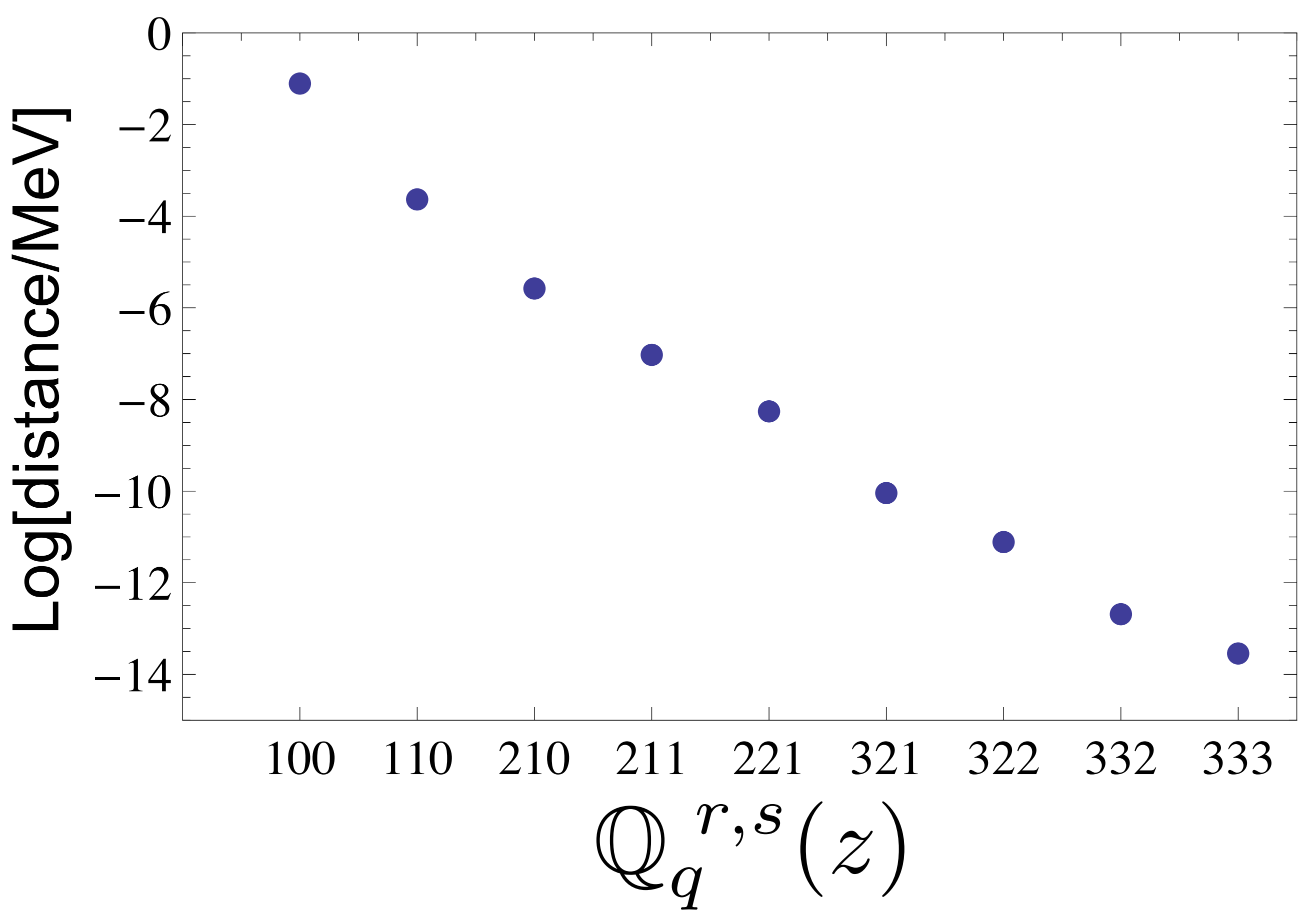
<u>Vector FF model</u>

$$F_V(s) = \frac{M_V^2}{M_V^2 - s + \frac{M_V \Gamma_V s}{\pi M_V^2}\left(-2\sigma(s)^2 - \sigma(s)^3 \ln\left(\frac{\sigma(s)-1}{\sigma(s)+1}\right)\right)}$$

$$m_\pi = 135 \text{ MeV}$$
$$M_V = 770 \text{ MeV}$$
$$\Gamma_V = 150 \text{ MeV}$$

Generate <u>derivatives</u> at 0.6 GeV^2 ⟶ fit $\mathbb{Q}_1^{1,0}(s)\,,\mathbb{Q}_2^{1,1}(s)\ \ldots$

1 pole ↗ 2 poles ↗



If the pole is OK, the phase is OK!

Quadratic Approximants

Examples

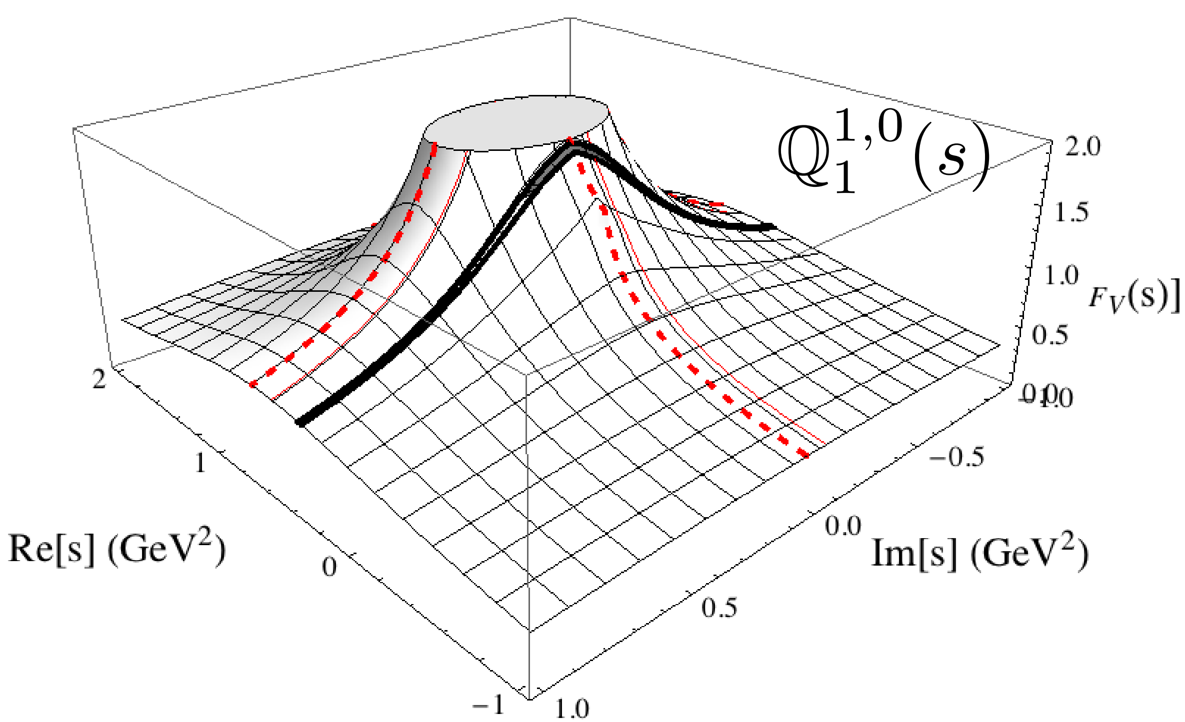
Vector FF model

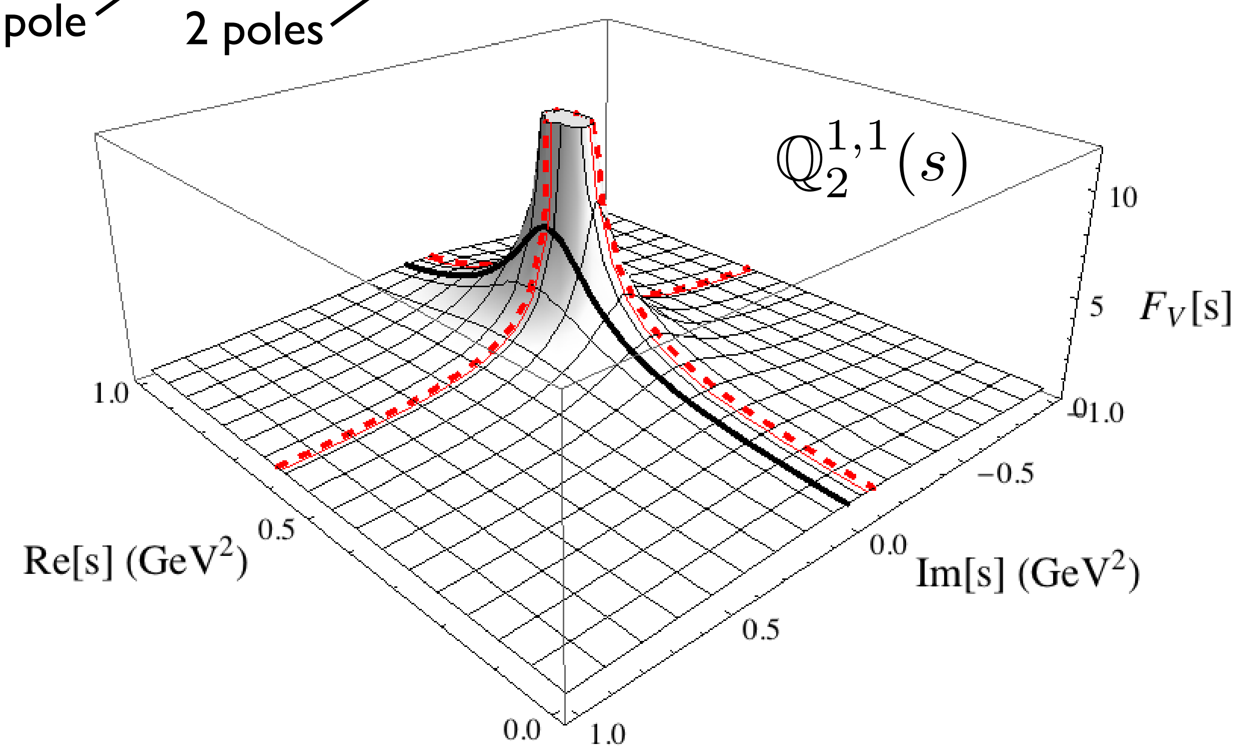
$$F_V(s) = \frac{M_V^2}{M_V^2 - s + \frac{M_V \Gamma_V s}{\pi M_V^2}\left(-2\sigma(s)^2 - \sigma(s)^3 \ln\left(\frac{\sigma(s)-1}{\sigma(s)+1}\right)\right)}$$

$$\begin{aligned} m_\pi &= 135 \text{ MeV} \\ M_V &= 770 \text{ MeV} \\ \Gamma_V &= 150 \text{ MeV} \end{aligned}$$

Generate Pseudodata in Space like ⟶ fit $\mathbb{Q}_1^{1,0}(s)\,,\mathbb{Q}_2^{1,1}(s)\;\ldots$

1 pole ↗ 2 poles ↗





Quadratic Approximants

Examples

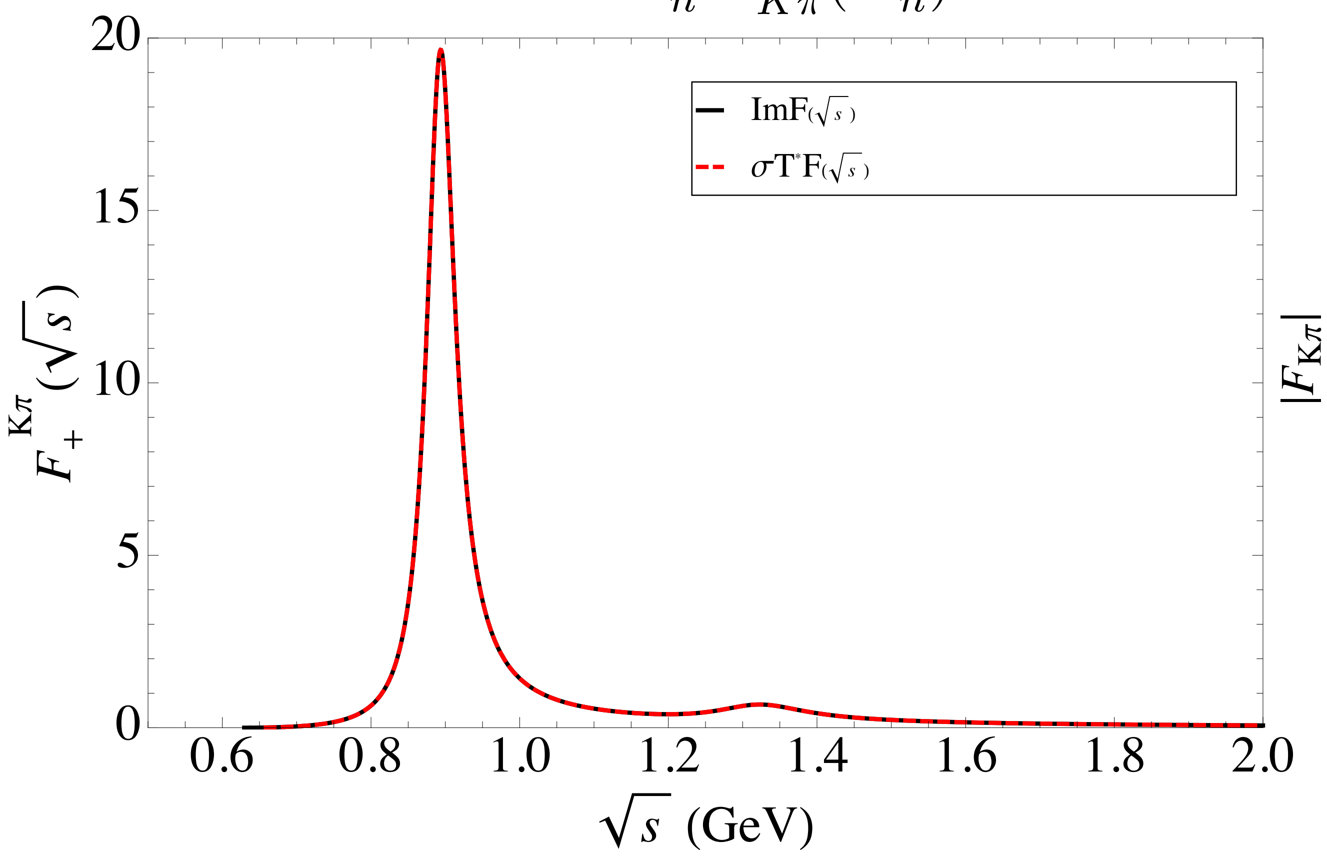
<u>Vector FF model</u> $\tau^- \to K_S \pi^- \nu_\tau$

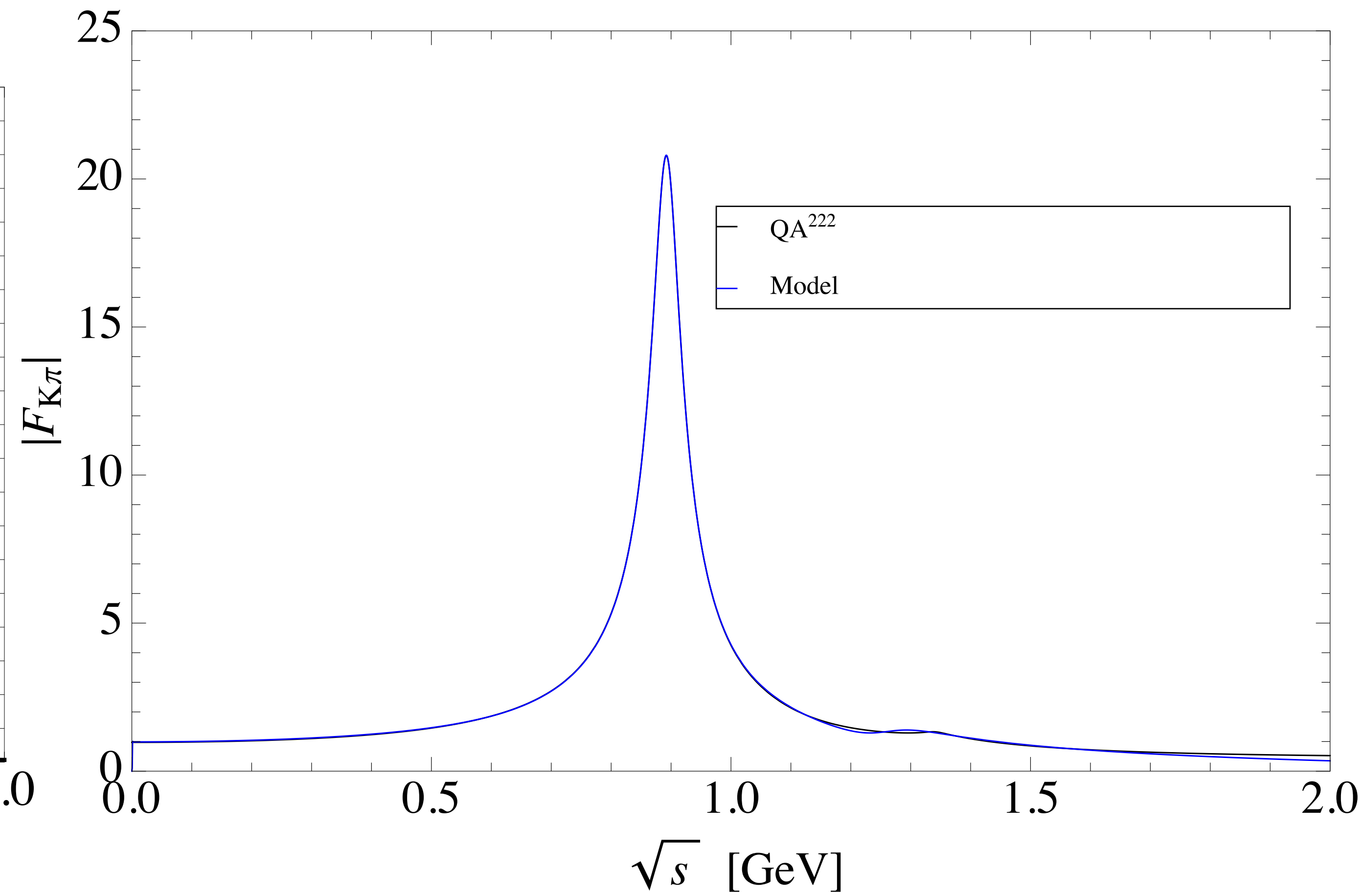
$$f_+^{K\pi}(s) = \left[\frac{m_{K^*}^2 + \gamma s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\gamma s}{D(m_{K^{*\prime}}, \gamma_{K^{*\prime}})} \right] \mathrm{e}^{\frac{3}{2}\mathrm{Re}\widetilde{H}_{K\pi}(s)}$$

$$M_{K^*} = 892\ \mathrm{MeV}\,,\ \Gamma_{K^*} = 46\ \mathrm{MeV}$$
$$M_{K^{*\prime}} = 1304\ \mathrm{MeV}\,,\ \Gamma_{K^{*\prime}} = 171\ \mathrm{MeV}$$

$$D(m_n, \gamma_n) = m_n^2 - s - i\, m_n \gamma_n(s)$$

$$\gamma_n(s) = \gamma_n \frac{s}{m_n^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_n^2)}$$





Quadratic Approximants

Examples

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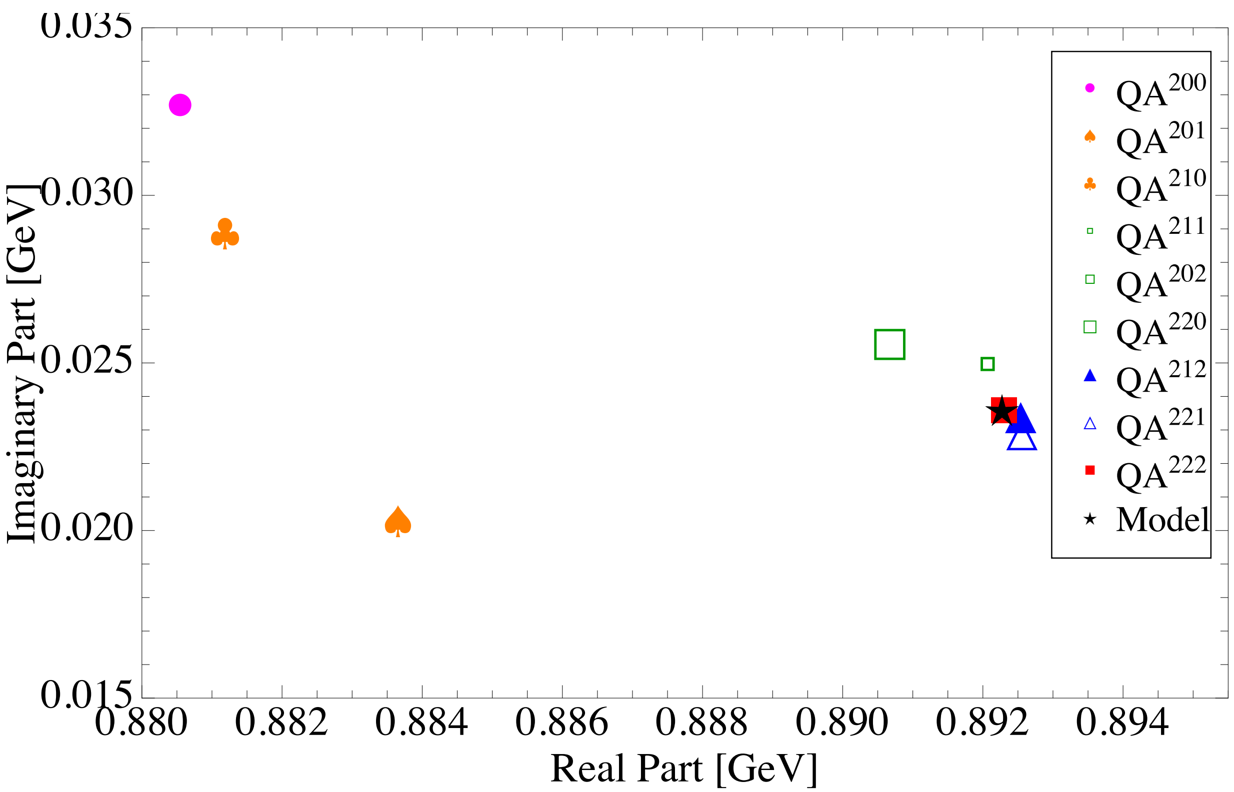
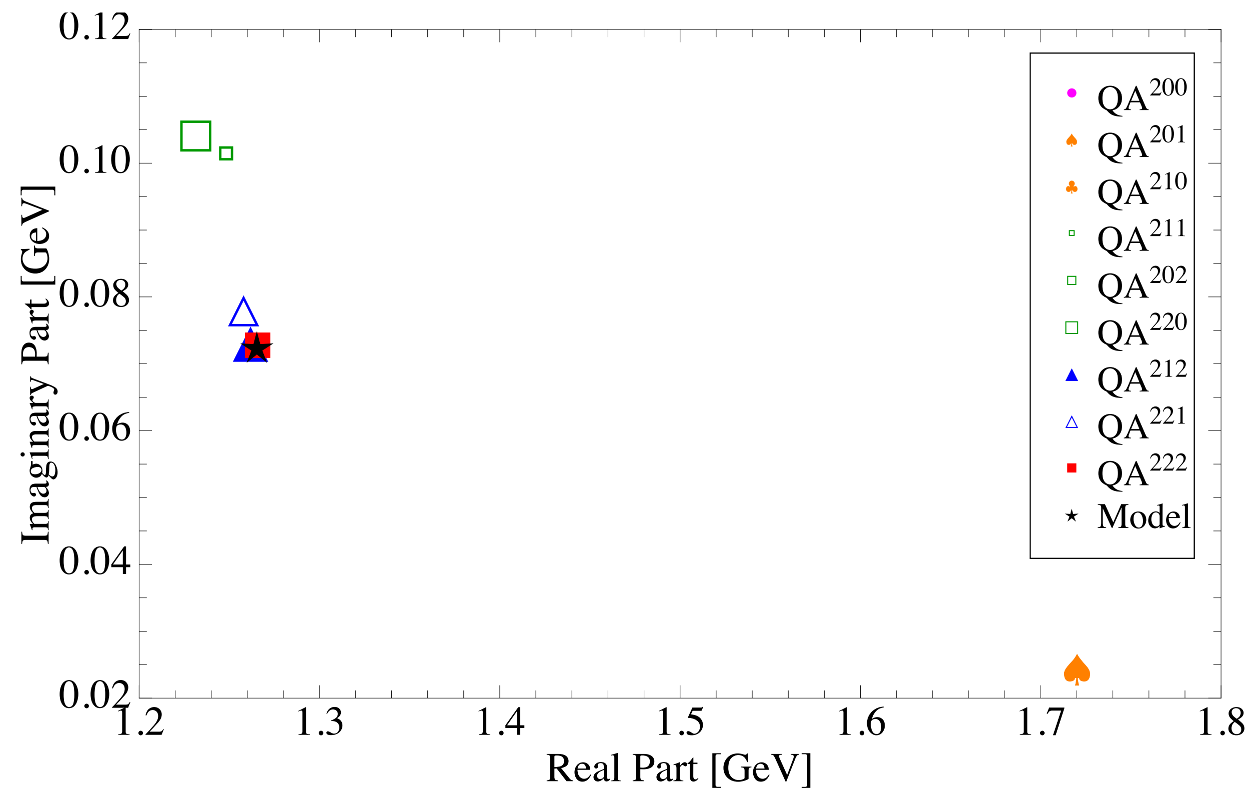
(derivatives)





Quadratic Approximants

Examples

<u>Vector FF model</u> $\tau^- \to K_S \pi^- \nu_\tau$

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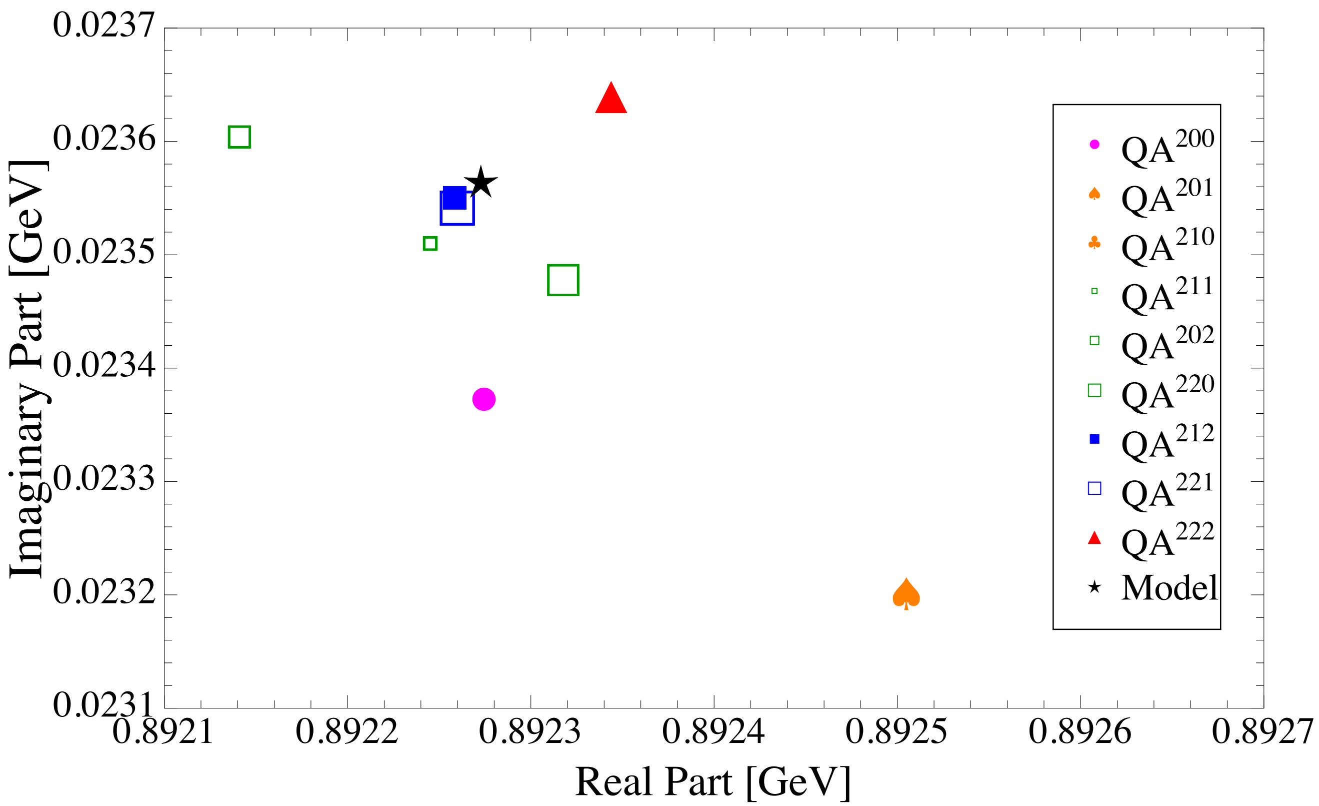
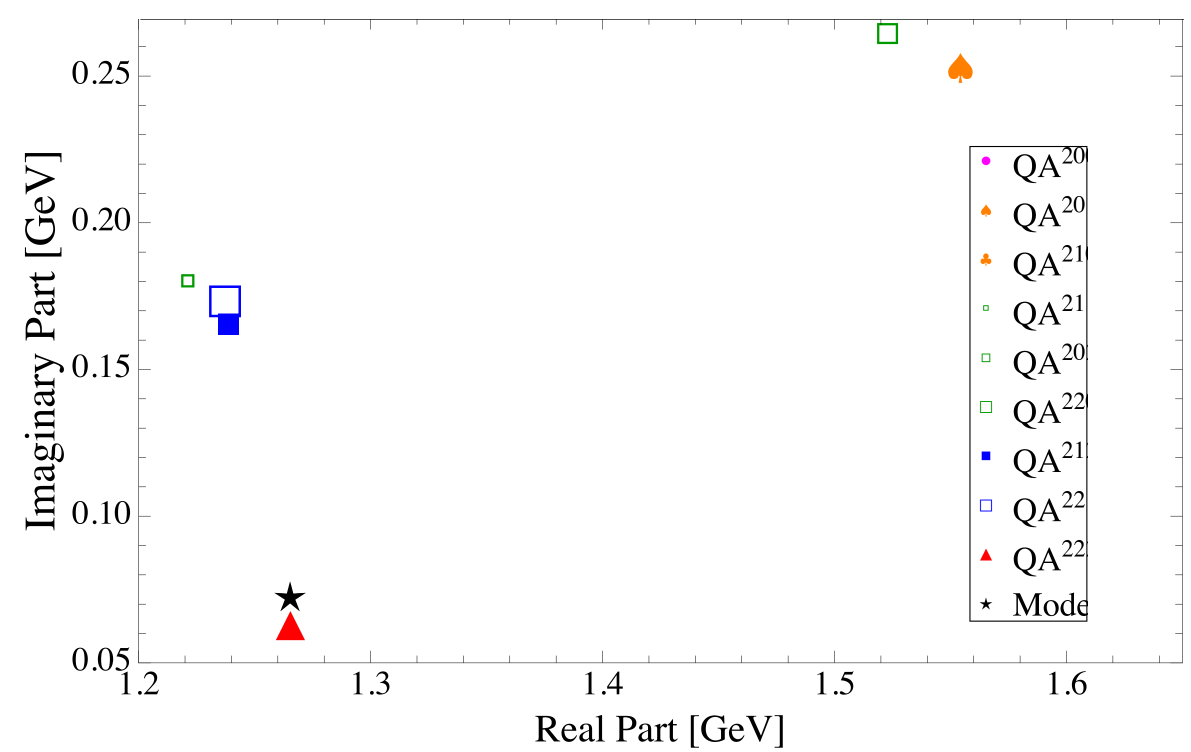
(pseudodata fit)





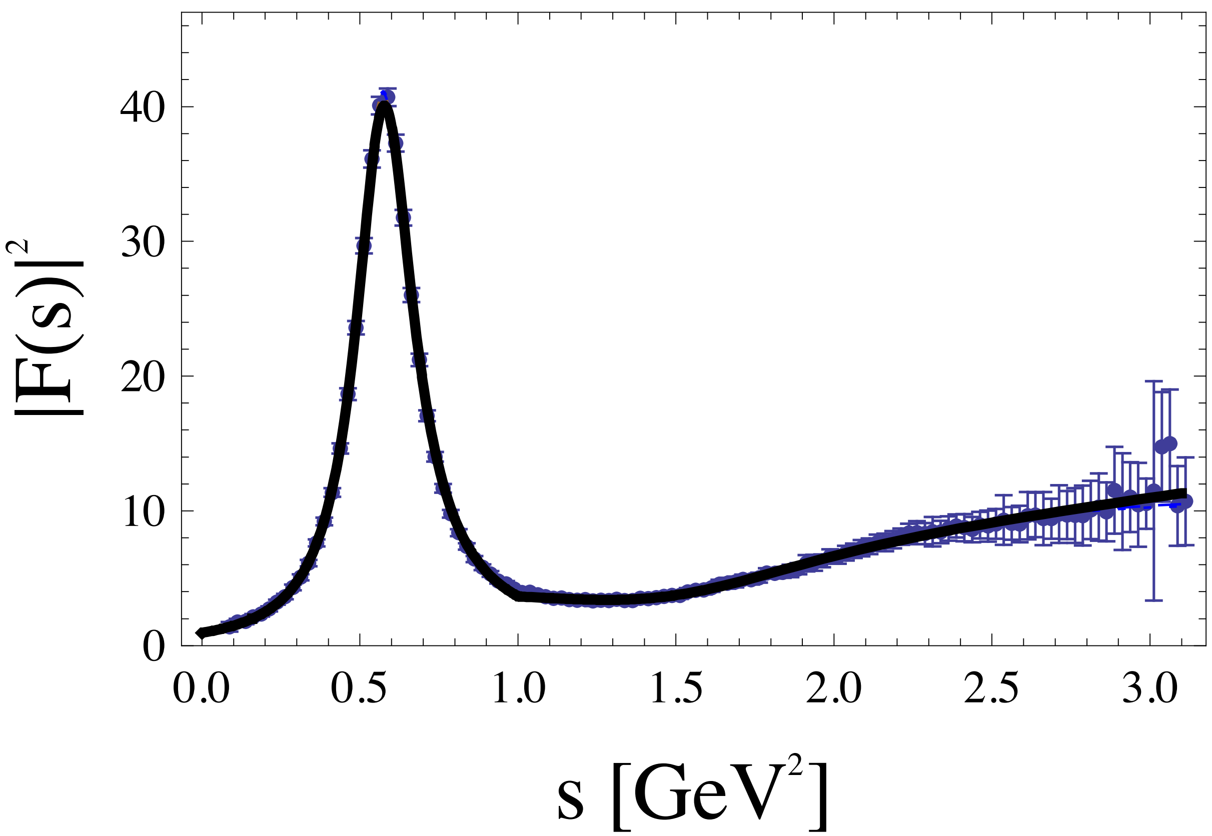
Quadratic Approximants

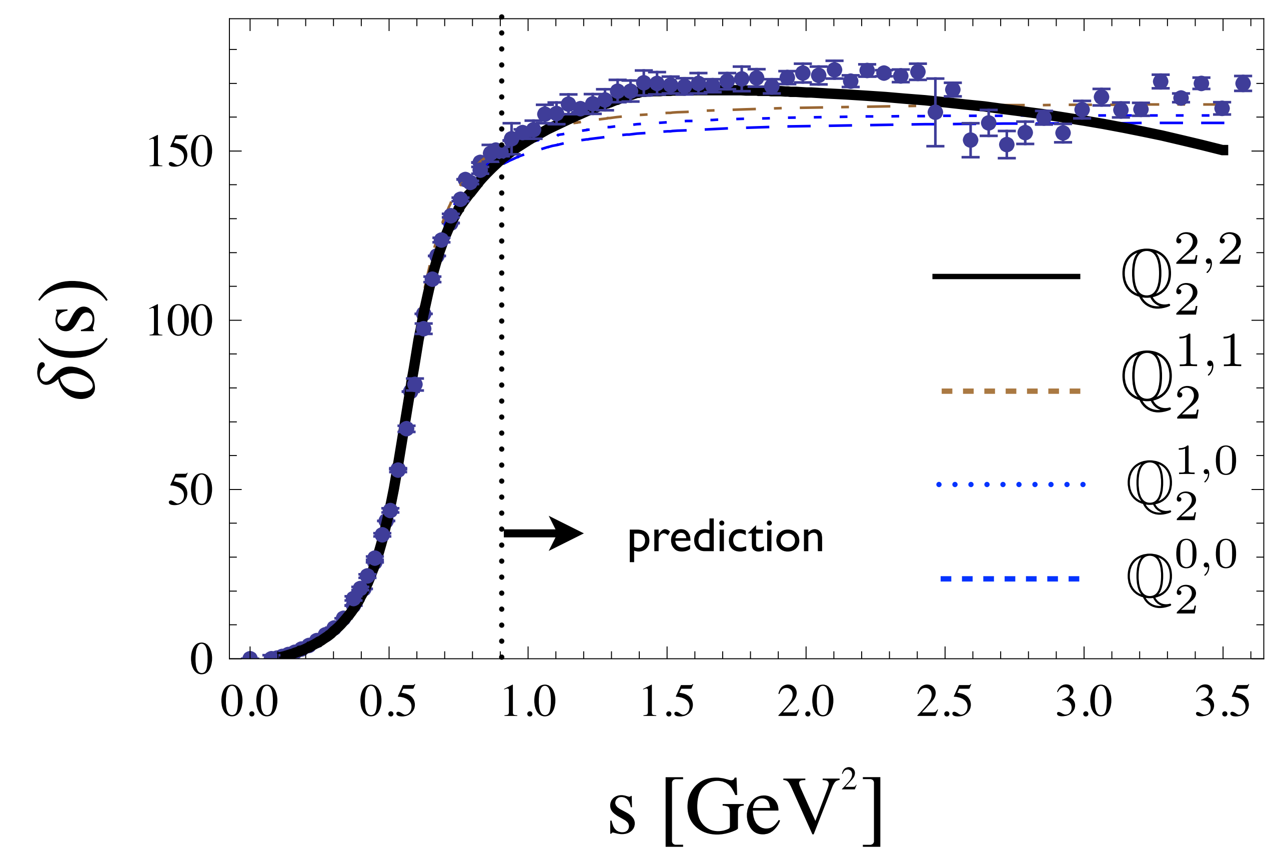
Preliminary: Real data (further details: P. Roig talk)

Vector FF model

Fit to ALEPH data + Phase-shift up to KK

Predict phase above KK!





Improvements:

- explore the symmetry z⇔1/z
- coupled channel (KK also)
- matching to ChPT
- Impose disp. rel. to the coefficients
- Include a third pole
- Use newer data + include space-like data

Conclusions

- We presented a method, a TOOLKIT
- based on analyticity and unitarity (convergence!)
- Simple method + flexible
- Approaches yes (improvable), assumptions no
- Systematic:
  - easy to update with new data (or derivatives)
  - error from approach
- Predictive (checkable)
- Useful for B decays! (can be easily extended to 2 vars)

Thanks!