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Local Quantum Gravity

N.C., Litim, Pawlowski:	Phys.Lett. B728, 2014
N.C., Knorr, Pawlowski, Rodigast:	Phys. Rev. D 93, 2016
N.C., Knorr, Meibohm, Pawlowski, Reichert:	Phys. Rev. D 92, 2015

Schladming
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Outline

- Introduction: Quantum Gravity and Asymptotic Safety
- Renormalization: Scale Evolution of Vertices
  - Vertices and Flow Equations
  - **Locality**
- Phase Diagram of Quantum Gravity
- Outlook

Classical Gravity

- classical gravity is described by general relativity on all scales observed so far

$$10^{-4}\mathrm{m} \quad \longrightarrow \quad 10^{26}\mathrm{m}$$

• no precision tests
• dark matter ?!

- Einstein Hilbert action

$$\boxed{S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^d x \sqrt{-\mathrm{det} g}\,(-R(g) + 2\Lambda)}$$

Newtons Constant

Ricci Scalar

Cosmological Constant

- dimensional analysis $d = 4$

$$\boxed{[G_{\mathrm{N}}] = \mathrm{energy}^{-2}} \qquad \boxed{[\Lambda] = \mathrm{energy}^{2}}$$

Perturbative Quantization

- relevant energy scale: Planck scale $M_{\mathrm{Pl}} \approx 10^{19}\mathrm{GeV}$

- expansion parameter for n-point Greens functions:

$$g \equiv G_{\mathrm{N}} E^2 = \frac{E^2}{M_{\mathrm{Pl}}^2}$$

dimensionless

energy scale

→ higher loop orders require higher derivative counterterms !

→ full theory is either divergent or includes infinitely many free parameters

(perturbatively) <u>non-renormalizable</u>

Asymptotic Safety in a Nutshell

- Construction of quantum gravity in asymptotic safety: lecture of A. Eichhorn

  (a) d.o.f. carried by the metric field

  (b) diffeomorphism invariance (subtleties: background field dependence, ...)

- Quantum fluctuations ⇨ $g_i \longrightarrow g_i(k)$ scale dependent couplings

  energy scale

- **UV fixed point:**

  $$\boxed{\lim_{k\to\infty} g_i(k) = g_{i,*}}$$

  finite fixed point value

- finite number of free parameters (predictive)

⇔ **Asymptotic Safety:**

Non-pert. Renormalization

Weinberg (1993)

- example: Asymptotic Freedom : $g_* = 0$ (perturbative)

Vertices and Flows

- **<u>Functional Renormalization Group</u>** Wetterich (1993)

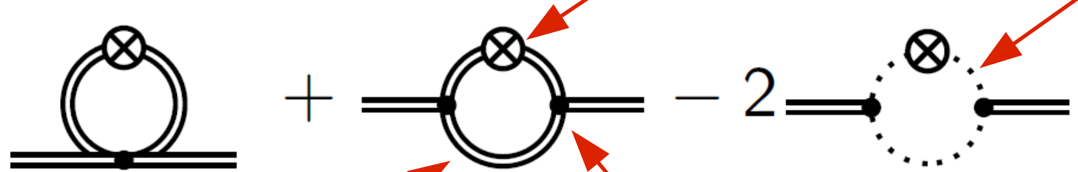
Scale dependence of full vertex functions $\Gamma^{(n)} = \dfrac{\delta^n \Gamma[\phi]}{\delta \phi^n}$

Non-Perturbative

regulator

ghost propagator

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma^{(2)} = -\frac{1}{2}$$

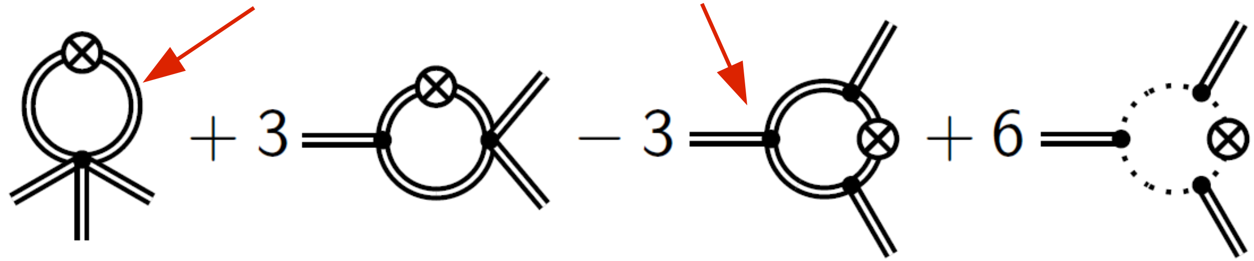


RG scale

full graviton propagators

full graviton vertices

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma^{(3)} = -\frac{1}{2}$$



$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma^{(n)} = \mathrm{Flow}[\Gamma^{(2)}, \ldots, \Gamma^{(n+2)}]$$

infinite hierarchy of flow equations

Locality I

- **Definition: Locality**

$$\lim_{t_i/k^2\to\infty} \frac{|\dot{\Gamma}_k^{(n)}(\mathbf{p})|}{|\Gamma_k^{(n)}(\mathbf{p})|} = 0\,, \quad \text{with} \quad \mathbf{p} = (p_1, ..., p_n)$$

t_i : momentum channels $\quad | \cdot |$: projection on tensor structure

$\triangleq$ separation of scales

- UV physics decouples from long distance physics

  e.g.: QFT in finite box: $\longrightarrow$ change length L $\longrightarrow$ UV fluct. unaffected

- Necessary property for well-defined RG-steps (block spinning)

  $\Rightarrow$ required by physics and mathematics

- pert. renormalizable theories: locality is trivial

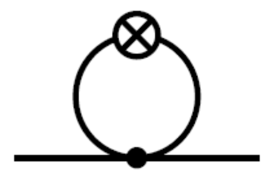
  pert. non-renormalizable theories: non-trivial cancellations necessary!

Locality II

- Locality in pert. renormalizable theories

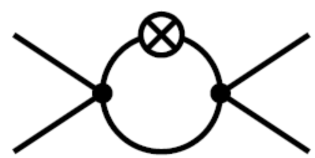
example: $\boxed{\phi^4\text{-theory:}}$

Vertices: $\Gamma^{(2)} \sim p^2$ and $\Gamma^{(4)} \sim p^0$

flow of two-point function: $\dot{\Gamma}^{(2)} \sim$  $\sim p^0$

$$\lim_{p\to\infty} \frac{\dot{\Gamma}^{(2)}}{\Gamma^{(2)}} = 0$$

momentum local!

flow of four-point function: $\dot{\Gamma}^{(4)} \sim$  $\sim p^{-2}$

$$\lim_{p\to\infty} \frac{\dot{\Gamma}^{(4)}}{\Gamma^{(4)}} = 0$$

momentum local!

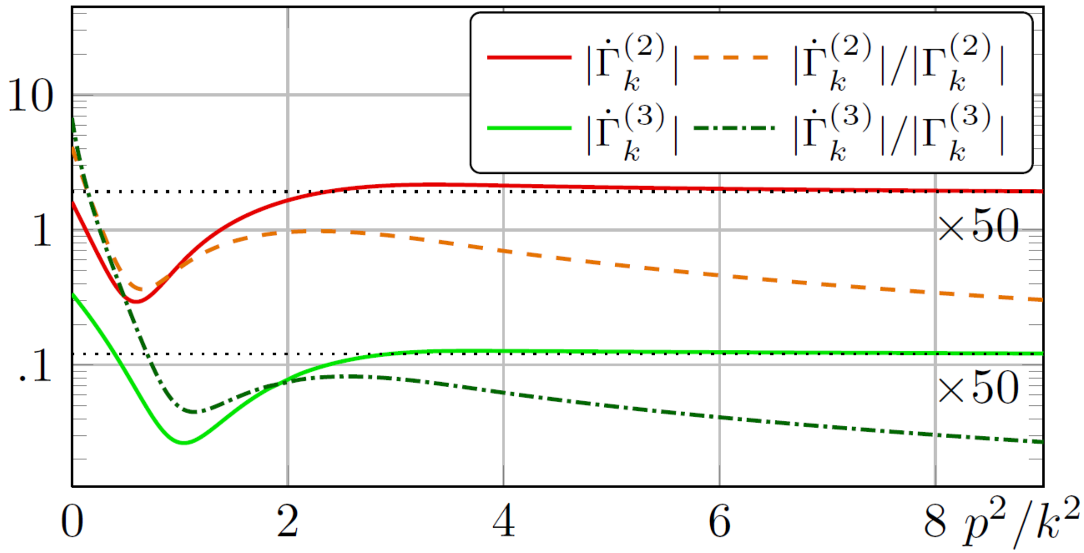
Locality III

- **What about gravity?**

Vertices: $\Gamma^{(n)} \sim p^2 \quad \forall\, n$ (Einstein-Hilbert)

power counting suggests: $\dot{\Gamma}^{(n)} \sim p^2$ non-local?

- **Explicit calculation:**



$\Gamma^{(2)}$ and $\Gamma^{(3)}$ momentum local!

highly non trivial cancelation of diagrams!

linked to diffeomorphism invariance!

Locality IV

- **Locality as a construction principle?**

  → (brave) proposal: only local terms allowed !!!

  → diffeomorphism invariance only necessary, not sufficient?

- Systematics: check locality property for higher-order operators in $\dot{\Gamma}^{(n)}$

  → two point function $n = 2$:

$$\int \omega R$$

momentum local!

$$\int \omega(R + R^2)$$

momentum local!

$$\int \omega(R + R^2 + C^2)$$

non-local!

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$$\int \omega R$$ ✓ momentum local!

$$\int \omega(R + R^2)$$ ✓ momentum local!

$$\int \omega(R + R^2 + C^2)$$ non-local!

The Running Couplings

- Fixed point condition of Asymptotic Safety

$$\longrightarrow \quad \beta_{g_i} = k\frac{\mathrm{d}}{\mathrm{d}k} g_i(k) \overset{!}{=} 0$$

UV attractive directions $\longrightarrow$ free parameter

UV repulsive directions $\longrightarrow$ parameter fixed from theory

- Beta Functions from Flows:

$\longrightarrow$ parameterization of vertices with running couplings

$\longrightarrow$ coupled system of flow equations for two and three point function

two point function $\Gamma^{(2)}$

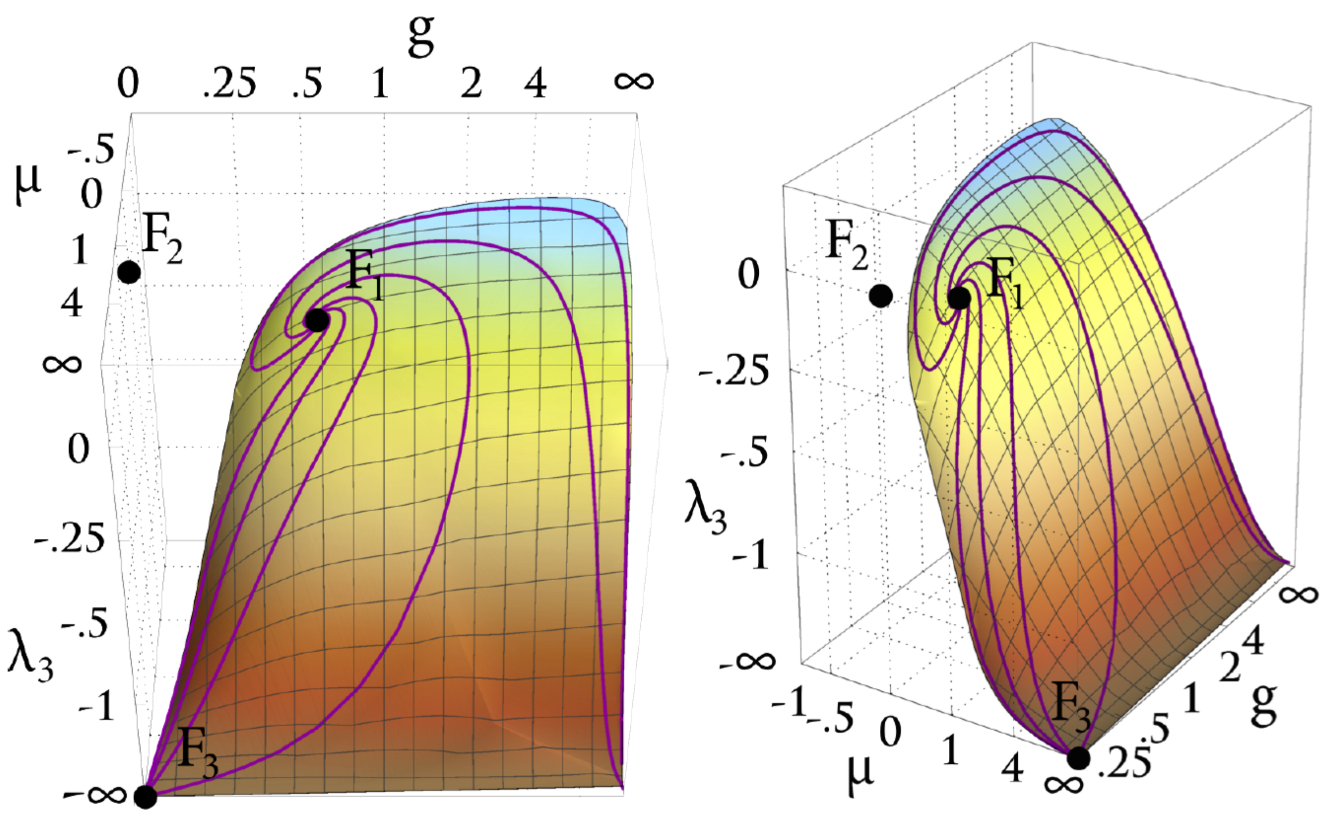
$$\eta(k,p) \;,\; \mu(k) = -2\lambda_2(k)$$

three point function $\Gamma^{(3)}$

$$G_N(k) \;,\; \lambda_3(k)$$

The Phase Diagram

- Phase Diagram: Solutions for different initial conditions



F_1: Non-Gaussian UV-FP

F_2: Gaussian FP

F_3: Non-Gaussian IR-FP

→ At the UV-FP: two attractive, one repulsive direction

→ Further evidence for asymptotic safety in quantum gravity

Summary and Outlook

- Non-Pert. Renormalization of Quantum Gravity

  → Asymptotic Safety

- Vertex Expansion and Vertex Flows in Quantum Gravity
- Locality of Renormalization Group Flows
- UV-fixed point with one irrelevant direction

<u>Outlook</u>

- understanding locality
- phase diagram with higher derivative operators
- curved backgrounds
- higher order correlation functions

Thank You!!!