A non-perturbative study of the correlation functions of three-dimensional Yang-Mills theory

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From Green functions to 'observables'

Basic building blocks of functional equations: n-point functions $\Gamma_{i_1 \dots i_n}$

Effective action: generating functional of 1PI Green functions

$$\Gamma[\Phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \Phi_1 \dots \Phi_n \Gamma_{i_1 \dots i_n}$$

$\longleftrightarrow$

The set of **all** Green functions describes the theory completely.

$$\Gamma_{ij} = \left.\frac{\delta^2 \Gamma[\Phi]}{\delta\Phi_i \delta\Phi_j}\right|_{\Phi=0},$$

$$\Gamma_{ijk} = \left.\frac{\delta^3 \Gamma[\Phi]}{\delta\Phi_i \delta\Phi_j \delta\Phi_k}\right|_{\Phi=0}, \quad \dots$$

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Green functions $\rightarrow$ 'observables'?

Examples:

- Bound state equations $\rightarrow$ masses and properties of hadrons
- (Pseudo-)Order parameters $\rightarrow$ Phases and transitions

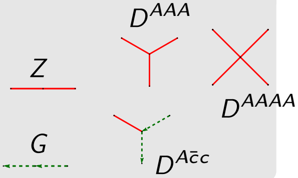
Landau gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

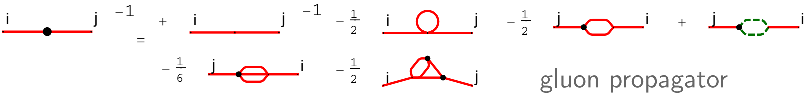
$$\mathcal{L} = \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\,g\,[\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}}\,(-\Box + g\,\boldsymbol{A}\,\times)\,\boldsymbol{c}$



The tower of DSEs



gluon propagator

ghost propagator

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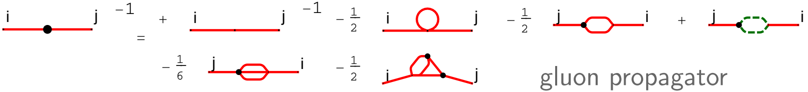

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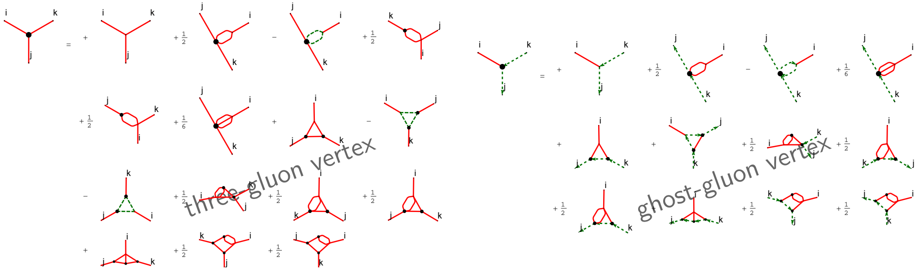

Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

Truncating the equations

Truncation

- Drop quantities (unimportant?)
- Model quantities (good models available? 'true' or 'effective'?)
- Use fits

Ideally: Find a truncation that has (I) no parameters and yields (II) quantitative results.

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Guides

- Perturbation theory
- Symmetries
- Lattice
- Analytic results

Practical obstacle: Manage the system of equations. $\rightarrow$ Automatization tools [Alkofer, MQH, Schwenzer '08; Braun, MQH '11; MQH, Mitter '11; `http://tinyurl.com/dofun2`; `http://tinyurl.com/crasydse`]

Truncation of Yang-Mills system

Neglect all non-primitively divergent Green functions.

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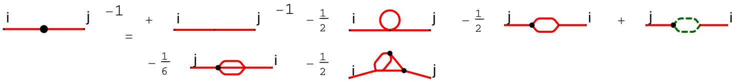
Full propagator equations (two-loop diagrams!):



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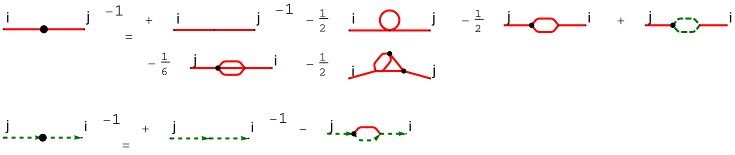
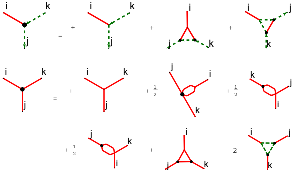
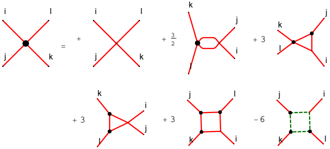


Truncated three-point functions:



Truncated four-gluon vertex:



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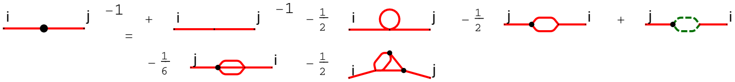

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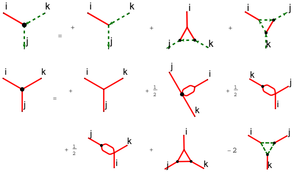

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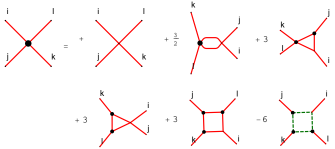

Technical questions: spurious divergences in gluon propagator, RG resummation

Yang-Mills theory in 3 dimensions

$$d = 3$$

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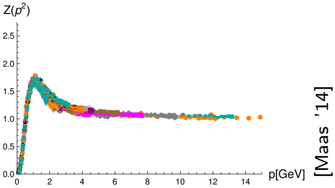
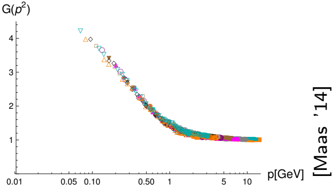
Historically interesting because cheaper on the lattice $\rightarrow$ easier to reach the IR, e.g., [Cucchieri '99; Cucchieri, Mendes, Taurines '03; Cucchieri, Maas, Mendes, '08; Maas '08, '14; Maas, Pawlowski, Spielmann, Sternbeck, von Smekal '09; Cucchieri, Dudal, Mendes, Vandersickel '11; Bornyakov, Mitrjushkin, Rogalyov '11, '13; Cucchieri, Dudal, Mendes, Vansersickel '16]





Continuum results:

- Coupled propagator DSEs: [Maas, Wambach, Grüter, Alkofer '04]
- (R)GZ: [Dudal, Gracey, Sorella, Vandersickel, Verschelde '08]
- YM + mass term: [Tissier, Wschebor '10, '11]
- DSEs of PT-BFM: [Aguilar, Binosi, Papavassiliou '10]

Yang-Mills theory in 3 dimensions: Why again?

NB: Numerically not cheaper for functional equations of 2- and 3-point functions.

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Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle

$\boxed{\Rightarrow \text{Many complications from } d = 4 \text{ absent!}}$

Subtraction of divergences of gluon propagator (d=4)

1. Logarithmic divergences handled by subtraction at p_0.
2. Quadratic divergences subtracted, coefficient C_{sub}.

$$Z(p^2)^{-1} := \underbrace{Z_\Lambda(p^2)^{-1}}_{\uparrow} - C_{\mathrm{sub}}(\Lambda)\left(\frac{1}{p^2} - \frac{1}{p_0^2}\right)$$

calculated right-hand side

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One-loop diagrams with model vertices: C_{sub} can be calculated anlytically, since it is a purely perturbative [MQH, von Smekal '14].

Dynamic vertices? Two-loop diagrams?

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Importance of spurious divergences

Simplification in $d = 3$:

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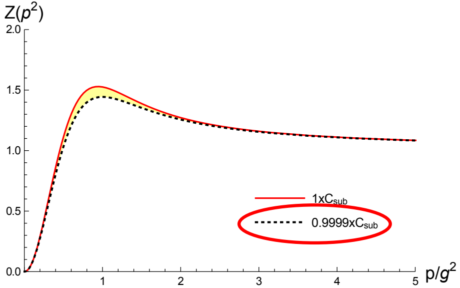
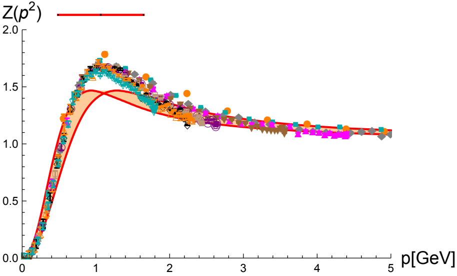
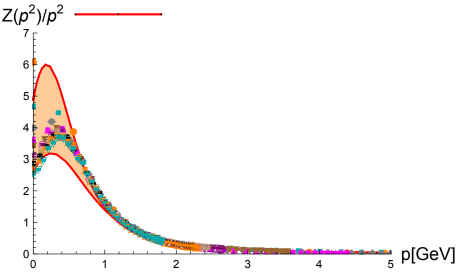
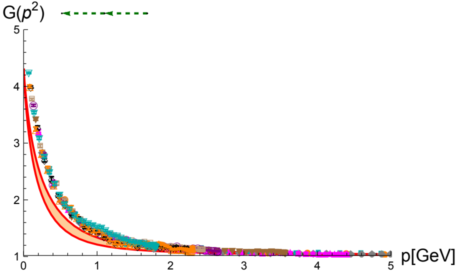


Small deviations $\rightarrow$ large effect.

Results: Propagators





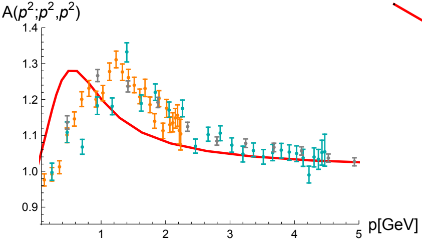


Bands from uncertainty in setting the physical scale.

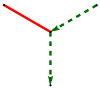
[lattice: Maas '14]

Results: Three-point functions

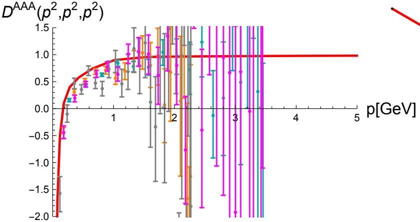
Dressings:



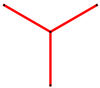
[lattice: Maas, unpublished]



- Maximum position shifted.
- Bump height ok.



[lattice: Cucchieri, Maas, Mendes '08]



- Good agreement with lattice data.
- Linear IR divergence.

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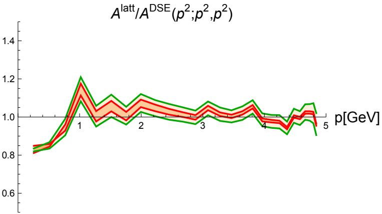
Ratio lattice/DSE results:



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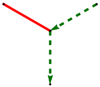

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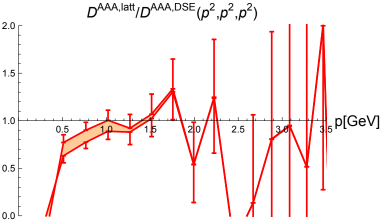

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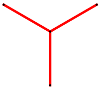

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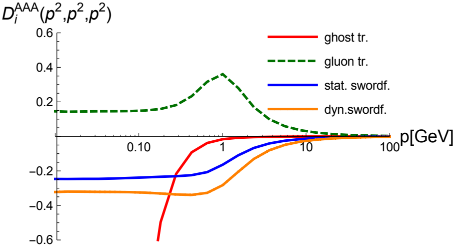
Cancellations in gluonic vertices

Three-gluon vertex:



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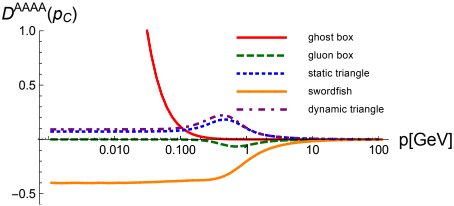

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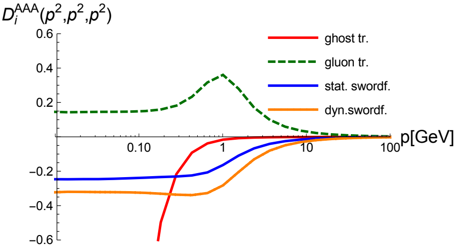

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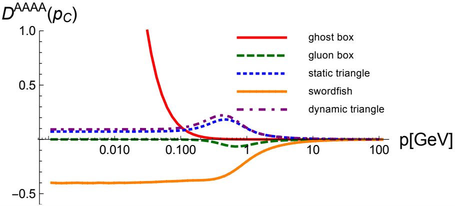

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$\Downarrow$

Higher contributions:

1. Small each or
2. cancellations again?

Non-perturbative gauge fixing

Gribov copies: Gauge equivalent configurations that fulfill the Landau gauge condition $\partial A = 0$.

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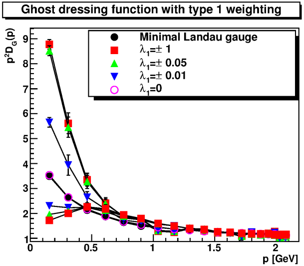
Another possibility: Absolute Landau gauge (global minimum of gauge fixing functional)

$\rightarrow$ Different solutions on the lattice,

e.g. [Maas '09, '11; Cucchieri '97; Bogolubsky et al. '05; Sternbeck, Müller-Preussker '12].



[Maas '13]

NB: Different solutions also from functional equations [Fischer, Maas, Pawlowski '08].

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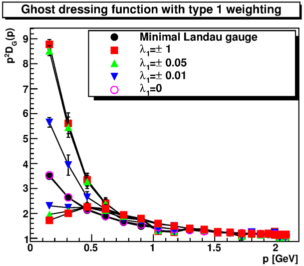

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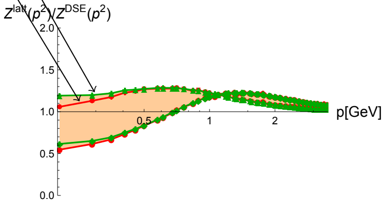

[lattice: Bornyakov, Mitrjushkin, Rogalyov '13]

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Solution from the 3PI effective action

Different set of functional equations:
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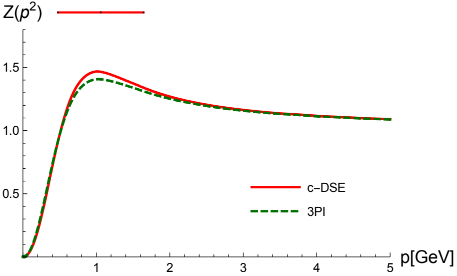

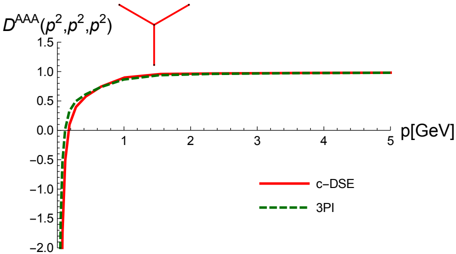

$\rightarrow$ Very similar results.

For yet another set of functional equations (functional RG for $d = 4$), see talk by Mitter and poster by Cyrol.

Comparison $d = 3$ and $d = 4$

- Two-loop diagrams important in propagators.

  [Blum, MQH, Mitter, von Smekal '14; Meyers, Swanson '14]

- Two-loop diagrams not important in three-gluon vertex.

  [Blum, MQH, Mitter, von Smekal '14; Eichmann, Williams, Alkofer, Vujinovic '14]

- Vertices deviate only mildly from tree-level above 1 GeV.

  [Blum, MQH, Mitter, von Smekal '14; Eichmann, Williams, Alkofer, Vujinovic '14; Binosi, Ibanez, Papavassiliou '14; Cyrol, MQH, von Smekal '14]

- RG improvement irrelevant in $d = 3$. Role in $d = 4$?

  [Eichmann, Williams, Alkofer, Vujinovic '14]

Summary and conclusions

Test truncation effects in $d = 3$, where spurious divergences and RG resummation are understood:

- Used a self-contained truncation $\rightarrow$ no model parameters.
- Truncation stable under all tested variations:
  - comparison with 3PI
  - changing the four-gluon vertex
  - different DSEs for the ghost-gluon vertex
- Direct relation between different solutions in continuum and on the lattice to be understood.

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