

Yang-Mills correlation functions from the functional renormalization group

Mario Mitter

Ruprecht-Karls-Universität Heidelberg

Schladming, February 2016

FWF

Der Wissenschaftsfonds.

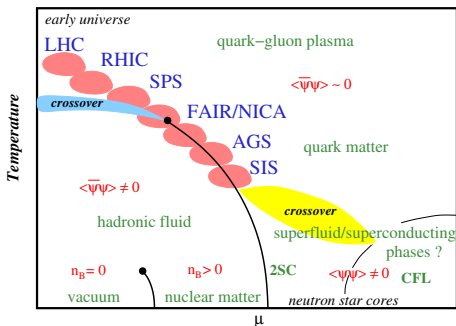


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fQCD collaboration - QCD (phase diagram) with FRG:

J. Braun, A. K. Cyrol, L. Fister, W. J. Fu, T. K. Herbst, MM
N. Müller, J. M. Pawłowski, S. Rechenberger, F. Rennecke, N. Strodthoff

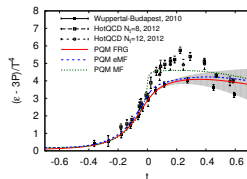


Outline

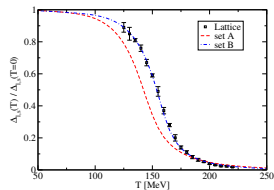
- 1 Why Yang-Mills theory in the vacuum?
- 2 Vertex Expansion of effective action with FRG
- 3 YM correlation functions
- 4 A glimpse at unquenching
- 5 Conclusion

QCD phase diagram with functional methods

- works well at $\mu = 0$: agreement with lattice



[Herbst, MM, Pawłowski, Schaefer, Stiele, '13]

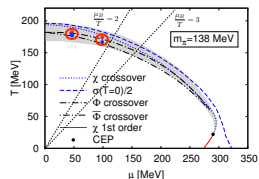


[Luecker, Fischer, Welzbacher, 2014]

[Luecker, Fischer, Fister, Pawłowski, '13]

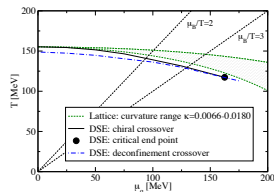
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- different results at large μ
(possibly already at small μ)



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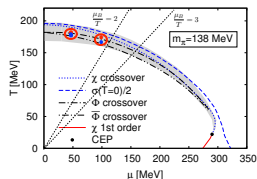
[Braun, Haas, Pawłowski, unpublished]



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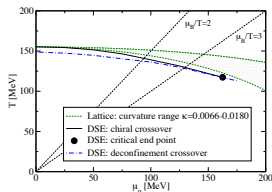
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- calculations need model input:
 - ▶ Polyakov-quark-meson model with FRG:
 - ★ initial values at $\Lambda \approx \mathcal{O}(\Lambda_{\text{QCD}})$
 - ★ input for Polyakov loop potential
 - ▶ quark propagator DSE:
 - ★ IR quark-gluon vertex



[Herbst, Pawłowski, Schaefer, 2013]

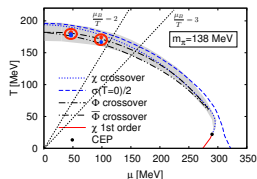
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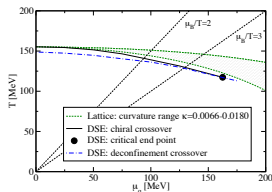
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 - quark propagator DSE:
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- possible explanation for disagreement:
- $\mu \neq 0$: relative importance of diagrams changes
 \Rightarrow summed contributions vs. individual contributions



[Herbst, Pawłowski, Schaefer, 2013]

[Braun, Haas, Pawłowski, unpublished]



[Luecker, Fischer, Fister, Pawłowski, '13]

Back to QCD in the vacuum (Wetterich equation)

- use only perturbative QCD input: $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$ (and $m_q(\Lambda)$)

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$$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2} \text{ (diagram with wavy loop)} - \text{ (diagram with straight loop)}$$

The diagram on the left shows a circular loop with wavy lines (representing gluons) and a vertex at the top with a cross inside a circle. The diagram on the right shows a similar circular loop but with straight lines (representing ghost loops) and the same vertex at the top.

$$\Rightarrow \text{effective action } \Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$$

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The diagram on the left is a circular loop with a wavy line, representing a ghost loop. The diagram on the right is a circular loop with a straight line, representing a fermion loop. Both loops have a cross in a circle at the top vertex.

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- ∂_k : integration of momentum shells controlled by regulator
- full field-dependent equation with $(\Gamma_k^{(2)}[\Phi])^{-1}$ on rhs

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- ∂_k : integration of momentum shells controlled by regulator
- full field-dependent equation with $(\Gamma_k^{(2)}[\Phi])^{-1}$ on rhs
- gauge-fixed approach (Landau gauge):
 - ▶ ghosts appear
 - ▶ gauge invariance: $\Gamma_{(k)}$ fulfills (modified) Slavnov-Taylor identities

Vertex Expansion

- approximation necessary - vertex expansion

$$\Gamma[\Phi] = \sum_n \int_{p_1, \dots, p_{n-1}} \Gamma_{\Phi_1 \dots \Phi_n}^{(n)}(p_1, \dots, p_{n-1}) \Phi^1(p_1) \cdots \Phi^n(-p_1 - \cdots - p_{n-1})$$

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- functional derivatives with respect to $\Phi_i = A, \bar{c}, c, \bar{q}, q$:
⇒ equations for 1PI n -point functions, e.g. gluon propagator:

The diagrammatic equation shows the derivative of the inverse gluon propagator with respect to the coupling t . On the left is ∂_t followed by a wavy line with a minus sign. This is equal to the sum of three diagrams: 1) a loop of wavy lines with a cross on top, connected to two external wavy lines; 2) a loop of dashed lines with a cross on top, connected to two external wavy lines, with a minus sign in front; 3) a loop of wavy lines with a cross on top, connected to two external wavy lines, with a plus sign and a fraction of 1/2 in front.

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- functional derivatives with respect to $\Phi_i = A, \bar{c}, c, \bar{q}, q$:
 \Rightarrow equations for 1PI n -point functions, e.g. gluon propagator:

The diagrammatic equation is enclosed in a blue box. On the left, a wavy line representing a gluon propagator is shown with a derivative symbol ∂_t to its left and a superscript -1 to its right. This is followed by an equals sign. To the right of the equals sign are three terms: 1) a circular loop of wavy lines with two external wavy lines attached to the left and right sides; 2) a minus sign followed by a circular loop of dashed lines with a cross on top and two external wavy lines attached to the left and right sides; 3) a plus sign followed by a fraction $\frac{1}{2}$ and a circular loop of wavy lines with a cross on top and two external wavy lines attached to the left and right sides.

- want “apparent convergence” of $\Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$

“Quenched” Landau gauge QCD

- two crucial phenomena: S_χ SB and confinement
- similar scales - hard to disentangle
- quenched QCD: allows separate investigation:

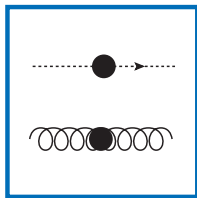
see e.g. [Williams, Fischer, Heupel, 2015]

“Quenched” Landau gauge QCD

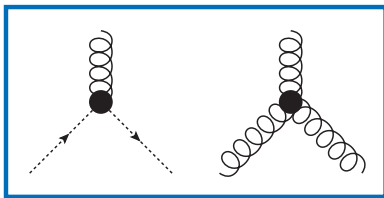
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- recent results for YM propagators [Cyrol, Fister, MM, Pawłowski, Strodthoff, to be published]
- matter part (only one slide) [MM, Strodthoff, Pawłowski, 2014]

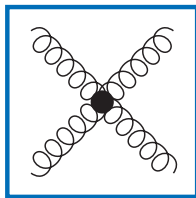
Vertex Expansion in YM theory [Cyrol, Fister, MM, Strodthoff, Pawłowski, to be published]



full. mom. dep.

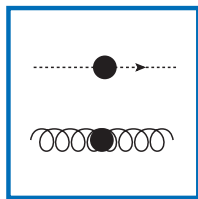


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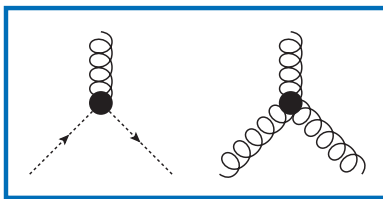


sym. point and
tadpole config.

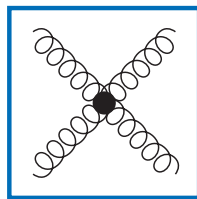
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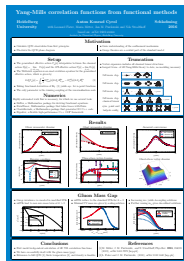
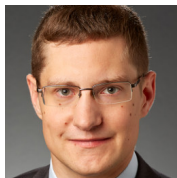


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⇒ cf. poster Anton K. Cyrol



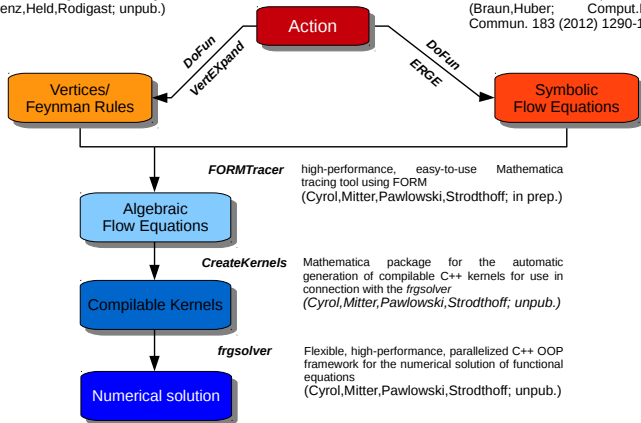
Derivation of equations

VertEXpand

Mathematica package for the derivation of vertices from a given action using FORM (Denz,Held,Rodigast; unpub.)

DoFun

Mathematica package for the derivation of functional equations (Braun,Huber; Comput.Phys. Commun. 183 (2012) 1290-1320)



[Cyrol, MM, Pawlowski, Strodthoff, 2013-2016]

(Truncated) Equations

[Cyrol, Fister, MM, Strodtzoff, Pawlowski, to be published]

$$\partial_t \text{---}^{-1} = \text{---} \otimes \text{---} + \text{---} \otimes \text{---}$$

$$\partial_t \text{---}^{-1} = \text{---} \otimes \text{---} - 2 \text{---} \otimes \text{---} + \frac{1}{2} \text{---} \otimes \text{---}$$

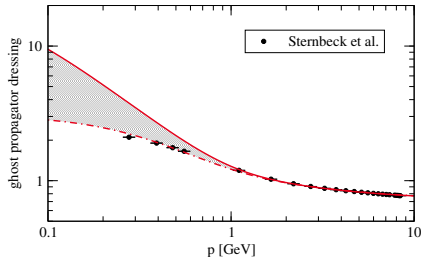
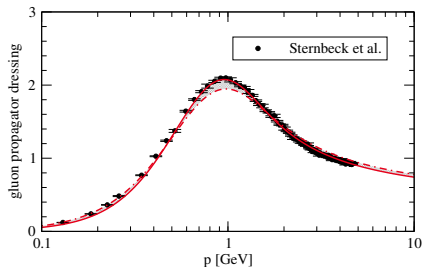
$$\partial_t \text{---} = - \text{---} - \text{---} + \text{perm.}$$

$$\partial_t \text{---} = - \text{---} + 2 \text{---} - \text{---} + \text{perm.}$$

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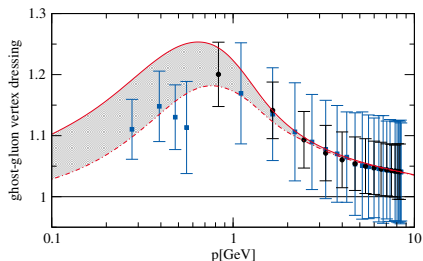
- $\Gamma_{AA}^{(2)}(p) \propto Z_A(p) p^2 (\delta^{\mu\nu} - p^\mu p^\nu / p^2)$

- $\Gamma_{\bar{c}c}^{(2)}(p) \propto Z_c(p) p^2$



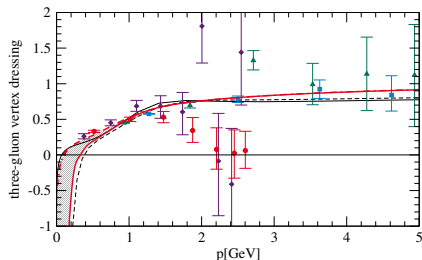
- band: family of decoupling solutions bounded by scaling solution

- comparison to Sternbeck '06

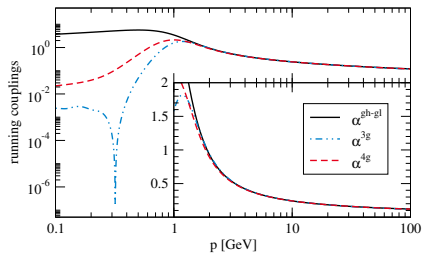
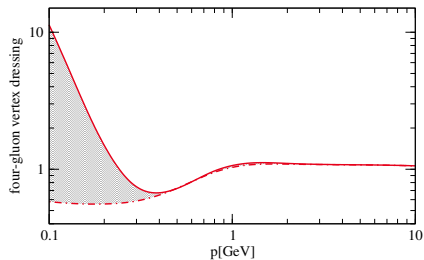


- comparison to
Cucchieri, Maas, Mendes, '08

Blum, Huber, MM, von Smekal '14
(black lines)

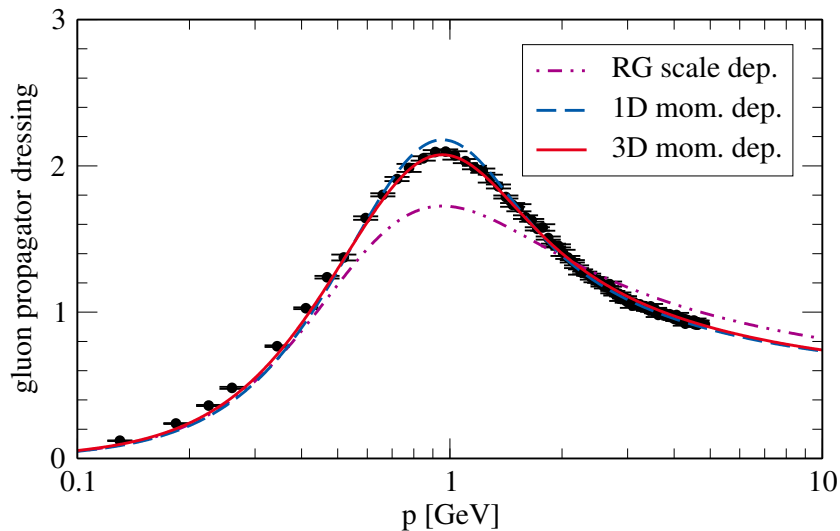


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Apparent Convergence

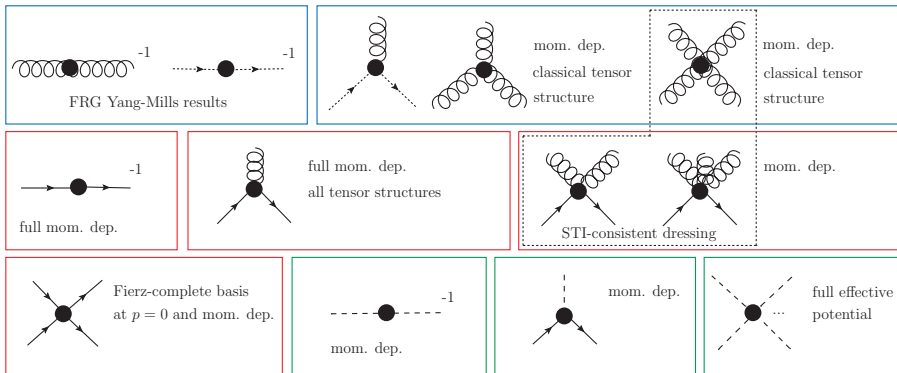
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Quenched quark propagator

[MM, Pawłowski, Strodthoff, 2014]

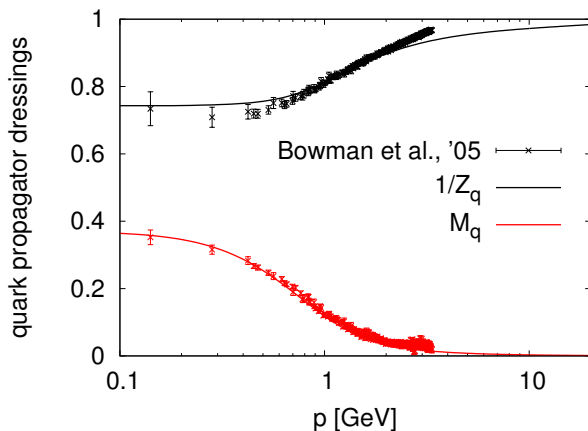
- $\Gamma_{\bar{q}q}^{(2)}(p) \propto Z_q(p) \not{p} + M(p)$



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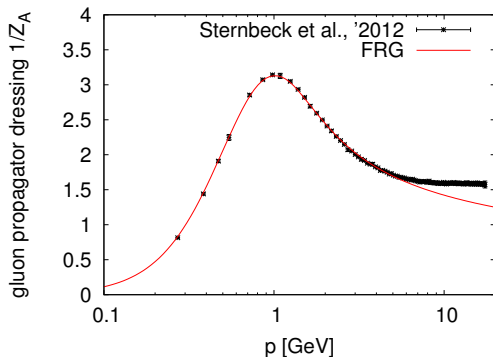
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- FRG vs. lattice: bare mass, quenched, scale
- agreement not sufficient: need apparent convergence at $\mu \neq 0$

Outlook: unquenched gluon propagator



- self-consistent solution of classical propagators and vertices (1D)
- massless quarks

[Cyrol, MM, Pawłowski, Strodthoff, in preparation]

Summary and Outlook

(quenched) QCD with functional RG

- QCD phase diagram: need for quantitative precision
- quenched QCD in vacuum:
 - ▶ sole input $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$ and $m_q(\Lambda = \mathcal{O}(10) \text{ GeV})$
 - ▶ good agreement with lattice simulations (sufficient?)

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 - ▶ good agreement with lattice simulations (sufficient?)
- unquenching (first results)
- finite temperature/chemical potential
- more checks on convergence of vertex expansion
- bound-state properties (form factor, PDA. . .)