Nonperturbative RG flow of the Higgs potential

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February 22

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RESEARCH TRAINING GROUP
QUANTUM AND GRAVITATIONAL FIELDS

LHC run II already started!

- search for new physics beyond the standard model

http://people.physics.tamu.edu/kamon/research/refColliders/LHC/LHC_is_back.html

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- detailed studies of the Higgs

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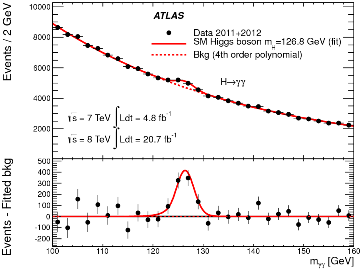
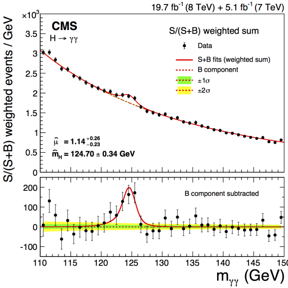
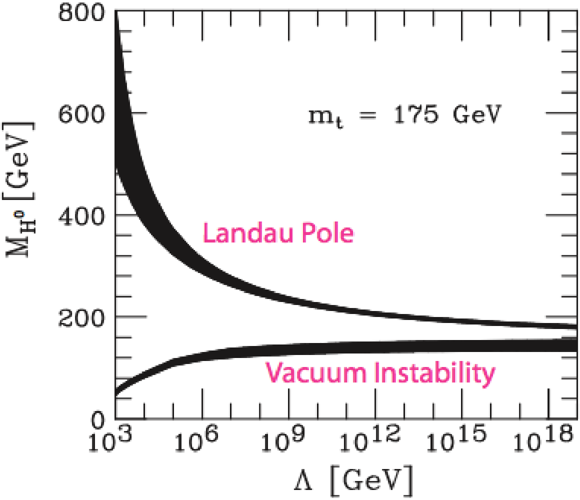
Highlight LHC run I: $m_{\mathrm{H}} = 125.09^{\pm 0.21(stat)}_{\pm 0.11(sys)}\,\mathrm{GeV}$





Higgs mass bounds & vacuum stability



Hagiwara et al '02

Krive, Linde '76
Maiani et al '78
Krasnikov '78
Politzer, Wolfram '78
Cabibbo et al. '79
Hung '79
Linde '80
Lindner '85
Wetterich '87
Lindner et al '89
Sher '89
Ford et al 93
Altarelli, Isidori '94
Espinosa, Quiros '95
Schremp, Wimmer '96
...

Higgs-Top model

- effective potential dominated by top fluctuations

$$S = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + U(\phi^2) + \bar{\psi} i \partial\!\!\!/ \psi + i h \phi \bar{\psi} \psi \right]$$

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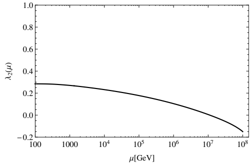
$$\mu \frac{d\lambda_2}{d\mu} \equiv \beta_{\lambda_2} = \frac{1}{4\pi^2}\left[3\lambda_2^2 + \lambda_2 h^2 - h^4\right]$$



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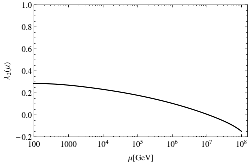

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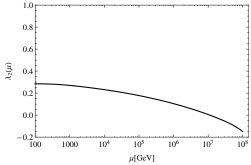

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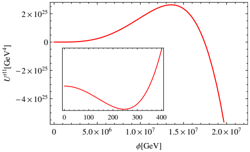

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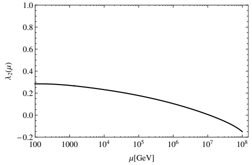

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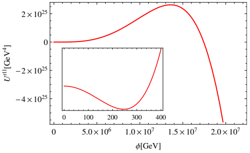


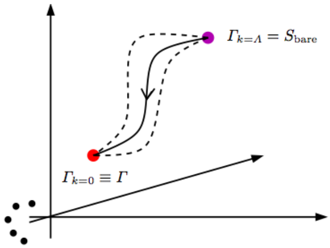
BUT:

- in contrast to lattice results
- interaction part of the fermion determinant is strictly positive Gies, RS '14
- multi-scale problem $U(\mu;\phi)$

Functional renormalization group

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k}\right]$$

Wetterich '93



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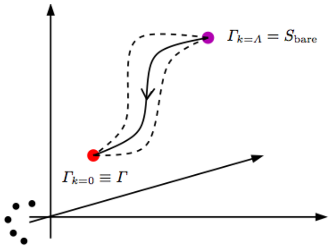

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$$\partial_k U(k,\phi) = \beta_U, \qquad \partial_k h^2 = \beta_{h^2}$$

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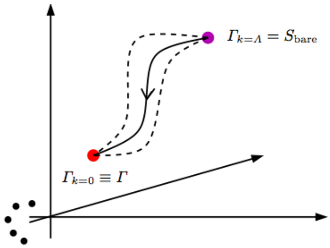

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$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 \qquad \textit{or} \qquad U_\Lambda = \frac{\lambda_{2\Lambda}}{8}(\phi^2 - v_\Lambda^2)^2$$

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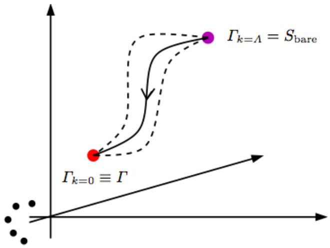

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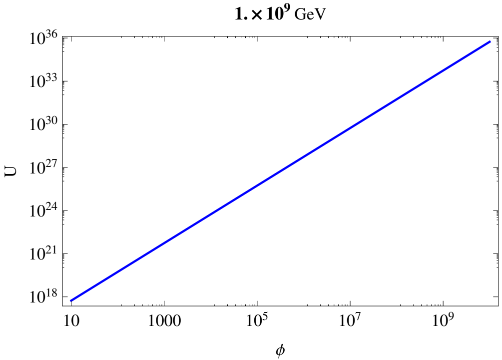
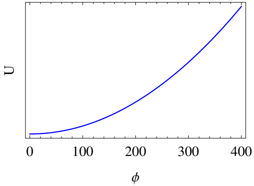
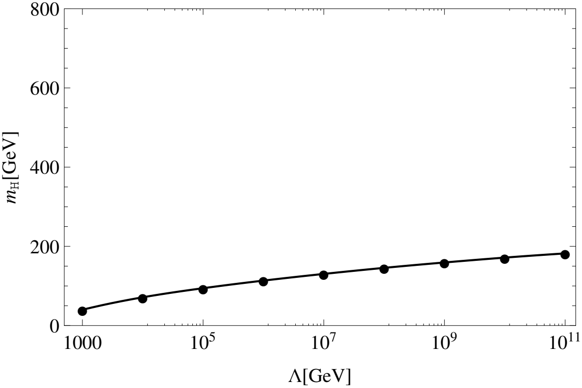
$$\lambda_{1\Lambda}\ (\text{or } v_\Lambda) \to v_0 = 246\,\mathrm{GeV}$$
$$h_\Lambda \to m_{\mathrm{top}} = 173\,\mathrm{GeV}$$

RG flow of the Higgs potential





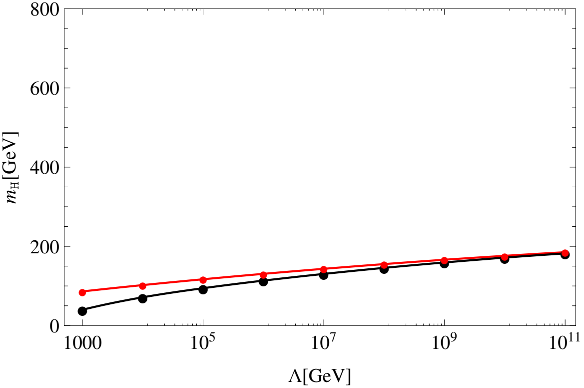
Nonperturbative Higgs mass bounds



Gies, Gneiting, RS '13

$\lambda_{2\Lambda} = 0$

Nonperturbative Higgs mass bounds



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$\lambda_{2\Lambda} = 0.1$

Gies, Gneiting, RS '13

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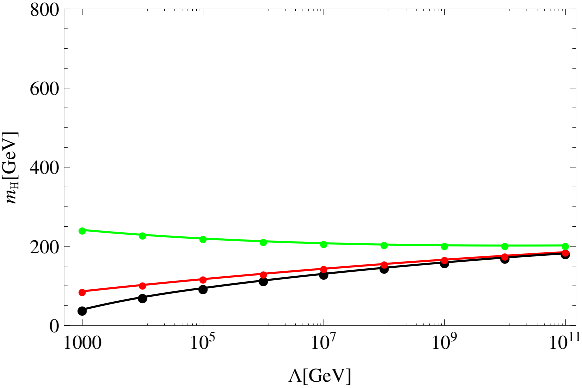

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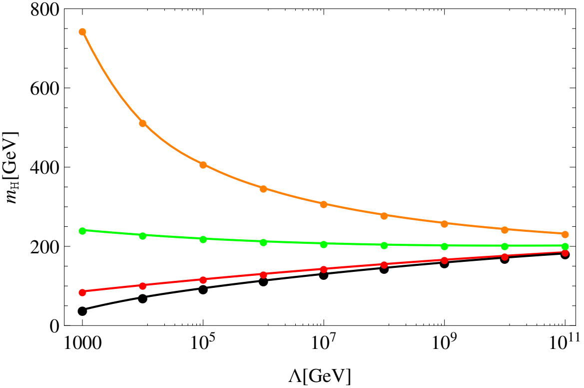

$\lambda_{2\Lambda} = 0$
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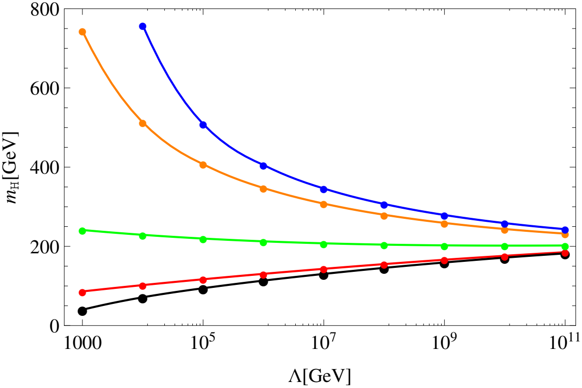

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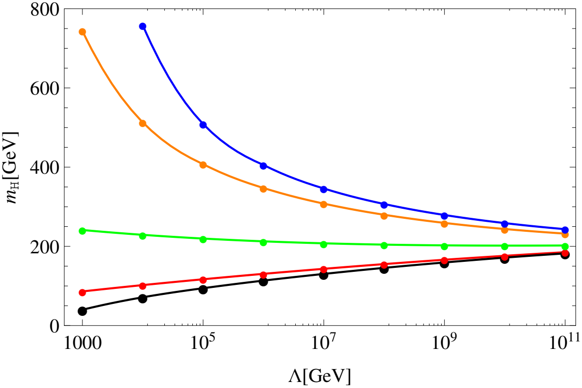

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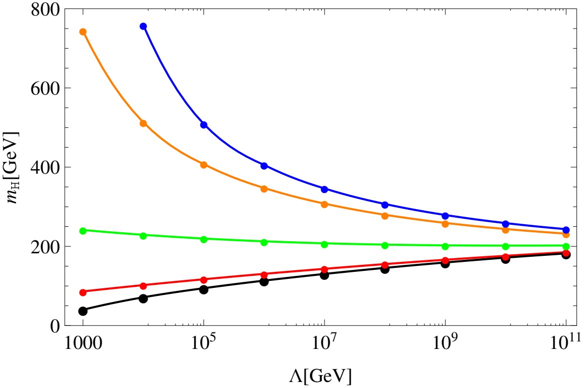

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Gies, Gneiting, RS '13

- $m_{\mathrm{H}}(\lambda_{2\Lambda})$ is monotonically increasing
- natural lower bound for quartic bare potentials $\lambda_{2\Lambda}\,\phi^4$ (cf. lattice simulations Gerhold et al. '07)

Generalized UV potentials

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{48\Lambda^2}\phi^6$$

- we can choose $\lambda_{2\Lambda} < 0$, if the potential is stabilized by $\lambda_{3\Lambda} > 0$

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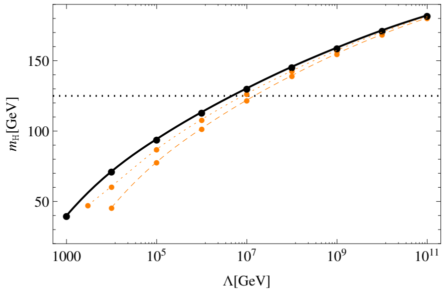

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$\lambda_{2\Lambda} = -0.05, \ \lambda_{3,\Lambda} = 3$
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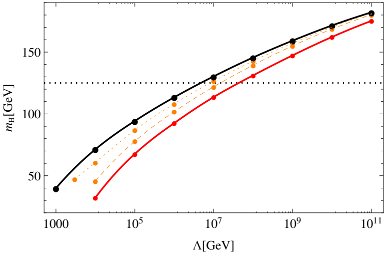

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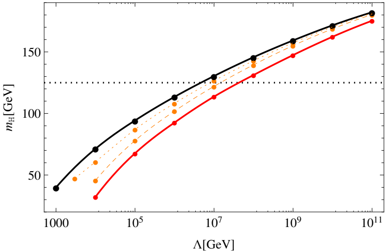

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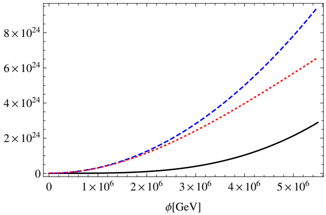

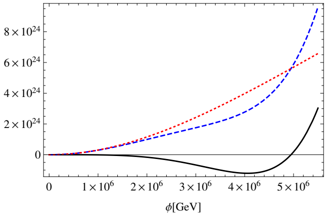

Borchardt, Gies, RS '16

cf. poster by J. Borchardt

Conclusions & Outlook

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Outlook

- Investigation of nonperturbative flow equations within the entire standard model.
- Influence of UV potentials beyond polynomial type?