QCD factorization in non-leptonic B decays: status of phenomenology and challenges

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"Future Challenges in Non-Leptonic B Decays",
Bad Honnef, February 10 - 12, 2016

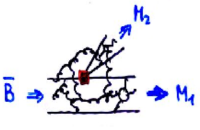
Outline

- Tree-dominated decays
- Penguin-dominated decays
- Polarisation

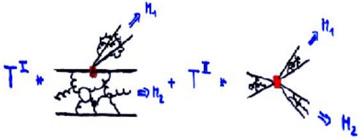
Hadronic matrix elements from QCD factorization [BBNS, 1999-2001]

Heavy quark limit: $m_b \gg \Lambda_{\rm QCD}$
Large-energy limit: $E_M \approx m_b/2 \gg \Lambda_{\rm QCD}$
Scales: m_b, $\sqrt{m_b\Lambda_{\rm QCD}}$, $\Lambda_{\rm QCD}$, $(M_{\rm EW}, \Lambda_{\rm NP})$

$\mu \approx m_b$



$\mu \approx 1\,\text{GeV}$



- Reduces $\langle M_1M_2|\mathcal{O}|B\rangle$ to simpler $\langle M|\mathcal{O}|B\rangle$ (form factors), $\langle 0|\mathcal{O}|B\rangle$, $\langle 0|\mathcal{O}|M\rangle$ (decay constants and distribution amplitudes).
- Calculation from first principles, but limited accuracy by $\Lambda_{\rm QCD}/m_b$ corrections.

QCDF analyses at NLO

Analyses of complete sets of final states

- PP, PV
  MB, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237
- VV
  MB, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237
- AP, AV, AA
  Cheng, Yang, 0709.0137, 0805.0329
- SP, SV
  Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403
- TP, TV
  Cheng, Yang, 1010.3309

Based on NLO hard-scattering functions.
Well-established successes and problems.

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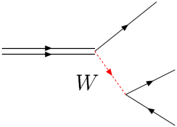
This talk: amplitudes and phenomenology with NNLO results (except polarisation)

Based on NLO hard-scattering functions.
Well-established successes and problems.

Tree-dominated modes

$[T \sim a_1, C \sim a_2]$

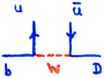


2-loop: Bell, 0705.3127, 0902.1915; Bell, Pilipp, 0910.1016; MB, Huber, Li, 0911.3655 + 1-loop spectator-scattering (2005)

Colour-allowed vs. colour-suppressed

$$H_{\rm eff} = C_1\,[\bar u_i b_i]_{\rm V-A}[\bar d_j u_j]_{\rm V-A} + C_2\,[\bar u_i b_j]_{\rm V-A}[\bar d_j u_i]_{\rm V-A}$$
$$= (C_1 + \frac{C_2}{N_c})\,[\bar u b]_{\rm V-A}[\bar d u]_{\rm V-A} + 2C_2\,[\bar u T^A b]_{\rm V-A}[\bar d T^A u]_{\rm V-A}$$
$$= (C_2 + \frac{C_1}{N_c})\,[\bar d b]_{\rm V-A}[\bar u u]_{\rm V-A} + 2C_1\,[\bar d T^A b]_{\rm V-A}[\bar u T^A u]_{\rm V-A}$$

$$C_1(xm_b) \sim 1.1 \qquad C_2(xm_b) \sim -0.3 \ldots -0.1$$



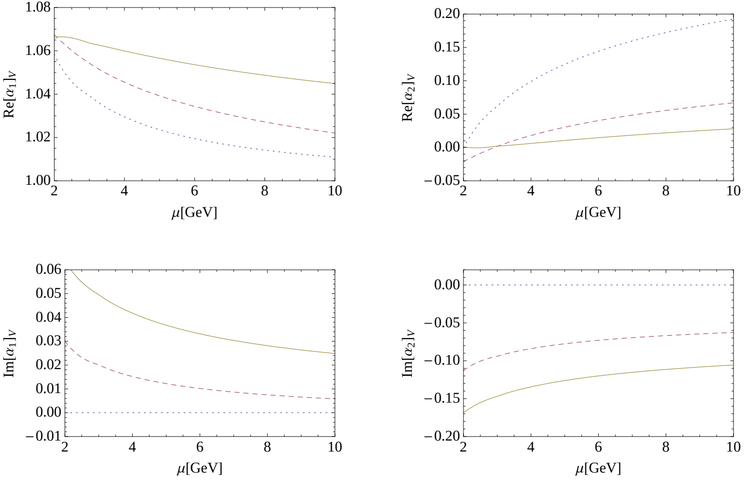
- Colour-allowed and colour-suppressed (topological) "tree" amplitude

$$T \sim a_1(\pi\pi) \,\propto\, f_\pi \Phi_\pi \left[(C_1 + \frac{C_2}{N_c} + \alpha_s 2C_2) f_+^{B\pi}(0) + \alpha_s 2C_2\, \frac{f_B f_\pi \Phi_\pi}{m_B \lambda_B} \right]$$
$$C \sim a_2(\pi\pi) \,\propto\, f_\pi \Phi_\pi \left[(C_2 + \frac{C_1}{N_c} + \alpha_s 2C_1) f_+^{B\pi}(0) + \alpha_s 2C_1\, \frac{f_B f_\pi \Phi_\pi}{m_B \lambda_B} \right]$$

In effect, (N)NLO is (N)LO for the colour-suppressed tree amplitude.

$$C \propto a_2(\pi\pi) = 0.220 - [0.179 + 0.077i]_{\rm NLO} + \left[\frac{r_{\rm sp}}{0.485}\right] \left\{ [0.123]_{\rm LOsp} + [0.072]_{\rm tw3} \right\}$$

Size of the 2-loop vertex correction



Numerical result (tree amplitudes)

$$
\begin{aligned}
T \equiv a_1(\pi\pi) &= 1.009 + [0.023 + 0.010i]_{\rm NLO} + [0.026 + 0.028i]_{\rm NNLO} \\
&\quad - \left[\frac{r_{\rm sp}}{0.485}\right] \Big\{ [0.015]_{\rm LOsp} + [0.037 + 0.029i]_{\rm NLOsp} + [0.009]_{\rm tw3} \Big\} \\
&= 1.00 + 0.01i \quad \rightarrow \quad 0.93 - 0.02i \quad (\text{if } 2 \times r_{\rm sp})
\end{aligned}
$$

$$
r_{\rm sp} = \frac{9 f_{M_1} \hat{f}_B}{m_b f_+^{B\pi}(0) \lambda_B}
$$

$$
\begin{aligned}
C \equiv a_2(\pi\pi) &= 0.220 - [0.179 + 0.077i]_{\rm NLO} - [0.031 + 0.050i]_{\rm NNLO} \\
&\quad + \left[\frac{r_{\rm sp}}{0.485}\right] \Big\{ [0.123]_{\rm LOsp} + [0.053 + 0.054i]_{\rm NLOsp} + [0.072]_{\rm tw3} \Big\} \\
&= 0.26 - 0.07i \quad \rightarrow \quad 0.51 - 0.02i \quad (\text{if } 2 \times r_{\rm sp})
\end{aligned}
$$

- Sizeable correction to imaginary part (phases), but cancellation between vertex and spectator-scattering.
- The colour-suppressed amplitudes are dominated by spectator-scattering. [But $\arg\,(C/T_{\pi\pi}) \lesssim 15^\circ$.]
- Qualitative understanding why colour-suppressed decay modes ($\pi^0\pi^0, \ldots$) can be large. Allows $|C/T|_{\pi\pi} \approx 0.7$, if λ_B is small.
  However, does not solve problems related to C (see below)

Branching fractions (tree-dominated decays) [MB, Huber, Li, 2009]

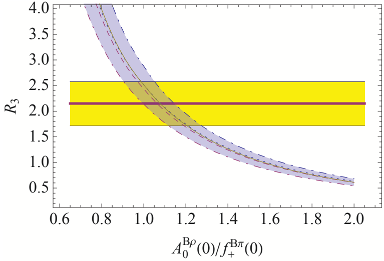
	Theory I		Theory II		Experiment
$B^- \to \pi^-\pi^0$	$5.43\,^{+0.06}_{-0.06}\,^{+1.45}_{-0.84}$	$(\star)$	$5.82\,^{+0.07}_{-0.06}\,^{+1.42}_{-1.35}$	$(\star)$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.37\,^{+0.86}_{-0.69}\,^{+1.22}_{-0.97}$	$(\star)$	$5.70\,^{+0.70}_{-0.55}\,^{+1.16}_{-0.97}$	$(\star)$	5.16 ± 0.22
$\bar{B}^0_d \to \pi^0\pi^0$	$0.33\,^{+0.11}_{-0.08}\,^{+0.42}_{-0.17}$		$0.63\,^{+0.12}_{-0.10}\,^{+0.64}_{-0.42}$		1.55 ± 0.19
			BELLE CKM 14:		0.90 ± 0.16
$B^- \to \pi^-\rho^0$	$8.68\,^{+0.42}_{-0.41}\,^{+2.71}_{-1.56}$	$(\star\star)$	$9.84\,^{+0.41}_{-0.40}\,^{+2.54}_{-2.52}$	$(\star\star)$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38\,^{+0.90}_{-0.77}\,^{+2.18}_{-1.41}$	$(\star)$	$12.13\,^{+0.85}_{-0.73}\,^{+2.23}_{-2.17}$	$(\star)$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\rho^-$	$17.80\,^{+0.62}_{-0.56}\,^{+1.76}_{-2.10}$	$(\star)$	$13.76\,^{+0.49}_{-0.44}\,^{+1.77}_{-2.18}$	$(\star)$	15.7 ± 1.8
$\bar{B}^0 \to \pi^-\rho^+$	$10.28\,^{+0.39}_{-0.39}\,^{+1.37}_{-1.42}$	$(\star\star)$	$8.14\,^{+0.34}_{-0.33}\,^{+1.35}_{-1.49}$	$(\star\star)$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08\,^{+0.27}_{-0.19}\,^{+3.82}_{-3.50}$	$(\dagger)$	$21.90\,^{+0.20}_{-0.12}\,^{+3.06}_{-3.55}$	$(\dagger)$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52\,^{+0.04}_{-0.03}\,^{+1.11}_{-0.43}$		$1.49\,^{+0.07}_{-0.07}\,^{+1.77}_{-1.29}$		2.0 ± 0.5
$B^- \to \rho^-_L\,\rho^0_L$	$18.42^{+0.23}_{-0.21}\,^{+3.92}_{-2.55}$	$(\star\star)$	$19.06^{+0.24}_{-0.22}\,^{+4.59}_{-4.22}$	$(\star\star)$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho^+_L\,\rho^-_L$	$25.98^{+0.85}_{-0.77}\,^{+2.93}_{-3.43}$	$(\star\star)$	$20.66^{+0.68}_{-0.62}\,^{+2.99}_{-3.75}$	$(\star\star)$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho^0_L\rho^0_L$	$0.39^{+0.03}_{-0.03}\,^{+0.83}_{-0.36}$		$1.05^{+0.05}_{-0.04}\,^{+1.62}_{-1.04}$		$0.55^{+0.22}_{-0.24}$

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05$, $A_0^{B\rho}(0) = 0.30 \pm 0.05$, $\lambda_B(1\,\text{GeV}) = 0.35 \pm 0.15$ GeV
Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\,\text{GeV}) = 0.20^{+0.05}_{-0.00}$ GeV

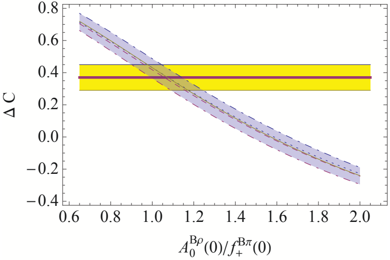
First error γ, $|V_{cb}|$. $|V_{ub}|$ uncertainty *not* included. Second error from hadronic inputs.
Brackets: form factor uncertainty not included.

Charged $\pi^{\mp}\rho^{\pm}$ modes

$$R_3 = \frac{\Gamma(\overline{B}_0 \to \pi^+\rho^-)}{\Gamma(\overline{B}_0 \to \pi^-\rho^+)}$$



$$\Delta C = \frac{1}{2}\left[C(\pi^-\rho^+) - C(\pi^+\rho^-)\right]$$



Both depend mainly on $f_\pi A_0^{B\to\rho}(0)/(f_\rho f_+^{B\to\pi}(0)) \times \alpha_1(\rho\pi)/\alpha_1(\pi\rho) = R\, e^{i\delta_T}$. E.g.,

$$\Delta C = \frac{1-R^2}{1+R^2} + \frac{4R^2}{(1+R^2)^2}\,(a\cos\delta_a + b\cos\delta_b)\cos\gamma + \ldots$$

Slightly smaller but compatible with $B \to \rho$ to $B \to \pi$ form factor ratio from QCD sum rules (≈ 1.2).

Factorization test

$$\frac{\Gamma(B^- \to \pi^-\pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+ l^- \bar{\nu})/dq^2\big|_{q^2=0}} = 3\pi^2 f_\pi^2\,|V_{ud}|^2\,|\alpha_1(\pi\pi)+\alpha_2(\pi\pi)|^2$$

- From exclusive semi-leptonic data [HFAG 2014]
  $|V_{ub}|f_+(0) = (9.23 \pm 0.24) \times 10^{-4}$
  equivalent to

$$|\alpha_1(\pi\pi)+\alpha_2(\pi\pi)|_{\rm exp} = 1.27 \pm 0.04$$

- to be compared to [$\lambda_B = 350\,\mathrm{MeV}$]

$$|\alpha_1(\pi\pi)+\alpha_2(\pi\pi)|_{\rm th} = 1.24^{+0.16}_{-0.10}$$

Leading uncertainties: λ_B (B LCDA), α_2^π (pion LCDA), power corrections.

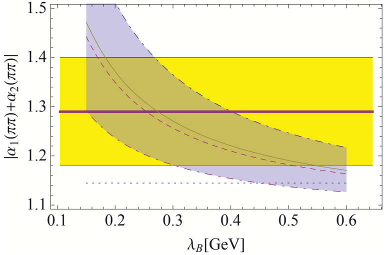


Figure from BHL2009 with obsolete data (yellow band), $|\alpha_1(\pi\pi)+\alpha_2(\pi\pi)|_{\rm exp} = 1.29 \pm 0.11$.

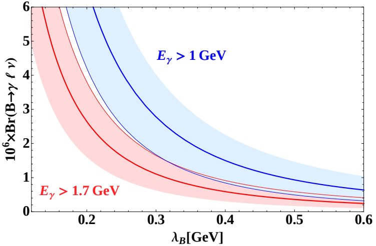
Colour-suppressed tree can be large only if it also has a large relative phase.

λ_B from $B \to \gamma\ell\nu$

$$iF_{\rm stat}(\mu)\Phi_{B+}(\omega,\mu) = \frac{1}{2\pi}\int dt\, e^{it\omega}\,\langle 0|(\bar q_s Y_s)(tn_-)\not{n}_-\gamma_5(Y_s^\dagger h_v)(0)|\bar B_v\rangle_\mu$$

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\,\Phi_{B+}(\omega,\mu)$$

$\Gamma(B\to\gamma\ell\nu) \propto 1/\lambda_B^2$. Dominant parametric dependence.



[MB, Rohrwild, 2011]

- NLL + tree-level $1/m_b$ [MB, Rohrwild, 2011]
- QCD sum rule estimate of power-suppressed soft form factor [Braun, Khodjamirian, 2012]
- BELLE [1504.05831] gives $\lambda_B > 217\,$MeV (with caveats). Promising for SuperKEKB

Tree amplitudes – summary

- NNLO corrections individually sizeable, but ultimately not large due to cancellation.
- Colour-allowed modes well described by factorization, less the purely colour-suppressed ones.
- C/T can be large but still need large phase for πK puzzle and other CP asymmetries (below). Apparent π, ρ non-universality.
- NNLO is end of the road at leading power

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- NNLO corrections individually sizeable, but ultimately not large due to cancellation.
- Colour-allowed modes well described by factorization, less the purely colour-suppressed ones.
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- NNLO is end of the road at leading power

Challenges

- Determine λ_B precisely to remove main parameter uncertainty at LP.
- Power corrections to the SCET matching of the colour-octet operators

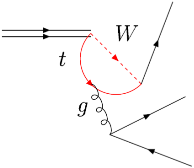
$$2C_1\,[\bar{d}T^A b]_{\mathrm{V-A}}[\bar{u}T^A u]_{\mathrm{V-A}},\;\; 2C_2\,[\bar{u}T^A b]_{\mathrm{V-A}}[\bar{d}T^A u]_{\mathrm{V-A}}$$

[cf. BBNS, hep-ph/0006124; MB, Vernazza, 0810.3575]

Affects both C and T, more C due to $C_1 \gg |C_2|$.

QCD penguin-dominated modes

$[P^{u,c} \sim \lambda_{u,c}^{(D)} \sum_q [\bar{q}_s q][\bar{q} D]]$



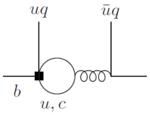
2-loop: Bell, MB, Huber, Li, 1507.03700 + 1-loop spectator-scattering (2006)

Penguin amplitudes

- Magnitude controls BR of most $b \to s$ modes.
- Interference of QCD penguin is main source of CP violation.

$$\left[\frac{P^c}{T}\right]_{\pi\pi}, \quad \left[\frac{T}{P^c}\right]_{\pi K}, \quad \left[\frac{P^u}{P^c}\right]_{\phi K}$$

- $$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V^*_{pD} \Big(C_1 \mathcal{O}^p_1 + C_2 \mathcal{O}^p_2 + \sum_{i=\rm pen} C_i \mathcal{O}_{i,\rm pen} \Big)$$



Two amplitudes $P^{u,c}$. Dominant contribution beyond tree-level from tree operators $\mathcal{O}^p_{1,2}$.

- Non-singlet amplitude. Very little known (experimentally) for singlet penguin $S^{u,c} \sim \lambda^{(D)}_{u,c} \sum_q [\bar{q}q][\bar{q}_s D]$. ($B \to \pi\phi$ in the absence of $\omega - \phi$ mixing.)

The $B \to \pi K$ system and its PV, VP, VV variants

$$\mathcal{A}_{B^- \to \pi^- \bar{K}^0} = \lambda_c^{(s)} [P_c - \frac{1}{3} P_c^{C,EW}] + \lambda_u^{(s)} [P_u - \frac{1}{3} P_u^{C,EW}]$$

$$\sqrt{2}\,\mathcal{A}_{B^- \to \pi^0 K^-} = \lambda_c^{(s)} [P_c + P_c^{EW} + \frac{2}{3} P_c^{C,EW}] + \lambda_u^{(s)} [T + C + P_u + P_u^{EW} + \frac{2}{3} P_u^{C,EW}]$$

$$\mathcal{A}_{\bar{B}^0 \to \pi^+ K^-} = \lambda_c^{(s)} [P_c + \frac{2}{3} P_c^{C,EW}] + \lambda_u^{(s)} [T + P_u + \frac{2}{3} P_u^{C,EW}]$$

$$\sqrt{2}\,\mathcal{A}_{\bar{B}^0 \to \pi^0 \bar{K}^0} = \lambda_c^{(s)} [-P_c + P_c^{EW} + \frac{1}{3} P_c^{C,EW}] + \lambda_u^{(s)} [C - P_u + P_u^{EW} + \frac{1}{3} P_u^{C,EW}]$$

Ratios with little dependence on γ, but sensitive to electroweak penguins.
CP asymmetry differences and sum rules.

$$R_{00} = \frac{2\Gamma(\bar{B}^0 \to \pi^0 \bar{K}^0)}{\Gamma(B^- \to \pi^- \bar{K}^0)} = |1 - r_{\rm EW}|^2 + 2\cos\gamma\,{\rm Re}\,r_C + \ldots$$

$$R_L = \frac{2\Gamma(\bar{B}^0 \to \pi^0 \bar{K}^0) + 2\Gamma(B^- \to \pi^0 K^-)}{\Gamma(B^- \to \pi^- \bar{K}^0) + \Gamma(\bar{B}^0 \to \pi^+ K^-)} = 1 + |r_{\rm EW}|^2 - \cos\gamma\,{\rm Re}(r_T\, r_{\rm EW}^*) + \ldots$$

$$\delta A_{\rm CP} = A_{\rm CP}(\pi^0 K^\pm) - A_{\rm CP}(\pi^\mp K^\pm) = -2\sin\gamma \Big({\rm Im}(r_C) - {\rm Im}(r_T\, r_{\rm EW})\Big) + \ldots$$

theory: $r_{\rm EW} \approx 0.12 - 0.01i$, $\quad r_C \approx 0.03[\times 2?] - 0.02i$, $\quad r_T \approx 0.18 - 0.02i$

$$r_{\rm EW} = \frac{3}{2}\,R_{\pi K}\,\frac{\alpha^c_{3,\rm EW}(\pi\bar K)}{\hat\alpha^c_4(\pi\bar K)} \qquad r_C = -R_{\pi K}\left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right|\frac{\alpha_2(\pi\bar K)}{\hat\alpha^c_4(\pi\bar K)} \qquad r_T = -\left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right|\frac{\alpha_1(\pi\bar K)}{\hat\alpha^c_4(\pi\bar K)}$$

where $R_{\pi K} = (f_\pi/f_K)\cdot(F_0^{B\to K}/F_0^{B\to\pi}) \approx 1.$

- Direct CP asymmetry difference

$$\delta A_{\rm CP} = A_{\rm CP}(\pi^0 K^\pm) - A_{\rm CP}(\pi^\mp K^\pm) = -2\sin\gamma\Big(\mathrm{Im}(r_C) - \mathrm{Im}(r_T\, r_{\rm EW})\Big) + \ldots$$

theory: $r_{\rm EW} \approx 0.12 - 0.01i$, $\quad r_C \approx 0.03[\times 2?] - 0.02i$, $\quad r_T \approx 0.18 - 0.02i$

	theory	data
$\delta A_{\rm CP}$	0.03 ± 0.03	0.122 ± 0.022

- Hadronic explanation needs large C *and* large phase. Phase in P_c does not work. Problem since 2003.

Explore this for πK^*, ρK, ρK^*! Larger effects expected and different signs, since P_c is strongly dependent on V or P.

Time-dependent CP asymmetry ΔS in $b \to s$

$$\Delta S_f = -\eta_f S_f - \sin(2\beta) = \frac{2\,\mathrm{Re}(d_f)\cos(2\beta)\sin\gamma + |d_f|^2\,(\sin(2\beta+2\gamma) - \sin(2\beta))}{1 + 2\,\mathrm{Re}(d_f)\cos\gamma + |d_f|^2}$$

$$d_f = \epsilon_{\mathrm{KM}} \hat{d}_f \quad \text{with} \quad \epsilon_{\mathrm{KM}} = \left|\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*}\right| \sim 0.025$$

$$\pi K_S \quad \hat{d}_f \sim \frac{[-P^u] + [C]}{[-P^c]} \qquad \rho K_S \quad \hat{d}_f \sim \frac{[P^u] - [C]}{[P^c]}$$

$$\eta' K_S \quad \hat{d}_f \sim \frac{[-P^u] - [C]}{[-P^c]} \qquad \phi K_S \quad \hat{d}_f \sim \frac{[-P^u]}{[-P^c]}$$

$$\eta K_S \quad \hat{d}_f \sim \frac{[P^u] + [C]}{[P^c]} \qquad \omega K_S \quad \hat{d}_f \sim \frac{[P^u] + [C]}{[P^c]}$$

[Quantities in square brackets have positive real part.]

$P^c(\pi K) \sim 2P^c(\rho K) \sim 0.4P^c(\eta' K) \sim 2.3P^c(\eta K) \sim 1.3P^c(\phi K) \sim 2.3P^c(\omega K)$

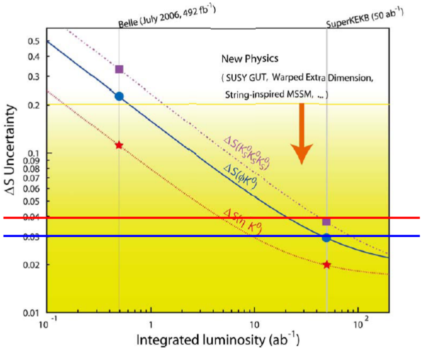
- $\epsilon_{\mathrm{KM}}\,|P^u/P^c| \approx 0.02$ are roughly independent of f
- Influence of C determines the difference between the different modes $\Longrightarrow$ need to know C well, here *real* part.

Precision matters for ΔS in $b \to s$

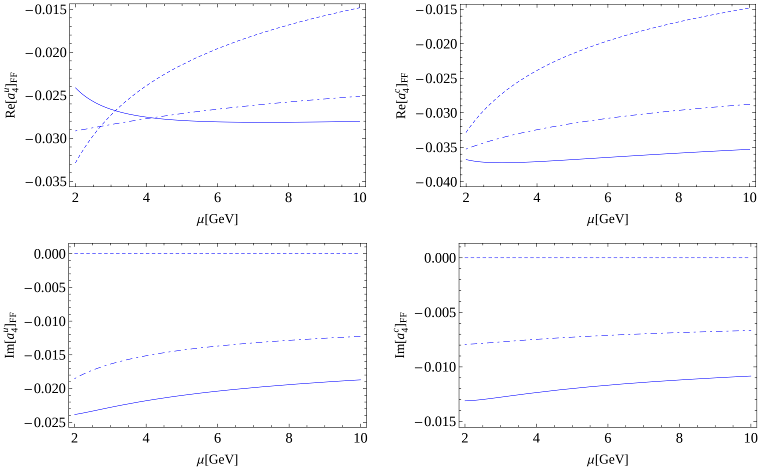
Mode	ΔS_f (Theory)	ΔS_f [Range*]
ϕK_S	$0.02^{+0.01}_{-0.01}$	$[+0.01, 0.05]$
$\eta' K_S$	$0.01^{+0.01}_{-0.01}$	$[+0.00, 0.03]$
$\pi^0 K_S$	$0.07^{+0.05}_{-0.04}$	$[+0.03, 0.13]$
$\rho^0 K_S$	$-0.08^{+0.08}_{-0.12}$	$[-0.29, 0.01]$
ηK_S	$0.10^{+0.11}_{-0.07}$	$[-0.76, 0.27]$
ωK_S	$0.13^{+0.08}_{-0.08}$	$[+0.02, 0.21]$

[MB; Cheng, Chua, Soni; Buchalla, Hiller, Nir, Raz; 2005]

- $\eta' K_S$ (red) has colour-suppressed tree contamination. Conservatively increase uncertainty.
- ϕK_S (blue) optimal. Theoretical uncertainty becomes limiting only with $\approx 50\,\text{ab}^{-1}$ at SuperKEKB.



Size of the 2-loop penguin vertex correction [$C_{1,2}$ only]



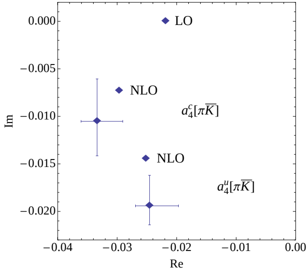
Numerical result (penguin amplitudes)

$$a_4^u(\pi\bar{K})/10^{-2} = -2.87 - [0.09+0.09i]_{V_1} + [0.49-1.32i]_{P_1} - [0.32+0.71i]_{P_2}$$
$$+ \left[\frac{r_{sp}}{0.434}\right]\left\{[0.13]_{LO} + [0.14+0.12i]_{HV} - [0.01-0.05i]_{HP} + [0.07]_{tw3}\right\}$$
$$= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i$$

$$r_{sp} = \frac{9 f_{M_1}\hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$$

$$a_4^c(\pi\bar{K})/10^{-2} = -2.87 - [0.09+0.09i]_{V_1} + [0.05-0.62i]_{P_1} - [0.77+0.50i]_{P_2}$$
$$+ \left[\frac{r_{sp}}{0.434}\right]\left\{[0.13]_{LO} + [0.14+0.12i]_{HV} + [0.01+0.03i]_{HP} + [0.07]_{tw3}\right\}$$
$$= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i$$

- Two-loop is 40% (15%) of the imaginary (real) part of $a_4^u(\pi\bar{K})$, and 50% (25%) in the case of $a_4^c(\pi\bar{K})$.
- Spectator-scattering not relevant.



$$\hat{P}^p \sim \alpha_4^p(M_1M_2) = a_4^p(M_1M_2) + \{1,-1,0\} \times r_\chi^{M_2} a_6^p(M_1M_2) + \underbrace{\beta_3^p(M_1M_2)}_{\approx 0.03}$$

$$\mathrm{PP} \sim \underbrace{a_4}_{\mathrm{V}\mp\mathrm{A}} + \underbrace{r_\chi a_6}_{\mathrm{S+P}}$$

$$\mathrm{PV} \sim a_4 \approx \frac{\mathrm{PP}}{3}$$

$$\mathrm{VP} \sim a_4 - r_\chi a_6 \sim -\mathrm{PV}$$

$$\mathrm{VV} \sim a_4 \sim \mathrm{PV}$$

Large NNLO correction to a_4^p diluted by important/dominant power-suppressed effects.

Use $B \to M_1^+ M_2^-$ (Br and $\mathrm{A_{CP}}$) to determine P_c/T.
Small phases ($\to$ CP asymmetries)

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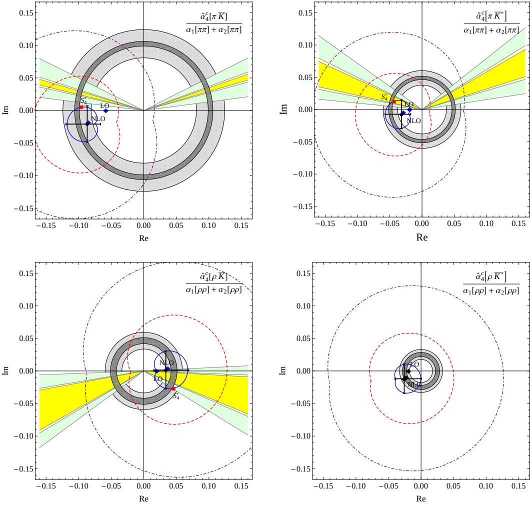

Direct CP asymmetries

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^0$	$0.71^{+0.13\,+0.21}_{-0.14\,-0.19}$	$0.77^{+0.14\,+0.23}_{-0.15\,-0.22}$	$0.10^{+0.02\,+1.24}_{-0.02\,-0.27}$	-1.7 ± 1.6
π^0K^-	$9.42^{+1.77\,+1.87}_{-1.76\,-1.88}$	$10.18^{+1.91\,+2.03}_{-1.90\,-2.62}$	$-1.17^{+0.22\,+20.00}_{-0.22\,-\ 6.62}$	4.0 ± 2.1
π^+K^-	$7.25^{+1.36\,+2.13}_{-1.36\,-2.58}$	$8.08^{+1.52\,+2.52}_{-1.51\,-2.65}$	$-3.23^{+0.61\,+19.17}_{-0.61\,-\ 3.36}$	-8.2 ± 0.6
$\pi^0\bar{K}^0$	$-4.27^{+0.83\,+1.48}_{-0.77\,-2.23}$	$-4.33^{+0.84\,+3.29}_{-0.78\,-2.32}$	$-1.41^{+0.27\,+5.54}_{-0.25\,-6.10}$	1 ± 10
$\delta(\pi\bar{K})$	$2.17^{+0.40\,+1.39}_{-0.40\,-0.74}$	$2.10^{+0.39\,+1.40}_{-0.39\,-2.86}$	$2.07^{+0.39\,+2.76}_{-0.39\,-4.55}$	12.2 ± 2.2
$\Delta(\pi\bar{K})$	$-1.15^{+0.21\,+0.55}_{-0.22\,-0.84}$	$-0.88^{+0.16\,+1.31}_{-0.17\,-0.91}$	$-0.48^{+0.09\,+1.09}_{-0.09\,-1.15}$	-14 ± 11
$\pi^-\bar{K}^{*0}$	$1.36^{+0.25\,+0.60}_{-0.26\,-0.47}$	$1.49^{+0.27\,+0.69}_{-0.29\,-0.56}$	$0.27^{+0.05\,+3.18}_{-0.05\,-0.67}$	-3.8 ± 4.2
π^0K^{*-}	$13.85^{+2.40\,+5.84}_{-2.70\,-5.86}$	$18.16^{+3.11\,+\ 7.79}_{-3.52\,-10.57}$	$-15.81^{+3.01\,+69.35}_{-2.83\,-15.39}$	-6 ± 24
π^+K^{*-}	$11.18^{+2.00\,+\ 9.75}_{-2.15\,-10.62}$	$19.70^{+3.37\,+10.54}_{-3.80\,-11.42}$	$-23.07^{+4.35\,+86.20}_{-4.05\,-20.64}$	-23 ± 6
$\pi^0\bar{K}^{*0}$	$-17.23^{+3.33\,+\ 7.59}_{-3.00\,-12.57}$	$-15.11^{+2.93\,+12.34}_{-2.65\,-10.64}$	$2.16^{+0.39\,+17.53}_{-0.42\,-36.80}$	-15 ± 13
$\delta(\pi\bar{K}^*)$	$2.68^{+0.72\,+5.44}_{-0.67\,-4.30}$	$-1.54^{+0.45\,+4.60}_{-0.58\,-9.19}$	$7.26^{+1.21\,+12.78}_{-1.34\,-20.65}$	17 ± 25
$\Delta(\pi\bar{K}^*)$	$-7.18^{+1.38\,+3.38}_{-1.28\,-5.35}$	$-3.45^{+0.67\,+9.48}_{-0.59\,-4.95}$	$-1.02^{+0.19\,+4.32}_{-0.18\,-7.86}$	-5 ± 45

Direct CP asymmetries

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^0$	$0.71^{+0.13+0.21}_{-0.14-0.19}$	$0.77^{+0.14+0.23}_{-0.15-0.22}$	$0.10^{+0.02+1.24}_{-0.02-0.27}$	-1.7 ± 1.6
π^0K^-	$9.42^{+1.77+1.87}_{-1.76-1.88}$	$10.18^{+1.91+2.03}_{-1.90-2.62}$	$-1.17^{+0.22+20.00}_{-0.22-6.62}$	4.0 ± 2.1
π^+K^-	$7.25^{+1.36+2.13}_{-1.36-2.58}$	$8.08^{+1.52+2.52}_{-1.51-2.65}$	$-3.23^{+0.61+19.17}_{-0.61-3.36}$	-8.2 ± 0.6
$\pi^0\bar{K}^0$	$-4.27^{+0.83+1.48}_{-0.77-2.23}$	$-4.33^{+0.84+3.29}_{-0.78-2.32}$	$-1.41^{+0.27+5.54}_{-0.25-6.10}$	1 ± 10
$\delta(\pi\bar{K})$	$2.17^{+0.40+1.39}_{-0.40-0.74}$	$2.10^{+0.39+1.40}_{-0.39-2.86}$	$2.07^{+0.39+2.76}_{-0.39-4.55}$	12.2 ± 2.2
$\Delta(\pi\bar{K})$	$-1.15^{+0.21+0.55}_{-0.22-0.84}$	$-0.88^{+0.16+1.31}_{-0.17-0.91}$	$-0.48^{+0.09+1.09}_{-0.09-1.15}$	-14 ± 11
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$\pi^0\bar{K}^{*0}$	$-17.23^{+3.33+7.59}_{-3.00-12.57}$	-1		
$\delta(\pi\bar{K}^*)$	$2.68^{+0.72+5.44}_{-0.67-4.30}$	–		
$\Delta(\pi\bar{K}^*)$	$-7.18^{+1.38+3.38}_{-1.28-5.35}$	–		

$\rho^-\bar{K}^0$	$0.38^{+0.07+0.16}_{-0.07-0.27}$	$0.22^{+0.04+0.19}_{-0.04-0.17}$	$0.30^{+0.06+2.28}_{-0.06-2.39}$	-12 ± 17
ρ^0K^-	$-19.31^{+3.42+13.95}_{-3.61-8.96}$	$-4.17^{+0.75+19.26}_{-0.80-19.52}$	$43.73^{+7.07+44.00}_{-7.62-137.77}$	37 ± 11
ρ^+K^-	$-5.13^{+0.95+6.38}_{-0.97-4.02}$	$1.50^{+0.29+8.69}_{-0.27-10.36}$	$25.93^{+4.43+25.40}_{-4.90-75.63}$	20 ± 11
$\rho^0\bar{K}^0$	$8.63^{+1.59+2.31}_{-1.65-1.69}$	$8.99^{+1.66+3.60}_{-1.71-7.44}$	$-0.42^{+0.08+19.49}_{-0.08-8.78}$	6 ± 20
$\delta(\rho\bar{K})$	$-14.17^{+2.80+7.98}_{-2.96-5.39}$	$-5.67^{+0.96+10.86}_{-1.01-9.79}$	$17.80^{+3.15+19.51}_{-3.01-62.44}$	17 ± 16
$\Delta(\rho\bar{K})$	$-8.75^{+1.62+4.78}_{-1.66-6.48}$	$-10.84^{+1.98+11.67}_{-2.09-9.09}$	$-2.43^{+0.46+4.60}_{-0.42-19.43}$	-37 ± 37

Penguin amplitudes – summary

- NNLO corrections individually sizeable, but ultimately not large due to dilution by power-suppressed effects.
- C_{3-6} and C_{8g} [Kim, Yoon, 1107.1601] still missing.

Penguin amplitudes – summary

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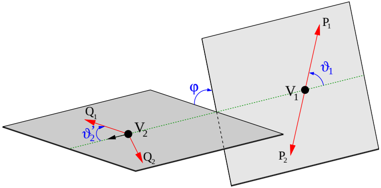
Challenges

- Determine NNLO correction to the NLP scalar penguin amplitude a_6 to complete the short-distance prediction.
- Combine QCDF + SU(3) + phenomenological parameterization of power corrections? What are the *quantitative* predictions of factorization?

Polarisation



MB, Rohrer, Yang, hep-ph/0512258 and hep-ph/0612290

More “kinematics”

- Vector-vector – three helicity amplitudes [$\rightarrow$ five observables]

$$\mathcal{A}_0,\ \mathcal{A}_-,\ \mathcal{A}_+$$

- Three-body

$$\frac{d^2\Gamma}{ds_{12}ds_{23}}$$

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$$\mathcal{A}_0,\ \mathcal{A}_-,\ \mathcal{A}_+$$

- Three-body

$$\frac{d^2\Gamma}{ds_{12}ds_{23}}$$

- Transverse amplitudes do not factorize even at leading power
- Large-angle three-body $s_{12} \sim s_{23} \sim m_b^2$ is power-suppressed compared to "side-bands". Sidebands too large for real m_b. Mainly physics of low-energy hadronic two-body scattering. [MB (2006); Stewart (2006), Kränkl, Mannel, Virto, 1505.04111]

VV transverse helicity amplitudes

- Parametric hierarchy

$$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ = 1 : \frac{\Lambda}{m_b} : \left(\frac{\Lambda}{m_b}\right)^2$$

due to $V-A$ weak interaction and helicity conservation of high-energy QCD (m_b/Λ).
$[\bar{\chi}\chi][\bar{\xi}h_v]$ operators:

$$\underbrace{\bar{\chi}\not{n}_-(1\mp\gamma_5)\chi}_{\mathcal{A}_0,\,\text{LP}},\; \underbrace{\bar{\chi}\not{n}_-\gamma_\perp^\mu(1\mp\gamma_5)\chi}_{\mathcal{A}_\pm\ \text{LP, not V}-\text{A}},\; \underbrace{\bar{\chi}\not{D}_\perp(1\mp\gamma_5)\chi}_{\mathcal{A}_-\ \text{NLP}}$$

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- Transverse amplitudes are power corrections. Spectator scattering does not factorize for $\mathcal{A}_-$
- Transverse penguin annihilation $\mathcal{O}(1)$ numerically

$$\frac{P^-}{P^0} \approx \frac{A^-_{\rho K*}}{A^0_{\rho K*}}\,\frac{\alpha_4^{c-}+\beta_3^{c-}}{\alpha_4^{c,0}} \approx \frac{0.05+[-0.04,0.10]}{0.12}$$

- Theoretically [Kagan, 2004; Rohrer, 2004; MB, Yang, Rohrer, 2006] expect and empirically find

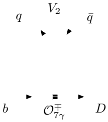
$\mathcal{A}_0 \gg \mathcal{A}_- \gg \mathcal{A}_+$ tree decays $\qquad$ $\mathcal{A}_0 \approx \mathcal{A}_- \gg \mathcal{A}_+$ penguin decays

Fit P_h^c to data. $T_h, P^c_{h,\mathrm{EW}}$ from theory.

Transverse polarization and electromagnetic dipole operators

Parametric hierarchy is violated by electromagnetic interactions [MB, Rohrer, Yang, 2005]

$$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ \qquad\qquad 1 : \frac{\Lambda}{m_b} : \frac{\Lambda^2}{m_b^2} \qquad \Rightarrow \qquad 1 : \frac{m_b}{\Lambda} : 1$$



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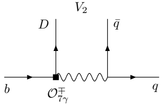

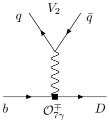

γ always off-shell, $q^2 \sim m_b^2$ $\qquad$ γ nearly on-shell, $q^2 = m_{V_2}^2 \sim \Lambda^2$

- V_2 longitudinal $\Rightarrow$ photon propagator is cancelled $\Rightarrow$ effective local four-quark interaction
- for V_2 transverse no cancellation $\Rightarrow$ local $b \to D\gamma$ transition followed by long-distance $\gamma \to V_2$ transition, enhanced by large photon propagator

New operator in SCET_I (leading in heavy quark power counting)

$$\left[\bar{\xi} W \gamma_{\perp\mu}(1 \mp \gamma_5) h_v\right](0) \left[W_\gamma^\dagger i D_{\gamma\perp}^\mu W_\gamma\right](0),$$

Consider electromagnetic dipole operators including both chiralities

$$Q_{7\gamma}^{\mp} = -\frac{e\bar{m}_b}{8\pi^2}\,\bar{D}\sigma_{\mu\nu}(1\pm\gamma_5)F^{\mu\nu}b,$$

Electroweak penguin coefficients

$$P_{\mp}^{\rm EW}(V_1V_2) = C_7 + C_9 + \frac{1}{N_c}(C_8+C_{10}) \;\mp\; \underbrace{\frac{2\alpha_{\rm em}}{3\pi}\,C_{7\gamma,{\rm eff}}^{\mp}R_{\mp}\,\frac{m_B\bar{m}_b}{m_{V_2}^2}}_{\text{dipole operator contribution}} \;+\ldots$$

with R_- a form factor ratio that equals 1 in the heavy-quark limit and $R_+ \sim m_b/\Lambda$.

Leading QCD penguin to EW penguin amplitude (in some units)

$$P_-(\rho K^*) \approx -1 \qquad\qquad P_-^{\rm EW}(\rho K^*) \approx -0.3 + 0.7\ \text{[dipole]}$$

For positive helicity $\quad 0.7 \to 0.7\times(10-20)\times\dfrac{C_{7\gamma}^{+}}{C_{7\gamma,{\rm eff}}^{-}}$

The $B \to \rho K^*$ system

$$
\begin{aligned}
\mathcal{A}_h(\rho^- \bar{K}^{*0}) &= P_h \\
\sqrt{2}\,\mathcal{A}_h(\rho^0 K^{*-}) &= [P_h + P_h^{EW}] + e^{-i\gamma}\,[T_h + C_h] \\
\mathcal{A}_h(\rho^+ K^{*-}) &= P_h + e^{-i\gamma}\,T_h \\
-\sqrt{2}\,\mathcal{A}_h(\rho^0 \bar{K}^{*0}) &= [P_h - P_h^{EW}] + e^{-i\gamma}\,[-C_h],
\end{aligned}
$$

T_h, C_h CKM suppressed.

Consider CP-averaged negative helicity decay rate ratio ($p_h^{EW} = P_h^{EW}/P_h$)

$$
R \equiv \frac{\bar{\Gamma}_-(\rho^0 \bar{K}^{*-})}{\bar{\Gamma}_-(\rho^0 \bar{K}^{*0})} = \left|\frac{1 + p_-^{EW}}{1 - p_-^{EW}}\right|^2 + \Delta = \begin{cases} 3.0 \pm 0.7 \\ 0.6 \pm 0.1 \;\; \text{without dipole operator} \end{cases}
$$

(Fit P_- to data, use QCDF for the other amplitudes)

The $B \to \rho K^*$ system

$$\begin{aligned} \mathcal{A}_h(\rho^- \bar{K}^{*0}) &= P_h \\ \sqrt{2}\,\mathcal{A}_h(\rho^0 K^{*-}) &= [P_h + P_h^{EW}] + e^{-i\gamma}\,[T_h + C_h] \\ \mathcal{A}_h(\rho^+ K^{*-}) &= P_h + e^{-i\gamma}\,T_h \\ -\sqrt{2}\,\mathcal{A}_h(\rho^0 \bar{K}^{*0}) &= [P_h - P_h^{EW}] + e^{-i\gamma}\,[-C_h], \end{aligned}$$

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(Fit P_- to data, use QCDF for the other amplitudes)

- $Q_{7\gamma}^+$ contribution to $\bar{\mathcal{A}}_+$ is suppressed only by $C_{7\gamma}^+/C_{7\gamma}^-$, while other contributions have additional Λ/m_b suppression $\Rightarrow$ Sensitivity to $C_{7\gamma}^+ \approx 0.1$ may be possible (or better?).
- An alternative to studies of photon polarization in $B \to K^\star \gamma$. Here the ρ meson (decay) acts as the *polarization analyzer.*

Summary

- (Soon) Ready for an update of QCDF predictions for (quasi-) two-body P and V modes (excluding polarisation)
  - NLO $\rightarrow$ NNLO
  - Improved input parameters
- Sets the reference for new ideas on power-suppressed effects.
  - Factorization obvious with tree-level accuracy.
  - Perturbative corrections not necessarily dominant [α_s vs. Λ/m_b].

New directions:

- Factorization techniques for electromagnetic effects.
- Three-body final states.