Lecture 3:
Gravity and the Puzzles of Naturalness

❀ Petr Hořava ❀

Berkeley Center for Theoretical Physics

54. Internationale Universitätswochen
für Theoretische Physik
Schladming, Styria, Austria
February 2016

Update on the status of Lifshitz gravity

Update on the status of Lifshitz gravity

Example: Lifshitz scalar field theory

Many interesting features can be illustrated by:

$$S = \frac{1}{2} \int dt\, d^D\mathbf{x} \left\{ \dot{\phi}^2 - (\Delta\phi)^2 \right\}$$

A theory closely related to the better-known

$$W = \frac{1}{2} \int d^D\mathbf{x}\, \partial_i\phi\partial_i\phi$$

The critical dimension has shifted:

$$[\phi] = \frac{D-2}{2};$$

ϕ is dimensionless in $2+1$ dimensions.

[Lifshitz,1941]

Gravity at a Lifshitz point

Minimal starting point: fields $g_{ij}(t, \mathbf{x})$ (the spatial metric), action $S = S_K - S_V$, with the kinetic term

$$S_K = \frac{1}{\kappa^2} \int dt\, d^D\mathbf{x} \sqrt{g}\, \dot{g}_{ij} G^{ijk\ell} \dot{g}_{k\ell}$$

where $G^{ijk\ell} = g^{ik} g^{j\ell} - \lambda g^{ij} g^{k\ell}$ is the De Witt metric, and the "potential term"

$$S_V = \frac{1}{4\kappa^2} \int dt\, d^D\mathbf{x} \sqrt{g}\, V(R_{ijk\ell})$$

containing all terms of the appropriate dimension.
Special case, theory in "detailed balance": $V = (\delta W / \delta g_{ij})^2$.

Extending the symmetries

A good starting point, but this action is only invariant under time-independent spatial diffeomorphisms, $\tilde{x}^i = \tilde{x}^i(x^j)$, and describes dynamical propagating components g_{ij} of the spatial metric.

Covariantization of the theory:

(1) Introduce ADM-like variables N (lapse) and N_i (shift), known from the space-time decomposition of the spacetime metric;

(2) Replace $\dot{g}_{ij} \to K_{ij} = \frac{1}{N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$,

$$\sqrt{g} \to N\sqrt{g}.$$

Gauge symmetries: Foliation-preserving diffeomorphisms $\mathrm{Diff}_{\mathcal{F}}(M)$,

$$\delta t = f(t),\ \delta x^i = \xi^i(t, x^j).$$

The transformation rules follow from a nonrelativistic contraction of spacetime diffeomorphisms; N and N_i are gauge fields of $\mathrm{Diff}_{\mathcal{F}}(M)$:

$$\delta N = \dot{f}(t)N + \ldots, \quad \delta N_i = \dot{\xi}_j + \ldots$$

In the minimal (="projectable") realization, N is a function of only t.

Symmetries reminiscent of the Causal Dynamical Triangulations (CDT) approach to quantum gravity on the lattice.

Simplest example: $z = 2$ gravity

The action is $S = S_K - S_V$, with

$$S_k = \frac{1}{\kappa^2} \int dt\, d^D\mathbf{x} \sqrt{g} N \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

and

$$S_V = \int dt\, d^D\mathbf{x} \sqrt{g} N \left(\alpha R_{ij} R^{ij} + \beta R^2 + \ldots \right).$$

Shift in the critical dimension, as in the Lifshitz scalar:

$$[\kappa^2] = 2 - D.$$

The minimal theory with $N(t)$ has the usual number of transverse-traceless graviton polarizations, plus an extra scalar DoF, all with the dispersion relation $\omega^2 \sim k^4$.

Two special values of λ: 1 and 1/D.

Another example: $z = 3$ gravity

The action is again $S = S_K - S_V$, with

$$S_K = \frac{1}{\kappa^2} \int dt\, d^D\mathbf{x} \sqrt{g} N \, \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

and

$$S_V = \int dt\, d^D\mathbf{x} \sqrt{g} N \, C_{ij} C^{ij}.$$

where $C^{ij} = \varepsilon^{ik\ell} \nabla_k (R^j_\ell - \frac{1}{4} R \delta^j_\ell)$ is the Cotton-York-ADM tensor. The shift of the critical dimension is

$$[\kappa^2] = 3 - D.$$

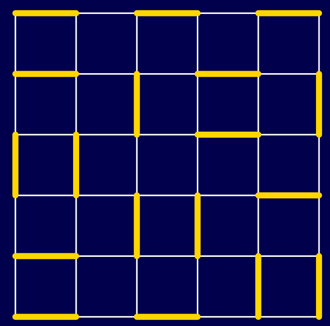
Anisotropic Weyl invariance eliminates the scalar graviton classically.

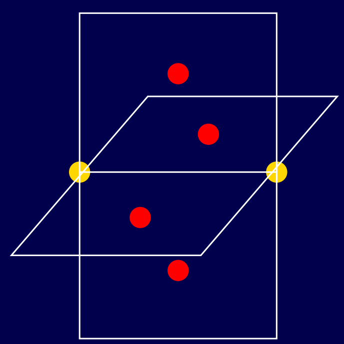
Emergent gravity at a Lifshitz point

[Cenke Xu and P.H., arXiv:1003.0009]

These models with $z = 2$ or $z = 3$ gravitons can emerge as IR fixed points on the fcc lattice. Emergent gauge invariance stabilizes new algebraic bose liquid phases.

Recall the emergence of $U(1)$ "photons" in dimer models [Fradkin,Kivelson,Rokhsar,...]:





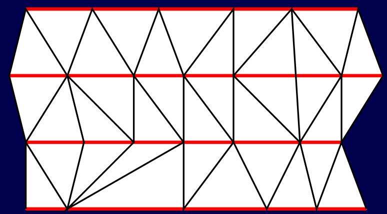
Lattice symmetries protect $z = 2$ or $z = 3$ in IR, forbid G_N.
But: interacting Abelian gravity is possible!

Gravity on the lattice

Causal dynamical triangulations approach [Ambjørn,Jurkiewicz,Loll] to $3+1$ lattice gravity:

Naive sum over triangulations does not work (branched polymers, crumpled phases).

Modify the rules, include a preferred causal structure:



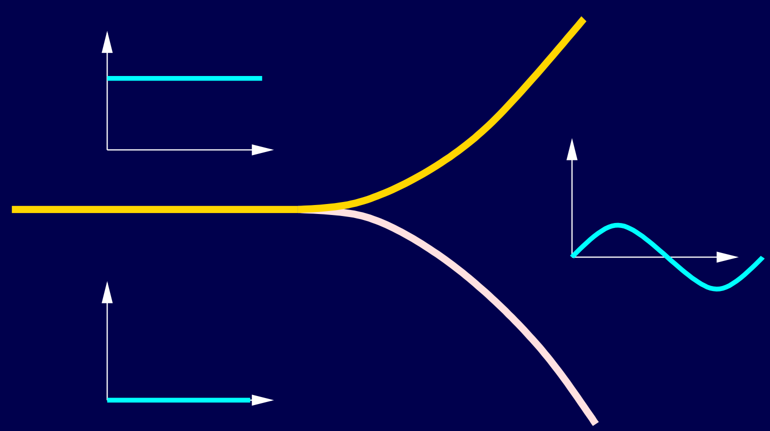
With this relevant change of the rules, a continuum limit appears to exist: The spectral dimension $d_s \approx 4$ in IR, and $d_s \approx 2$ in UV. Continuum gravity with anisotropic scaling: $d_s = 1 + D/z$. ([Benedetti,Henson,2009]: works in $2+1$ as well.)

Relevant deformations, RG flows, phases

The Lifshitz scalar can be deformed by relevant terms:

$$S = \frac{1}{2}\int dt\, d^D\mathbf{x}\, \left\{\dot{\phi}^2 - (\Delta\phi)^2 - \mu^2\partial_i\phi\partial_i\phi + m^4\phi^2 - \phi^4\right\}$$

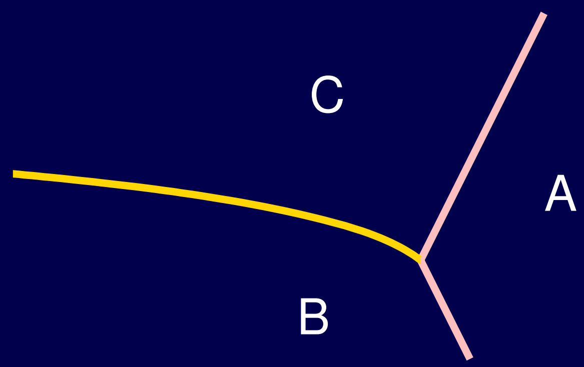
The undeformed $z=2$ theory describes a tricritical point, connecting three phases – disordered, ordered, spatially modulated ("striped") [A. Michelson, 1976]:



Phase structure in the CDT approach

Compare the phase diagram in the causal dynamical triangulations:

[Ambjørn et al, 1002.3298]



Note: $z = 2$ is sufficient to explain three phases.
Possibility of a nontrivial $z \approx 2$ fixed point in $3 + 1$ dimensions?

RG flows in gravity: $z = 1$ in IR

Theories with $z > 1$ represent candidates for the UV description. Under relevant deformations, the theory will flow in the IR. Relevant terms in the potential:

$$\Delta S_V = \int dt\, d^D\mathbf{x}\sqrt{g}N \left\{ \ldots + \mu^2(R - 2\Lambda) \right\}.$$

the dispersion relation changes in IR to $\omega^2 \sim k^2 + \ldots$
the IR speed of light is given by a combination of the couplings
μ^2 combines with $\kappa, \ldots$ to give an effective G_N.

Sign of k^2 in dispersion relation is opposite for the scalar and the tensor modes! Can we classify the phases of gravity? Can gravity be in a modulated phase?

My final two lectures are based on:

T. Griffin, K.T. Grosvenor, P.H. and Z. Yan,
Multicritical Symmetry Breaking and Naturalness of Slow Nambu-Goldstone Bosons,
arXiv:1308.5967, Phys. Rev. **D88** (2013) 101701;

T. Griffin, K.T. Grosvenor, P.H. and Z. Yan,
Scalar Field Theories with Polynomial Shift Symmetries,
arXiv:1412.1046, Commun. Math. Phys. **340** (2015) 1720;

T. Griffin, K.T. Grosvenor, P.H. and Z. Yan,
Cascading Multicriticality in Nonrelativistic Spontaneous Symmetry Breaking,
arXiv:1507.06992, Phys. Rev. Lett. **115** (2015) 241601;

T. Griffin, K.T. Grosvenor, P.H., C. Mogni and Z. Yan,
in progress.

Puzzles of Naturalness

Some of the most fascinating open problems in modern physics are all problems of naturalness:

- The cosmological constant problem
- The Higgs mass hierarchy problem
- The linear resistivity of strange metals, the regime above T_c in high-T_c superconductors [Bednorz&Müller '86; Polchinski '92]

In addition, the first two •s – together with the recent experimental facts – suggest that we may live in a strangely simple Universe . . .

Naturalness is again in the forefront
(as are its possible alternatives: landscape? . . .?)
If we are to save naturalness, we need new surprises!

What is Naturalness?

Technical Naturalness: 't Hooft (1979)

"The concept of causality requires that macroscopic phenomena follow from microscopic equations."

"The following dogma should be followed: At any energy scale μ, a physical parameter or a set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system."

Example: Massive $\lambda\phi^4$ in $3+1$ dimensions.

$$\lambda \sim \varepsilon, \quad m^2 \sim \mu^2 \varepsilon, \quad \mu \sim m/\sqrt{\lambda}.$$

Symmetry: The constant shift $\phi \to \phi + a$.

"Pursuing naturalness beyond 1000 GeV will require theories that are immensely complex compared with some of the grand unified schemes."

Gravity without Relativity

(a.k.a. gravity with anisotropic scaling, or Hořava-Lifshitz gravity)

Gravity on spacetimes with a preferred time foliation (cf. FRW!)

Opens up the possibility of new RG fixed points, with improved UV behavior due to anisotropic scaling.

Field theories with anisotropic scaling:

$$x^i \to \lambda x^i, \quad t \to \lambda^z t.$$

z: dynamical critical exponent – characteristic of RG fixed point. Many interesting examples in condensed matter, dynamical critical phenomena, quantum critical systems, . . ., with $z = 1,\ 2,\ \ldots,\ n,\ \ldots$, or fractions ($z = 3/2$ for KPZ surface growth in $D = 1$), . . ., continous families . . .

. . . and now gravity as well, with propagating gravitons, formulated as a quantum field theory of the metric.

Lifshitz spacetime

Just to be clear about terminology:

"Lifshitz" used for many different things in physics, historically and even more so now.

By the Lifshitz spacetime we will mean $\mathbf{R}^{D+1}$ with the flat metric $g_{ij} = \delta_{ij}$, $N = 1$, $N_i = 0$.

By Lifshitz symmetry we will mean the isometries of the Lifshitz spacetime: Spatial rotations plus spacetime translations,

$$x^i \to \Lambda^i_j x^j + b^i, \qquad t \to t + b.$$

If a QFT with Lifshitz symmetries is at an RG fixed point, it develops an extra symmetry, anisotropic conformal symmetry:

$$x^i \to \lambda x^i, \qquad t \to \lambda^z t.$$

Spontaneous Symmetry Breaking

Global internal symmetry breaking leads to Nambu-Goldstone modes. Phenomenon is remarkably universal, across many fields dealing with many-body systems.

But how many NG modes, and what is their low-energy dispersion relation?

- Relativistic case: All questions answered by Goldstone's theorem: One NG per broken generator, gapless=massless, $z = 1$ dispersion $\omega = k$.

- Nonrelativistic case: Classify NG modes by classifying their low-energy effective QFTs [Murayama&Watanabe, '12,'13].

Let's focus for definiteness on systems with Lifshitz symmetries. Write down possible EFT's for NG modes π^I.

Nonrelativistic Goldstone Theorem?

Assume Lifshitz symmetry. Then [Murayama&Watanabe]: the EFTs are

$$S = \int dt\, d^D\mathbf{x} \left(\Omega_I(\pi)\dot{\pi}^I + g_{IJ}\dot{\pi}^I\dot{\pi}^I - h_{IJ}\partial_i\pi^I\partial_i\pi^J + \ldots\right).$$

Hence, this yields two types of NG modes:

• Type A, $z = 1$ dispersion $\omega = ck$ (those unpaired by Ω, with no T-reversal breaking). As in the relativistic case, one Type A NG mode per one broken generator.

• Type B, dispersion $\omega \sim k^2$. Each associated with a *pair* of broken symmetry generators, as paired by Ω. Minimal T-reversal symmetry is broken.

Anything else would be fine tuning . . . or would it?