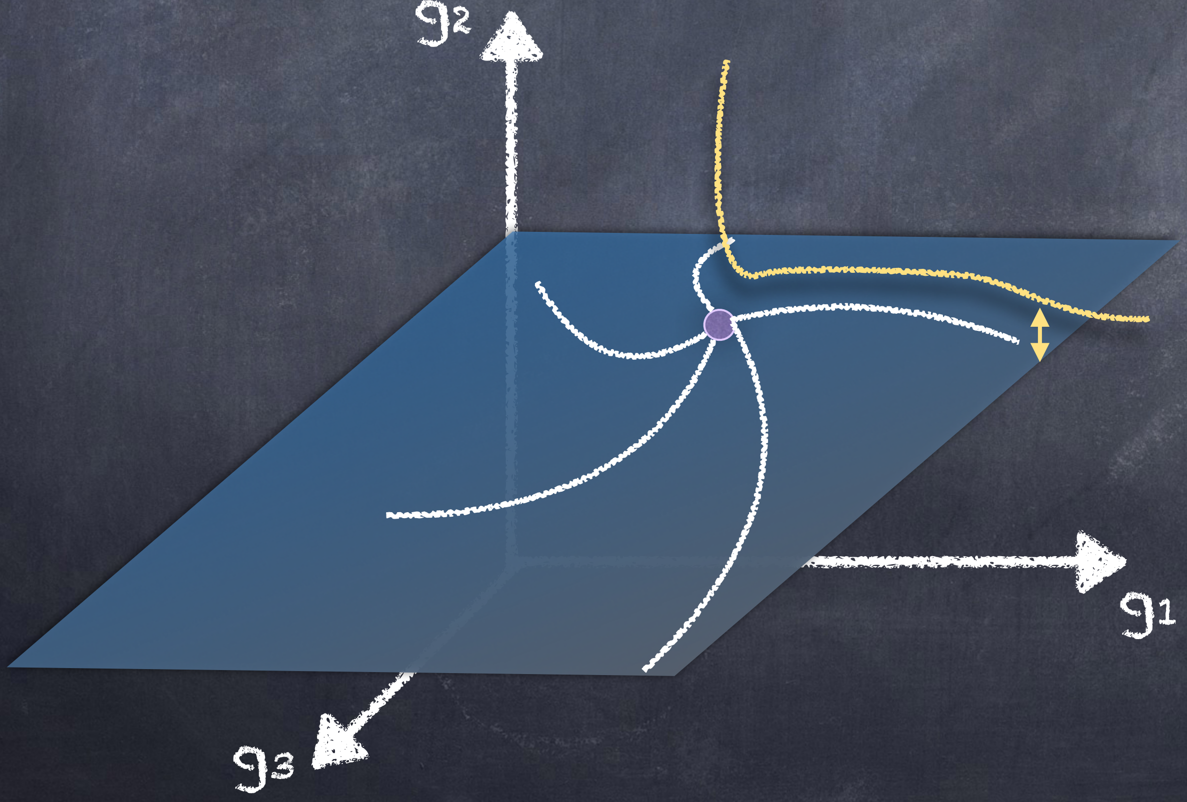
Asymptotic safety in a nutshell

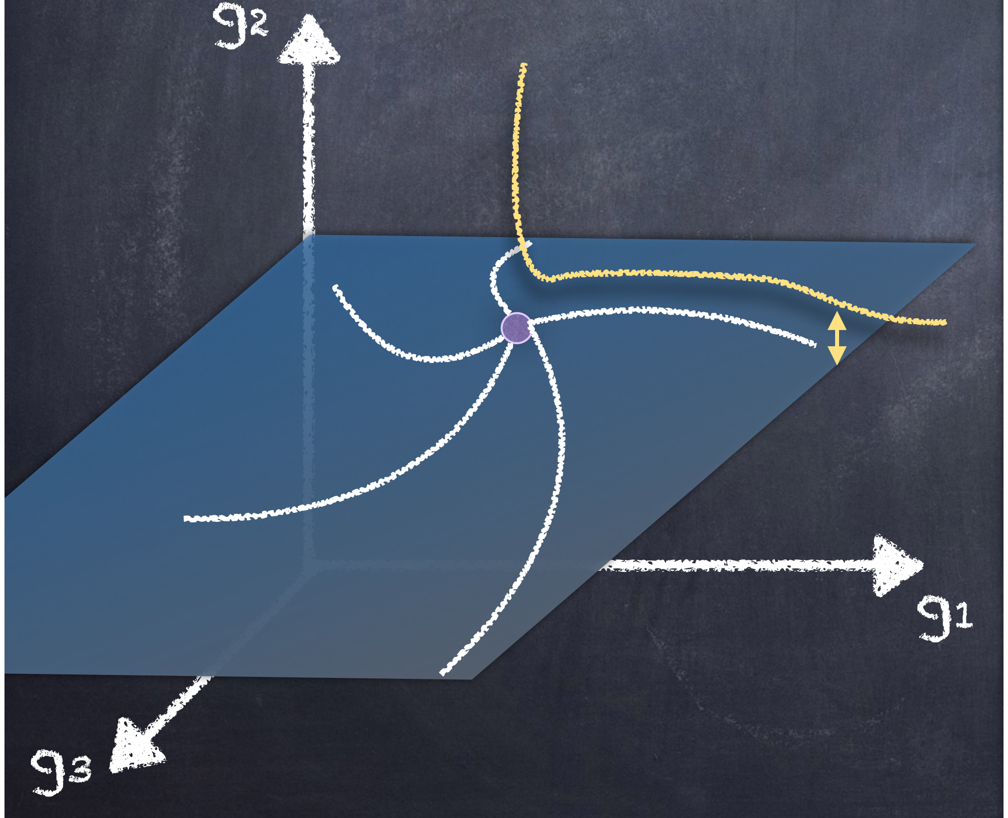
Interacting Renormalisation Group fixed point
(-> quantum scale invariance)
with finite number of UV-attractive (relevant) directions
(-> predictive)



Predictivity

If a fixed point has a finite-dimensional UV- critical surface, the corresponding theory has a finite number of free parameters

linearize around fixed point at $g_j = g_{j\,*}$



$$\theta_I = -\mathrm{eig}\left(\frac{\partial \beta_{g_n}}{\partial g_j}\bigg|_{g_j=g_{j\,*}}\right)$$

$$g_i(k) = g_{i\,*} + \sum_I C_I\, V_i^I \left(\frac{k}{k_0}\right)^{-\theta_I}$$

$\theta_I > 0$ relevant
UV-attractive -> free parameter

$\theta_I < 0$ irrelevant
UV-repulsive -> no free parameter

$\theta_I = 0$ marginally (ir)relevant
-> beyond linear approx.

At the Gaussian fixed point

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$$g_i = \bar{g}_i k^{-d_{g_i}}$$

dimensionality of $\bar{g}_i$

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Example: non- Abelian gauge theories

$g_n \left(F^a_{\mu\nu} F^{a\,\mu\nu}\right)^n$

-> g_n for $n \geq 2$

generated towards IR,
but determined in terms of g_1

At a non-Gaussian fixed point

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- stability matrix is (generically) off-diagonal

$$-\frac{\partial \beta_{g_i}}{\partial g_m}\Big|_{g_n = g_{n\,*}} = -\beta_{m,i} g_{i\,*} + \ldots$$

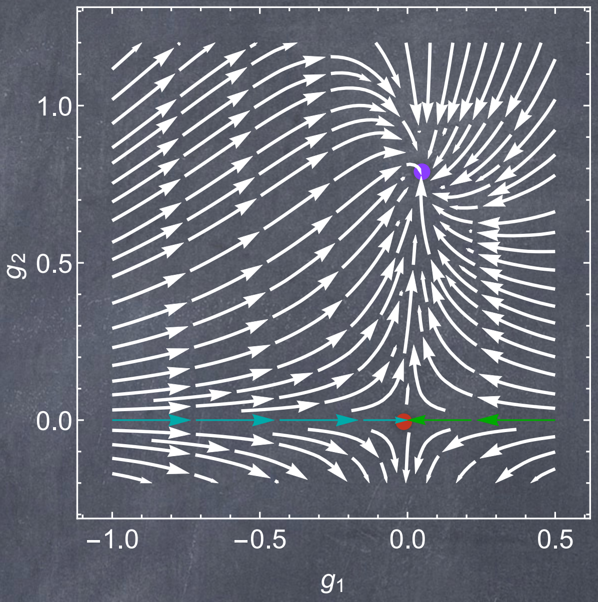
-> (ir)relevant operators are superpositions

Examples:

toy model:

$$\beta_{g_1} = -2g_1 + \frac{g_2}{2\pi} - \frac{8}{3\pi} g_1\, g_2$$

$$\beta_{g_2} = 2g_2 - \frac{23}{3\pi} g_2^2 \left(1 + \frac{26}{23} g_1\right)$$



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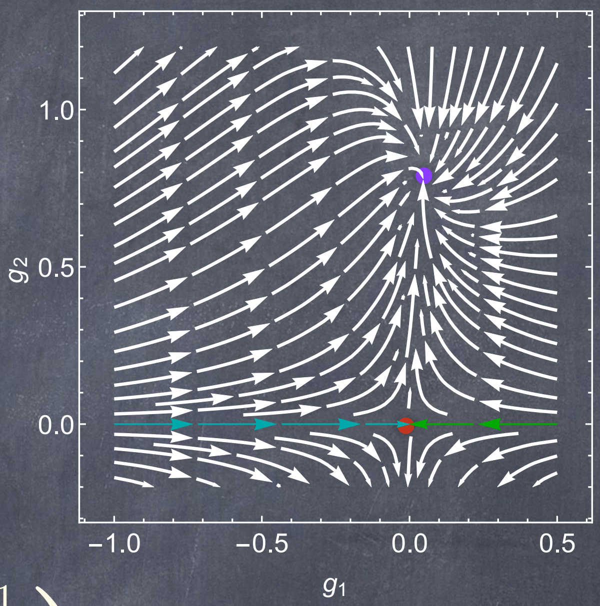

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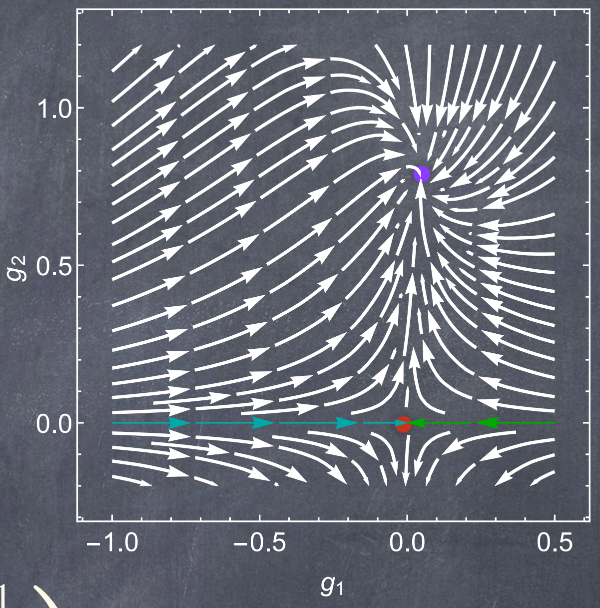

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Non- Gaussian fixed point:

$$\frac{\partial \beta_{g_i}}{\partial g_j}\Big|_{g_n = g_{n\,*}} = \begin{pmatrix} -2 & -1.67 \\ 0.12 & -2.66 \end{pmatrix} \rightarrow \theta_{1,2} = 2.33 \pm i0.30$$

-> UV attractive in two directions, RG flow spirals

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Non-Abelian gauge fields

-> antiscreening vacuum

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$$

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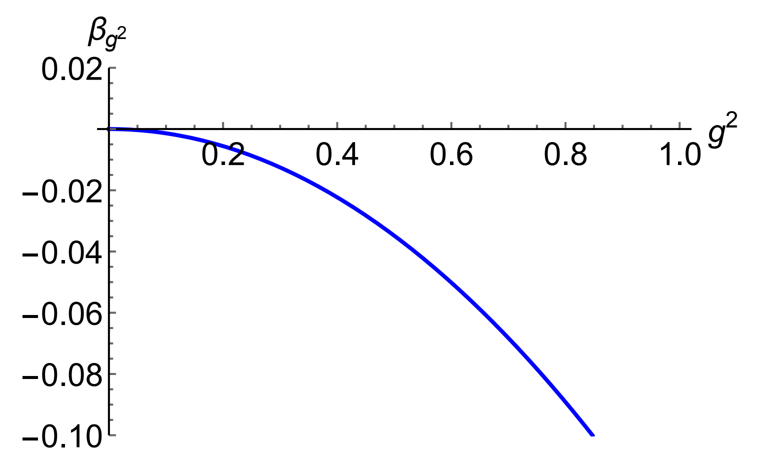

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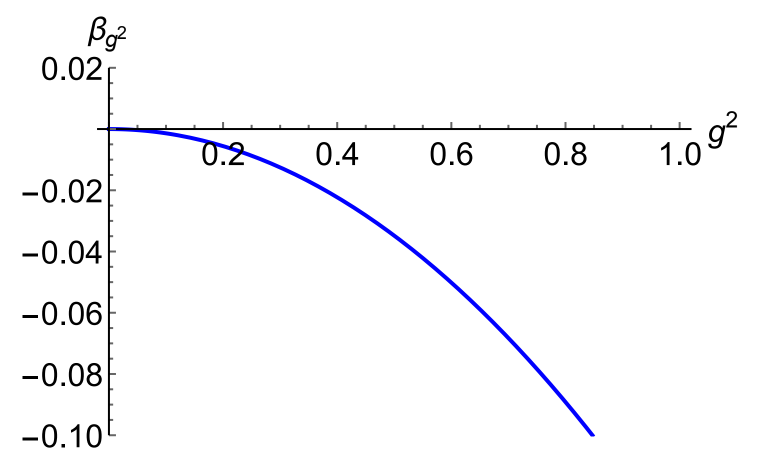

[Peskin '80; ε^4 Morris '04; FRG: Gies '03]

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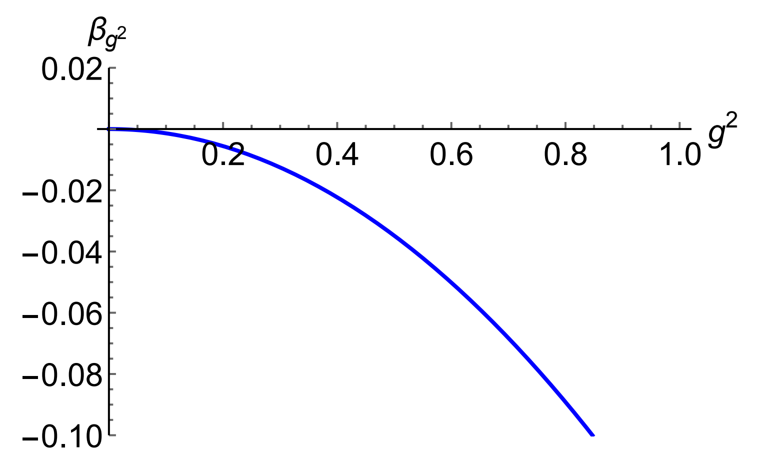

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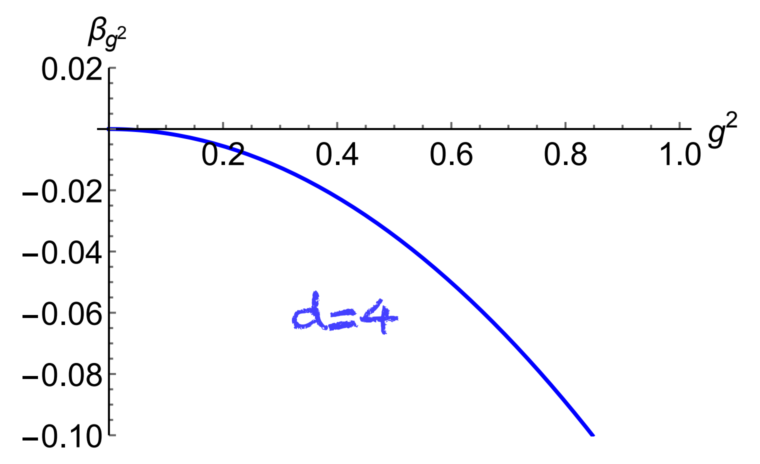

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quantum fluctuations

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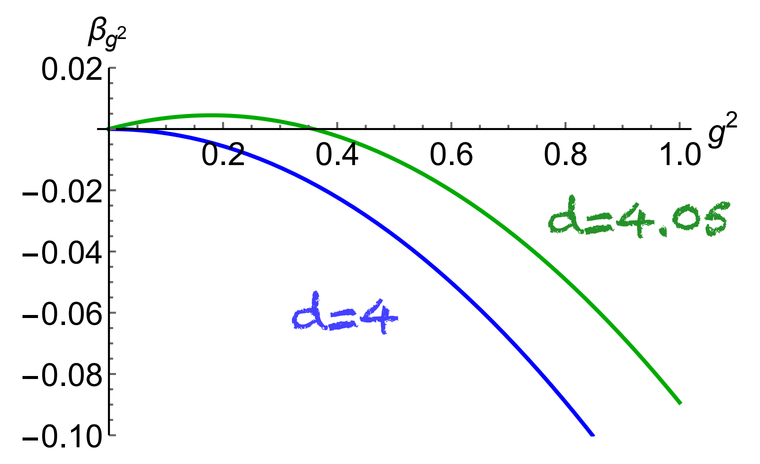

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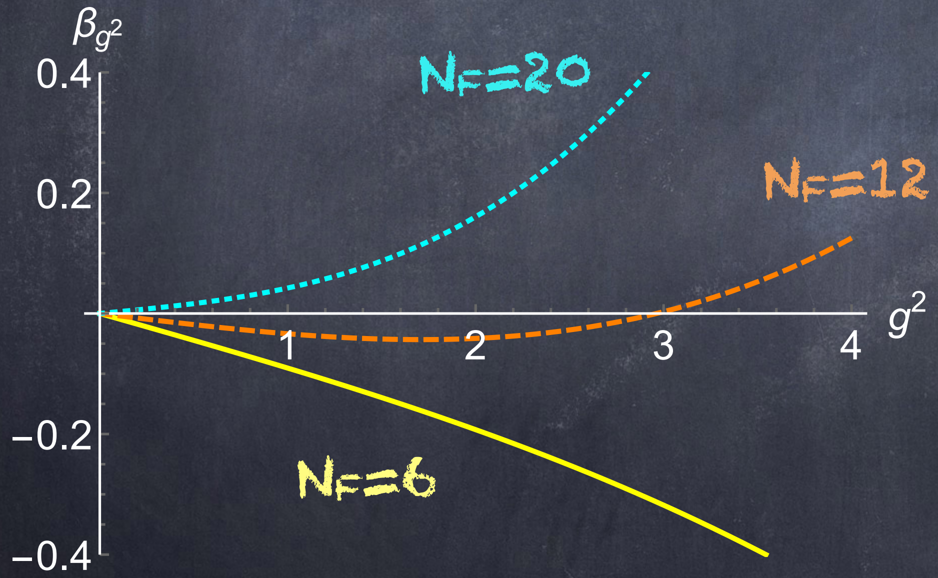

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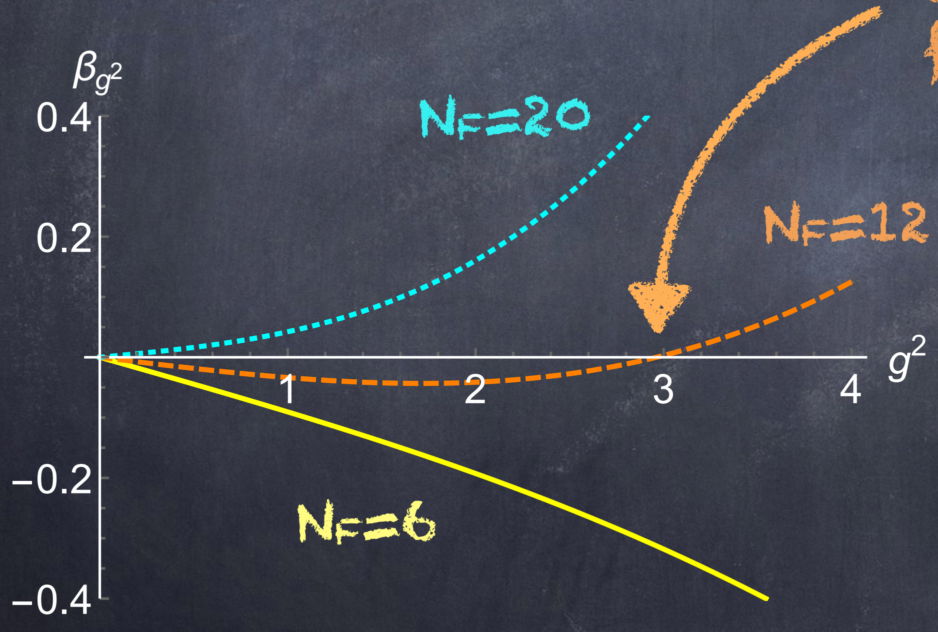

gauge boson and fermion fluctuations balance

[Caswell '74; Banks, Zaks '82]

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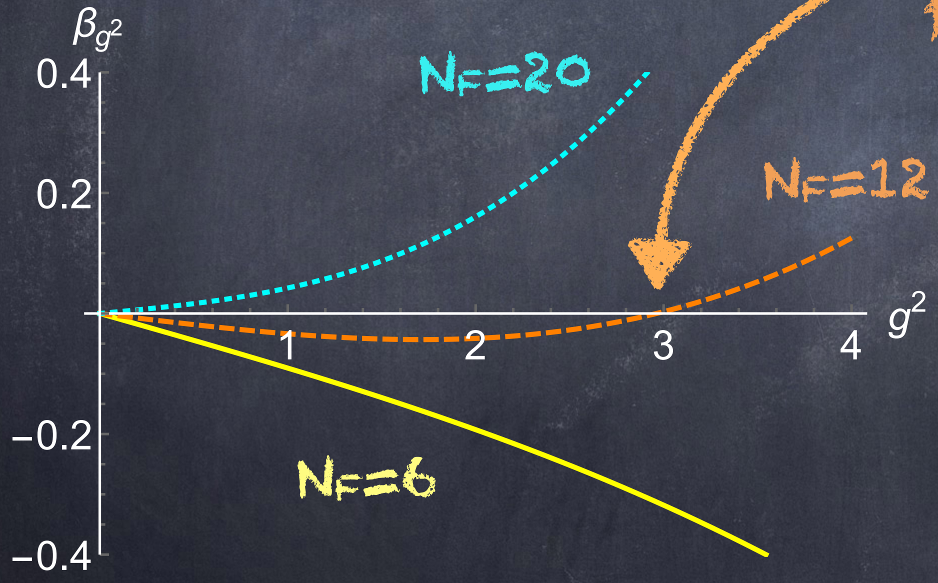

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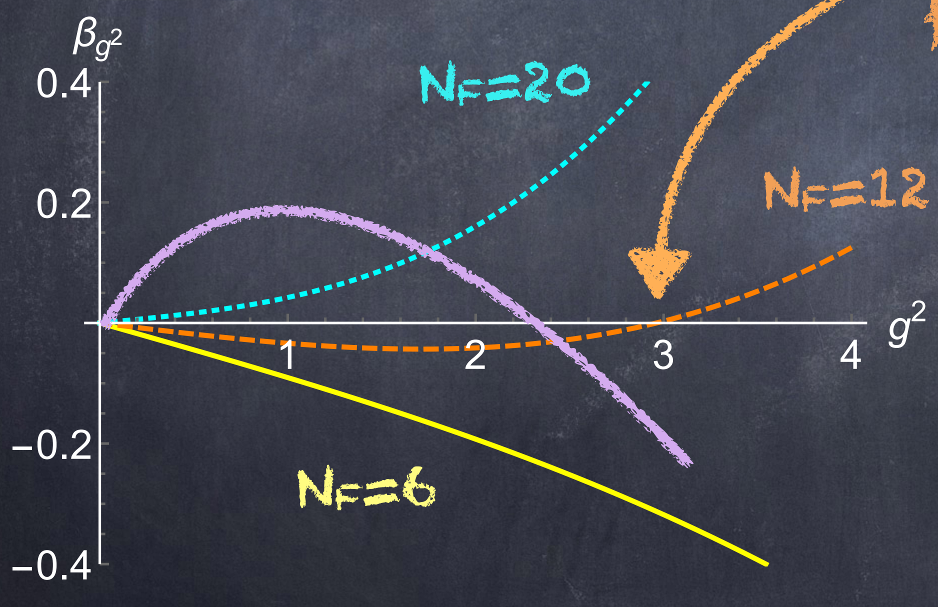

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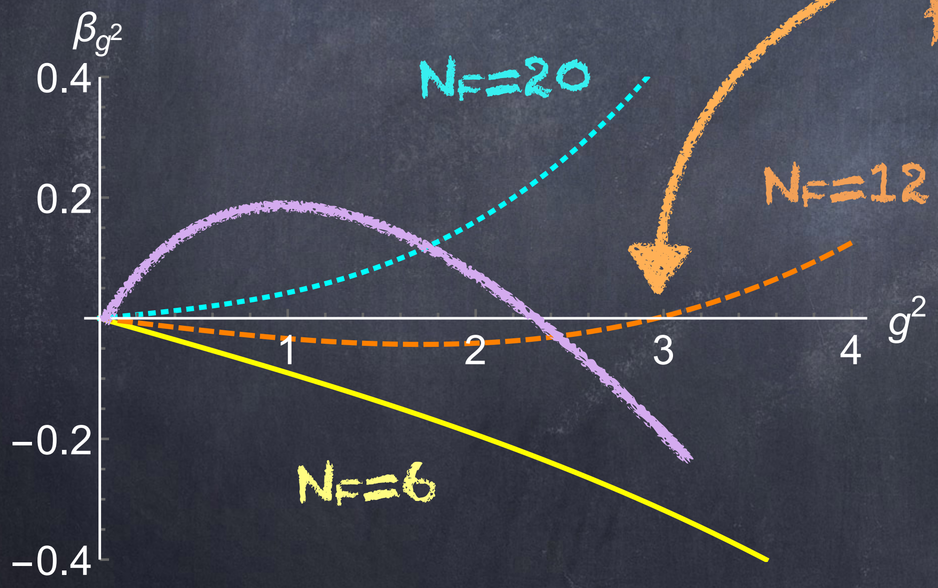

gauge boson and fermion fluctuations balance

[Caswell '74; Banks, Zaks '82]

-> fixed point is IR-attractive

use N$_F$ > 11/2 N$_c$

-> induce fixed point

through Yukawa coupling of N$_F$^{2} uncharged scalars

[Litim, Sannino '14]

Asymptotic safety in gravity

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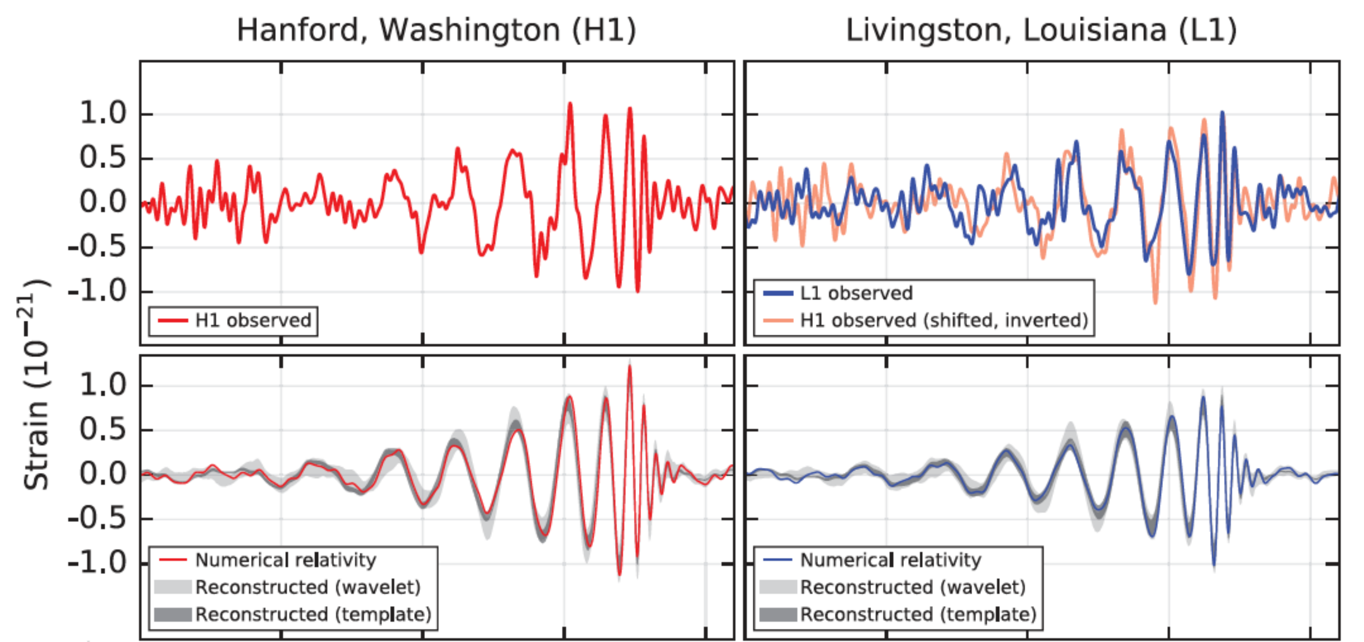

[LIGO collaboration '16]

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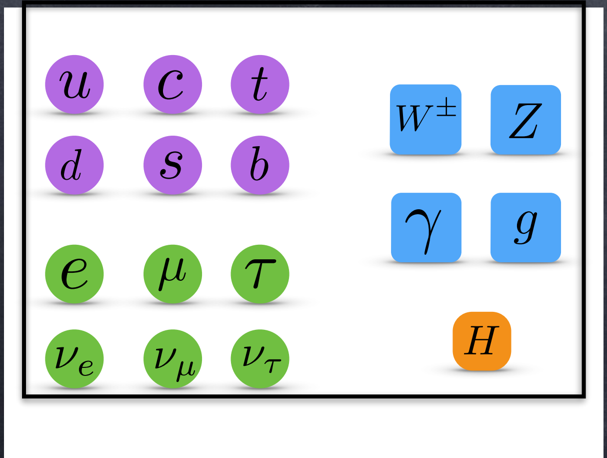

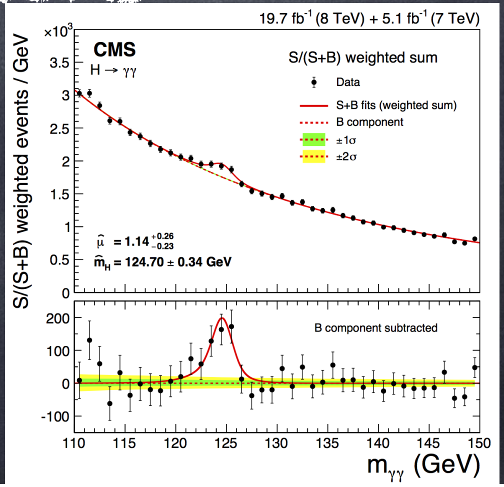

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-> (arguably) most conservative approach to quantum gravity

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expansion in $d = 2 + \epsilon$

[Weinberg '79; Christensen, Duff '78; Gastmans, Kallosh and Truffin '78; Kawai, Kitazawa, Ninomiya '93, '96]

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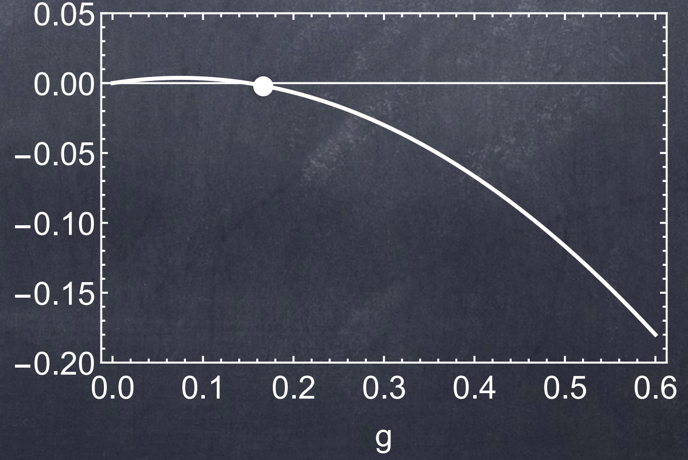
interacting fixed point at positive g

-> balancing of dimensional and fluctuation terms



$\epsilon = 0.1$

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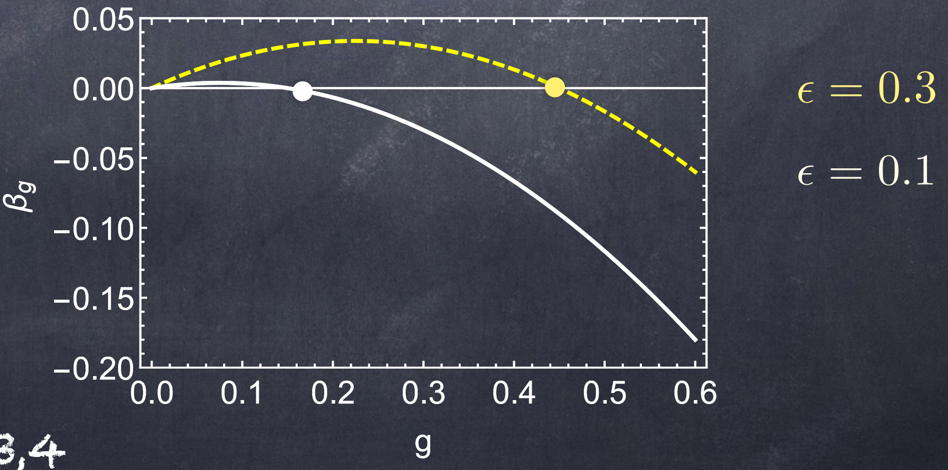

-> beyond perturbative regime for d->3,4

Asymptotic safety in gravity
- tools in d=4

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Asymptotic safety in gravity - tools in d=4

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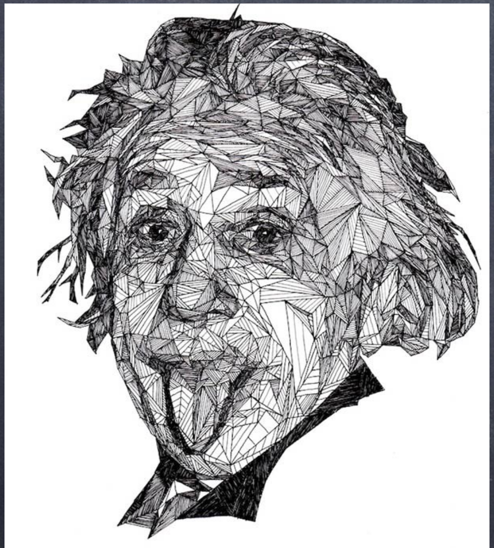

Asymptotic safety in gravity - tools in d=4

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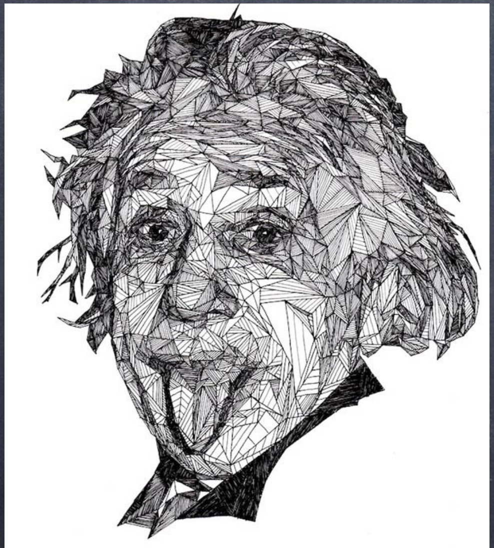
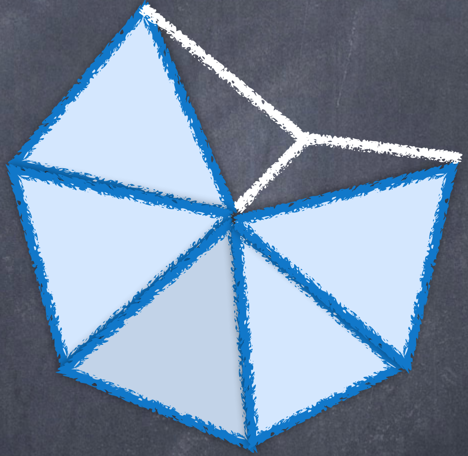
discrete analogue of Einstein action





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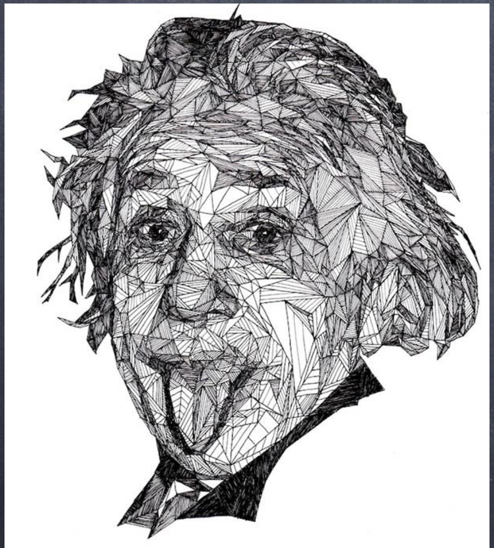

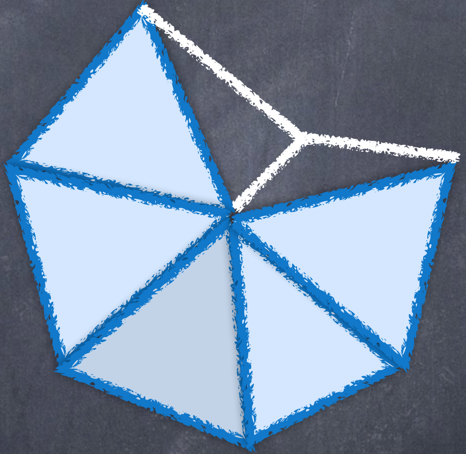

- Regge calculus [Hamber, Williams]
- Causal/ Euclidean Dynamical Triangulations [Ambjorn, Jurkiewicz, Loll]

Asymptotic safety in gravity
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continuum:

Functional Renormalisation Group:

Continuum path-integral
in momentum-shell-wise fashion

Functional Renormalisation Group

For details see: [Wetterich ‘93]

[Berges, Tetradis, Wetterich ’02; Pawlowski ’07;
Delamotte ’07; Gies ’12; Rosten ’12; Braun ‘12]

Main idea: Do path-integral
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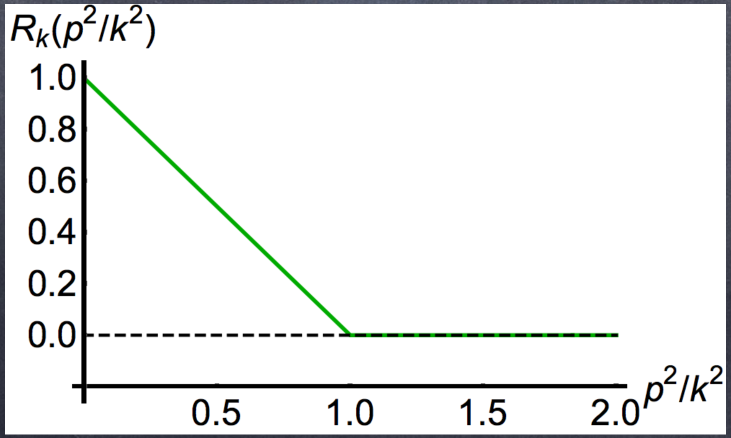
Main idea: Do path-integral momentum-shell wise

$$e^{-\Gamma_k[\phi]}{}^{*}= \int_\Lambda \mathcal{D}\varphi\, e^{-S[\varphi]-\frac{1}{2}\int\varphi(p)R_k(p^2)\varphi(-p)}$$

contains effect of qm fluc's at p > k



$\phi = \langle\varphi\rangle$

microscopic equations of motion:

$$\frac{\delta S}{\delta\varphi} = 0$$

quantum (eff.) equations of motion:

$$\frac{\delta\Gamma}{\delta\phi} = 0$$

---

* $\Gamma_k = \sup_J\left(\int J\cdot\phi - \ln\int\mathcal{D}\varphi\, e^{-S[\varphi]+\int J\cdot\varphi-\frac{1}{2}\int_p\varphi(-p)R_k(p)\varphi(p)}\right) - \frac{1}{2}\int_p\phi(-p)R_k(p)\phi(p)$

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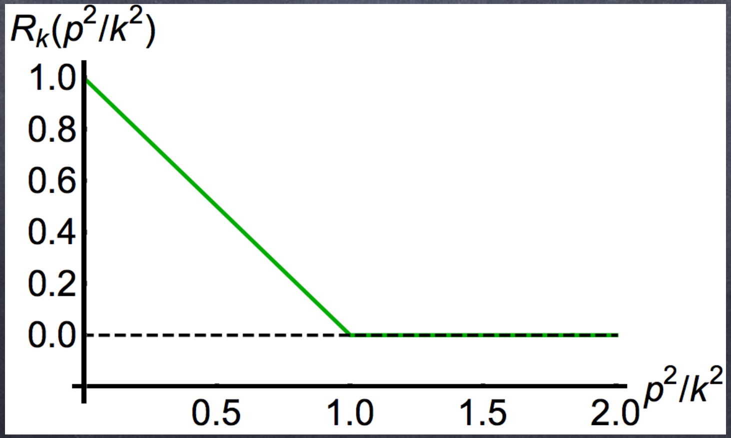

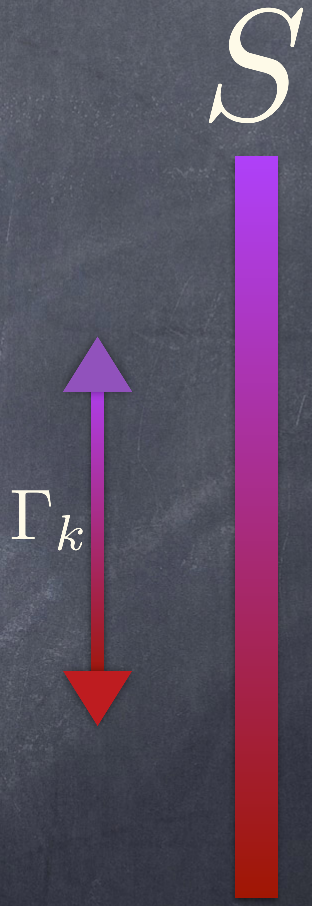

S

Γ_k

$\Gamma_{k\to 0} = \Gamma$

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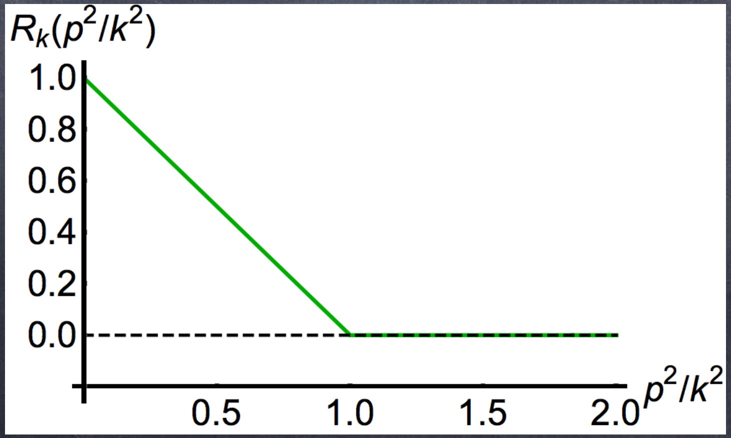

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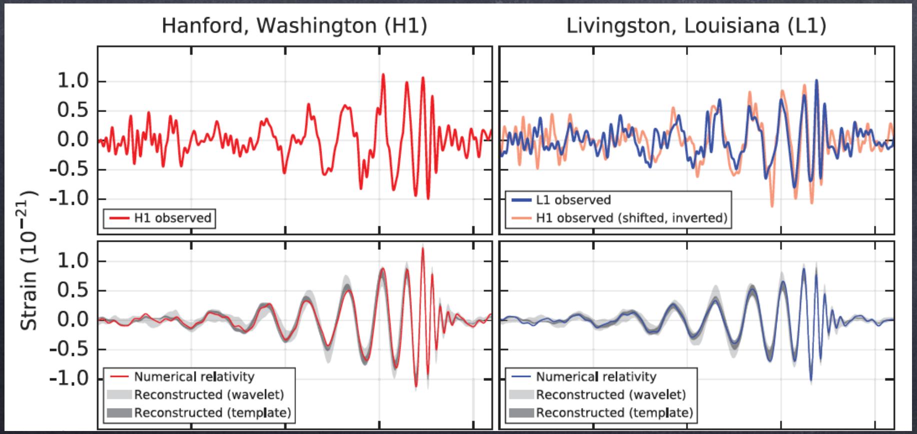
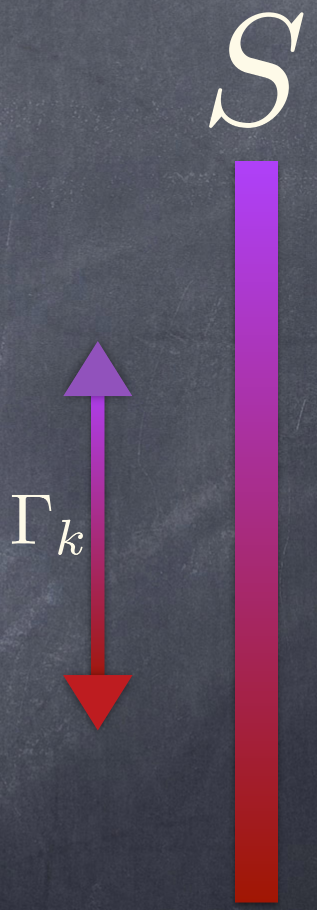
In quantum gravity:

Einstein-Hilbert action:



[LIGO collaboration '16]



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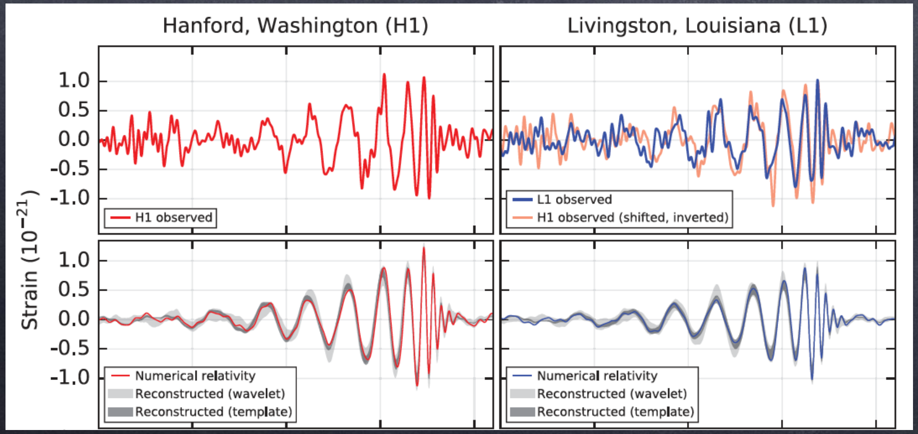

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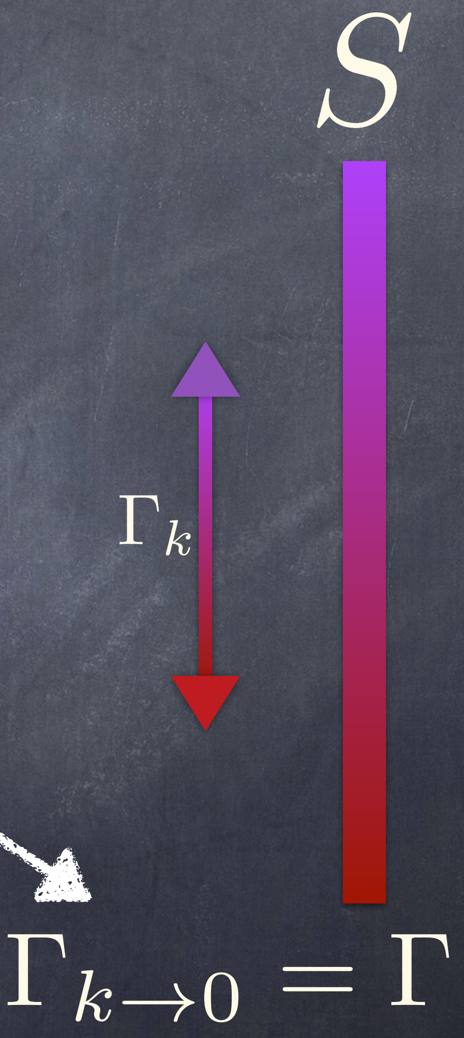

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In quantum gravity:

microscopic dynamics:
no experimental hints!

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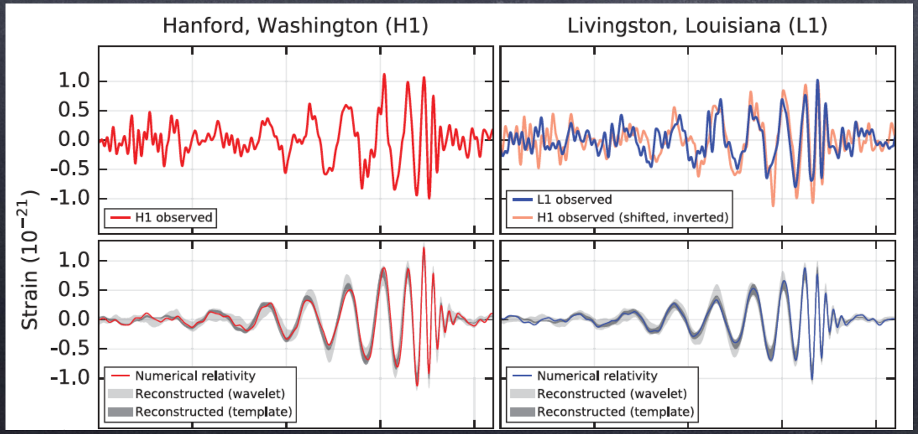

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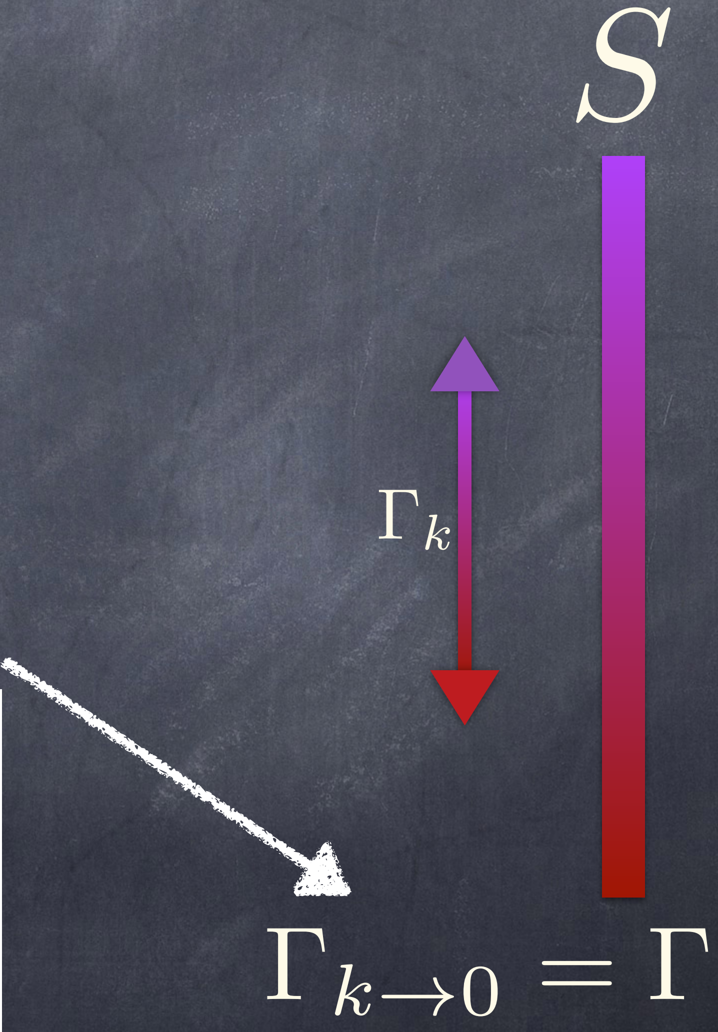

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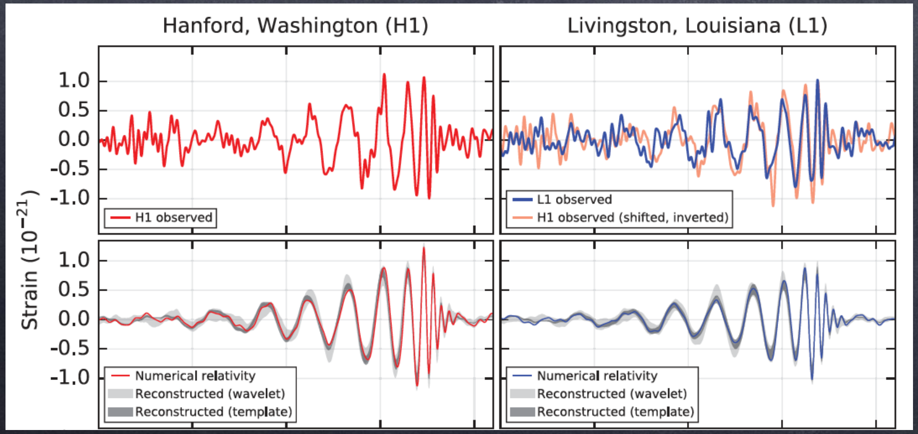

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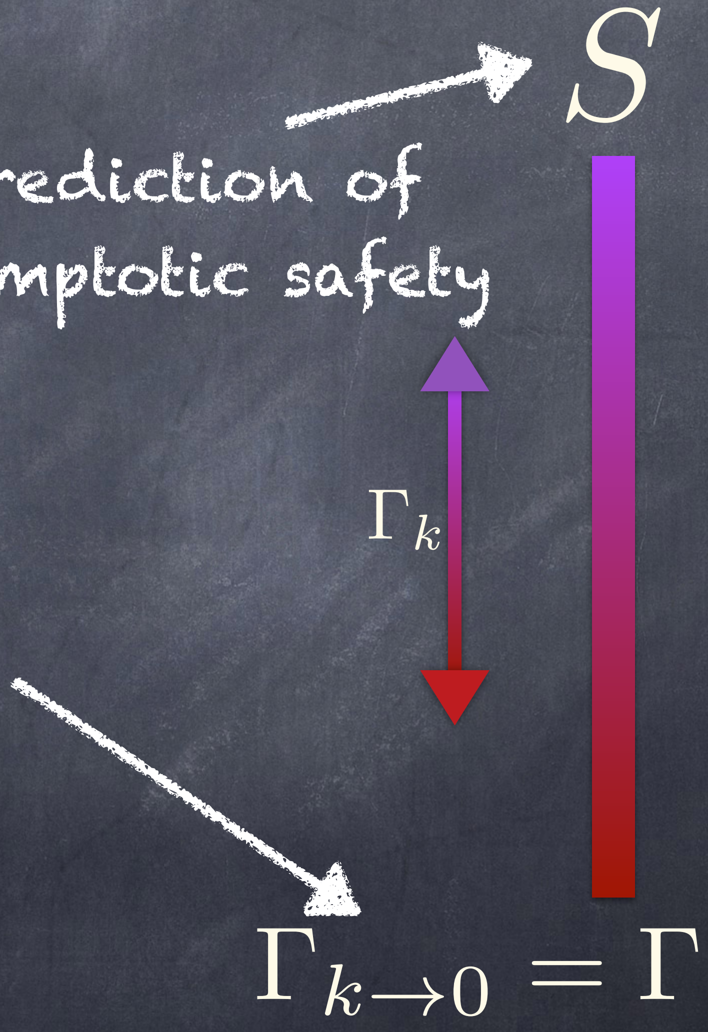

S

prediction of asymptotic safety

Γ_k

$\Gamma_{k\to 0} = \Gamma$

Functional Renormalisation Group

$$\Gamma_k = \sum_i \bar{g}_i(k) \mathcal{O}_i$$

$\beta_{g_i} = k\partial_k g_i(k)$ can be extracted from $k\partial_k \Gamma_k|_{\mathcal{O}_i}$

advantage of Functional Renormalisation Group:
(Exact) one-loop equation for $k\partial_k \Gamma_k$

-> can search for asymptotic safety in QFTs!

Functional Renormalisation Group

$$\partial_t \Gamma_k = k \partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k$$

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$$\frac{\delta^2}{\delta \Phi_I \delta \Phi_J} \Gamma_k [\Phi]$$

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regularised
(nonperturbative)
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summation over eigenvalues
of propagator,
(flat space: $\int dp\,p^{d-1}$)
internal and spacetime
indices and trace in
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$$\partial_t \Gamma_k = k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\boxed{\left(\boxed{\Gamma_k^{(2)}} + R_k\right)^{-1}}\partial_t R_k = \frac{1}{2}$$

summation over eigenvalues of propagator,
(flat space: $\int dp\, p^{d-1}$)

$$\frac{\delta^2}{\delta\Phi_I \delta\Phi_J}\Gamma_k[\Phi]$$

regularised
(nonperturbative)
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internal and spacetime indices and trace in field space

Structurally: One-loop

Encodes non-perturbative effects

=> searches for asymptotic safety possible!

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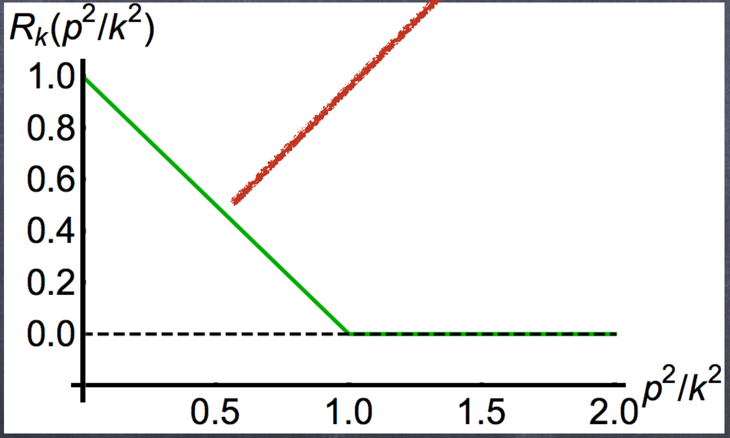
$$\int dp\, p^{d-1}$$

IR-finite



$$S$$

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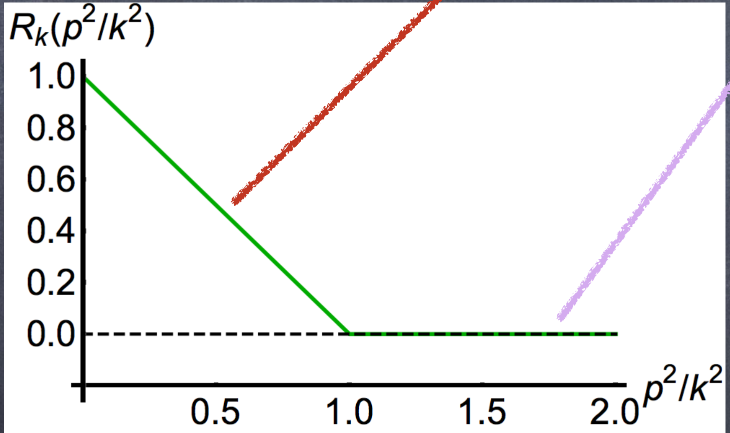
IR-finite

UV-finite



S

$\partial_t \Gamma_k$

-> path integral momentum-shell wise

-> alternative definition of QFT

(need no reference to path integral)

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in theory space:

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in theory space:

$$\Gamma_k = ... + \frac{\lambda}{12}\int d^4x\,\phi^4 \qquad \rightarrow \Gamma_k^{(2)} \sim \lambda\phi^2$$

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...

$$\beta_{\lambda_6} \sim \lambda_4^3 + ...$$

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...

$$\partial_t \Gamma_k \sim \frac{1}{1 + \lambda\phi^2 + \lambda_6\phi^4}$$

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$$\beta_{\lambda_4} \sim \lambda_4^2 + \#\lambda_6 + ...$$

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...

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$$\beta_{\lambda_4} \sim \lambda_4^2 + \# \lambda_6 + \ldots$$

-> tower of coupled equations for running couplings

Strategies for fixed-point searches

1. select a truncation (should contain relevant op's)
2. evaluate beta functions
3. find Fps and evaluate universal quantities (critical exponents)
4. enlarge truncation and repeat
5. convergence of results?

Functional RG and Quantum Gravity

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Main challenge: How to set a scale, when scales are fluctuating?

Renormalisation Group:

need to separate "high-momentum" and "low-momentum" modes!

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$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ (linear split)

Background field method:

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-> background sets the scale: $h_{\mu\nu} R_k^{\mu\nu\kappa\lambda}(-\bar{D}^2) h_{\kappa\lambda}$