Some Recent Developments in 4D CFTs

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Overview of the talk

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Part I

CFT in a non-lagrangian approach

Operators in CFT

Operators in CFT $\mathcal{O}_{\Delta}^{(\ell,\bar{\ell})}$ are labeled (follows form group theory) by their spin and non-perturbative dimension. Primary Operators - operators with the lowest dimension in the infinite series of operators. The scaling dimension:

$$\mathcal{O}_{\Delta}^{(\ell,\bar{\ell})}(\lambda x) = \lambda^{-\Delta}\mathcal{O}_{\Delta}^{(\ell,\bar{\ell})}(x)$$

$$\mathcal{O}_{\Delta}^{(\ell,\bar{\ell})}(x,s,\bar{s}) \equiv s^{\alpha_1}\dots s^{\alpha_\ell}\bar{s}_{\dot{\beta}_1}\dots\bar{s}_{\dot{\beta}_{\bar{\ell}}}\mathcal{O}_{\Delta}(x)_{\alpha_1\dots\alpha_\ell}^{\dot{\beta}_1\dots\dot{\beta}_{\bar{\ell}}},$$

where s^{α_1} and $\bar{s}_{\dot{\beta}_{\bar{\ell}}}$ are just some constant complex vectors. (The trick of removing explicit indices is called index-free formalism)

The observables of the theory

$$\langle\mathcal{O}_{\Delta_1}^{(\ell_1,\bar{\ell}_1)}(x_1,s_1,\bar{s}_1)\dots\mathcal{O}_{\Delta_n}^{(\ell_n,\bar{\ell}_n)}(x_n,s_n,\bar{s}_n)\rangle.$$

Three-point functions

For simplicity we will label the operators just by one index

$$\mathcal{O}_i \equiv \mathcal{O}_{\Delta_i}^{(\ell_i,\bar{\ell}_i)}(x_i, s_i, \bar{s}_i)$$

Three-point function

$$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = \mathcal{K}_3 \sum_{a=1}^{N_3} \lambda_{ijk}^a \tau_3^a,$$

$\mathcal{K}_3$ are "kinematic factors" which account for the fields scaling,
τ_3 are tensors which account for the fields spin structure,
λ^a are free parameters characterizing the dynamics.

Three-point functions have been computed for trace-less symmetric operators ($\ell = \bar{\ell}$) by [Costa, Penedones, Poland, Rychkov '11] and for arbitrary spins by [Elkhidir, DK, Serone '14].

Defining a CFT

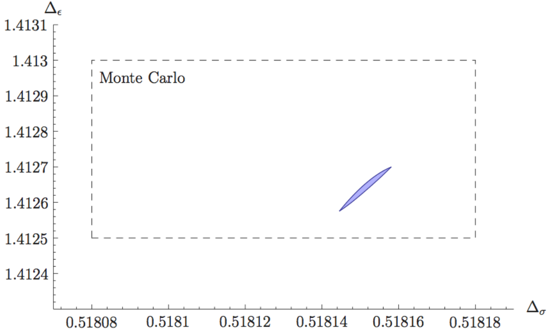
CFT is defined by a set of Primary Relevant Operators $\mathcal{O}_i$ and the CFT data (the set of parameters $\{\Delta_i, \lambda_{ijk}\}$) which describes the dynamics.

Conformal Bootstrap

Is the method of constraining the CFT data using the Conformal Symmetry only. The constraints arise from the the explicit expressions of 4-point functions.

Success: the 3D Ising model

The Ising model can be defined as a CFT with 2 relevant scalar operators σ and ϵ which are oppositely "charged" under the Z_2 symmetry. Numerically it was solved by [El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi '12 and '14] and improved in consequent papers. The plot was taken from [Simmons-Duffin '15].



Obtained by "bootstrapping" the correlators : $\langle\sigma\sigma\sigma\sigma\rangle$, $\langle\epsilon\epsilon\epsilon\epsilon\rangle$ and $\langle\sigma\sigma\epsilon\epsilon\rangle$.

Part II

Applications to phenomenology

$$\langle \psi_\alpha \bar{\psi}^{\dot{\alpha}} \psi_\beta \bar{\psi}^{\dot{\beta}} \rangle$$

In the composite higgs models one can construct "hyper-mesons" (composite higgses) $M \equiv \psi\bar{\psi}$ and "hyper-baryons" (top-partners) $B \equiv \psi\psi\psi$. One is interested to know the relation between Δ_M and Δ_B.

To put constraints on the "central charges": a and c

$$\langle T_{\mu\nu} T_{\rho\sigma} T_{\kappa\eta} T_{\lambda\omega} \rangle$$

QCD conformal window

If one ever attempts to use the bootstrap for studying the fixed point of massless QCD in the conformal window, one needs to go to non-zero spin.

Computing 4-point functions

One can compute a 4-point function using the OPE:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle =$$

$$\sum_{\mathcal{O}} \int D^4 x_0 \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}(x_0)\rangle \langle \widetilde{\mathcal{O}}(x_0)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle \Big|_M =$$

$$\sum_{\mathcal{O}} \sum_{i=1}^{N_3^{12\mathcal{O}}} \sum_{j=1}^{N_3^{34\overline{\mathcal{O}}}} \lambda_i^{12\mathcal{O}} \lambda_j^{34\overline{\mathcal{O}}} W^{ij}$$

The fundamental object W^{ij} is called **conformal partial wave.** For trace-less symmetric operators the author[Costa, Penedones, Poland, Rychkov '11] found a way to write

$$W^{ij} = \mathbb{D}^{ij} W_{seed}.$$

In [Echeverri, Elkhidir, DK, Serone '15] it was generalized to arbitrary spin. One can see a tremendous simplification just by looking at these numbers

$N_4^{spin\ \frac{1}{2}} = 6,\ N_4^{spin\ 1} = 70,\ N_4^{spin\ 2} = 1107.$

Seed Conformal Blocks

The seed **conformal partial waves** are labeled by a non-negative integer p:

$$W_{seed}^{(p)} = \mathcal{K}_4 \sum_{e=0}^{p} G_e^{(p)}(u,v)\mathcal{I}_1^e \mathcal{I}_2^{p-e},$$

where the objects $G_e^{(p)}(u,v)$ are called (seed) **conformal blocks**.

How many $W^{(p)}_{seed}$ ($G^{(p)}_e$) do we need to compute?

- For 4-ϕ correlator: we need to know $p = 0$
- For 4-ψ correlator: we need to know $p = 0, 2$
- For 4-J_μ correlator: we need to know $p = 0, 2, 4$
- For 4-$T_{\mu\nu}$ correlator: we need to know $p = 0, 2, 4, 6, 8$

The case $p = 0$ was computed and given in a remarkably simple form by [Francis Dolan and Hugh Osborn '00], it was further investigated in the consequent papers [Dolan and Osborn '04] and [Dolan and Osborn '11].

In the paper [Echeverri, Elkhidir, DK, Serone '16] it was obtained a general expression for $W^{(p)}_{seed}$ ($G^{(p)}_e$):

$$G^{(p)}_e(z,\bar{z}) = \left(\frac{z\bar{z}}{z-\bar{z}}\right)^{2p+1} \sum_{m,n} c^e_{m,n}\Big(k^{(a_e,b_e;c_e)}_{\rho_1+m}(z)k^{(a_e,b_e;c_e)}_{\rho_2+n}(\bar{z}) - (z \leftrightarrow \bar{z})\Big).$$

Conclusions

- I argued that there is a powerful non-lagrangian method of defining and studding CFTs called **conformal bootstrap**.
- Conformal bootstrap is based on the knowledge of explicit expressions of 4-point functions which are given by **conformal partial waves** or **conformal blocks**.
- I argued that **conformal partial waves** (**conformal blocks**) for operators with spin are very much needed and briefly described the current status of this problem.

Thank you!