Gravity and the Quantum

❀ Petr Hořava ❀

Berkeley Center for Theoretical Physics

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Outline

I. Why Gravity and the Quantum?

II. Holography for Non-Relativistic Systems

III. Gravity and and Puzzles of Naturalness

IV. Surprises with Non-Relativistic Naturalness

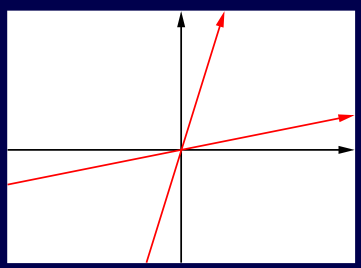
Later parts based on work with:
Charles Melby-Thompson, Tom Griffin, Kevin Grosvenor,
Ziqi Yan, Chris Mogni, . . .

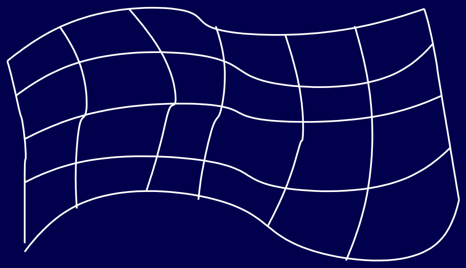
I.
Why Gravity and the Quantum?

Fundamental physics in the 21st Century

Built on two paradigms of the 20th century:

Relativity – first special, unifying space and time via Lorentz symmetries, characterized by the speed of light c,
. . . – then general, unifying gravity and spacetime geometry:





describes the universe at large scales (black holes, big-bang, . . .)

and then there is Quantum mechanics, characterized by the Planck constant $\hbar$, measuring the uncertainty between coordinates q and momenta p; describes our worlds well at microscopic scales (everything except gravity).

Reasons for unification of QM and GR

Why to look for quantum gravity?

1. Conceptual unity of “fundamental” interactions.

There is also condensed matter (many-body physics in fixed spacetime), with fascinating “derived” or “emergent” collective phenomena.

2. History of unifications – as explanations of dimensionful constants of Nature – because Newton’s constant remains unexplained, one more revolution is left! (also – new twist to this puzzle: the cosmological constant Λ)

3. Human curiosity: Which paradigm is more fundamental? Relativity, or “quantum”?

Early attempts to find quantum gravity

Classical gravity is described by an action principle,

$$S_{EH} = \frac{1}{16\pi G_N} \int_M d^4x \, \sqrt{g} \, (R - 2\Lambda),$$

which enjoys a local "gauge invariance" – under spacetime diffeomorphisms $\mathrm{Diff}(M)$.

So, let's just apply techniques of relativistic quantum field theory, which worked so well for Yang-Mills and matter!

Problems with gravity: Non-renormalizable ($=$ not "UV complete"), hence only an effective theory, predicting its own limits and eventual demise, around (or way before!) the characteristic scale, the Planck scale.

First "Coincidences"

In the Einstein-Hilbert action in D spacetime dimensions, G_N is dimensionless in $D = 2$ (gravity wants to live in two dimensions?)

The Hamiltonian is a sum of constraints, associated with gauge symmetries:

$$H = \int_\Sigma d^{D-1}x \, (N^\perp \mathcal{H}_\perp + N^i \mathcal{H}_i)$$

with N, N_i the lapse and shift, and $\mathcal{H}_\perp$, $\mathcal{H}_i$ the superhamiltonian and supermomentum. Their algebra is *not* a Lie algebra:

$$[\mathcal{H}_\perp(\mathbf{x}), \mathcal{H}_\perp(\mathbf{y})] \sim g^{ij} \partial_i \mathcal{H}_j$$

The RHS can only be made field-independent in $D = 2$.

String theory takes advantage of these coincidences.

Puzzles of (quantum) gravity

The effective, semiclassical theory of gravity has raised lots of fascinating questions, some old and some new:

• is gravity really just the dynamics of spacetime geometry?

• why do we live in a huge universe?

• what becomes of spacetime at shortest distances?

• is there a statistical explanation of black-hole entropy?

• is spacetime physics holographic?

• what is the nature of dark matter and dark energy – is it exotic matter, exotic gravity, or perhaps a mixture of both?

• in the end, do we modify gravity and relativity, or do we modify quantum mechanics, or perhaps both?

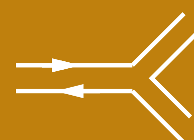
So, why Gravity and the Quantum?

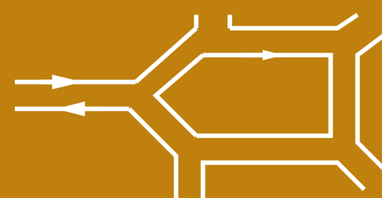
Besides the phenomenology and the fact that we live in a gravitating universe, there is a very compelling conceptual reason why to marry gravity and the quantum:

Structure of QFT at large N

Consider a theory of $N \times N$ matrices M,

$$S = \frac{1}{g^2} \int \mathrm{Tr} \left\{ (\dot{M})^2 + M^2 + M^3 + \dots \right\} .$$

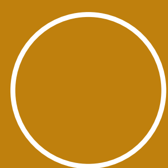
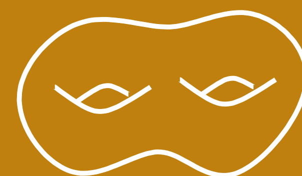
Feynman diagrams:  $\sim g^2$,  $\sim g^{-2}$,

while a closed loop brings  $\sim N$.

Define 't Hooft coupling $\lambda \equiv g^2 N$. Each diagram contributes

$$g^{2(\#P-\#V)} N^{\#L} = N^{\#V-\#P+\#L} \lambda^{\#P-\#V} = \left(\frac{1}{N}\right)^{2h-2} \lambda^{\#P-\#V}.$$

The sum over all Feynman diagrams organizes itself into a string theory expansion!

$\mathcal{A} = \frac{1}{g_s^2}$  $+$  $+ g_s^2$  $+ \ldots,$

where the role of the string coupling constant is played by $g_s = \frac{1}{N}$,

$$\mathcal{A} = \sum_{h=0}^{\infty} g_s^{2h-2} \mathcal{F}_h(\lambda, \ldots).$$

But: which string theory??

Which string theory??

Only three short steps to see how gravity inevitably fits in!

1: recall that QFT is not about Lagrangians, it is all about symmetries and renormalization-group flows.

Wilsonian paradigm: Organize nature scale-by-scale.
So, classify fixed points of RG first: CFTs? (=simplest QFTs)

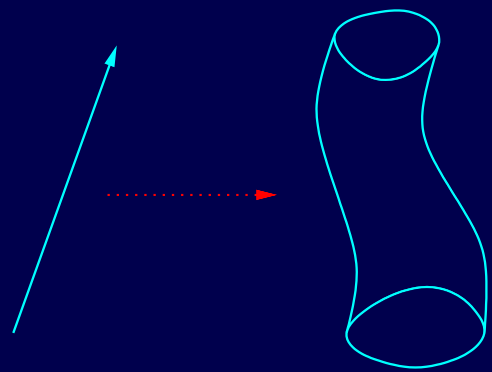
2: observe that symmetries (such as conformal $SO(D,2)$, superconformal, . . . should be realized on the string side by isometries: The string-theory dual of a CFT lives on AdS space!

3: basics of AdS/CFT dictionary: Bulk-boundary correspondence. Local operators od conformal dimension Δ vs. bulk fields of mass m.

Existence of conserved local $T^{\mu\nu}$ in QFT implies existence of a massless spin-two field g_{MN} in the bulk: **GRAVITY!!**

Superstring theory

Basic idea almost embarrassingly simple: Replace point-like particles with extended objects, strings:



• For the first time, we have a mathematically consistent quantum theory which (automatically) includes gravity!

Answer to our basic question: It is *quantum mechanics* that wins, general relativity is modified.

Many successes and exciting results; for example, space(time?) might be an "emergent property" of matter.

String theory: the successes

Simply too many to give an exhaustive list!

• Matter and spacetime geometry intimately tied to each other: stringy excitations create the background in which they propagate, giving new insights into the nature of spacetime.

• Besides conventional matter, new "stringy" defects: D-branes.

• The entropy of at least some black holes/branes explained microscopically!

• Spacetime, or at least space, can be "emergent"!

• A powerful tool for generating important "spin-off" ideas in physics and mathematics (SUSY, extra dimensions, braneworlds, gauge/gravity duality, etc)

String theory: current limitations?

Very good at understanding supersymmetric vacua and supersymmetric states. This has lead to dualities, microscopic understanding of entropy for supersymmetric black holes, uniqueness of the theory (= M-theory) etc.

Not so good at describing time-dependent phenomena, such as cosmology, even the simplest cosmological spacetime – the de Sitter space (= vacuum solution with positive Λ).

Very beautiful and rich, web of dualities, engineering of SUSY QFT's, AdS/CFT correspondence; landscape of vacua (populated by eternal inflation?), . . .

perhaps too rich and too complex for addressing the most basic questions? Compare QCD: Embeddable into string theory, but independently UV complete. What about gravity?

Is there a "smaller" quantum gravity?

String theory is a beautiful theory of quantum gravity, but it appears both "too large" and "too small."

Lessons from string theory:

Quantum mechanics is absolute, but GR undergoes corrections.

Lorentz symmetry unlikely to be fundamental, if space is emergent. (Once accepted, this scenario leads to systematic energy-dependent violations of Lorentz invariance.)

Motivation for string theory:

Reaching configurations far from equilibrium, far from static/stationary?

Gravity without Relativity

(a.k.a. gravity with anisotropic scaling, or Hořava-Lifshitz gravity)

Gravity on spacetimes with a preferred time foliation (cf. FRW!)

Opens up the possibility of new RG fixed points, with improved UV behavior due to anisotropic scaling.

Field theories with anisotropic scaling:

$$x^i \to \lambda x^i, \quad t \to \lambda^z t.$$

z: dynamical critical exponent – characteristic of RG fixed point. Many interesting examples in condensed matter, dynamical critical phenomena, quantum critical systems, ..., with $z = 1,\ 2,\ \ldots,\ n,\ \ldots$, or fractions ($z = 3/2$ for KPZ surface growth in $D = 1$), ..., continous families ...

... and now gravity as well, with propagating gravitons, formulated as a quantum field theory of the metric.

Gravity with anisotropic scaling

(also known as Hořava-Lifshitz gravity)

Evgenii Mikhailovich Lifshitz (1915 – 1985)

Why is this interesting?

(i) Phenomenology of gravity in our Universe, $3+1$ dimensions. How close can this resemble GR in IR? The multicritical universe scenario;

(ii) Gravity duals of field theories in AdS/CFT; in particular, candidates for duals of nonrelativistic field theories;

(iii) Useful also in conventional Einstein gravity, in spacetimes which are asymptotically anisotropic!

(iv) Analytic tool for understanding numerical results of lattice quantum gravity;

(v) Gravity on worldvolumes of branes;

(vi) Mathematical applications (theory of the Ricci flow);

(vii) Emergent Gaussian IR fixed points in lattice systems of condensed matter.

Comparison to Asymptotic Safety

Search for a UV fixed point in gravity:

Asymptotic safety: looking for relativistic, nontrivial RG fixed points. [Weinberg,. . .]

Gravity with anisotropic scaling: looking for nonrelativistic, often Gaussian fixed points. (These can be UV, leading to improved short-distance behavior of gravity; or IR, emergent in condensed matter systems.)

Price paid for improved UV behavior: Anisotropy between space and time (or even spatial) and absence of Lorentz symmetry at short distances.

Flow between UV and IR: from $z > 1$ to $z = 1$. Lorentz symmetry must be emergent at low energies, with systematic energy-dependent Lorentz-violating corrections.