Charmless $B \to M_1 M_2$ in QCDF: weak annihilation from data

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Future challenges in non-leptonic *B* decays
Theory and Experiment
Bad Honnef
2016

Motivation for charmless hadronic $B \to M_1 M_2$ decays

"Trees" and "Loops" in $b \to (d, s)\, \bar{q}q$ transitions

1) Test of standard model (=SM) CKM picture
2) Improve understanding of QCD dynamics of B-meson 2-body decays
3) Search for new physics (=NP)

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Status

- Various decays measured by Babar & Belle I ($\Leftarrow$ final data sets) and LHCb

  $B_{u,d,s} \to PP,\ PV,\ VV$ [see talk V. Chobanova]

- High statistics in the future at LHCb & Belle II
- Theory beyond naive factorization

  A) Light-Cone Sum Rules (=LCSR) [Khodjamirian et al. hep-ph/0012271, 0304179, 0509049]

$\Rightarrow$ expansion in $\Lambda_{\rm QCD}/m_b$

  B) QCD Factorisation (=QCDF) / Soft-Collinear Effective Theory (=SCET)

  [see talks by G. Bell and M. Beneke]

✔ in terms of $|\Delta B| = 1$ Wilson coeff's $\Rightarrow$ depend on NP parameters
✔ universal non-perturbative input (form factors, distribution amplitudes) at LO in $\Lambda_{\rm QCD}/m_b$

✗ at subleading order "QCD"-model-dep. due to end-point divergences (not yet resolved) from weak annihilation and higher-twist hard spectator scattering

Observables

CP-averaged branching ratios

$$\overline{Br}(B \to f) = \frac{1}{2}\left[Br\left(\bar{B} \to \bar{f}\right) + Br\left(B \to f\right)\right]$$

Direct CP-asymmetry

$$A_{\rm CP}(B \to f) = \frac{Br\left(\bar{B} \to \bar{f}\right) - Br\left(B \to f\right)}{Br\left(\bar{B} \to \bar{f}\right) + Br\left(B \to f\right)}$$

Mixing-induced CP-asymmetry

$$\frac{Br\left(\bar{B}(t) \to \bar{f}\right) - Br\left(B(t) \to f\right)}{Br\left(\bar{B}(t) \to \bar{f}\right) + Br\left(B(t) \to f\right)} \propto S \sin(\Delta m_B\, t) - C \cos(\Delta m_B\, t)$$

$$(A_{\rm CP} = -C)$$

Ratios of Br's

Here: Only experiments can account for common systematic uncertainties and should provide the errors on these ratios

$\Rightarrow$ **Belle II**

$$R_n^B = \frac{1}{2}\frac{\overline{Br}(B^0 \to K^+\pi^-)}{\overline{Br}(B^0 \to K^0\pi^0)}$$

$$R_c^K = 2\frac{\tau_0}{\tau_-}\frac{\overline{Br}(B^+ \to K^+\pi^0)}{\overline{Br}(B^0 \to K^+\pi^-)}$$

$$R_c^\pi = \frac{\tau_0}{\tau_-}\frac{\overline{Br}(B^+ \to K^0\pi^+)}{\overline{Br}(B^0 \to K^+\pi^-)}$$

Differences of CP-asymmetries

$$\Delta A_{\rm CP}^- = A_{\rm CP}(B^- \to K^-\pi^0) - A_{\rm CP}(\bar{B}^0 \to K^-\pi^+)$$

$$\Delta A_{\rm CP}^0 = A_{\rm CP}(B^- \to \bar{K}^0\pi^-) - A_{\rm CP}(\bar{B}^0 \to \bar{K}^0\pi^0)$$

Observables

Angular distributions in $B \to VV$

Polarization fractions and relative phases for $\overline{B}$ decays (transversity ampl's A_h with $h = L, \perp, \parallel$)

$$f_h^{\overline{B}} = \frac{|\overline{A}_h|^2}{\sum_h |\overline{A}_h|^2} \qquad\qquad \phi_{\parallel,\perp}^{\overline{B}} = \arg \frac{\overline{A}_{\parallel,\perp}}{\overline{A}_L}$$

CP-averaged and CP-asymmetric

$$f_h = \frac{1}{2}\left(f_h^{\overline{B}} + f_h^{B}\right) \qquad\qquad A_{\mathrm{CP},\,h} = \frac{f_h^{\overline{B}} - f_h^{B}}{f_h^{\overline{B}} + f_h^{B}}$$

CP-averaged and CP-asymmetric

$$\phi_h = \frac{1}{2}\left(\phi_h^{\overline{B}} + \phi_h^{B}\right) - \pi\,\mathrm{sgn}\left(\phi_h^{\overline{B}} + \phi_h^{B}\right)\theta\left(\phi_h^{\overline{B}} - \phi_h^{B} - \pi\right) \qquad\qquad \Delta\phi_h = \frac{1}{2}\left(\phi_h^{\overline{B}} - \phi_h^{B}\right) - \pi\,\theta\left(\phi_h^{\overline{B}} + \phi_h^{B}\right)$$

B-meson decays are a multi-scale problem . . .

. . . with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in *B* restframe	$\gg$	QCD-bound state effects
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		$\Lambda_{\rm QCD} \approx 0.5$ GeV

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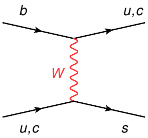
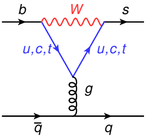
$$\mathcal{L}_{\text{eff}} \sim G_F\, V_{\text{CKM}} \times \Big[\sum_{p=u,c} \sum_{i=1,2} C_i\, \mathcal{O}_i^p + \sum_{3,\dots,6} C_i\, \mathcal{O}_i + \sum_{7,\dots,10} C_i\, \mathcal{O}_i + \sum_{7\gamma,\, 8g} C_i\, \mathcal{O}_i \Big]$$

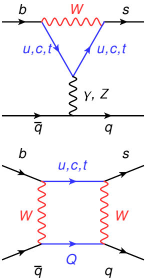
charged current



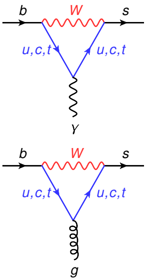
QCD-penguin



EW-penguin



electro- & chromo-mgn



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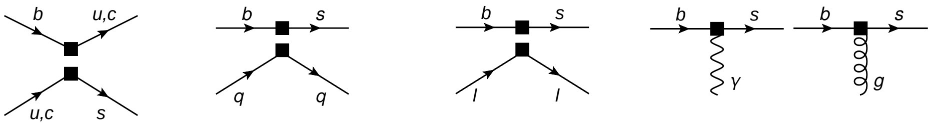
charged current | QCD-penguin | EW-penguin | electro- & chromo-mgn



C_i = Wilson coefficients: contains short-dist. pmr's (heavy masses $m_t, \ldots$ – CKM factored out) and leading logarithmic QCD-corrections to all orders in α_s

$\Rightarrow$ in SM known up to next-to-next-to-leading order

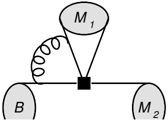
$\mathcal{O}_i$ = higher-dim. operators: flavour-changing coupling of light quarks and leptons

$B \to M_1 M_2$ matrix elements in QCDF

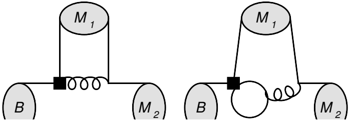
[Beneke/Buchalla/Neubert/Sachrajda hep-ph/0104110]

Classes of diagrams can be calculated perturbatively due to large momentum transfers

Vertex corr's

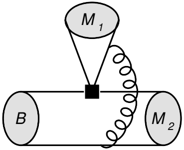


Penguin corr's



spectator quark not involved

Hard spectator scatt. (HS)



$$\langle M_1 M_2 | \mathcal{O}_i | B \rangle \approx F_j^{B\to M_1}\, T_{ij}^{I} * f_{M_2}\Phi_{M_2} \;+\; T_{ij}^{II} * f_B \Phi_B * f_{M_1}\Phi_{M_1} * f_{M_2}\Phi_{M_2} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

perturbatively calculable

- "kernels": $T_{ij}^{I,II}$

known up to nnLO QCD

non-perturbative quantities (sum rules, measurements, lattice)

- decay constants: f_{B,M_1,M_2}
- form factors: $F_j^{B\to M_1}$
- convolutions ($*$) of distribution amplitudes: Φ_{B,M_1,M_2}

Sub-leading order (=sLO) in $\Lambda_{\rm QCD}/m_b$ & endpoint divergences

[Beneke/Buchalla/Neubert/Sachrajda hep-ph/0104110]

... **arise in HS** from higher twist LCDA's Φ_{m1}

$$\int_0^1 \frac{dy}{1-y}\Phi_{m1}(y) \equiv \Phi_{m1}(1)\, X_H + \int_0^1 \frac{dy}{[1-y]_+}\Phi_{m1}(y)$$

with $\Phi_{m1}(y) \neq 0$ for $y \to 1$

$\Rightarrow$ phenomenological parameter of soft-gluon interaction with spectator quark

$$X_H \equiv (1+\rho_H)\ln\frac{m_b}{\Lambda_{\rm QCD}}, \qquad \rho_H \equiv |\rho_H|\, e^{i\phi_H} \in \mathbb{C}$$

$\to$ for $\rho_H = 0$ $\quad\Rightarrow\quad$ $X_H \approx \ln 10 \approx 2.3$

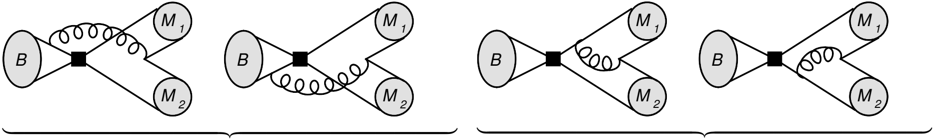
- X_H represents soft-gluon interaction with spectator quark
- one expects $X_H \sim \ln(m_b/\Lambda_{\rm QCD})$, because perturbative calculation of soft interactions at latest regulated at scale $\Lambda_{\rm QCD}$ via new effective degrees of freedom
- $(1+\rho_H)$ parameterizes some hadronic matrix elements inclusive strong phases

  $\Rightarrow$ should not be too large, usually $\rho_H \lesssim 2$

  $\Rightarrow$ affect CP-asymmetries

Another sLO contribution: Weak annihilation (=WA)

- **sub-leading** in $\Lambda_{\rm QCD}/m_b$, but **chirality-enhanced** [Keum/Li/Sanda hep-ph/0004004, 0004173]
- non-singlet annihilation amplitudes generated via CC-, QCD-, QED-penguin operators



$\underbrace{\qquad\qquad}_{A^i_{1,2,3}}$ $\qquad$ $\underbrace{\qquad\qquad}_{A^f_3 \quad (A^f_{1,2} = 0)}$

$$b_1 \propto C_1 A^i_1 \qquad b^p_3 \propto C_3 A^i_1 + C_5 (A^i_3 + A^f_3) + N_c C_6 A^f_3 \qquad b^p_4 \propto C_4 A^i_1 + C_6 A^i_2$$

$$b_2 \propto C_2 A^i_1 \qquad b^p_{3,\rm EW} \propto C_9 A^i_1 + C_7 (A^i_3 + A^f_3) + N_c C_8 A^f_3 \qquad b^p_{4,\rm EW} \propto C_{10} A^i_1 + C_8 A^i_2$$

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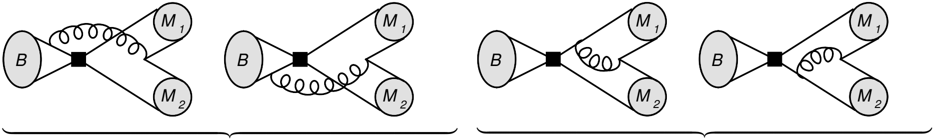

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This form obtained in QCDF

- ignoring soft endpoint divergences
- ”hardscattering kernels” convoluted with LCDA’s from chirally-enhanced twist-3 projections

[Beneke/Buchalla/Neubert/Sachrajda hep-ph/0104110, Beneke/Neubert hep-ph/0308039, Beneke/Rohrer/Yang hep-ph/0612290]

Can this parameterisation cover main features — is it physical — of not-yet known true result?
Or could you imagine completely different dependence on Wilson coefficients?

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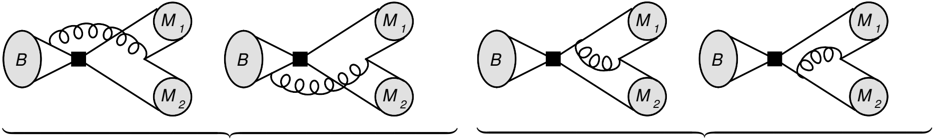

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Fits of Topological amplitude always fit product: "C × hadr. matrix element"

- C contains hard physics above $\mu \sim m_b$ and NP CP violating phases
- hadr. matrix element is more or less pure QCD

[Ciuchini/Franco/Martinelli/Pierini/Silvestrini 0811.0341, Cheng/Chiang/Kuo 1409.5026]

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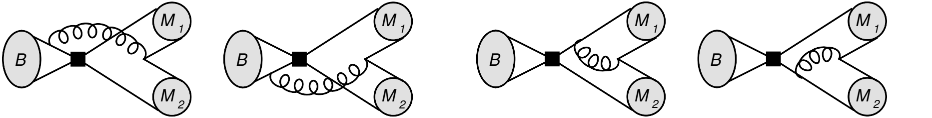

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classes of decays dominated by different b_i and $A_k^{(i,f)}$

- penguin $b \to s\bar{q}q$ decays: b^p_3
- purely WA $B_d \to K^+K^-, B_s \to \pi^+\pi^-$: b^p_4, b_1

NLO SM Wilson coefficients at $\mu = m_b$

C_3	C_4	C_5	C_6
0.014	−0.036	0.009	−0.042
C_7/α	C_8/α	C_9/α	C_{10}/α
−0.011	0.060	−1.254	0.223

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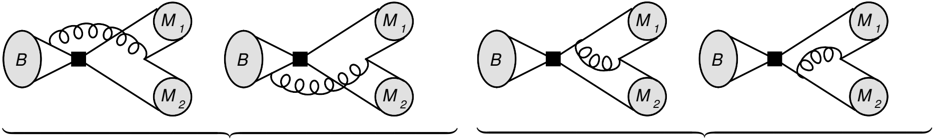

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WA in LCSR $B \to \pi\pi$

- takes similar form for $b_{1,3,4}$
- provides also result of “building blocks” $A_k^{(i,f)}$ with similar size used in QCDF predicitons
- in LCSR A^f_3 arises at order α_s^0, in QCDF at α_s

[Khodjamirian hep-ph/0012271, Khodjamirian/Mannel/Melic 0304179, Khodjamirian/Mannel/Melcher/Melic 0509049]

WA "building blocks" $A_k^{i,f}$ in QCDF

$A_k^{(i,f)}$ are convolution of "kernels" and LCDA's $\Rightarrow$ **endpoint divergent** and regularised by

$$\int_0^1 \frac{dy}{y} \to X_A, \qquad\qquad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2}(X_A)^2$$

- $X_{A,k}^{(i,f)}$ in principle different for each $B_{u,d,s}$ and $M_1 M_2$, and 3 possible $\Gamma_1 \otimes \Gamma_2$:
  k = 1: $(V-A) \otimes (V-A)$ **k = 2**: $(V-A) \otimes (V+A)$ **k = 3**: $(S-P) \otimes (S+P)$
  $\Rightarrow A_k^{i,f}$ with $A_1^f = A_2^f = 0$ and A_3^i negligible for $B \to PP$ [Beneke/Neubert hep-ph/0308039]
- assumption of isospin-symmetry might be justified:
  same $A_k^{i,f}$'s for each system $K\pi = (K^+\pi^-, K^+\pi^0, K^0\pi^+, K^0\pi^0), \ldots$

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Original approach to assign error from $1/m_b$: [Beneke/Buchalla/Neubert/Sachrajda hep-ph/0104110]

- ad hoc assumption of same X_A for each $A_k^{i,f}$ $\Rightarrow$ $\boxed{A_k^{i,f} = A_k^{i,f}(X_A)}$
- to agree with data need $|\rho_A| \simeq 1$, vary $0 \le \phi_A \le 2\pi$

$\Rightarrow$ usually large errors in predictions of direct CP-asymmetries

Fit of WA
in $b \to (d, s) + \bar{q}q$
QCD & EW-penguin decays

[CB/Gorbahn/Vickers 1409.3252]

WA contribution in $B \to K\pi$

$$\mathcal{A}_{\bar{B}^- \to \bar{K}^0 \pi^-} = A_{\pi\bar{K}} \left[\delta_{pu}\beta_2 + \alpha_4^p + \beta_3^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_{3,\mathrm{EW}}^p \right]$$

α_i = all but WA, β_i = WA

$$\sqrt{2}\mathcal{A}_{\bar{B}^- \to \bar{K}^- \pi^0} = A_{\pi\bar{K}} \left[\delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p + \beta_3^p + \alpha_{4,\mathrm{EW}}^p + \beta_{3,\mathrm{EW}}^p \right] + A_{\bar{K}\pi} \left[\delta_{pu}\,\alpha_2 + \frac{3}{2}\alpha_{3,\mathrm{EW}}^p \right]$$

$$\mathcal{A}_{\bar{B}^0 \to K^- \pi^+} = A_{\pi\bar{K}} \left[\delta_{pu}\alpha_1 + \alpha_4^p + \beta_3^p + \alpha_{4,\mathrm{EW}}^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p \right]$$

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Dominant WA contribution via $\hat{\alpha}_4^c \equiv \alpha_4^p + \beta_3^p = |\hat{\alpha}_4^c| \exp(i\,\hat{\phi}_4^c)$ in observables

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$$\mathcal{A}_{\bar{B}^0 \to K^- \pi^+} = A_{\pi\bar{K}} \left[\delta_{pu}\alpha_1 + \alpha_4^p + \beta_3^p + \alpha^p_{4,\mathrm{EW}} - \frac{1}{2}\beta^p_{3,\mathrm{EW}} \right]$$

$$\mathcal{A}_{\bar{B}^0 \to \bar{K}^0 \pi^0} = A_{\pi\bar{K}} \left[-\alpha_4^p - \beta_3^p + \frac{1}{2}\alpha^p_{4,\mathrm{EW}} + \frac{1}{2}\beta^p_{3,\mathrm{EW}} \right] + A_{\bar{K}\pi}\left[\delta_{pu}\,\alpha_2 + \frac{3}{2}\alpha^p_{3,\mathrm{EW}} \right]$$

Dominant WA contribution via $\hat{\alpha}_4^c \equiv \alpha_4^p + \beta_3^p = |\hat{\alpha}_4^c| \exp(i\,\hat{\phi}_4^c)$ in observables

$$Br(B \to K\pi) \propto \left| \hat{\alpha}_4^c \times \left[1 + \mathcal{O}(r_i)\right] \right|^2$$

$$R^{B,K,\pi}_{c,n} \simeq 1 + \cos\hat{\phi}_4^c \sum_i c_i \mathrm{Re}(r_i) + \ldots$$

$$-A_{\mathrm{CP}}(B^- \to K^-\pi^0) \simeq 2\,\mathrm{Im}(r_\mathrm{T} + r_\mathrm{T}^\mathrm{C})\sin\gamma$$

$$-A_{\mathrm{CP}}(\bar{B}^0 \to K^-\pi^+) \simeq 2\,\mathrm{Im}(r_\mathrm{T})\sin\gamma$$

$$-\Delta A_{\mathrm{CP}} \simeq 2\,\mathrm{Im}(r_\mathrm{T}^\mathrm{C})\sin\gamma$$

$$\beta_3^c \propto b_3^c \simeq \ldots N_c C_6 A_3^f$$

$$\mathrm{Im}(r_\mathrm{T}) \propto -\frac{\mathrm{Re}(\alpha_1)}{|\hat{\alpha}_4^c|}\sin\hat{\phi}_4^c + \frac{\mathrm{Im}(\alpha_1)}{|\hat{\alpha}_4^c|}\cos\hat{\phi}_4^c$$

$$\mathrm{Im}(r_\mathrm{T}^\mathrm{C}) \propto -\frac{\mathrm{Re}(\alpha_2)}{|\hat{\alpha}_4^c|}\sin\hat{\phi}_4^c + \frac{\mathrm{Im}(\alpha_2)}{|\hat{\alpha}_4^c|}\cos\hat{\phi}_4^c$$

$r_i = (r_\mathrm{T}, r_\mathrm{T}^\mathrm{C}, r_\mathrm{EW}, r_\mathrm{EW}^\mathrm{C}, r_\mathrm{EW}^\mathrm{A})$ suppressed contributions

[Hofer/Scherer/Verrnazza arXiv:1011.6319]

Fit of X_A from data

In literature usually central values for $|\rho_{A,H}|$ and $\phi_{A,H} = 0$ chosen
and errors estimated from variation

$$X_A = (1 + \rho_A) \ln \frac{m_b}{\Lambda_{\rm QCD}} \qquad \rho = |\rho| e^{i\phi} \qquad \text{with} \qquad |\rho| \in 1 \ldots 2 \text{ and } \phi \in [0, 2\pi]$$

$\boxed{\text{In the following fit the } \rho_A \in \mathbb{C} \text{ with prior } |\rho_A| \leq 8}$

Questions:

- can ρ_A be fitted separately for each $B \to M_1 M_2$ $(M_1 M_2 = K\pi, K\rho, \ldots)$ with current data?
- is the size of ρ_A consistent with naive expectations from QCDF?
- what are the preferred regions for $\rho_{A,H}$ from Br's and CP-asymmetries?
- what about composed observables R and $\Delta A_{\rm CP}$?

Assumptions:

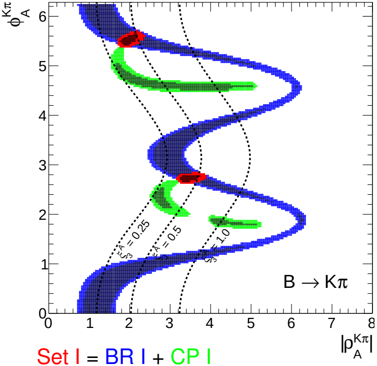
- nLO QCDF for leading order in $\Lambda_{\rm QCD}/m_b$
- one ρ_A for each system $B \to M_1 M_2$ $(M_1 M_2 = K\pi, K\rho, \ldots)$ with current data
- “everything” unknown attributed to WA in the fit

Determine relative size WA and HS contributions compared to leading order

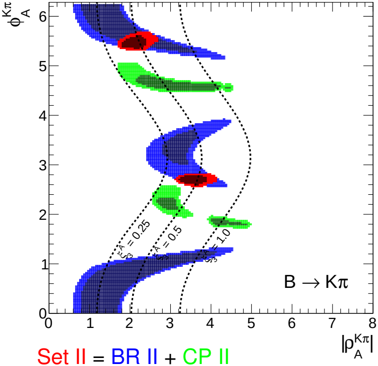
$$\xi_i^A(\rho_A) = \left| \frac{\beta_i(\rho_A)}{\alpha_{(i+\delta_{i3}),\rm I}} \right| \qquad\qquad \xi_i^H(\rho_H) = \left| \frac{\alpha_{i,\rm II}^{\rm tw-3}(\rho_H)}{\alpha_{i,\rm I} + \alpha_{i,\rm II}^{\rm tw-2}} \right|$$

Fit $B \to K\pi$

$|\rho_A| - \arg(\rho_A)$



Set I = BR I + CP I



Set II = BR II + CP II

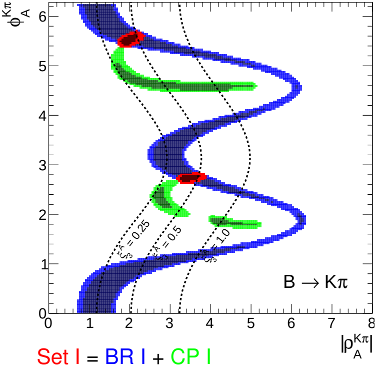
Use 2 sets of observables for $M_1 M_2 = (K\pi,\ K\rho,\ K^*\pi)$

Set	Observables
Set I	$Br(B^- \to \bar{K}^0\pi^-)$, $Br(B^- \to K^-\pi^0)$, $Br(\bar{B}^0 \to K^-\pi^+)$, $Br(\bar{B}^0 \to \bar{K}^0\pi^0)$
	$A_{\rm CP}(B^- \to \bar{K}^0\pi^-)$, $A_{\rm CP}(B^- \to K^-\pi^0)$, $A_{\rm CP}(\bar{B}^0 \to K^-\pi^+)$, $A_{\rm CP}(\bar{B}^0 \to \bar{K}^0\pi^0)$
Set II	$Br(B^- \to \bar{K}^0\pi^-)$, R_n^B, R_c^K, R_c^π
	$A_{\rm CP}(B^- \to \bar{K}^0\pi^-)$, $A_{\rm CP}(\bar{B}^0 \to K^-\pi^+)$, $A_{\rm CP}(\bar{B}^0 \to \bar{K}^0\pi^0)$, $\Delta A_{\rm CP}^-$

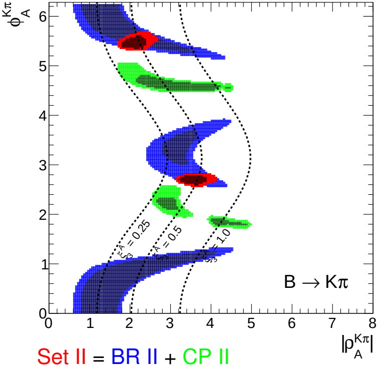
Fit $B \to K\pi$

$|\rho_A| - \arg(\rho_A)$

$\xi_3^A \in [0.37, 0.54]$
$\xi_3^A \in [0.34, 0.69]$
@ 68, 95 % CL



Set I = BR I + CP I



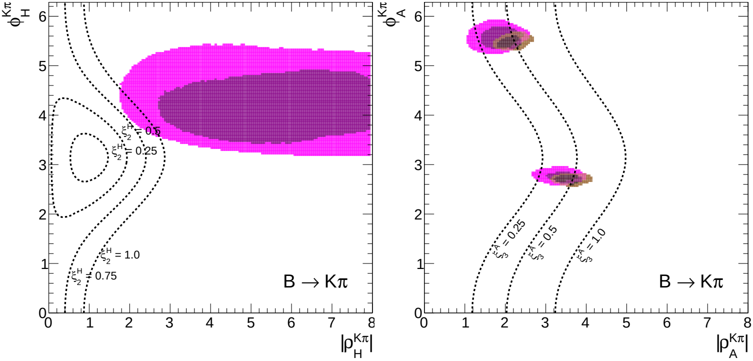
Set II = BR II + CP II

⇒ p-Value	44 %	4 %
⇒ Best-FP	(3.4, 2.7)	(2.3, 2.7)
⇒ Pull's @ BFP		
R_n^B	–	-1.9σ
R_c^π	–	$+0.9\sigma$
$C_{K^-\pi^0}/\Delta A_{\rm CP}^-$	-2.1σ	-2.8σ
$C_{\bar{K}^0\pi^-}$	$+1.0\sigma$	$+1.0\sigma$

Fit $B \to K\pi$

$|\rho_H| - \arg(\rho_H)$
$|\rho_A| - \arg(\rho_A)$

@ 68, 95 % CL



purple = Fit of ρ_H and ρ_A
brown = Fit only ρ_A (previous slide)

- Fit Set II without $A_{\rm CP}(\bar{B}^0 \to \bar{K}^0\pi^0)$
- $\Delta A_{\rm CP} \propto \alpha_{2,\rm I} + \alpha_{2,\rm II}^{\rm tw-2} + \alpha_{2,\rm II}^{\rm tw-3}(\rho_H) + \beta_2(\rho_A^i)$

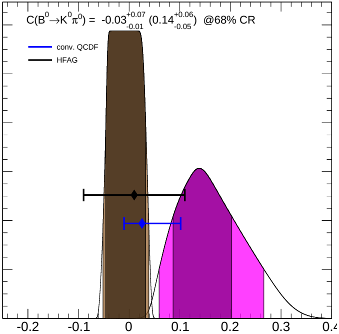
  Note: $\beta_2 \ll \alpha_{2,\rm II}^{\rm tw-3}$ unless $\rho_H \ll \rho_A^i$

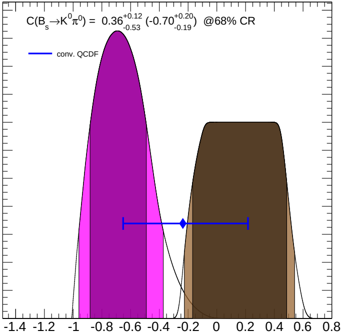
  $\Rightarrow$ can assume $\rho_A^i = \rho_A$
- even smaller ξ_3^A allowed

Fit $B \to K\pi$

$|\rho_H| - \arg(\rho_H)$
$|\rho_A| - \arg(\rho_A)$

@ 68, 95 % CL





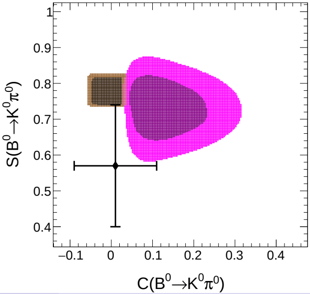
purple = Fit of ρ_H and ρ_A
brown = Fit only ρ_A (previous slide)

- can test "large HS" scenario via measurement of

1) $A_{\mathrm{CP}}(\bar{B}^0 \to \bar{K}^0\pi^0)$

2) $A_{\mathrm{CP}}(\bar{B}_s \to \bar{K}^0\pi^0)$

3) correlation of S and $C = -A_{\mathrm{CP}}$ in $\bar{B}^0 \to \bar{K}^0\pi^0$



Available experimental input from HFAG 2014 and ...

$B \to PP$

$b \to s$				
$B \to K\pi$	$B \to K\eta$	$B \to K\eta'$	$B_s \to KK$ [LHCb]	$B_s \to \pi\pi$
$K^0\pi^0$: $Br,\ C,\ S$	$K^0\eta$: Br	$K^0\eta'^0$: $Br,\ C,\ S$	K^+K^- : $Br,\ C,\ S$	$\pi^+\pi^-$: Br
$K^+\pi^-$: $Br,\ C$	$K^+\eta$: $Br,\ C$	$K^+\eta'^0$: $Br,\ C$		
$K^+\pi^0$: $Br,\ C$				
$K^0\pi^+$: $Br,\ C$				
$b \to d$				
$B \to KK$	$B_s \to K\pi$			
$K^0\bar{K}^0$: Br	$K^+\pi^-$: Br			
K^+K^- : Br				
K^+K^0 : $Br,\ C$				

$B \to PV$

$b \to s$					
$B \to K^*\pi$	$B \to K\rho$	$B \to K^*\eta$	$B \to K^*\eta'$	$B \to K\phi$	$B \to K\omega$
				[Babar, Belle, CDF, LHCb]	[Babar, Belle]
$K^{*0}\pi^0$: $Br,\ C$	$K^0\rho^0$: $Br,\ C,\ S$	$K^{*0}\eta$: $Br,\ C$	$K^{*0}\eta'$: $Br,\ C$	$K^0\phi$: $Br,\ C,\ S$	$K^0\omega$: $Br,\ C,\ S$
$K^{*+}\pi^-$: $Br,\ C$	$K^+\rho^-$: $Br,\ C$	$K^{*+}\eta$: $Br,\ C$	$K^{*+}\eta'$: $Br,\ C$	$K^+\phi$: $Br,\ C$	$K^+\omega$: $Br,\ C$
$K^{*+}\pi^0$: $Br,\ C$	$K^+\rho^0$: $Br,\ C$				
$K^{*0}\pi^+$: $Br,\ C$	$K^0\rho^+$: $Br,\ C$				

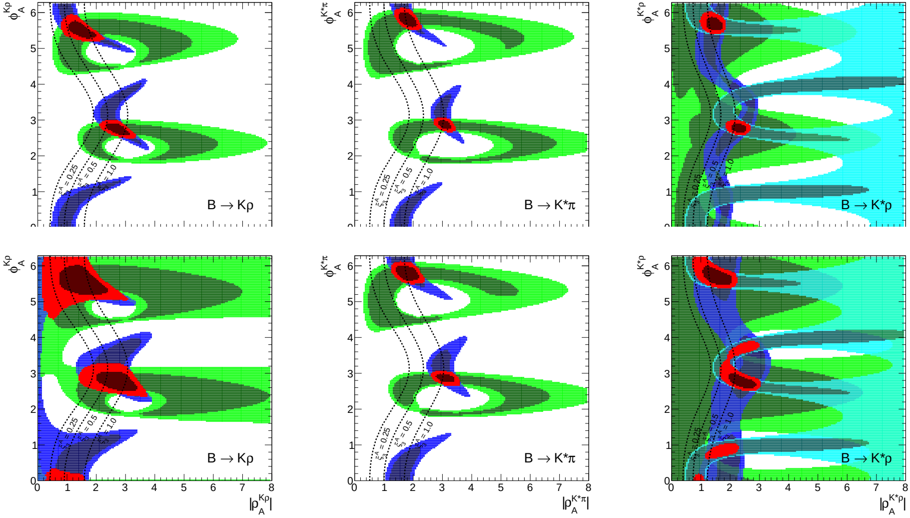
Available experimental input from HFAG 2014 and . . .

$B \to VV$

$b \rightharpoonup s$				
$B \to K^* \rho$	$B \to K^* \phi$ [Babar, Belle, LHCb]	$B \to K^* \omega$	$B_s \to \phi\phi$	$B_s \to K^* K^*$ [LHCb]
$K^{*0}\rho^0$: $Br,\ C,\ f_L$ $K^{*+}\rho^-$: $Br,\ C,\ f_L$ $K^{*+}\rho^0$: $Br,\ C,\ f_L$ $K^{*0}\rho^+$: $Br,\ C,\ f_L$	$K^{*0}\phi$: $Br,\ C,\ C_{L,\perp},\ f_{L,\perp},\ \phi_{\parallel,\perp}$ $K^{*+}\phi$: $Br,\ C,\ C_{L,\perp},\ f_{L,\perp},\ \phi_{\parallel,\perp}$	$K^{*0}\omega$: $Br,\ C,\ f_L$ $K^{*+}\omega$: $Br,\ C,\ f_L$	$\phi\phi$: $Br,\ f_L$	$K^{*0}K^{*0}$: $Br,\ f_L$
$b \rightharpoonup d$				
$B_s \to K^* \phi$	$B \to K^* K^*$			
$K^{*+}\phi$: $Br,\ f_L$	$K^{*0}K^{*0}$: $Br,\ f_L$ $K^{*+}K^{*0}$: $Br,\ f_L$			

$B \to K\rho$, $B \to K^*\pi$ and $B \to K^*\rho$

Fits of ρ_A to Set I [upper] and Set II [lower]



- $\alpha_4(M_1 M_2) = a_4 \pm r_\chi^{M_2} a_6$ with "+" for $M_1 M_2 = PP, PV$ and "−" for $M_1 M_2 = VP, VV$
- no tree-level contribution to a_6 for $M_2 = V$

$\Rightarrow$ ξ_3^A in general larger for $MM = PV, VP, VV$ than for $MM = PP$

Fit of ρ_A and ratio ξ_3^A in penguin dominated $B \to MM$

ρ_H treated as nuisance parameter: $|\rho_H| = 1$, $0 \le \phi_H \le 2\pi$

$MM = PP$

		$B \to K\pi$	$B \to K\eta$	$B \to K\eta'$	$B \to KK$	$B_s \to KK$	$B_s \to \pi\pi$	$B_s \to K\pi$
ξ_3^A	BFP	0.39	0.08	1.83	0.58	1.83	–	0.96
	68% CR	[0.37; 0.54]	[0.00; –]	[0.18; 3.25]	[0.00; 2.07]	[0.02; 2.09]	–	[0.56; 1.54]
	95% CR	[0.34; 0.69]	[0.00; –]	[0.16; 3.34]	[0.00; 2.10]	[0.00; 2.13]	–	[0.44; 1.83]
$\lvert\rho_A\rvert$	lower	> 1.8	> 0	> 0.9	$> 0\,(0.9)$	> 0	> 3.4	> 2.3
	upper	< 3.9	–	< 7.7	$< 6.1\,(8.6)$	< 5.5	< 10.9	< 4.8

$MM = PV$

		$B \to K^*\pi$	$B \to K\rho$	$B \to K^*\eta$	$B \to K^*\eta'$	$B \to K\phi$	$B \to K\omega$
ξ_3^A	BFP	0.89	0.78	2.74	0.48	0.50	2.7
	68% CR	[0.75; 1.40]	[0.39; 1.55]	[0.71; 3.77]	[0.02; 7.84]	[0.40; 2.41]	[0.63; 2.88]
	95% CR	[0.69; 1.56]	[0.16; 2.18]	[0.64; 5.06]	[0.02; 8.41]	[0.32; 2.54]	[0.57; 2.88]
$\lvert\rho_A\rvert$	lower	> 1.4	> 0.8	> 1.1	> 0	> 0.8	> 1.3
	upper	< 3.4	< 3.4	< 4.4	< 6.1	< 3.6	< 4.3

$MM = VV$

		$B \to K^*\rho$	$B \to K^*\phi$	$B_s \to K^*\phi$	$B \to K^*K^*$	$B_s \to K^*K^*$	$B_s \to \phi\phi$	$B \to K^*\omega$
ξ_3^A	BFP	1.33	0.38	1.53	1.84	3.01	0.50	0.91
	68% CR	[0.84; 1.94]	[0.31; 0.43]	[0.30; 2.05]	[0.85; 2.68]	[1.94; 3.79]	[0.49; 1.11]	[0.20; 1.39]
	95% CR	[0.56; 2.33]	[0.25; 0.50]	[0.10; 2.15]	[0.09; 2.90]	[0.96; 4.17]	[0.41; 1.38]	[0.09; 1.46]
$\lvert\rho_A\rvert$	lower	> 1.0	> 0.6	> 0.3	> 1.2	> 1.6	> 0.7	> 0.3
	upper	< 2.9	< 1.8	< 3.2	< 3.0	< 3.6	< 2.3	< 2.4

$\Rightarrow$ data does not require huge WA: at 68% CR always $\xi_3^A < 1$ possible (except $B_s \to K^*K^*$)

BUT usually $\xi_3^A > 0$ at 95% $\Rightarrow$ WA necessary for agreement with data

Purely WA decays

$B_{d,s} \to \pi\pi, KK$

[CB/Gorbahn/Vickers 1409.3252]

Purley WA decays $B \to K^+K^-$ & $B_s \to \pi^+\pi^-$

$$\mathcal{A}(B \to K^+K^-) \simeq f_{B_d} f_K^2 \sum_p \lambda_p^{(d)} B^p_{K^+K^-}$$

$$\mathcal{A}(B_s \to \pi^+\pi^-) \simeq f_{B_s} f_\pi^2 \sum_p \lambda_p^{(s)} B^p_{\pi^+\pi^-}$$

with

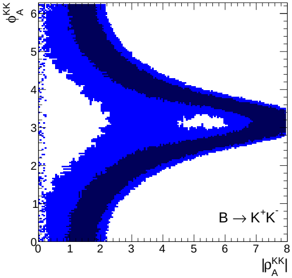
$$B^p_{M_1 M_2} = \left(\delta_{pu} b_1 + 2 b_4^p + \frac{1}{2} b^p_{4,\mathrm{EW}}\right)$$

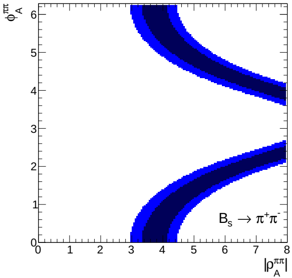
- depend only on building block $A^i_{1,2}$ — off initial state
- parametric uncertainties small: independent of λ_B and form factors

If $A_1^i \approx A_2^i$ only one strong phase $\Rightarrow$ CP-asymmetries should be tiny

$$B^p_{PP(PV)} \approx \frac{C_F}{N_c^2} A_1^i \left(\delta_{pu} C_1 + 2(C_4 \pm C_6) + \frac{C_{10} \pm C_8}{2}\right)$$

Important to measure CP asymmetries and other purely WA decays





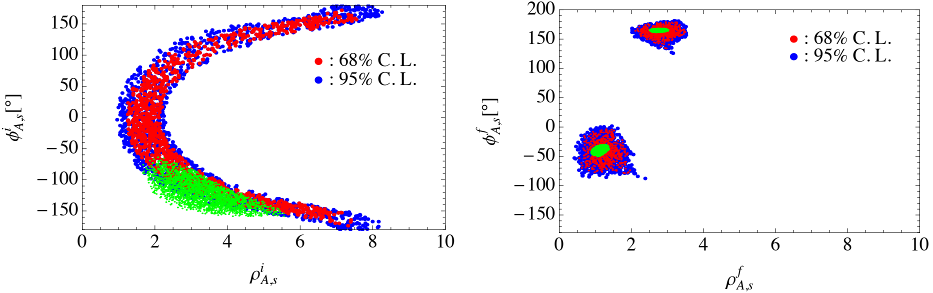
X_A^i and X_A^f from $B \to PP, PV$

[Chang/Sun/Yang/Li 1409.2995, Sun/Chang/Hu/Yang 1412.2334]

X_A^i and X_A^f in $B \to PP$ — 1

- use QCDF at nLO for $B_{u,d,s} \to \pi\pi, \pi K, KK$
- assume $X^i_{A,s}$ different from $X^i_{A,d}$
- assume X_A^f different for $B_{u,d}$ and B_s
- assume $X_H = X_A^i$ in fits
- pure annihilation $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ $\Rightarrow$ determine X_A^i

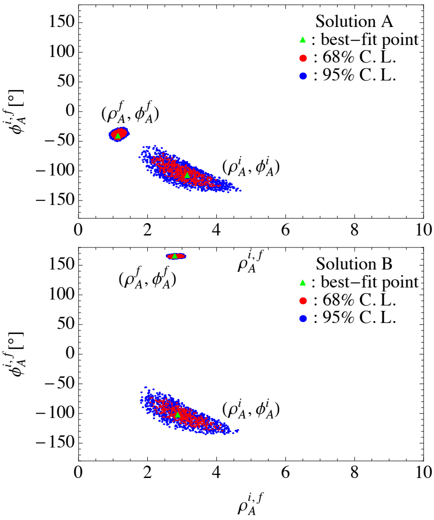
$\Rightarrow$ Fit of $X^{i,f}_{A,s}$ from B_s-decays (68% CL, 95% CL) and $X^{i,f}_{A,d}$ from $B_{u,d}$-decays (68% CL)

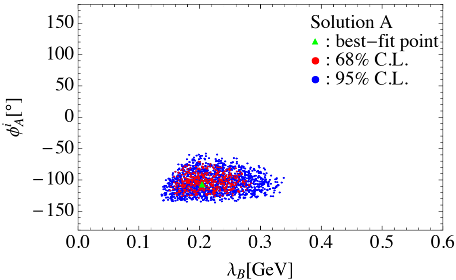


$\Rightarrow$ current data still allows for $X^i_{A,s} \simeq X^i_{A,d}$ and universal $X^f_{A,s} \simeq X^f_{A,d}$

X_A^i and X_A^f in $B \to PP$ — 2

- assume $X^i_{A,s} \simeq X^i_{A,d}$ and $X^f_{A,s} \simeq X^f_{A,d}$ since seem justified
- fit (ρ^i_A, ϕ^i_A), (ρ^f_A, ϕ^f_A) and λ_B (1st inv. moment of B-LCDA) from 16 decays, 42 observables





- two solutions A and B, which give same annihilation ampl.
- $\lambda_B = 0.20^{+0.10}_{-0.05}$ similar to Beneke/Huber/Li 0911.3655 from $B \to \pi\pi, \rho\pi, \rho\rho$

$\Rightarrow$ predictions for not-yet measured modes

$\Rightarrow$ no indication of flavour symmetry breaking

X_A^i and X_A^f in $B \to PV$ $= \rho\pi, \rho K, \phi K, \pi K^*$

Study 3 cases

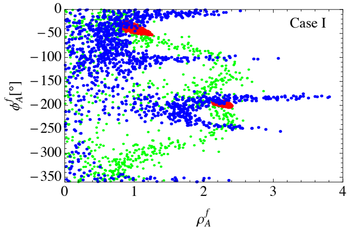
I) fit $X_A^{i,f}$ and λ_B separately from ($X_H = X_A^i$)
A) $B \to \pi K^*, \rho K$, B) $B \to \rho\pi$, C) $B \to \phi K$

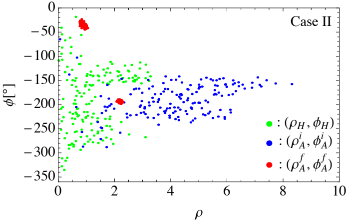
II) fit $X_A^{i,f}$ and X_H, using $\lambda_B = 0.19^{+0.09}_{-0.04}$
combining all modes

III) fit $X_A^{i,f}$ with $X_H = X_A^i$ from all modes

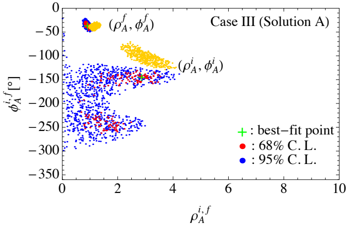
$\Rightarrow$ only discrepancy $A_{\rm CP}(B^- \to K^{*-}\pi^0)$ with Babar

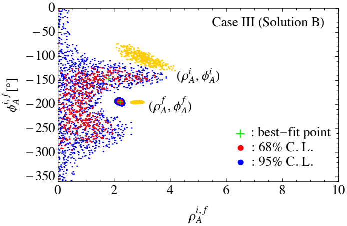
$B_s \to PV$ discussed in [Chang/Hu/Sun/Yang 1504.04907]





results from
$B \to PP$





Summary

If the general form of WA amplitudes b_i (slide 7) in QCDF from chirally-enhanced twist-3 projections covers main features of not-yet known true result,

then

- building blocks A_k^i and A_k^f represent hadronic quantities, expressed in terms of $X_{A,k}^{i,f}$
- ad hoc simplifying assumptions to reduce their number

  1) within each decay $MM = K\pi, \ldots$ system all same

  2) different X_A^i and X_A^f for several $B \to PP, PV$

  ...

and fits show

- chirally-enhanced power corr's are required to find agreement with data
- size of WA contributions not larger than leading amplitude
- in $B \to PP$ and $B \to PV$ data

  A) strongly fixes X_A^f

  B) gives quite loose constraints on X_A^i, allowing for $X_A^i = X_H$

  C) $B \to PP$ preference for $\lambda_B \simeq 200$ MeV

$\Rightarrow$ improved measurements of $B \to MM$ — also purely WA mediated decays — will allow to further test the consistency

Backup Slides

Fit method and treatment of theory uncertainties

Parameters of interest

$$\vec{\theta} = (C_i, \rho_{A,H}, \lambda_B)$$

Fit method and treatment of theory uncertainties

Parameters of interest

$$\vec{\theta} = (C_i, \rho_{A,H}, \lambda_B)$$

Nuisance parameters

$\vec{\nu}$

1) process-specific

   FF's, decay const's,
   LCDA pmr's,
   renorm. scales: $\mu_{b,0}$

2) general

   quark masses, CKM, . . .

Fit method and treatment of theory uncertainties

Parameters of interest

$$\vec{\theta} = (C_i, \rho_{A,H}, \lambda_B)$$

Nuisance parameters

$\vec{\nu}$

1) process-specific

FF's, decay const's,
LCDA pmr's,
renorm. scales: $\mu_{b,0}$

2) general

quark masses, CKM, ...

Observables

1) observables

$$O_i(\vec{\theta}, \vec{\nu})$$

depend usually on sub-set of $\vec{\theta}$ and $\vec{\nu}$

2) experimental data for each observable

$$\text{pdf}(O_i = o)$$

$\Rightarrow$ probability distribution of values o

Fit method and treatment of theory uncertainties

Parameters of interest

$$\vec{\theta} = (C_i, \rho_{A,H}, \lambda_B)$$

Nuisance parameters

$\vec{\nu}$

1) process-specific
   - FF's, decay const's,
   - LCDA pmr's,
   - renorm. scales: $\mu_{b,0}$
2) general
   - quark masses, CKM, ...

Observables

1) observables

$$O_i(\vec{\theta}, \vec{\nu})$$

depend usually on sub-set of $\vec{\theta}$ and $\vec{\nu}$

2) experimental data for each observable

$$\text{pdf}(O_i = o)$$

$\Rightarrow$ probability distribution of values o

Fit strategy: Put theory uncertainties in likelihood using RFit-scheme and ...

- sample $\vec{\theta}$-space (Markov Chains: BAT = Bayesian Analysis Tool)
- theory uncertainties of O_i due to $\vec{\nu}$ at each $(\vec{\theta})_a$:
  vary $\vec{\nu}$ in some ranges and combine uncertainties in quadrature $\Rightarrow \Delta_{i,\text{th}}[(\vec{\theta})_a]$
- use Bayesian method $\Rightarrow$ 68 & 95 % (CL or probability) regions of $\vec{\theta}$ with flat priors

Likelihood in Rfit scheme

Likelihood = product of pdf's of measurements O_i
$p[O_i = o_i]$ = probability of O_i having value o_i

$$\mathcal{L}(\vec{\theta}) = \prod_{i \in \text{data}} p\left[O_i = o_i(\vec{\theta})\right]$$

$$\to \exp\left[-\frac{1}{2} \sum_{i \in \text{data}} \left(\chi_i(\vec{\theta})\right)^2\right]$$

most O_i gaussian distributed

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$$\chi_i(\vec{\theta}) = \begin{cases} \dfrac{|O_{i,\text{th}} - O_{i,\text{exp}}| - \Delta^+_{i,\text{th}}}{\sigma^-_{i,\text{exp}}} & \text{if} \quad O_{i,\text{exp}} \geq O_{i,\text{th}} + \Delta^+_{i,\text{th}} \\ \dfrac{|O_{i,\text{th}} - O_{i,\text{exp}}| - \Delta^+_{i,\text{th}}}{\sigma^+_{i,\text{exp}}} & \text{if} \quad O_{i,\text{exp}} \leq O_{i,\text{th}} - \Delta^-_{i,\text{th}} \\ 0 & \text{else} \end{cases}$$

experimental measurement: $\left(O_{i,\text{exp}}\right)^{+\sigma^+_{i,\text{exp}}}_{-\sigma^-_{i,\text{exp}}}$ theory prediction: $\left(O_{i,\text{th}}\right)^{+\Delta^+_{i,\text{th}}}_{-\Delta^-_{i,\text{th}}}$

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Theory uncertainties for O_i:

$$\Delta^\pm_{i,\text{th}} = \sqrt{\Sigma_a \left(\Delta^\pm_{i,a,\text{th}}\right)^2}$$

$\Rightarrow$ each $\nu_a \in (\nu_{a,\text{min}}, \nu_{a,\text{cen}}, \nu_{a,\text{max}})$:

$$\Delta^+_{i,a,\text{th}} = o_i(\nu_{a,\text{max}}) - o_i(\nu_{a,\text{cen}}), \qquad \Delta^-_{i,a,\text{th}} = o_i(\nu_{a,\text{cen}}) - o_i(\nu_{a,\text{min}})$$

if $o_i(\nu_{a,\text{min}}) \leq o_i(\nu_{a,\text{cen}}) \leq o_i(\nu_{a,\text{max}})$, else ...