Predictiveness of inflation with the Higgs boson

Jacopo Fumagalli
Nikhef, Amsterdam

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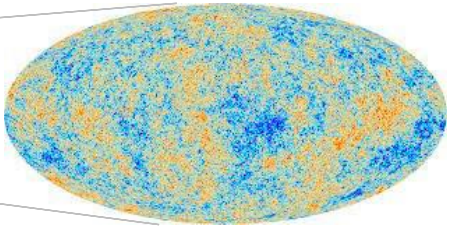
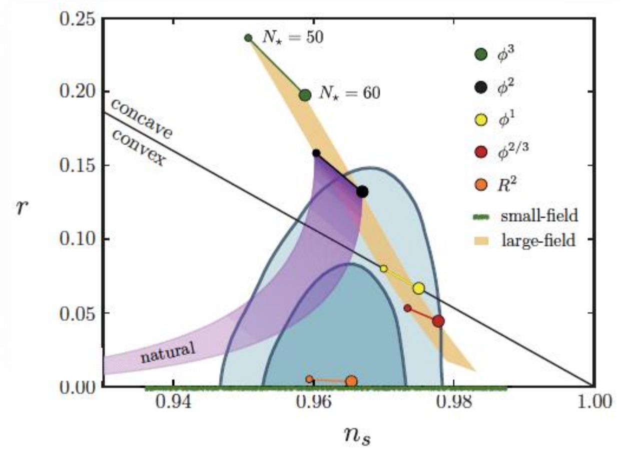
Motivation

- Inflation $\ddot{a}(t) > 0$

  A.Guth, A.Linde, P. Steinhard'80

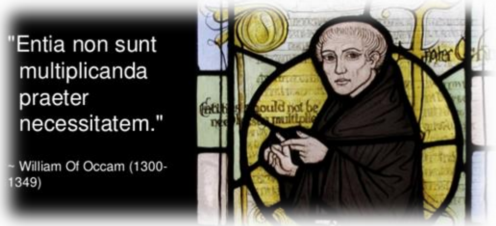


- Who is the Inflaton?



- Request for simplicity, $\Longrightarrow$ Only one scalar field in the SM

  LHC , PLANCK



"Entia non sunt multiplicanda praeter necessitatem."

~ William Of Occam (1300-1349)

- Consistent simplicity

Overview

- Higgs Inflation
  - Tree level
  - Quantum aspects (Unitarity, renormalizability ..)
- UV sensitivity
  - UV completion
  - RG flow and renormalization scale
  - Results for the CMB parameters

Higgs Inflation

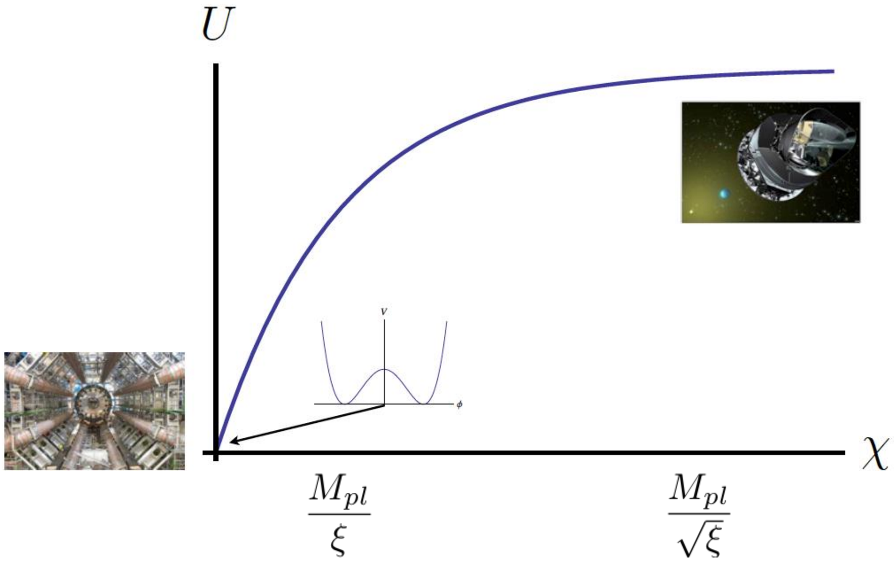
Bezrukov-Shaposhnikov
'08

$$S = \mathcal{S}_{SM} + \frac{M_{pl}^2}{2}\int\sqrt{-g}\,\underbrace{\left(1+\frac{2}{M_{pl}^2}\xi\mathcal{H}^t\mathcal{H}\right)}\,R$$

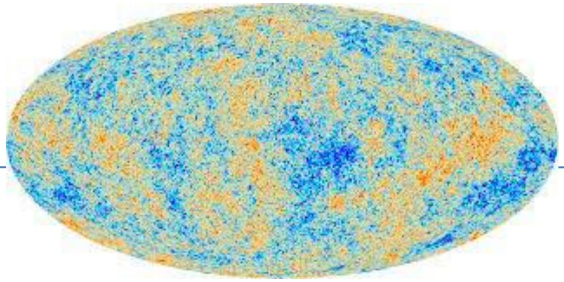
- Einstein frame $$g_{E\,\mu\nu} = \Omega^2 g_{\mu\nu} \qquad \gamma(\phi)(\partial_\mu\phi)^2 = (\partial_\mu\chi)^2$$

$$S = \frac{1}{2}\int\sqrt{-g_E}R(g_E) - \int\sqrt{-g_E}\left[\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + U(\chi)\right]$$

$$U(\chi) = \frac{\lambda(\phi^2(\chi) - v_{\mathtt{ew}}^2)}{4\Omega^4(\chi)}$$



Density perturbations



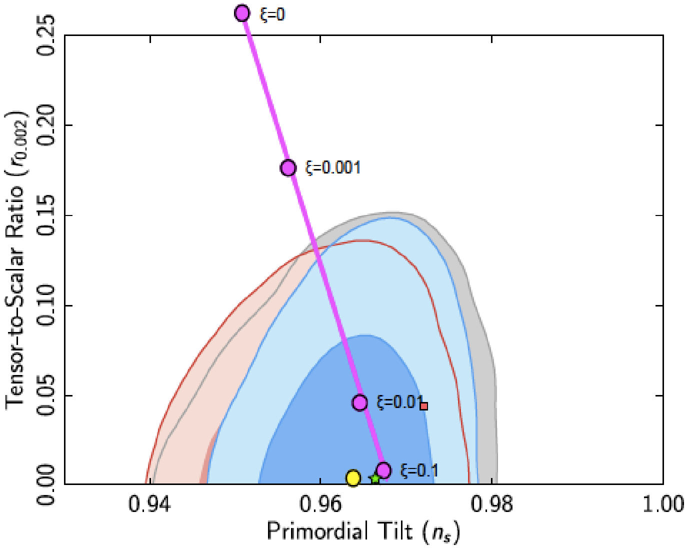
$$N_\star = \int_{\chi_E}^{\chi_\star} \frac{U}{dU/d\chi}\frac{d\chi}{M_{pl}} \implies \phi_\star \simeq 9.13 M_{pl}/\sqrt{\xi}$$

$$\Delta_{\mathcal{R}} \left(\propto \frac{\lambda}{\xi^2}\right) \simeq 2.2 \cdot 10^{-9} \qquad \implies \xi \simeq 47000\sqrt{\lambda}$$

$$n_s \simeq 1 - \frac{2}{N_\star} - \frac{3}{N_\star^2} \simeq 0.967$$

$$r = 16\epsilon_\star \simeq (1 + \tfrac{1}{6\xi_\star})\frac{12}{N_\star^2} \simeq 0.0031$$

Bezrukov’14



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Unitarity bound

$$S \supset \sqrt{-g}\,\frac{\xi}{M_{pl}^2}\phi^2 R \;\xrightarrow[g_{\mu\nu}=\eta_{\mu\nu}+M_{pl}^{-1}h_{\mu\nu}]{}\; \frac{1}{M_{pl}/\xi}\phi\Box h_{\mu\nu}$$

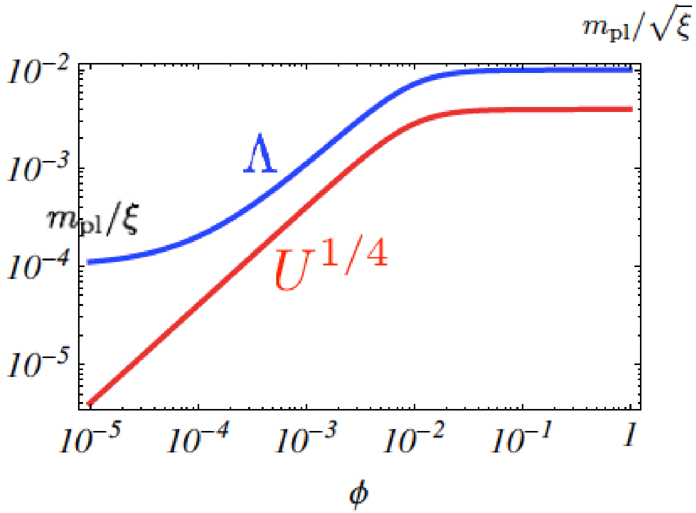
Burgess ‘09, Barbon ’09, Hertzberg ’10

- Cutoff field dependent

- Considering Gauge interaction (Goldston

$$\mathcal{M}(\theta\theta \to \theta\theta) > 1$$

$$\Lambda_{gauge}(\phi_0) \sim \left(\frac{M_{pl}}{\xi}, \phi_0, \frac{M_{pl}}{\sqrt{\xi}}\right)$$

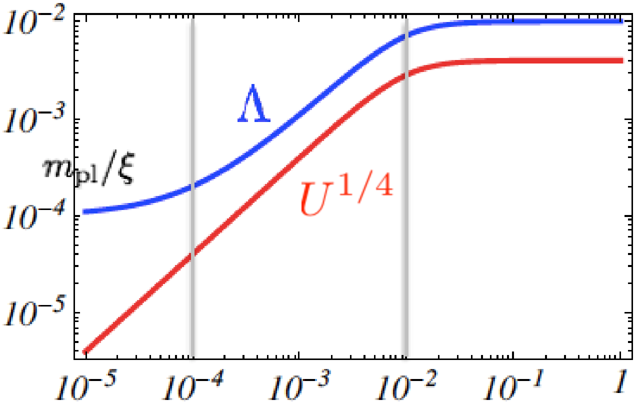


Bezrukov, Magnin, Shaposhnikov, Sibiryakov ’11,Burgess ’14, Prokopec & Weenink ’14

Renormalizability in EFT sense

D.George, S.Mooij, M.Postma '14 '15

- Small regime $\phi \ll \frac{M_{pl}}{\xi}$ $\qquad \delta_s = \xi\phi$
- Mid regime $\frac{M_{pl}}{\xi} \ll \phi \ll \frac{M_{pl}}{\sqrt{\xi}}$ $\qquad \xi \to \delta_m^{-2}\xi,\ \phi \to \delta_m^{\frac{3}{2}}\phi$
- Large regime $\phi \gg \frac{M_{pl}}{\sqrt{\xi}}$ $\qquad \delta = 1/\xi\phi$



Demand: at every order a finite number of counter terms

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UV completion

$$\frac{\mathcal{L}_{HI}}{\sqrt{-g}} + \sum_i \frac{c_i}{\Lambda^n} \mathcal{O}_i^{n+4}$$

Bezrukov, Rubio and Shaposhnikov.’14,
Burgess, Patil, Trott ’14

- Which shape for the suppression scale?

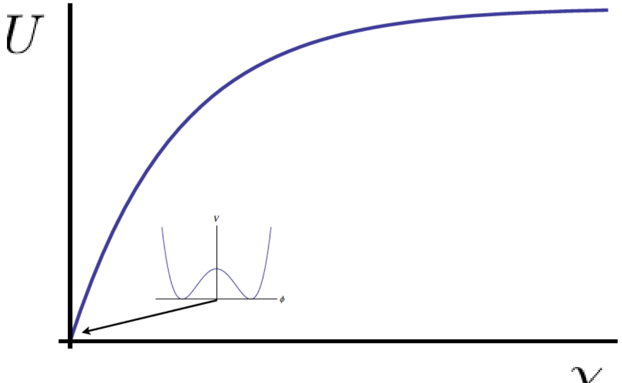
$$\Lambda = \Lambda_{gauge}(\phi)$$

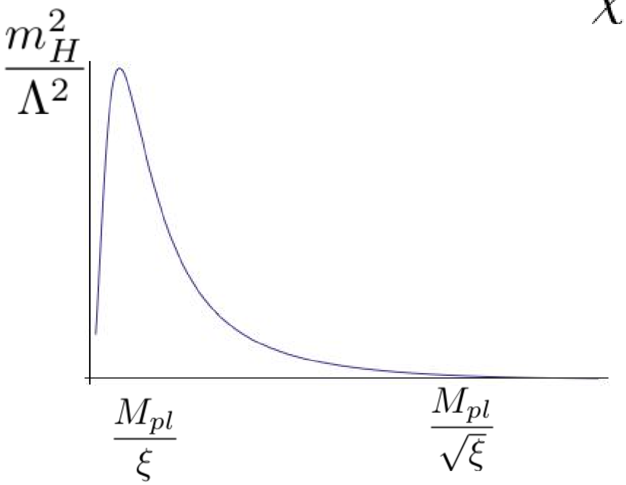
- Which form for the higher d operators?

$$\mathcal{L}_{new} \supset \cancel{\frac{(\mathcal{H}^\dagger \mathcal{H})^3}{\Lambda^2}} + .. \qquad \mathcal{L}_{new} \supset \frac{m_h^2 \mathcal{O}^{(4)}}{\Lambda^2}$$

Grzadkowski,Iskrzynsi,Misiak,Rosiek‘10

  - Preserving the quasi-shift symmetry
  - Effect only where really needed!





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RG flow

$$\beta_i = \mu \frac{\partial \lambda_i}{\partial \ln \mu} = \beta_i^{HI} + \delta\beta_i$$

Barvinsky, Kamenshchik, Kiefer, Starobinsky, ‘08
Simone, Hertzberg, Wilzcek ’09
Bezrukov, Magnin, Shaposhnikov ’10
George,Mooij,Postma ‘14 ’15
Jenkins, Manohar, Trott ‘13

- Corrected Potential

$$U = \frac{\lambda \phi^4}{4\left(1 + \frac{\xi \phi^2}{M_{pl}^2}\right)} + \boxed{U^{(1)}} + \texttt{2-loop} + ..$$

$$\supset (-1)^{f_i} c_i \frac{m^4(\phi)}{64\pi^2} \ln\left(\frac{m^2(\phi)}{\mu^2}\right)$$

$$m(\phi) = \frac{m_{\texttt{SM}}(\phi)}{\Omega(\phi)}$$

- RG improvement

$$U(\phi, g_i, \mu) = U(\phi, g_i(t), \mu(t))$$

$$\mu(t) = \mu e^t \quad , \quad \frac{dg_i(t)}{dt} = \beta_i(g_j(t))$$

$$\mu(t) \sim \frac{\phi}{\Omega} \equiv \frac{\phi}{\sqrt{1 + \xi(t)\phi^2}}$$

RG flow

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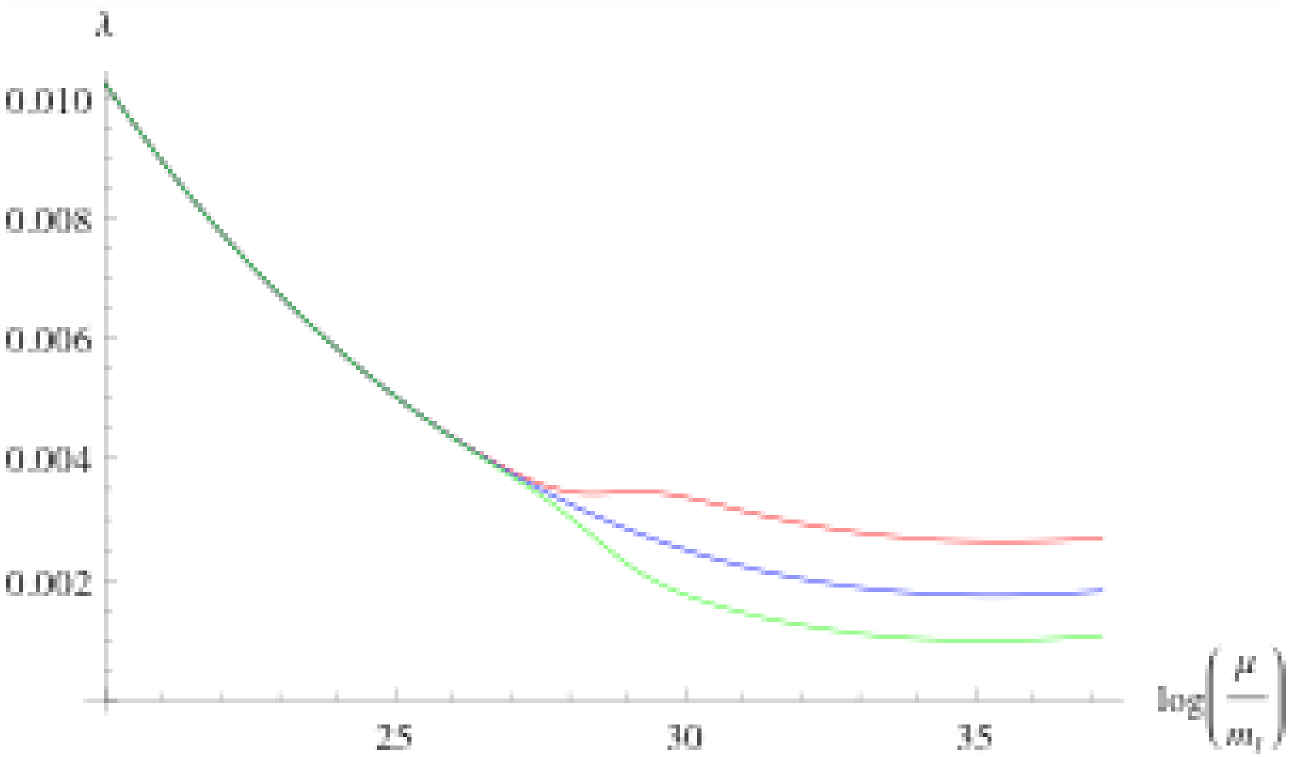

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Effect on the CMB observables – Analitical results

$$U_{eff} = \frac{\lambda(t(\phi))\phi^4}{4\left(1 + \frac{\xi(t(\phi))\phi^2}{M_{pl}^2}\right)}$$

$$t = \ln\left(\frac{\mu}{m_t}\right) = \ln\left(\frac{\phi}{m_t\sqrt{1+\xi(t)\phi^2}}\right)$$

J.F., Postma 1602.07234

Inflationary regime $\phi^2 \gg M_{pl}^2/\xi \longrightarrow \boxed{\delta = \frac{1}{\xi\phi^2} \ll 1}$

$$\eta = -\frac{4}{3}\delta F + O(\delta^2)$$

$$\downarrow$$

$$\frac{(1+\frac{1}{4}\frac{\beta_\lambda}{\lambda})}{(1+\frac{1}{2}\frac{\beta_\xi}{\xi})(1+\frac{1}{6\xi})}$$

$$\epsilon = \frac{4}{3}\delta^2 F^2\left(1 + \frac{1}{6\xi}\right) + O(\delta^3)$$

$$n_s \simeq 1 - \frac{2}{N_\star} + O(N_\star^{-2})$$

$$r \simeq \frac{12}{N_\star^2}\left(1 + \frac{1}{6\xi_\star}\right) + O(N_\star^{-3})$$

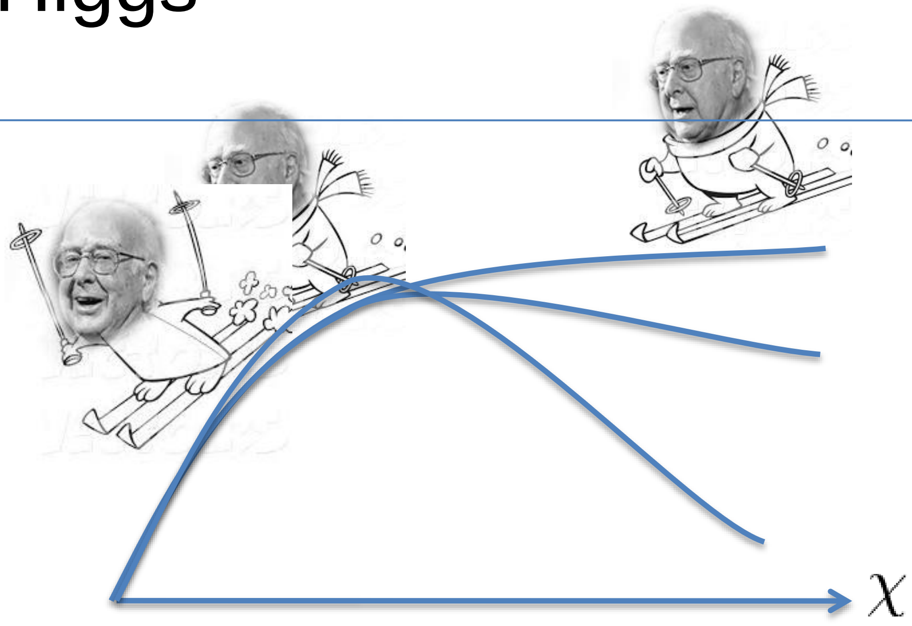
Same as tree level results

!

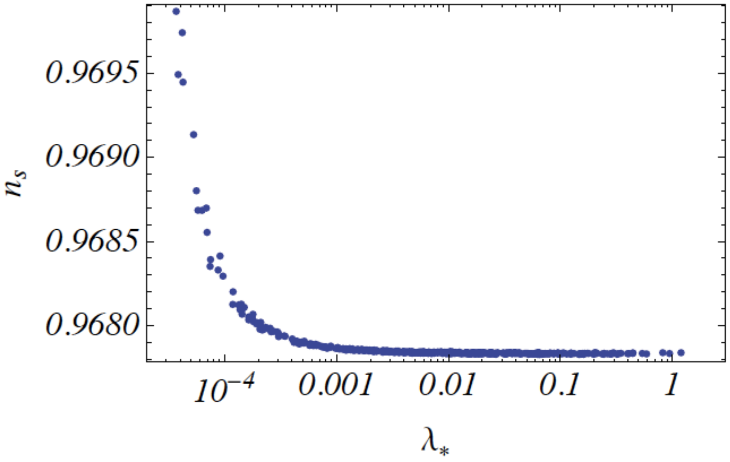
UV (in)sensitivity of Higgs inflation

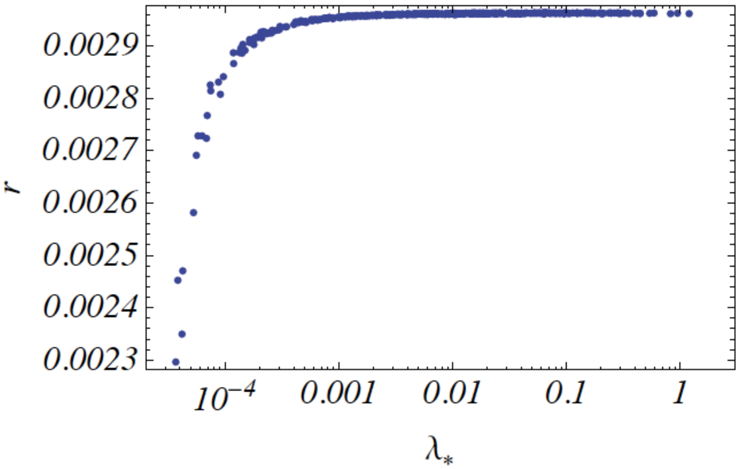
- $U_\chi \propto F(\beta_\lambda, \beta_\xi)$

$F \ll \delta_*$



- Numerical results, example





- h.o. corrections $\sim O(10^{-4})$ and ξ^{-1} suppressed

J.F., Postma 1602.07234

Conclusions

- Higgs inflation at tree level gives great prediction for (n_s, r)
- The theory is not renormalizable , we need at least a particular UV completion
- Threshold corrections to the slow roll parameter cancel in the expression for n_s and r independetly of the form of this UV completion
- Results confirmed numerically

RG flow

$$\Gamma[\phi_{cl}] = S[\phi_{cl}] + \Gamma^{\texttt{1-loop}} + \dots$$

$$V_{CW} = V_t(\phi_{cl}) + \frac{1}{64\pi^2}\sum_i (-1)^{f_i} s_i m_i^4(\phi_{cl}) \ln\left(\frac{m_i^2(\phi_{cl})}{\mu^2} - c_i\right)$$

Coleman-Weinberg ’73

$$g_E = \Omega^2 g_J$$

Quantum corrections two routes:

Jordan

$$V_J(\phi) + V_{J\,CW}^{(1)} \xrightarrow{E} \frac{V_J(\phi)}{\Omega^4} + \frac{V_J^{(1)}}{\Omega^4}$$

Einstein

$$V_E(\phi) + V_{CW}^{(1)} = \frac{V_J}{\Omega^4} + V_E^{(1)}$$

$\Longrightarrow$

- Diffrent results in the GB sector
- Back reaction negligible only in the Einstein frame $\sim O(\epsilon), \epsilon \ll \eta$
- Approx. FLRW

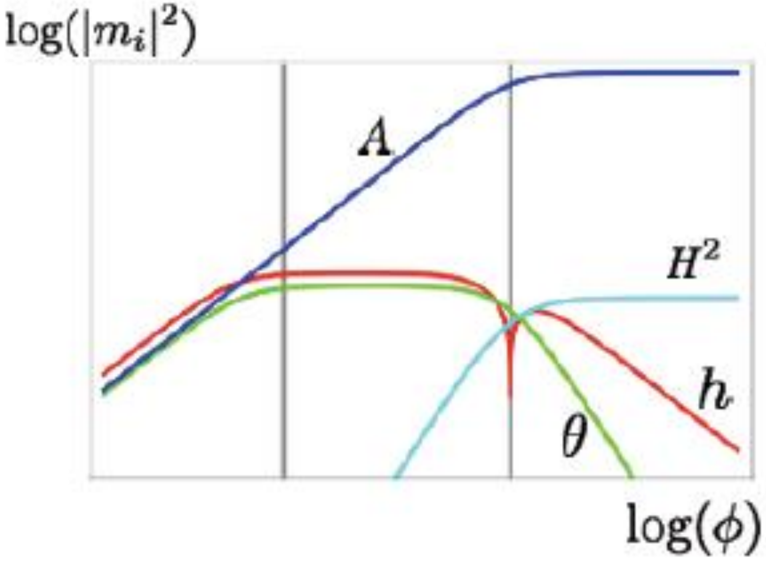
S.Mooij, M.Postma ’11
D. George, S.Mooij,

M.Postma ‘14

D. George, S.Mooij,

M.Postma ‘15

Masses



Higgs instability

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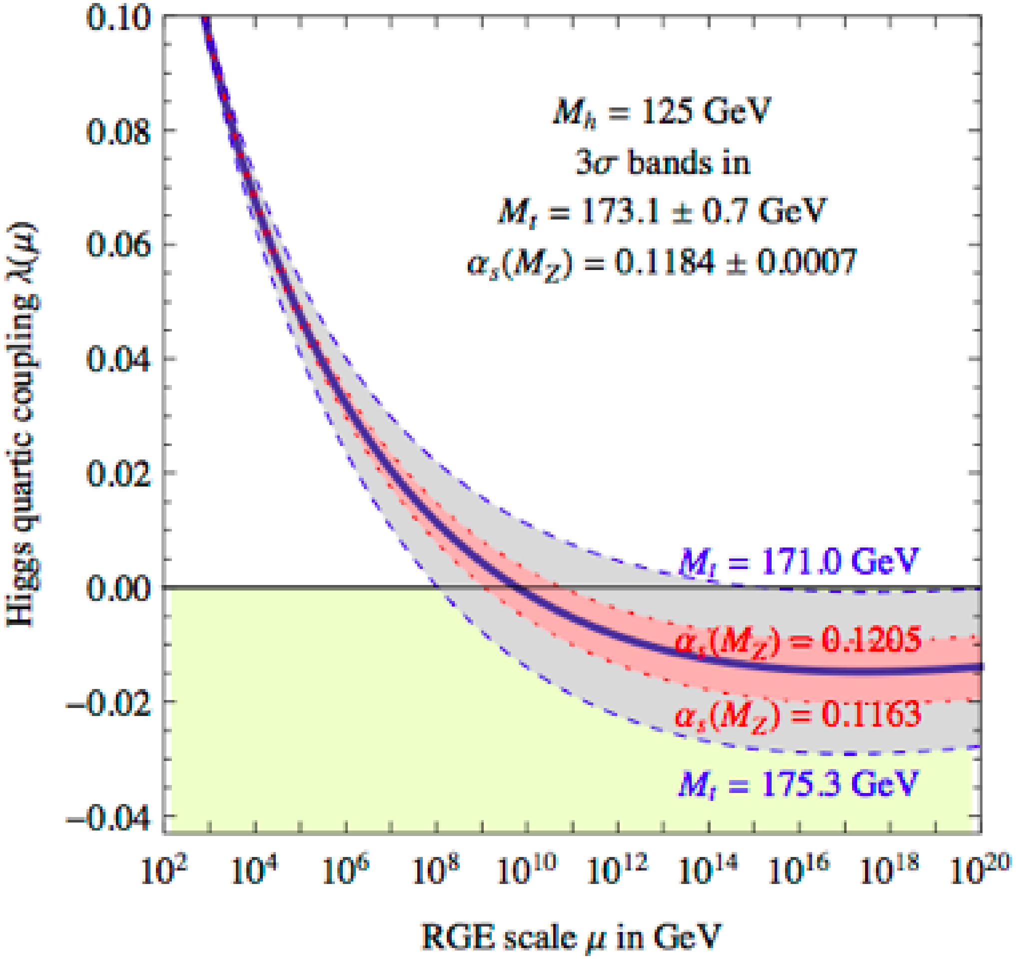
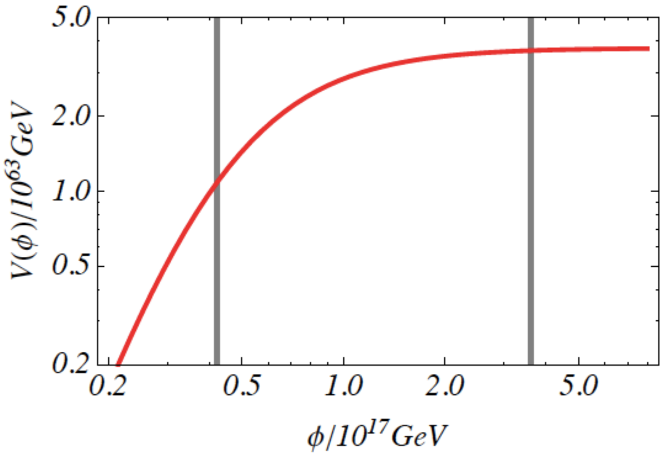


Running insensitivity

- $U_\chi \propto F$

$$F > \delta_\star$$



$$F < \delta_\star$$