Asymptotic safety in the Einstein- Hilbert truncation

$$\Gamma_k = -\frac{1}{16\pi G_N}\int d^4x\,\sqrt{g}\,(R-2\Lambda)$$

$$\partial_t\Gamma_k = k\partial_k\Gamma_k = \frac{1}{2}\mathrm{STr}\left(\Gamma_k^{(2)}+R_k\right)^{-1}\partial_t R_k$$

-> RG flow from metric fluctuations $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

-> need to gauge fix

-> background field gauge $S_{\mathrm{gf}} = \frac{1}{\alpha\,32\pi\,G}\int d^4x\sqrt{\bar{g}}\bar{g}^{\mu\nu}F_\mu F_\nu$

$$F_\mu = \bar{D}^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar{D}_\mu h^\nu_\nu$$

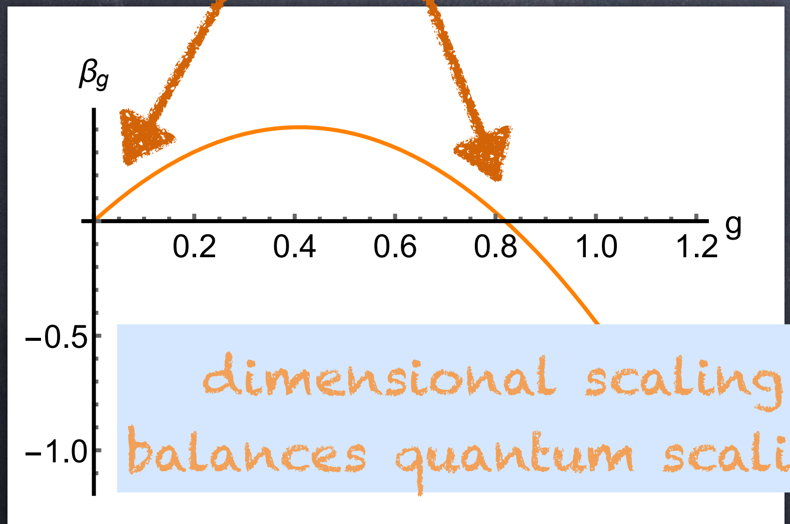
Asymptotic safety in the Einstein- Hilbert truncation

Litim cutoff [Litim, '01] type IIa [Codello, Percacci, Rahmede '08]

$$\Gamma_k = -\frac{1}{16\pi G_N}\int d^4x\,\sqrt{g}\,(R \qquad)$$

$$g = G_N k^2$$

$$\beta_g = 2g - g^2\frac{92}{12\pi}$$



dimensional scaling
balances quantum scaling

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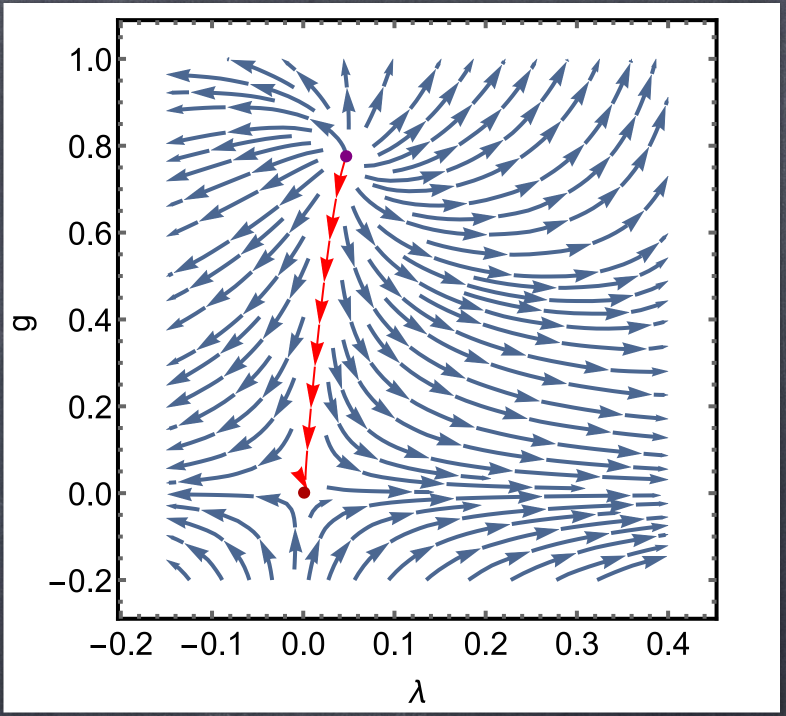

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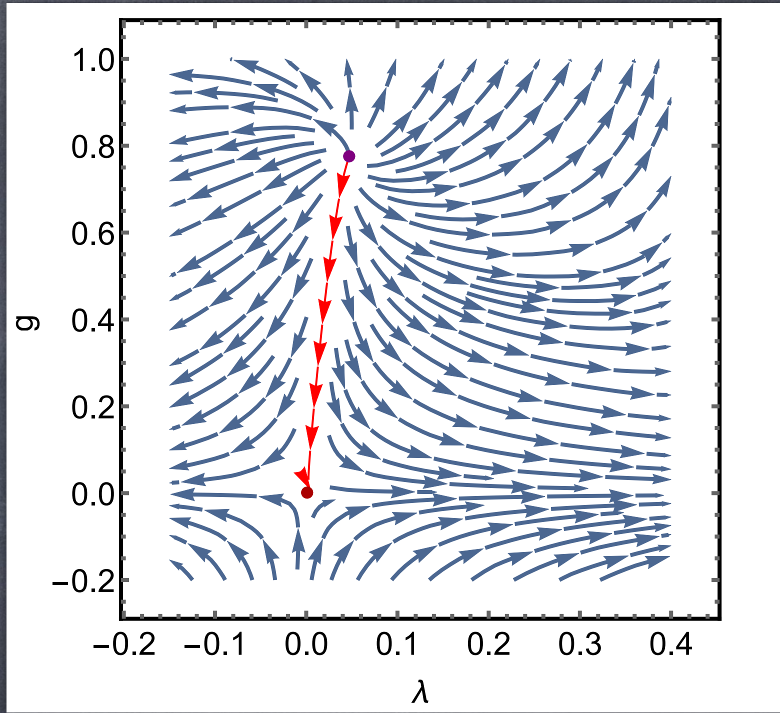

two relevant directions:

$$\theta_{1,2} = 2.31 \pm i\,0.38$$

Dimensionful Newton coupling in asymptotic safety

$$G_N(k) = \frac{g(k)}{k^2}$$

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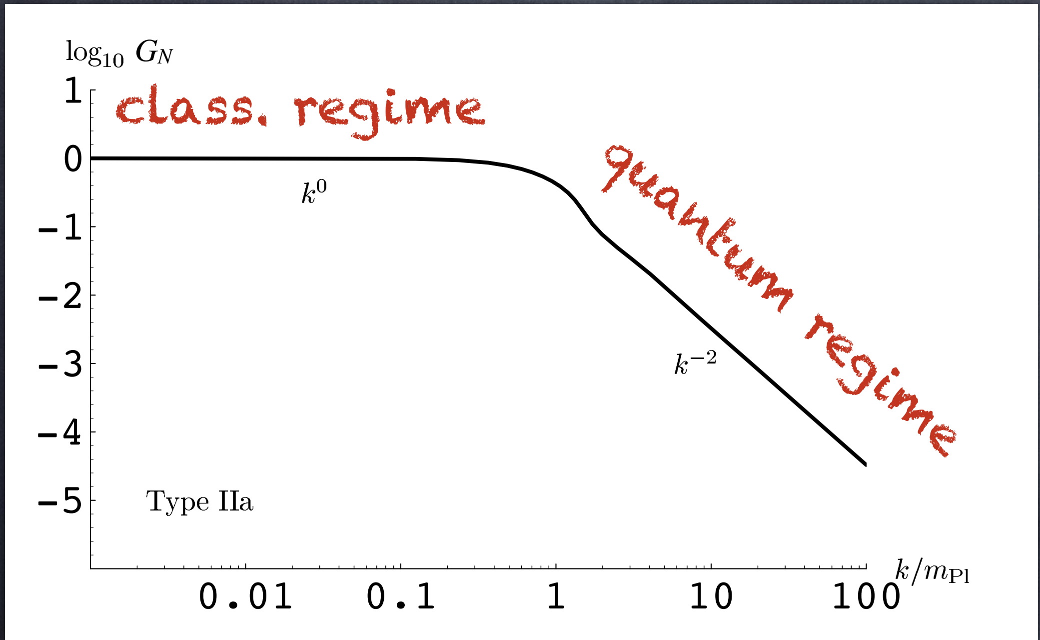

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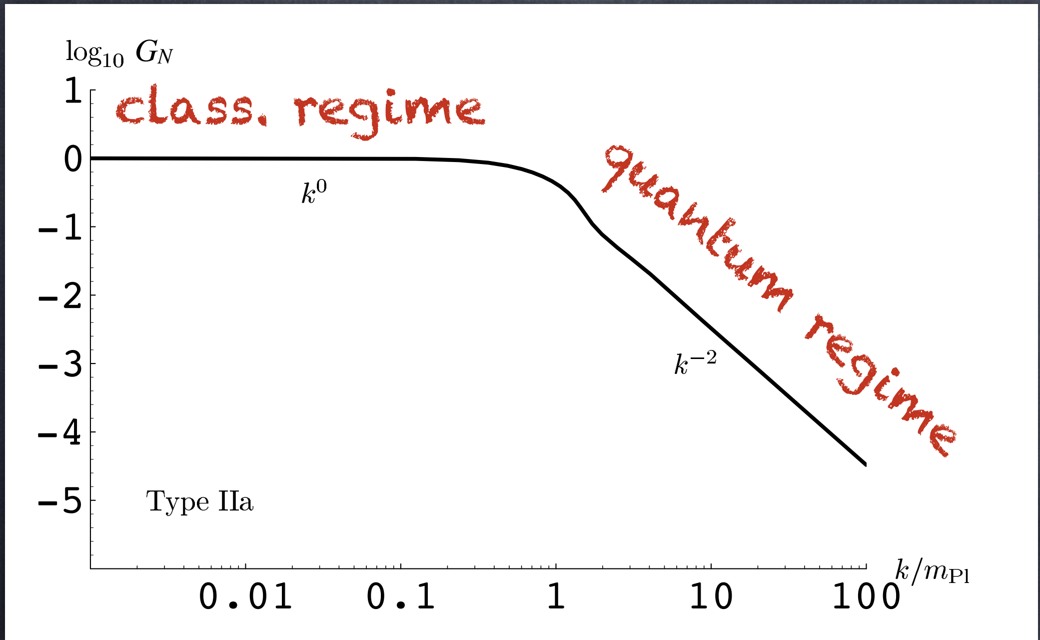

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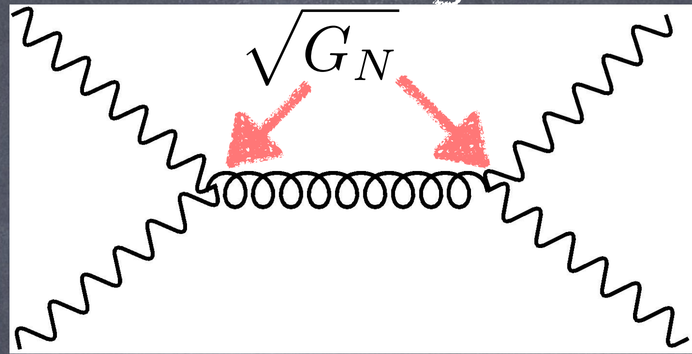

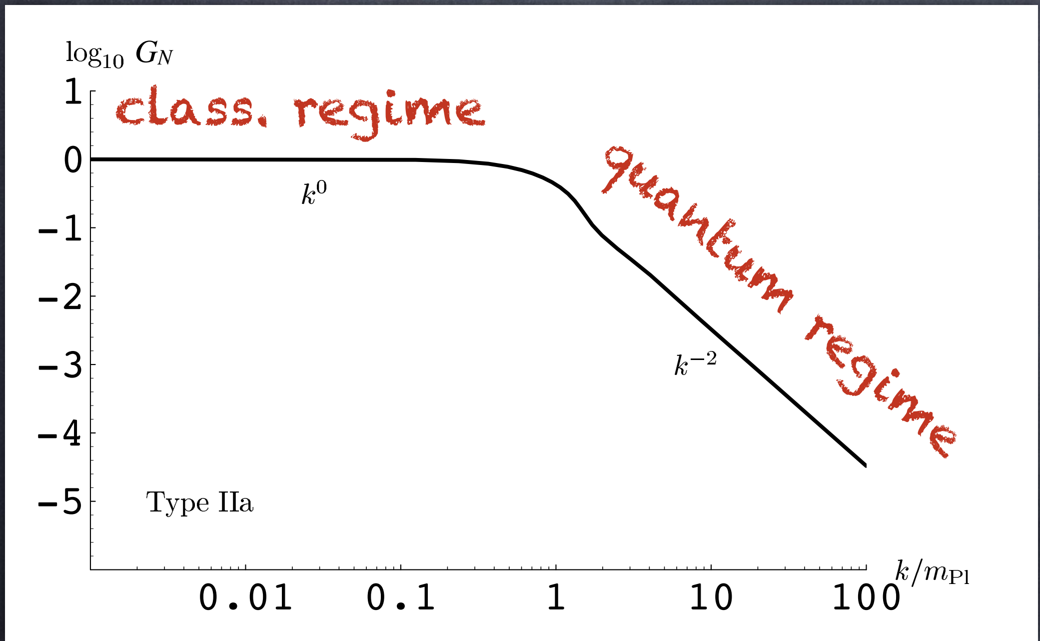

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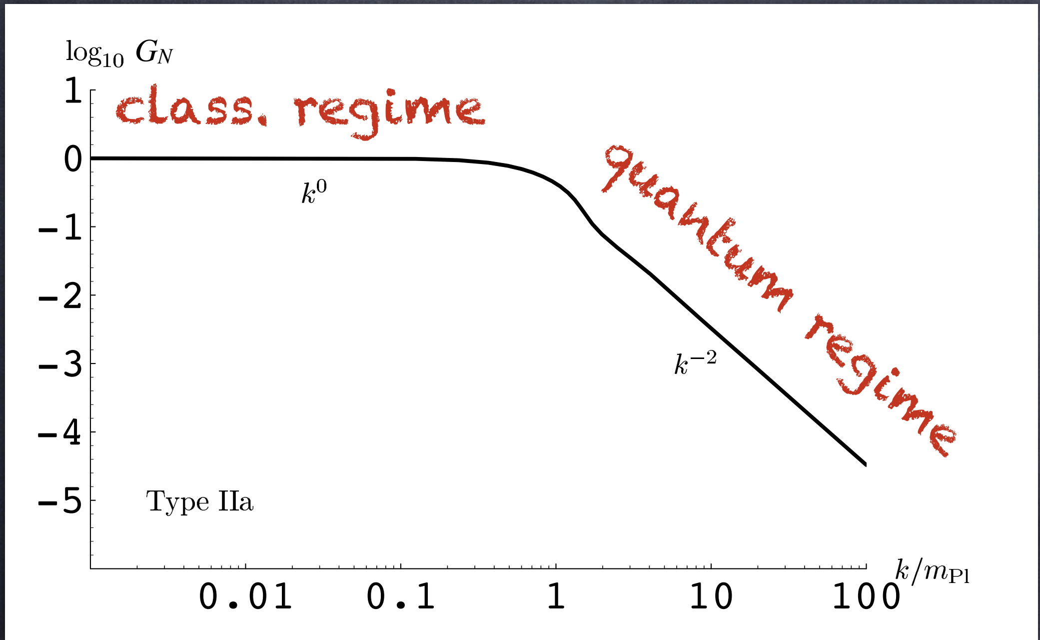

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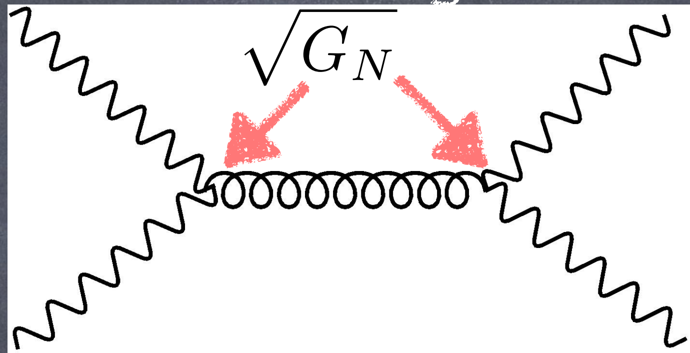

$$\frac{d\sigma}{d\Omega} \to E^2 G_N(E)^2 f(\theta)$$

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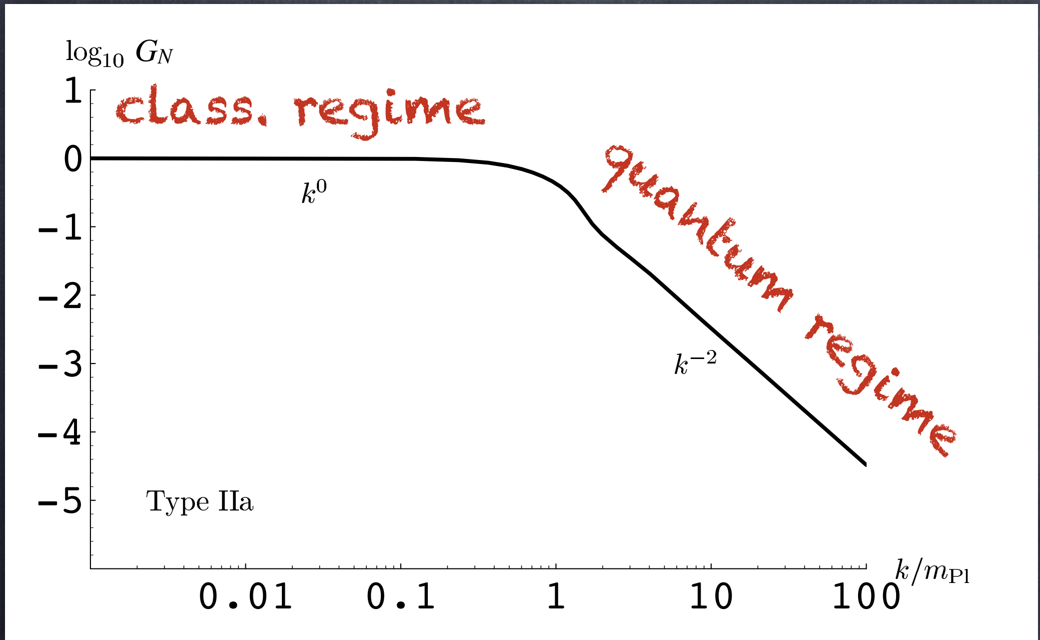

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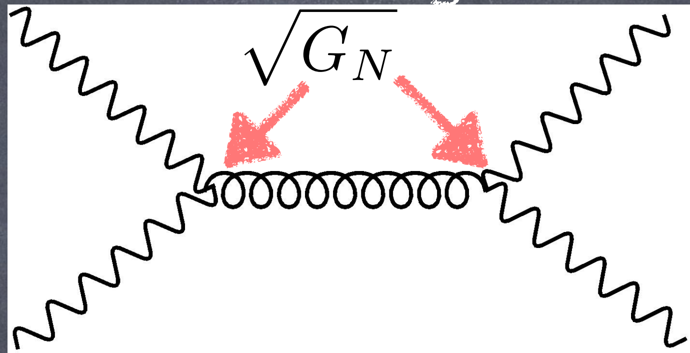

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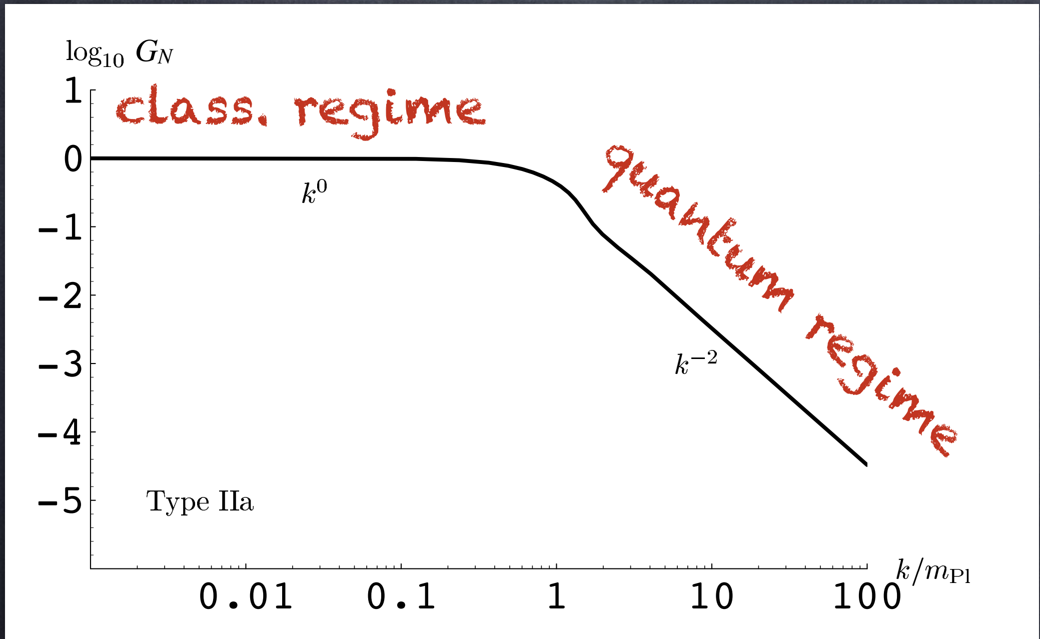

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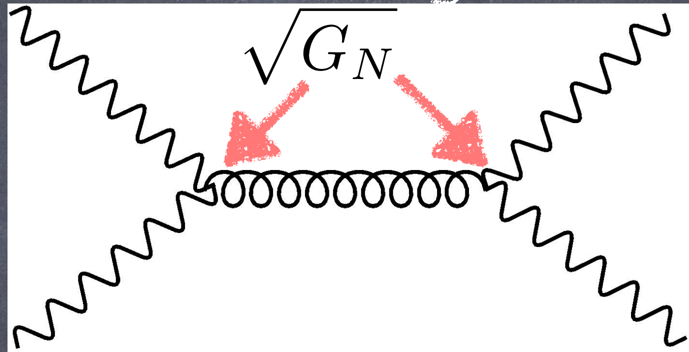

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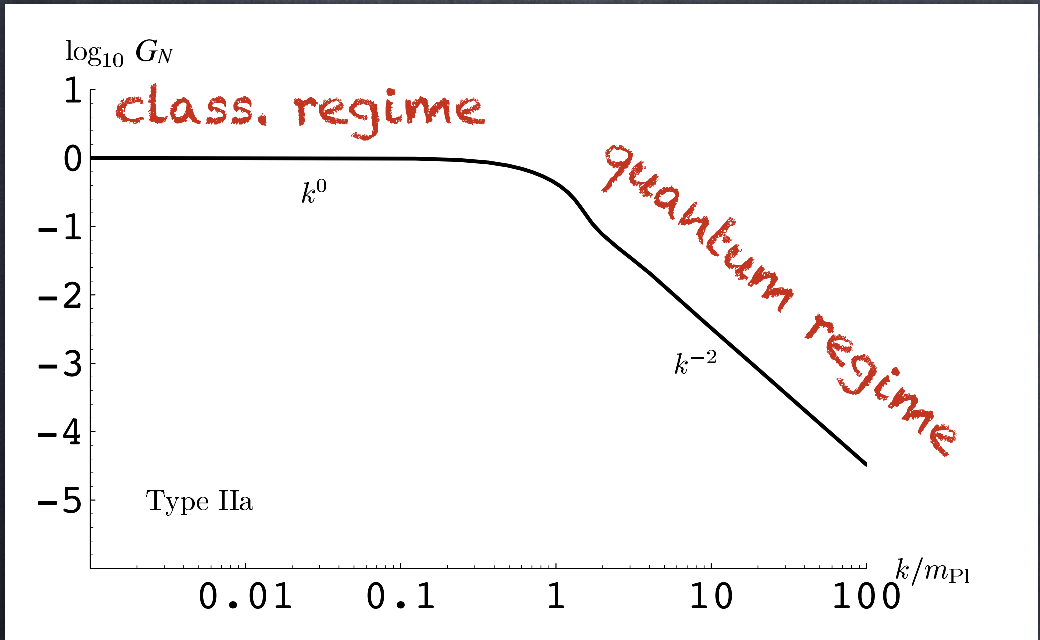

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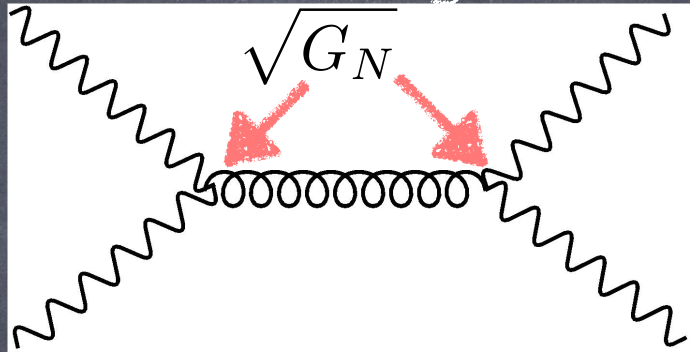

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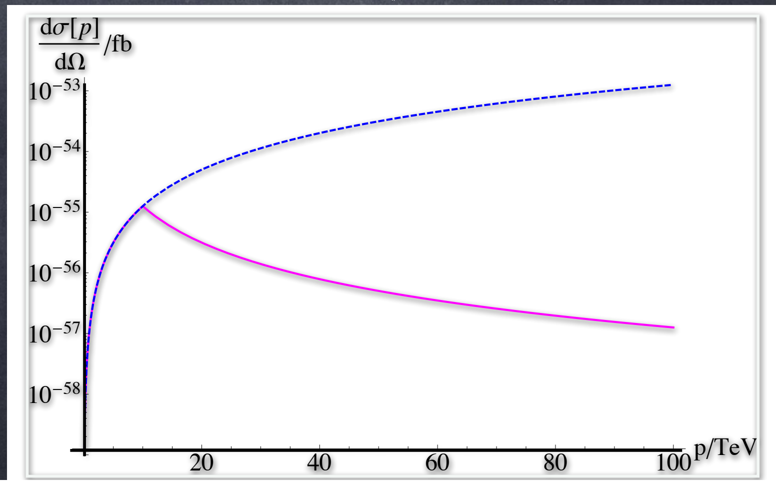

"Minimal length" from asymptotic safety

Quantum gravity should/could have a "minimal length":

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Hint: Black holes

$$S_{\rm BH} = k_B \frac{A}{4 l_{\rm Pl}^2}$$


[Interstellar]

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Many quantum-gravity models incorporate this idea:

- strings
- Loop Quantum Gravity (spectrum of area operator)
- causal sets
- ...

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Is asymptotic safety irreconcilable with these ideas?

"Minimal length" from asymptotic safety

$$G_N(k) = \frac{g(k)}{k^2} \to \frac{g_*}{k^2} \qquad G_N \sim \frac{1}{M_{\rm Pl}^2}$$

measure momenta in Planck units

-> can we take $\frac{k}{M_{\rm Pl}} \to \infty$?

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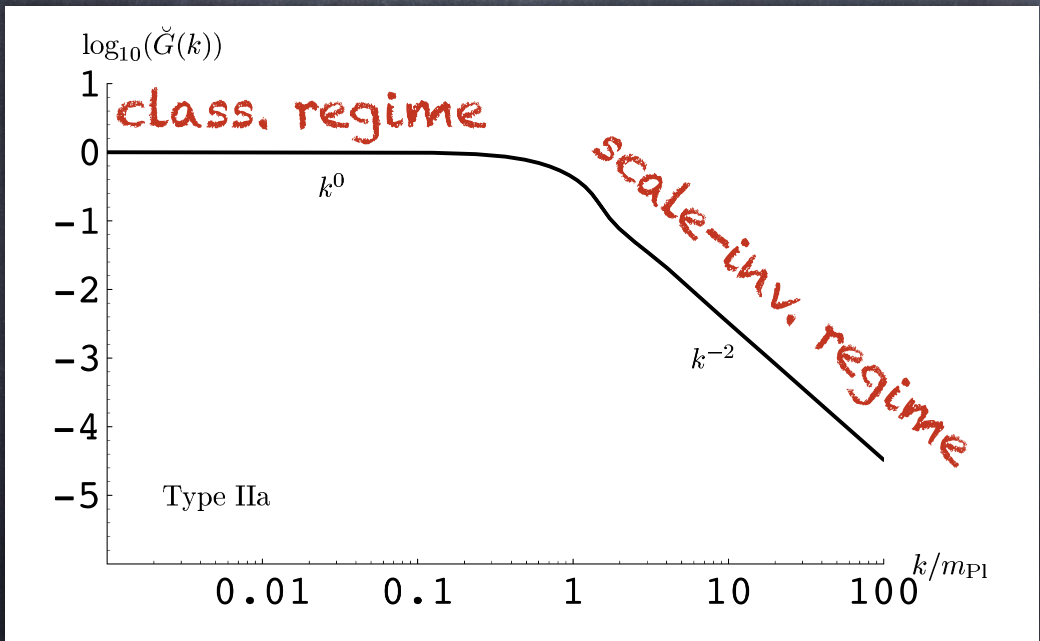

[Reuter, Saueressig '01]

$$\frac{k}{M_{\text{Pl}}(k)} = \frac{k}{k\, m_{\text{Pl}}(k)} \to \frac{1}{m_{\text{Pl}\,*}}$$

increases in classical regime

[Percacci, Vacca '10; Reuter, Schwindt '05, '06]

Planck scale "maximal" momentum

Converging to quantum gravity from different directions?

Different approaches to quantum gravity are based on different assumptions

Converging to quantum gravity from different directions?

Different approaches to quantum gravity are based on different assumptions

-> maybe difference only on mathematical level (cf. Schrödinger's and Heisenberg's formulations of Quantum Mechanics)

-> physics of quantum spacetime encoded in different models might (partially) agree (e.g. question of minimal length can be subtle)

-> if different QG models agree on some property of quantum spacetime, this might be "true property of nature"?

Spectral dimension in asymptotic safety

$\langle g_{\mu\nu} \rangle_k$ scale-dependent

-> what are the small-scale properties of quantum spacetime?

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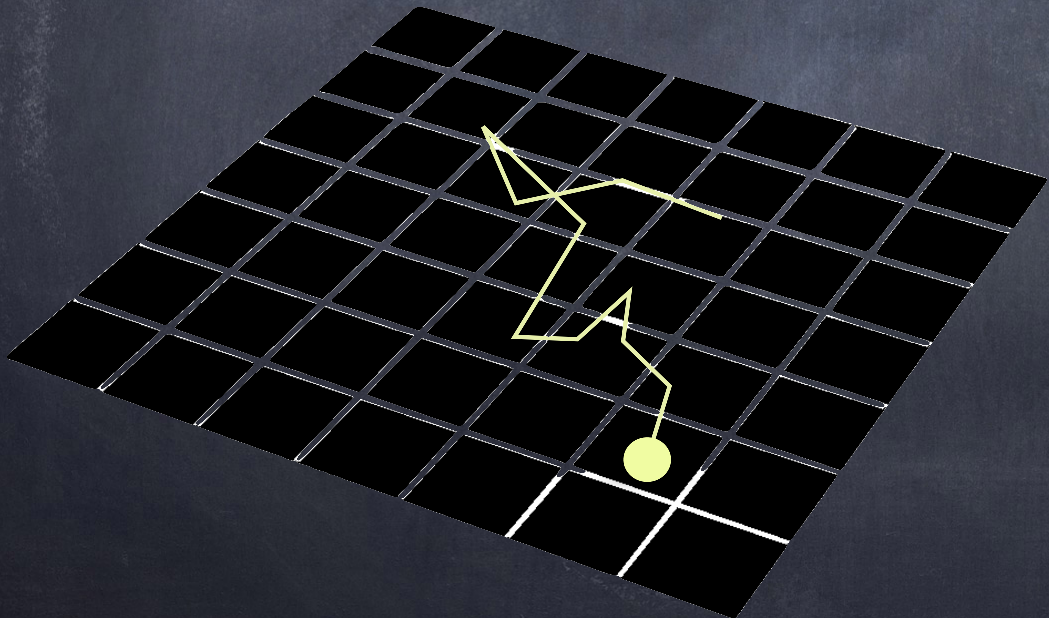

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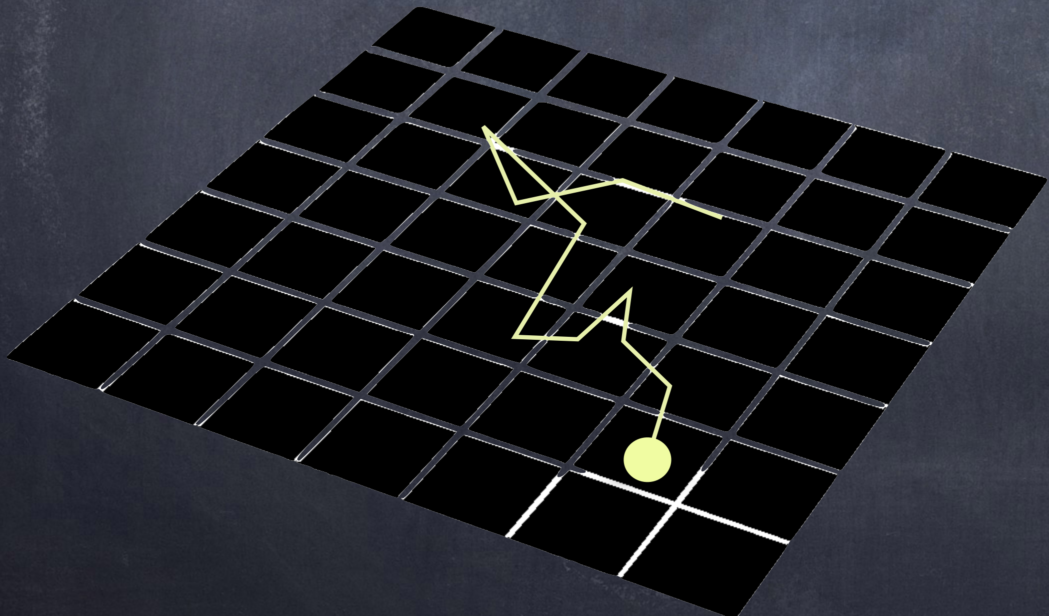

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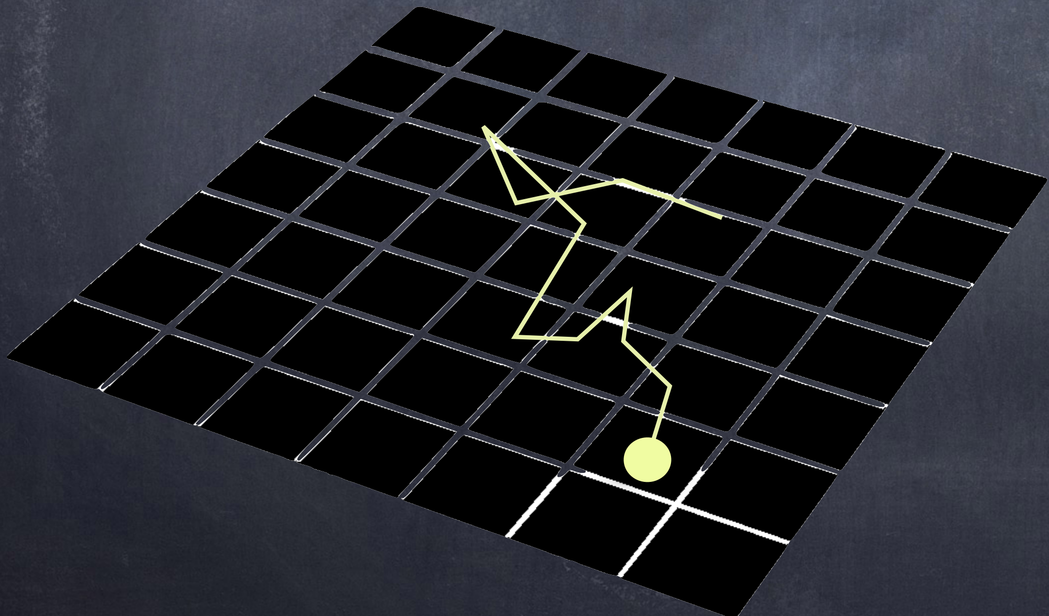

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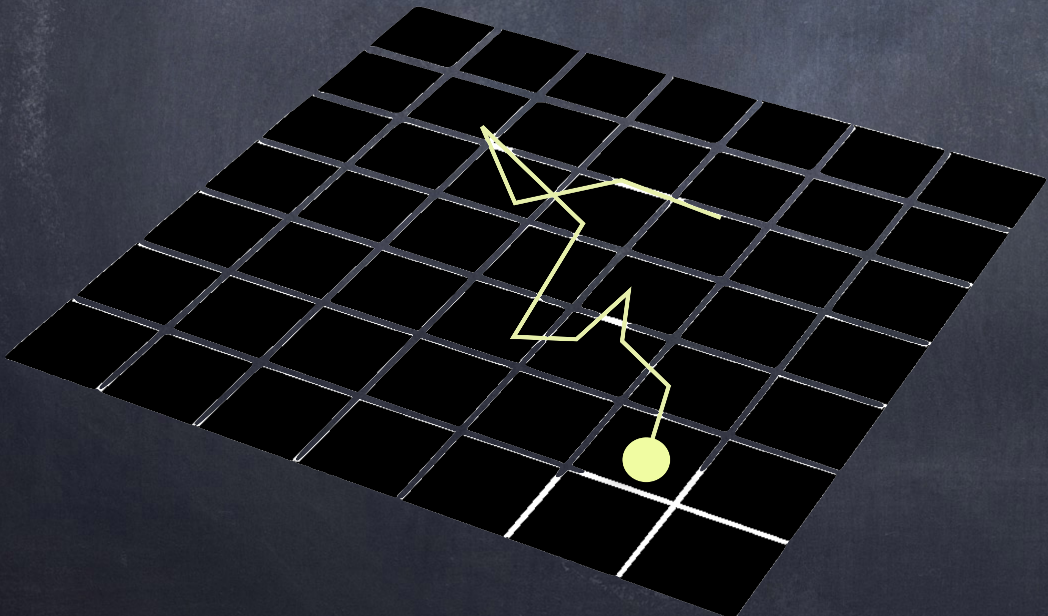

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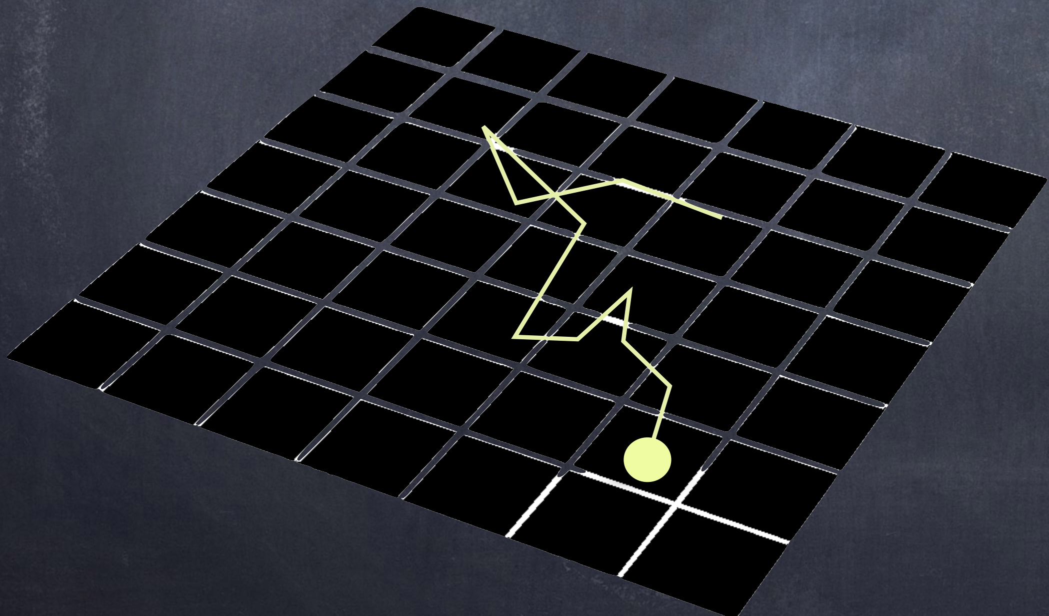

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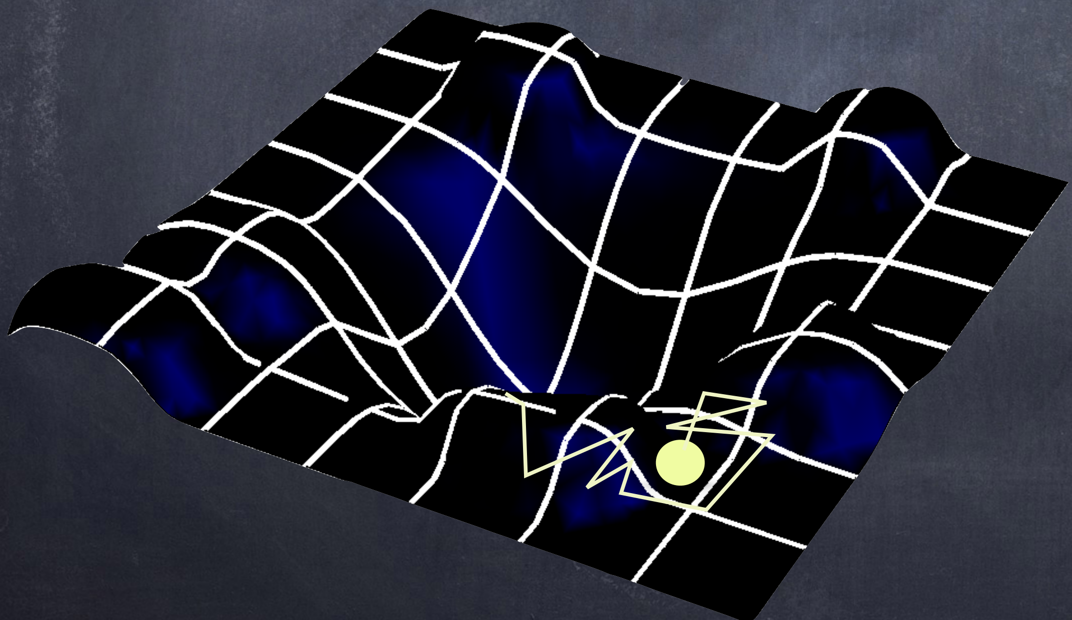

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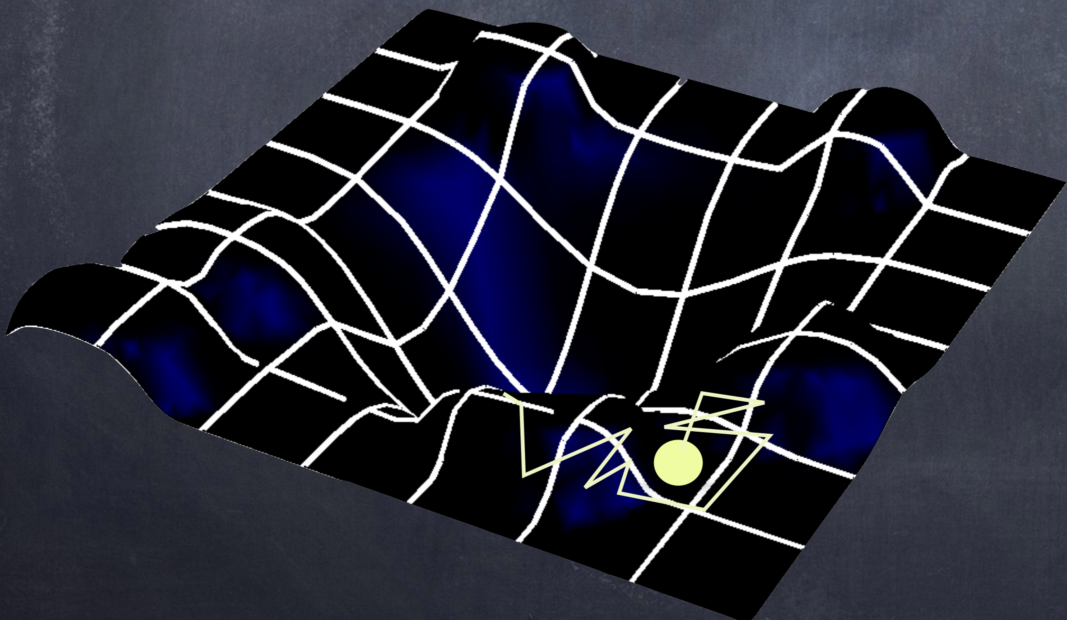

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Spectral dimension in asymptotic safety

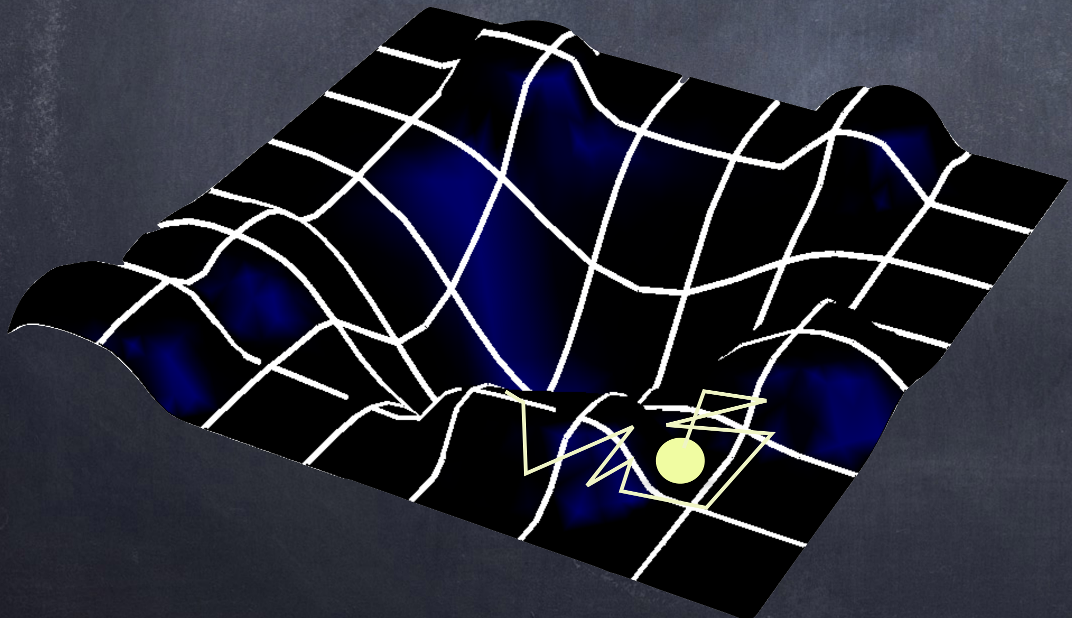
$$R_{\mu\nu}(\langle g_{\mu\nu}\rangle_k) = \Lambda(k)\,\langle g_{\mu\nu}\rangle_k = \lambda_* k^2\,\langle g_{\mu\nu}\rangle_k$$

$$R_{\mu\nu}(\langle g_{\mu\nu}\rangle_{k_0}) = \lambda_* k_0^2 \langle g_{\mu\nu}\rangle_{k_0} = \lambda_* k_0^2 f(k/k_0)\langle g_{\mu\nu}\rangle_k$$

$$\rightarrow \langle g^{\mu\nu}\rangle_k = \frac{k^2}{k_0^2}\langle g^{\mu\nu}\rangle_{k_0}$$

[Lauscher, Reuter '05]

(beyond truncation: this scaling follows directly from fixed point)



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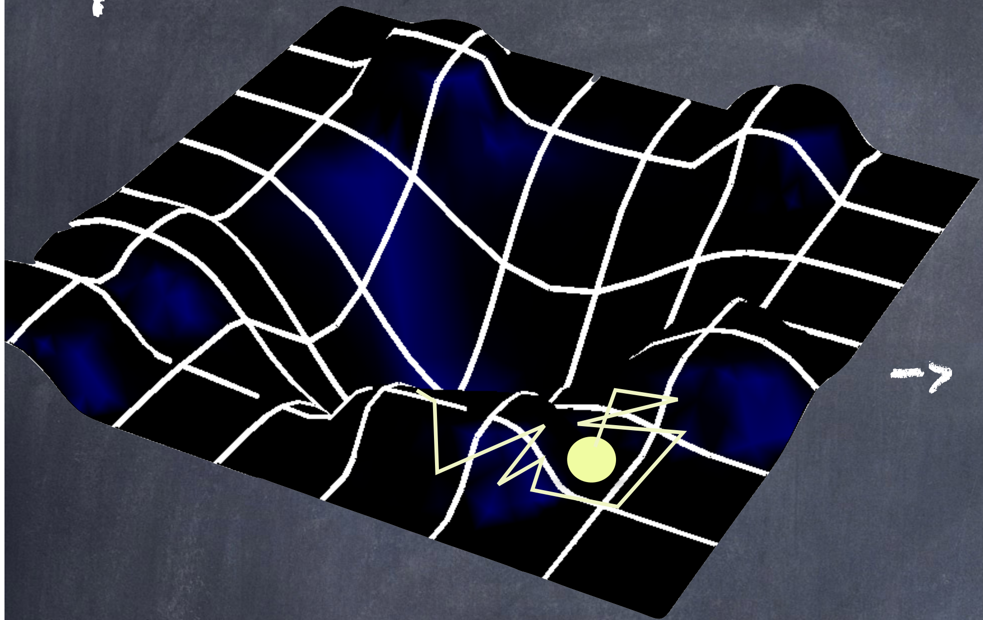

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scales of the diffusion process:

- momentum of the diffusing particle
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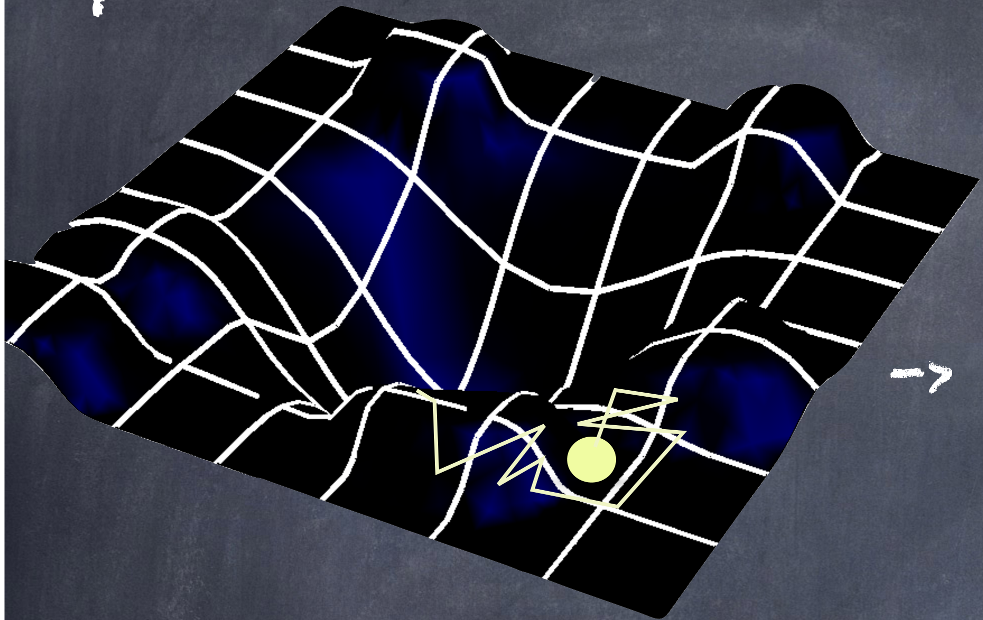

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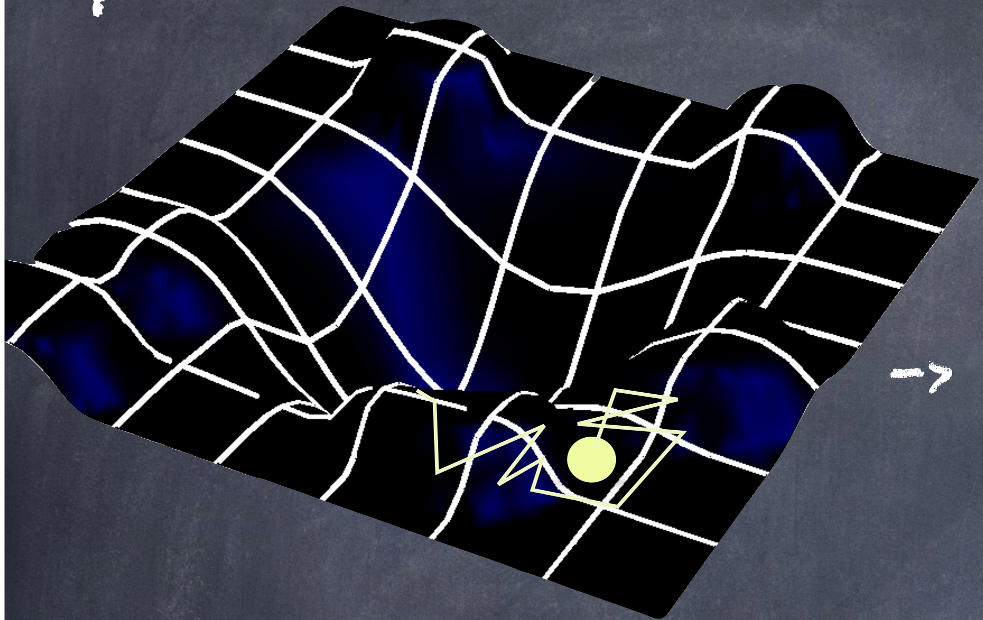

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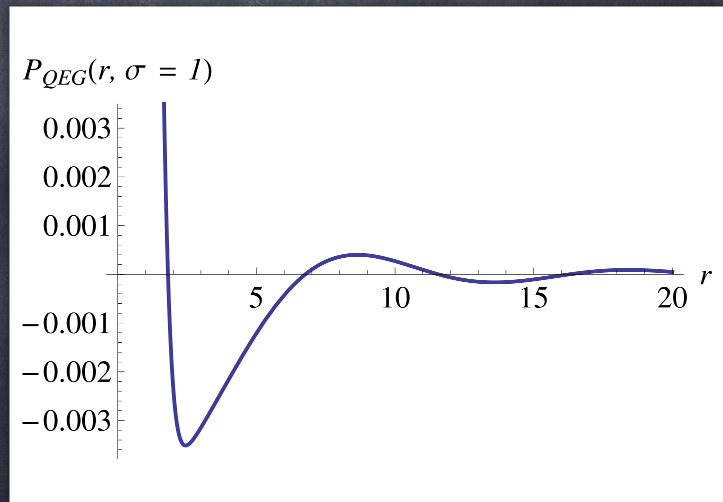
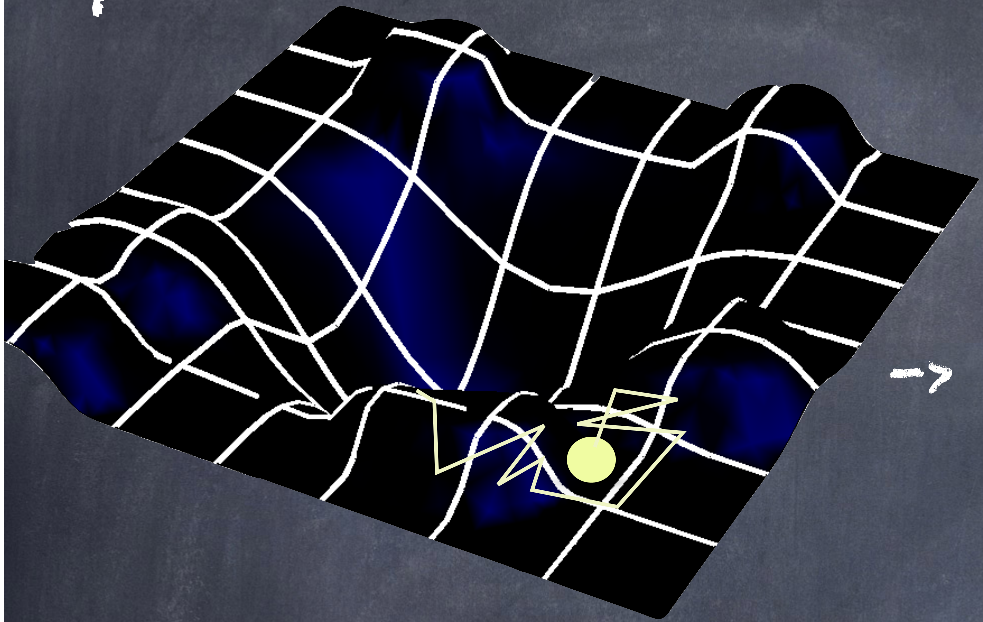
problem: higher-order equation:
solution not positive semi-definite



but P probability density?!

Spectral dimension in asymptotic safety



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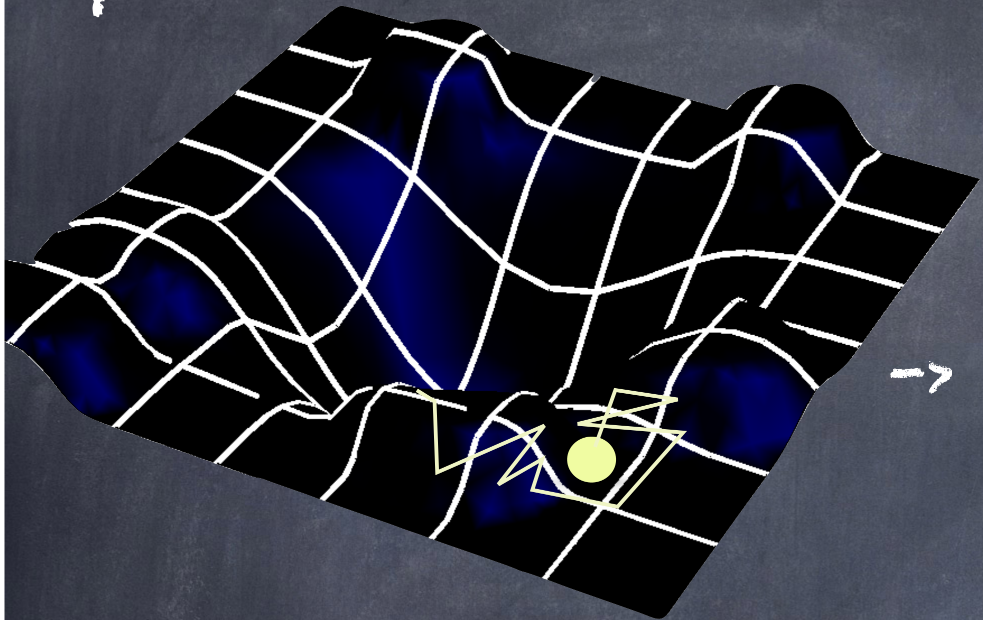

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[Calcagni, AE, Saueressig '13]

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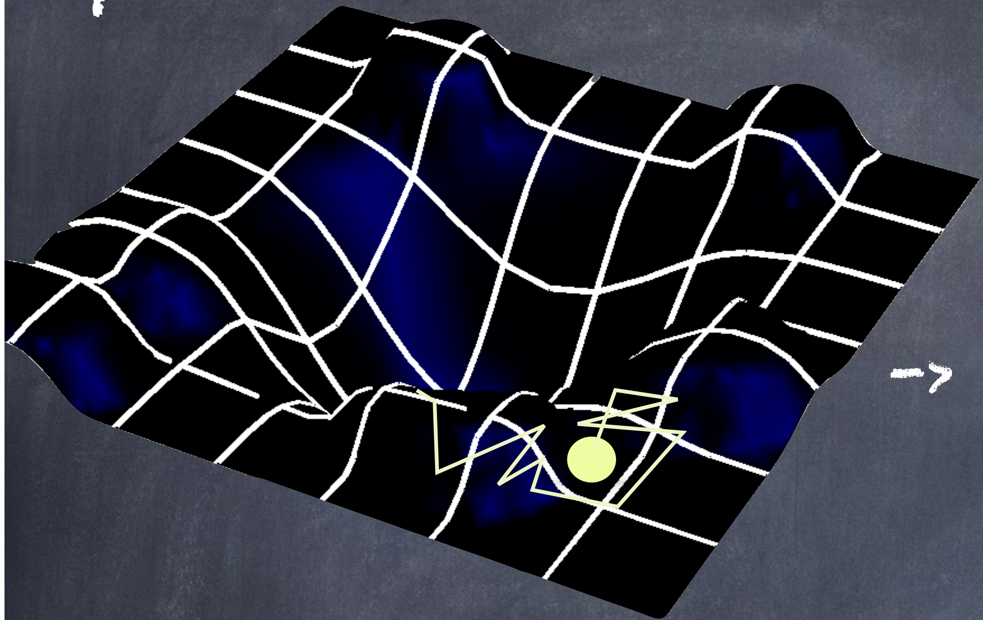

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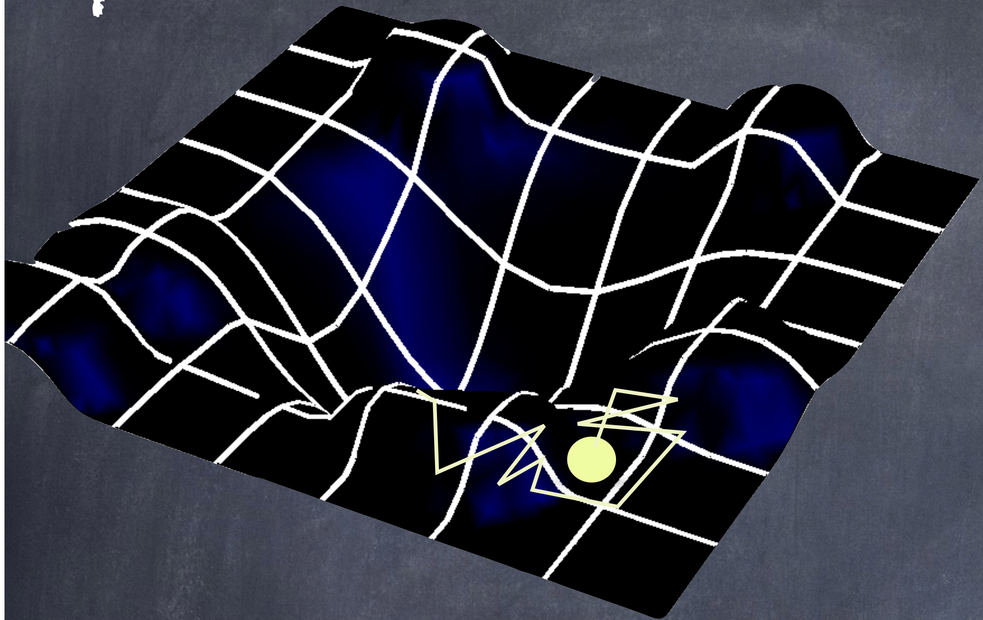
[Lauscher, Reuter '05; Reuter, Saueressig '12; Rechenberger, Saueressig '12; Calcagni, AE, Saueressig '13]

Spectral dimension in asymptotic safety



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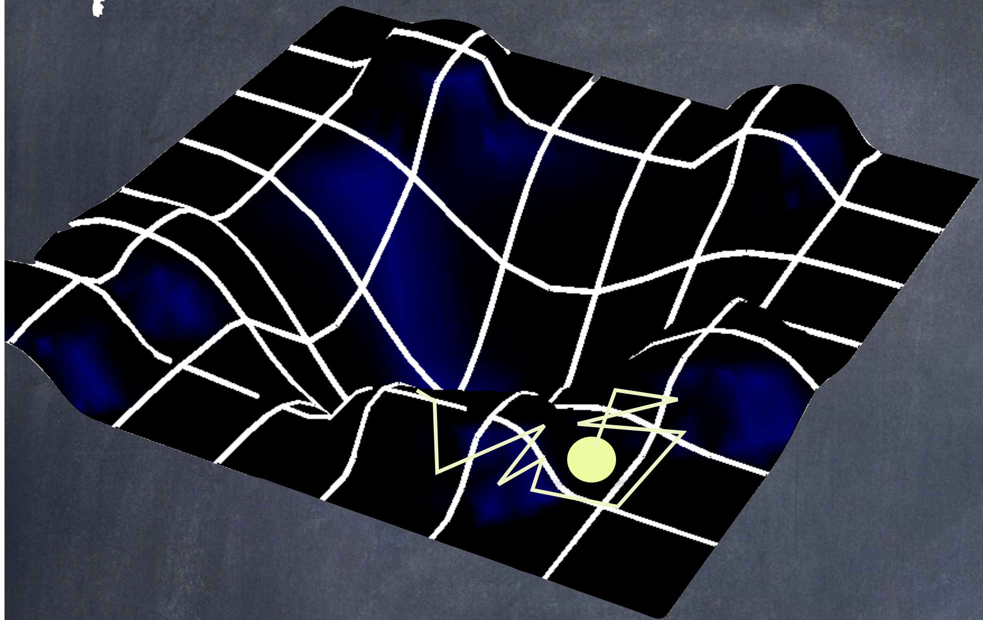

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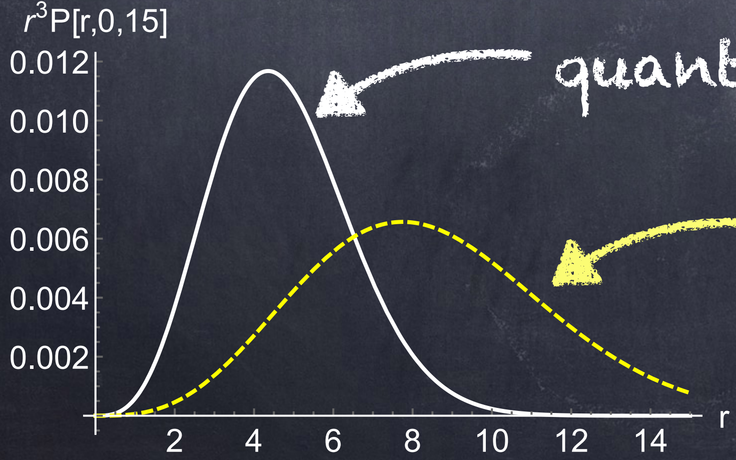
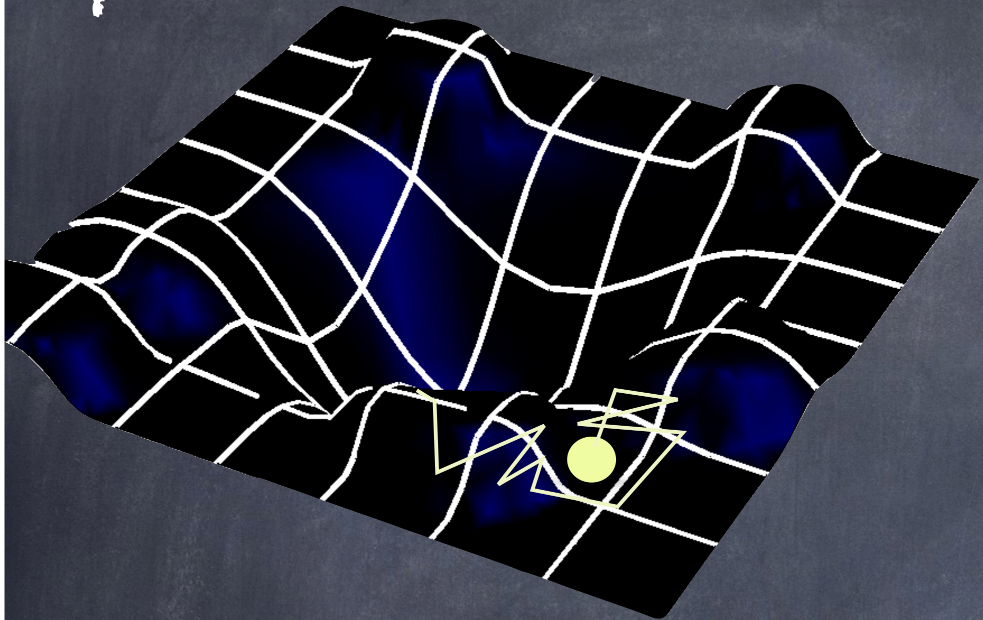


quantum diffusion

classical diffusion

-> subdiffusion:
quantum fluctuations of spacetime slow down the diffusion process

Spectral dimension in asymptotic safety



$$-2\frac{d\ln P(x,x,\sigma)}{d\ln\sigma} = d_s = 2$$

[Lauscher, Reuter '05; Reuter, Saueressig '12; Rechenberger, Saueressig '12; Calcagni, AE, Saueressig '13]

-> dynamical dimensional reduction

universal property of (d=4) quantum spacetime?

Causal Dynamical Triangulations [Ambjorn, Jurkiewicz, Loll '05]

Loop Quantum Gravity [Modesto '05]

Horava-Lifshitz gravity [Horava '09; Sotiriou, Visser, Weinfurtner '11]

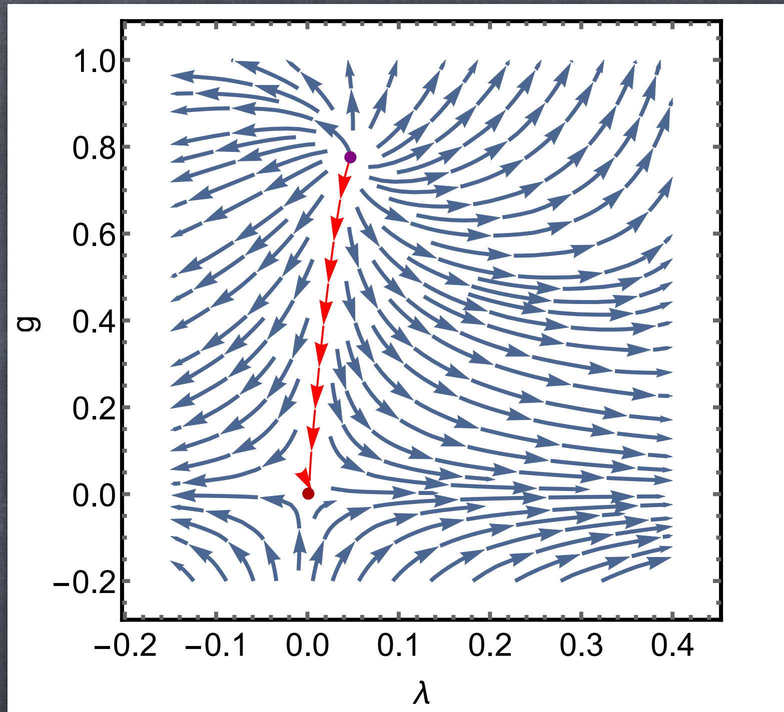
Wheeler -deWitt equation [Carlip '09]

Models of non-commutative spacetime [Benedetti '08; Arzano, Trześniewski '14]

Asymptotic safety in the Einstein- Hilbert truncation

Litim cutoff [Litim, '01] type IIa [Codello, Percacci, Rahmede '08]

$$\Gamma_k = -\frac{1}{16\pi G_N}\int d^4x\,\sqrt{g}\,(R-2\Lambda)$$



$$\beta_g = 2g - g^2\frac{92-80\lambda}{12\pi(1-2\lambda)}$$

$$\beta_\lambda = -2\lambda + g\frac{1+8\lambda}{2\pi(1-2\lambda)}$$

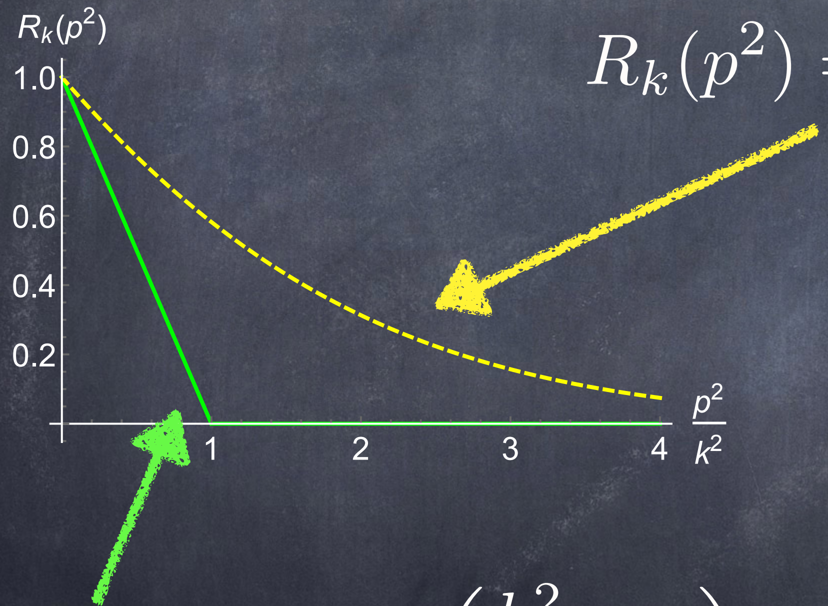
$$-g\,\lambda\frac{92-80\lambda}{12\pi(1-2\lambda)}$$

two relevant directions:

$$\theta_{1,2} = 2.31 \pm i\,0.38$$

Universality

Different choices for $R_k(p)$ possible



$$R_k(p^2) = p^2 \frac{1}{\exp\left[\frac{p^2}{k^2}\right] - 1}$$

$$R_k(p^2) = p^2 \left(\frac{k^2}{p^2} - 1\right) \theta(k^2 - p^2)$$

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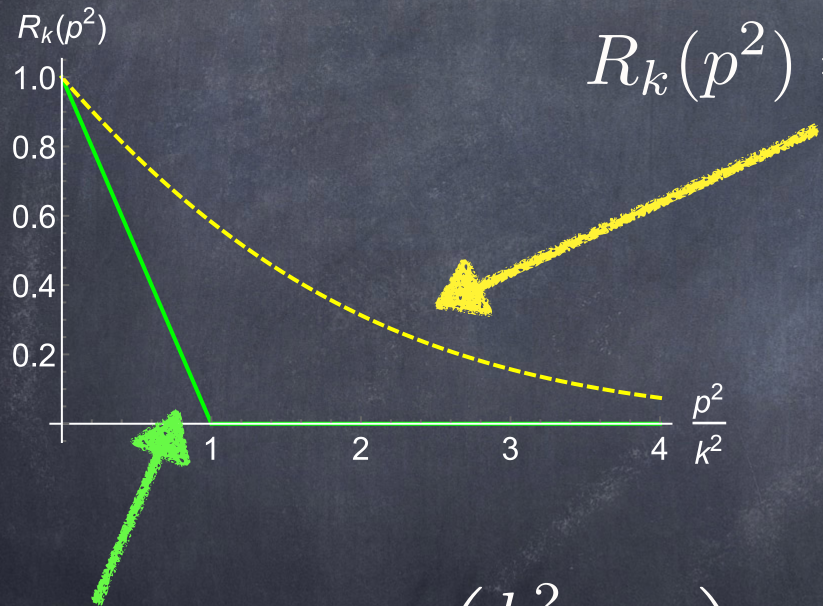

$$R_k(p^2) = p^2 \frac{1}{\exp\left[\frac{p^2}{k^2}\right] - 1}$$

-> RG flow will depend on the choice

-> Which result to trust?

$$R_k(p^2) = p^2 \left(\frac{k^2}{p^2} - 1\right) \theta(k^2 - p^2)$$

Universality

$$\beta_g = d_g g + \beta_1 g^2 + \beta_2 g^3 + \ldots$$

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Non-/universality of one-loop coefficient for dimensionfull/less couplings

$$\beta_{\tilde{g}} = f'(g)\beta_g$$

$$= \left(1 + f_1\, 2\, g + f_2\, 3g^2 + ...\right)\left(d_g\, g + \beta_1 g^2 + \beta_2 g^3 ...\right)$$

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for $d_g = 0$

universality at 1-loop

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-> fixed-point values in asymptotic safety are non- universal

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However: critical exponents are universal $-\frac{\partial \beta_g}{\partial g} = -\frac{\partial \beta_{\tilde{g}}}{\partial \tilde{g}}$

-> physical observables must be universal
(but couplings are not directly physical observables)

Critical exponents are universal, but truncating removes contributions

-> can test quality of truncations by considering scheme dependence of universal quantities

Einstein-Hilbert truncation

$$\Gamma_k = -\frac{1}{16\pi G}\int d^4x\,\sqrt{g}\,(R-2\Lambda)$$

$$S_{\rm gf} = \frac{1}{\alpha\, 32\pi\, G}\int d^4x\sqrt{\bar g}\bar g^{\mu\nu}F_\mu$$

$$F_\mu = \bar D^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar D_\mu h^\nu_{\ \nu}$$

$$R_k = Z_k\left(-\bar D^2+\gamma\bar R\right) r_k\left(\frac{-\bar D^2+\gamma\bar R}{k^2}\right)$$

type I: $\gamma = 0$

type II: $\gamma \neq 0$

type III: $R_k = Z_k\left(-\bar D^2+\gamma\bar R+\gamma_2\lambda_k\right) r_k\left(\frac{-\bar D^2+\gamma\bar R+\gamma_2\lambda_k}{k^2}\right)$

Einstein-Hilbert truncation

λ_*	G_*	$\lambda_* G_*$	$\mathrm{Re}[\theta_{1,2}]$	$\mathrm{Im}[\theta_{1,2}]$	cutoff	α	β	
0.35	0.27	0.10	1.55	3.84	exp. I	1	1	[Lauscher, Reuter '02]
0.19	0.71	0.14	1.48	3.04	opt. I	1	1	[Codello, Percacci, Rahmede '07]
0.09	0.56	0.05	2.43	1.27	opt. II	1	1	
0.27	0.33	0.09	1.75	2.07	opt. III	1	1	
0.25	1.17	0.30	1.67	4.31	opt. I	0	1	[Litim '04]
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strong scheme dependence

weaker scheme dependence

see also [Gies, Knorr, Lippoldt '15]...

fixed point exists in different schemes

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-> larger truncations required

f(R) truncations

$$\Gamma_k = \int d^4x\sqrt{g} f_k(R) = \sum_i a_i(k) \int d^4x\sqrt{g}\, R^i$$

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classical equivalence

But: not on the quantum level

$$\int \mathcal{D}g_{\mu\nu} \to \int \mathcal{D}\tilde{g}_{\mu\nu}f'(R)$$

see also [Benedetti, Guarnieri '14]...

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0.13	1.57	0.0015	-	1.38	26.9	-	opt.		[Codello, Percacci, Rahmede '07] $i_{max}=2$

see also: [Machado, Saueressig '07; Codello, Percacci, Rahmede '08; Falls, Litim, Nikolakopoulos, Rahmede '13; AE '15]

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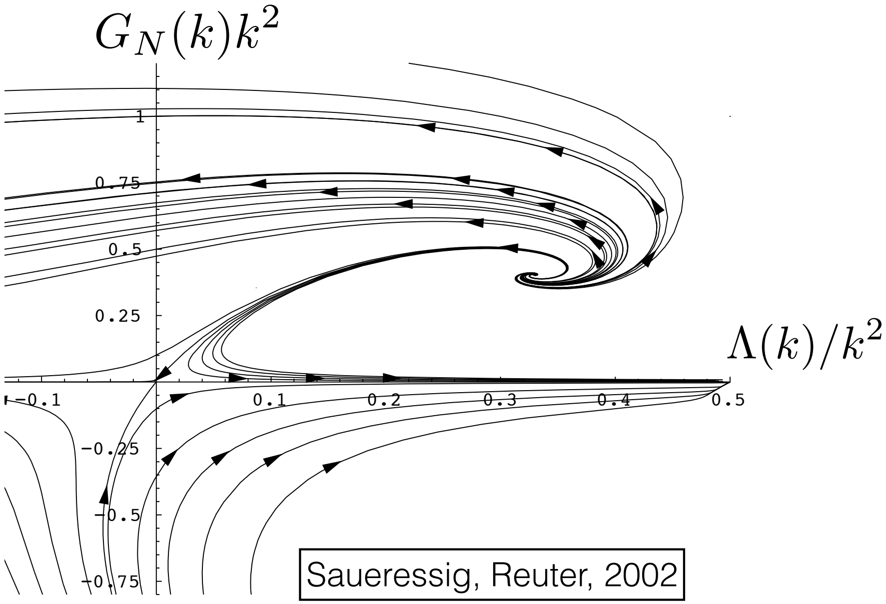

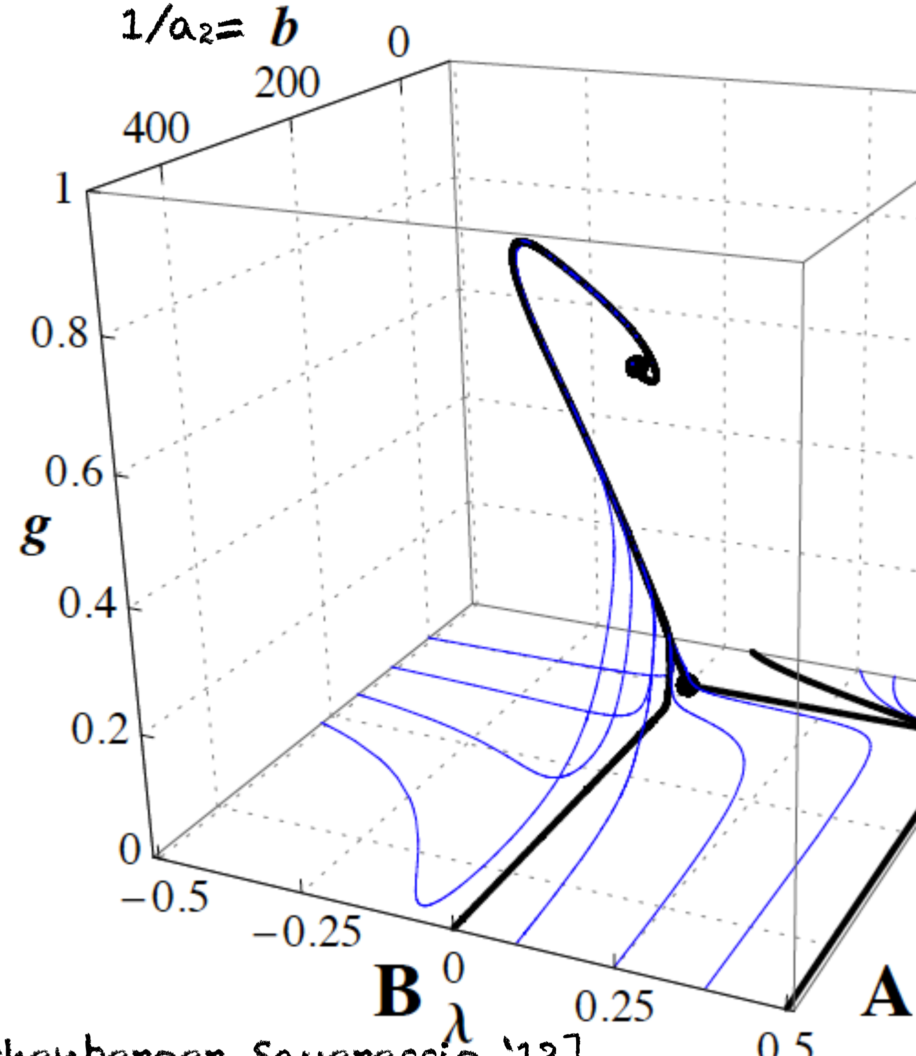

see also: [Machado, Saueress

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$[R^2] = 4$

-> $d_{a_2}=0$

Huge departures from canonical scaling?

-> Which couplings can become relevant?

-> What is a good truncation?

see also: [Machado, Saueressig '07; Codello, Percacci, Rahmede '08; Falls, Litim, Nikolakopoulos, Rahmede '13; AE '15]

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0.12	0.97	0.0003	-0.009	2.51	1.59	-3.93	opt.		[Falls, Litim, Nikolakopoulos, Rahmede '14] $i_{max}=35$

see also: [Machado, Saueressig '07; Codello, Percacci, Rahmede '08; Falls, Litim, Nikolakopoulos, Rahmede '13; AE '15]

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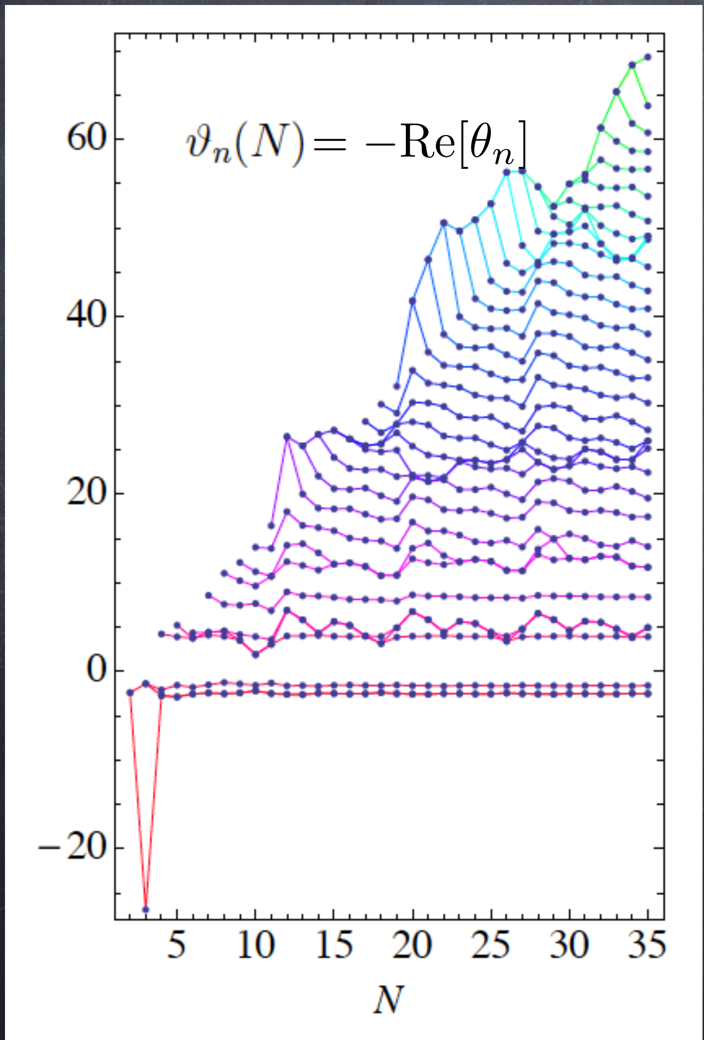

[Falls, Litim, Nikolakopoulos, Rahmede '14]

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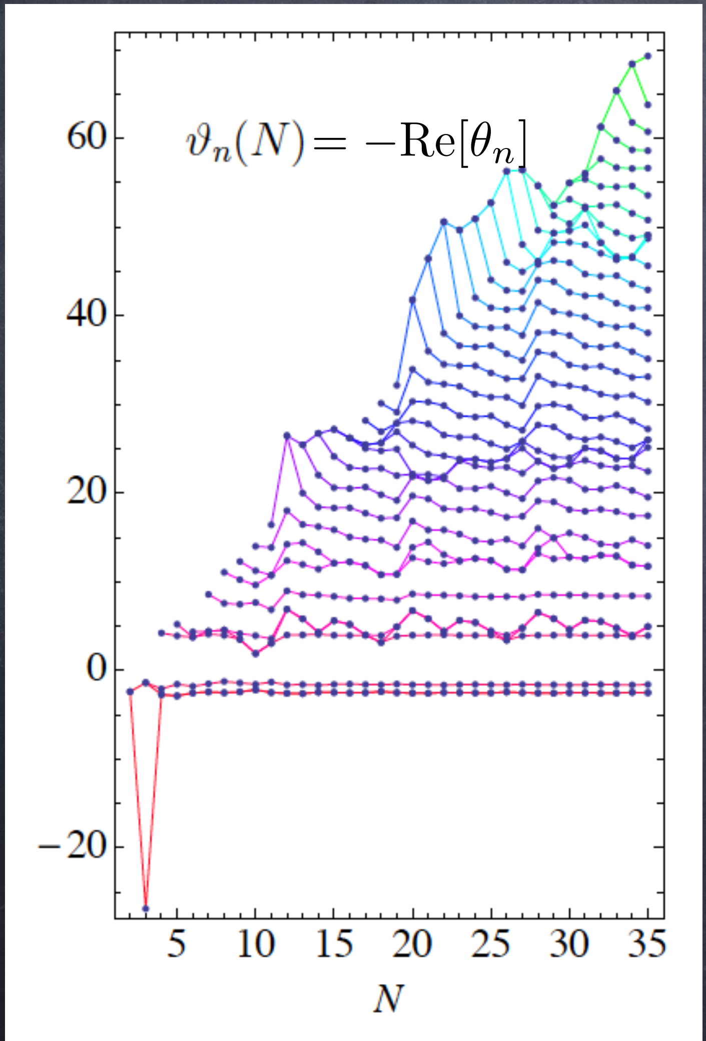

- three relevant directions

[Falls, Litim, Nikolakopoulos, Rahmede '14]

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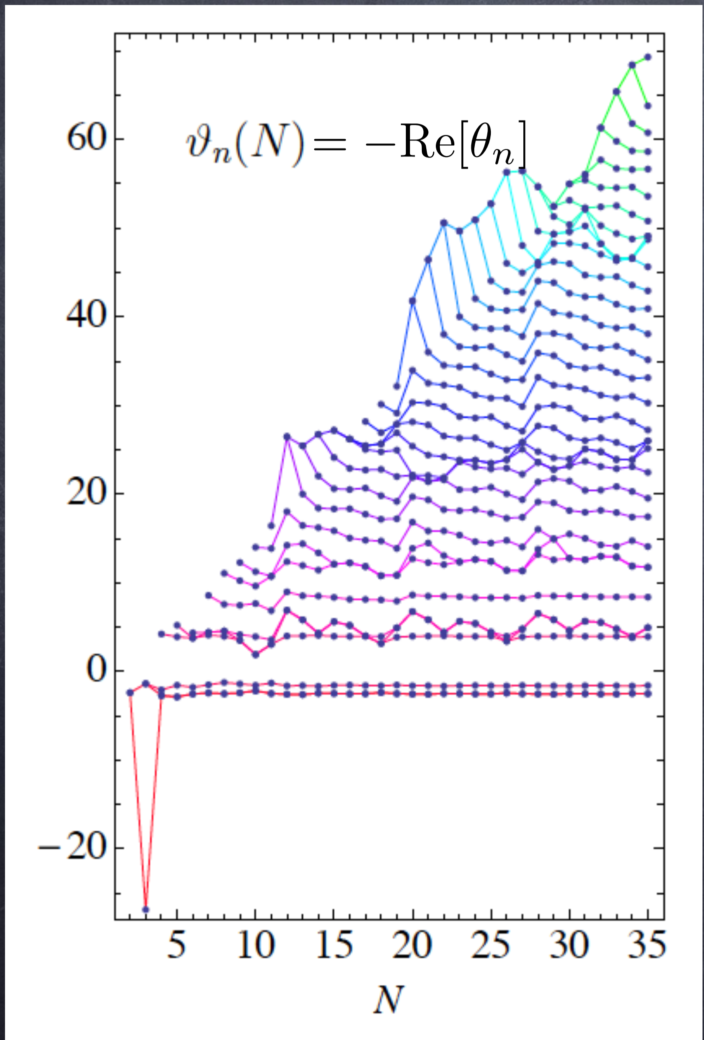

- three relevant directions
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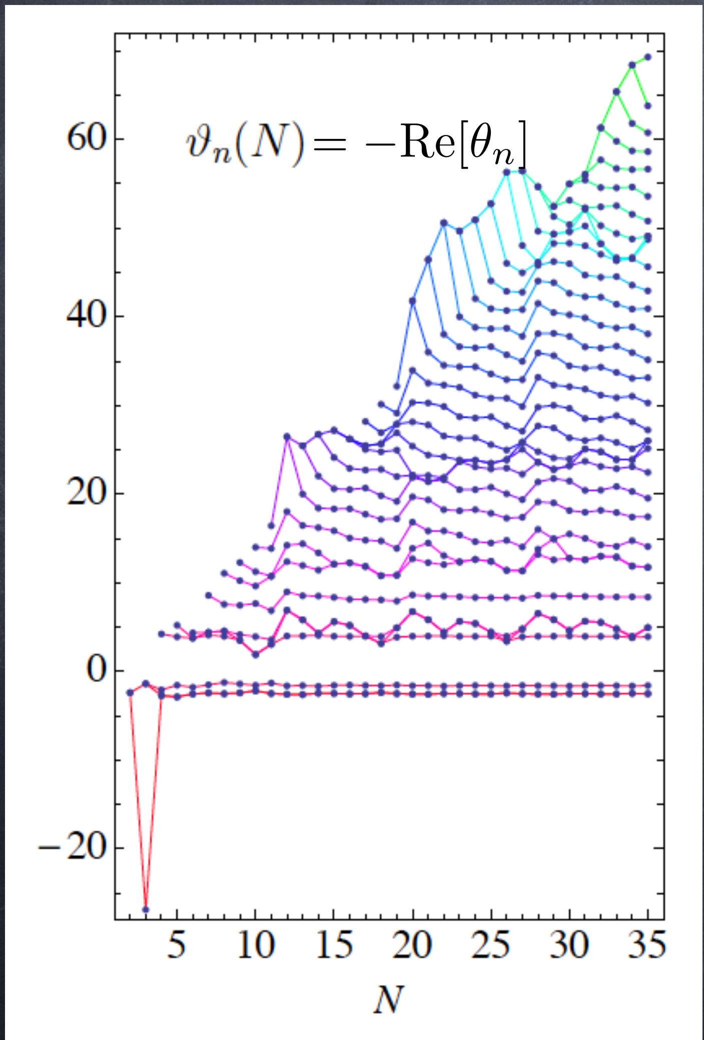

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$[R^n] = 2\, n$

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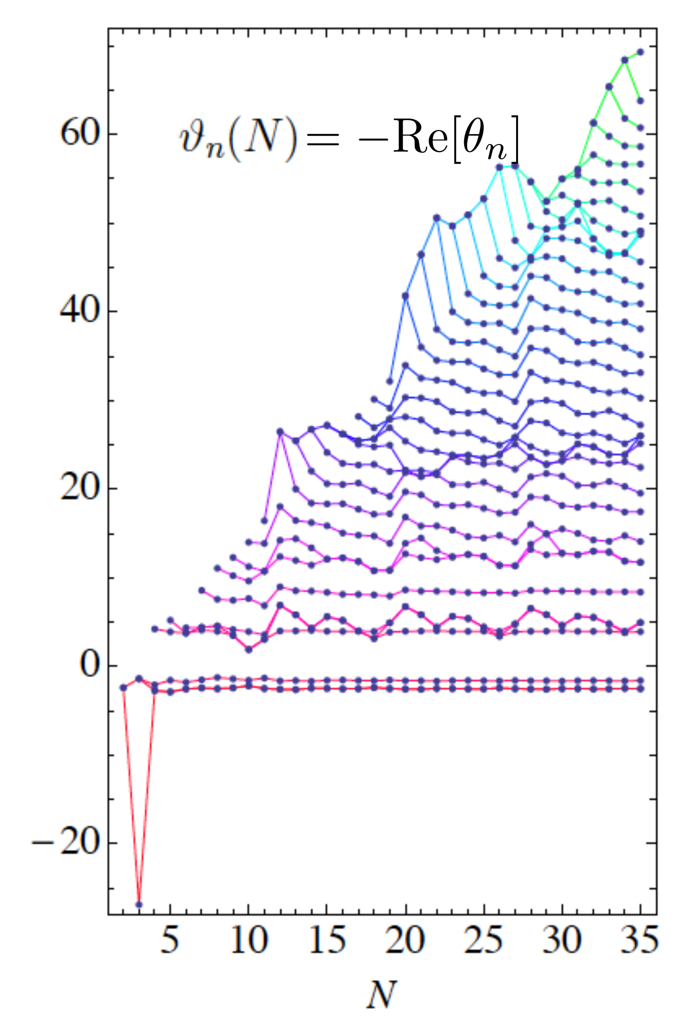

- three relevant directions
- near-Gaussian scaling for irrelevant directions:

$[R^n] = 2n \;\; \rightarrow d_n = 4-2n$

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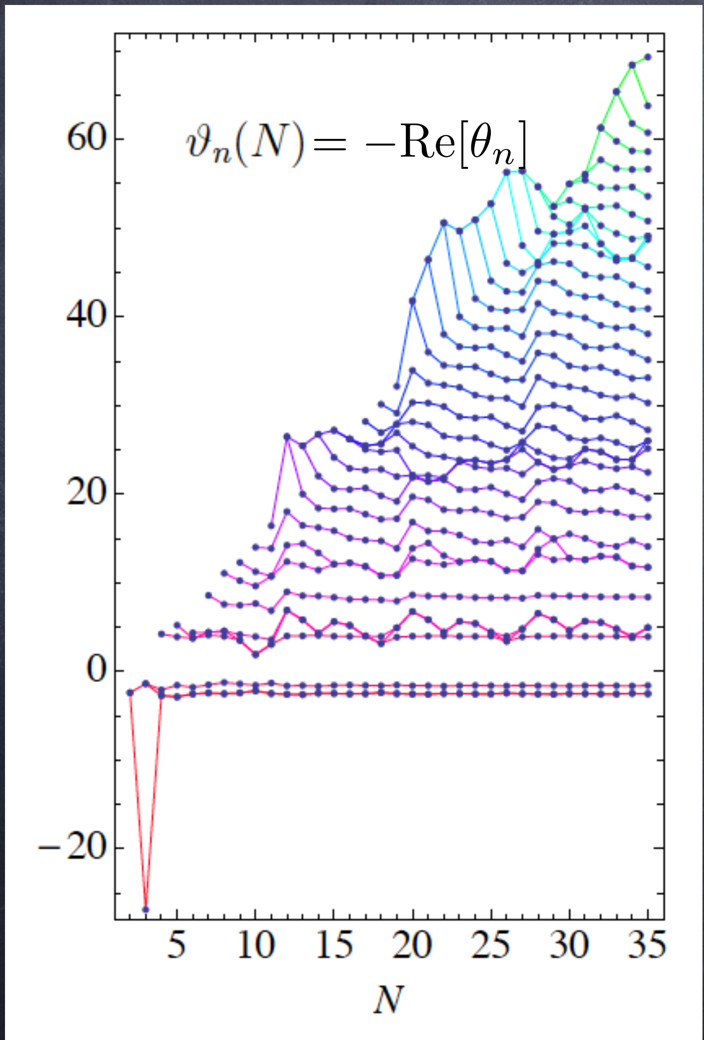

- three relevant directions
- near-Gaussian scaling for irrelevant directions:
  $[R^n] = 2\,n \quad \to d_n = 4 - 2n$

$$\theta_n = 4.06 - 2.17\,n$$

-> canonical dimensionality useful guide for further truncations

[Falls, Litim, Nikolakopoulos, Rahmede '14]

Tensor structures beyond R

$$\Gamma_k = \int d^4x \sqrt{g} \left(u_0 + u_1 R + (u_2 - \frac{2}{3} u_3) R^2 + 2 u_3 \, R_{\mu\nu} R^{\mu\nu} \right)$$

[Benedetti, Machado, Saueressig '09]

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-> structurally similar to results in f(R) truncations

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$$\Gamma_k = -\frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,(R - 2\Lambda) + \bar{\sigma}\int d^4x\sqrt{g}C_{\alpha\beta}{}^{\mu\nu}C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\alpha\beta}$$

[Gies, Knorr, Lippoldt, Saueressig '16]

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$$\theta_I = \{1.48 \pm 3.04\, i,\, -79.39\}$$

[Gies, Knorr, Lippold, Saueressig '16]

-> structurally similar to results in f(R) truncations