Effects of Element Misalignments on Accelerator Performance

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2nd PACMAN Workshop - Debrecen, Hungary - June 13-15, 2016

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Recap of beam dynamics (I) : Forces and kicks

Phase-space coordinates of a single particle:

$$\begin{pmatrix} x, & x', & y, & y', & \Delta t, & \delta = \frac{P-P_0}{P_0} \\ \text{[m]} & \text{[rad]} & \text{[m]} & \text{[rad]} & \text{[m/c]} & \text{[\#]} \end{pmatrix}^T$$

with the transverse angles defined as:

$$x' = \frac{P_x \text{ [MeV/c]}}{P_z \text{ [MeV/c]}}; \quad y' = \frac{P_y \text{ [MeV/c]}}{P_z \text{ [MeV/c]}}$$

Recall the Lorentz force, $F = q\,(E + v \times B)$. A transverse force over a length Δs imparts a transverse momentum $\Delta P_\perp = F_\perp \Delta t$:

$$\Delta P_\perp \text{ [MeV/c]} = F_\perp \text{ [MeV/m]} \frac{1}{V_z \text{ [c]}} \Delta s \text{ [m]}$$

which translates into a transverse kick, e.g. $\Delta x'$ along the x axis:

$$\boxed{x'_{(s+\Delta s)} = \frac{P_x + \Delta P_x}{P_z} = x'_{(s)} + \Delta x'} \qquad \text{with} \qquad \boxed{\Delta x' \text{ [rad]} = \frac{\Delta P_x \text{ [MeV/c]}}{P_z \text{ [MeV/c]}}}$$

Recap of beam dynamics (II) : Twiss parameters

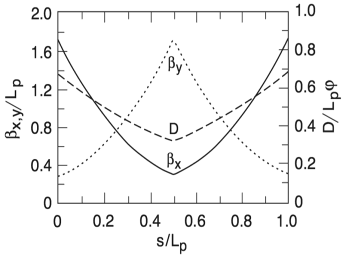
The particle motion in a periodic lattice is described via the solution of the Hill's equation:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos(\mu(s) + \mu_0)$$

- $\beta(s)$, beta function, and $\mu(s)$, phase advance (see Twiss parameters), are *lattice properties*:

$$\mu(s) = \int_0^s \frac{ds'}{\beta(s')}$$

- μ_0, the initial phase, and ε, the Courant-Snyder invariant (or "action"), are a *particle properties*



Particle transport:

$$\vec{X}_j = \underbrace{\begin{pmatrix} \sqrt{\frac{\beta_j}{\beta_i}}(\cos\mu + \alpha_i \sin\mu) & \sqrt{\boldsymbol{\beta_j \beta_i}} \sin\mu \\ \frac{(\alpha_i - \alpha_j)\cos\mu - (1 + \alpha_i\alpha_j)\sin\mu}{\sqrt{\beta_j\beta_i}} & \sqrt{\frac{\beta_i}{\beta_j}}(\cos\mu - \alpha_j \sin\mu) \end{pmatrix}}_{M_{i\to j} \text{ (actually a } 6\times 6 \text{ matrix)}} \cdot \left[\vec{X}_i + \begin{pmatrix} 0 \\ \Delta x' \\ 0 \\ \Delta y' \\ 0 \\ 0 \end{pmatrix} \right]_i$$

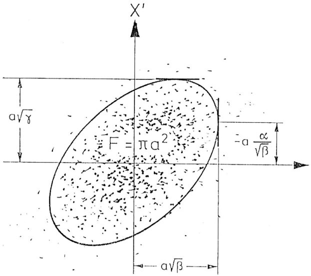
IMPORTANT: the β-function amplifies unwanted transverse kicks.

Recap of beam dynamics (III) : Beam emittance

The Twiss parameters define the beam ellipse:

$$\varepsilon = \gamma\, x^2 + 2\alpha\, x\, x' + \beta\, x'^2$$

- The ellipse amplitude, ε, , is a *particle property* (called Courant-Snyder invariant or "action")
- For an ensemble of particles we define the (geometric) *beam emittance*, ϵ, a quantity proportional to the area of the ellipse



The geometric emittance, ϵ, is defined as:

$$\epsilon_{\mathbf{geom}} = \sqrt{\det\left(\mathrm{cov}\left(\mathbf{x},\, \mathbf{x}'\right)\right)} = \sqrt{\langle x^2\rangle \langle x'^2\rangle - \langle x\, x'\rangle^2}$$

From which we can compute beam size and divergence:

$$\sigma_x = \sqrt{\beta \epsilon_{\mathbf{geom}}}$$

$$\sigma_{x'} = \sqrt{\frac{\epsilon_{\mathbf{geom}}}{\beta}}$$

(recall the *normalised* emittance, $\epsilon_{\mathbf{norm}} = \beta_{\mathbf{rel}}\, \gamma_{\mathbf{rel}}\, \epsilon_{\mathbf{geom}}$)

Effects of kicks on the emittance

Nominal emittance:

$$\epsilon_0 = \sqrt{\langle x^2\rangle \langle x'^2\rangle - \langle x\, x'\rangle^2} = \sqrt{\langle x^2\rangle \langle x'^2\rangle} \qquad \text{(if x and } x' \text{ are uncorellated)}$$

In presence of transverse kicks, $\Delta x'$, the emittance transforms $\epsilon_0 \to \epsilon$:

$$\epsilon = \sqrt{\langle x^2\rangle \left\langle (x' + \Delta x')^2 \right\rangle} \approx \sqrt{\underbrace{\langle x^2\rangle \langle x'^2\rangle}_{\epsilon_0^2} + \langle x^2\rangle \langle \Delta x'\rangle^2}$$

$$= \epsilon_0 \sqrt{1 + \frac{\cancel{\langle x^2\rangle} \langle \Delta x'\rangle^2}{\cancel{\langle x^2\rangle} \langle x'^2\rangle}}$$

$$\approx \epsilon_0 \left(1 + \frac{1}{2} \frac{\langle \Delta x'\rangle^2}{\langle x'^2\rangle}\right)$$

From which derives the emittance growth:

$$\frac{\Delta\epsilon}{\epsilon_0} = \frac{\epsilon - \epsilon_0}{\epsilon_0} = \frac{1}{2}\frac{\sigma^2_{\Delta x'}}{\sigma^2_{x'}} \quad \Rightarrow \quad \boxed{\Delta\epsilon \propto \sigma^2_{\Delta x'}}$$

For example a quadrupole:

$$\Delta x' = K_1 L_q\, \Delta x_{\text{misalign}} \equiv \frac{\Delta x_{\text{misalign}}}{f_{\text{length}}} \quad \Rightarrow \quad \boxed{\Delta\epsilon \propto \sigma^2_{\text{misalign}}}$$

Why is the emittance important?

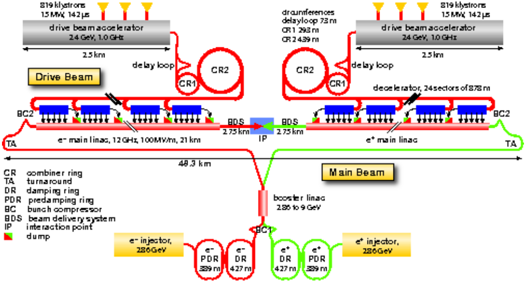
- In a **particle collider** the number of collision is given by the **luminosity:**

$$L = H_D \frac{N^2 n_b f_r}{4\pi \sigma_x^\star \sigma_y^\star}$$

with

$$\sigma_\perp^\star = \sqrt{\beta_\perp^\star \, \epsilon_{\perp \mathbf{geom}}}$$

$$\perp = x \text{ or } y$$



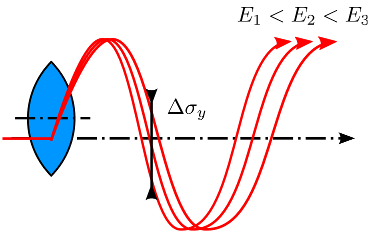
- H_D, **disruption parameter (enhances the luminosity)** ; N, **number of particles per bunch** ; n_b, **number of bunches per train** ; f_r, **repetition frequency** ; $\sigma^\star_{x,y}$ **transverse beam sizes at the interaction point (IP)**

- CLIC emittance growth budgets, in the main linac, due to static misalignments: $\boxed{\Delta\epsilon_x{=}60 \text{ nm}}$ and $\boxed{\Delta\epsilon_y{=}10 \text{ nm}}$ in the horizontal and vertical axes, respectively.

Effect of magnet misalignments

- "Feed-down" effect: A misaligned magnet of order **N** behaves like a magnet of order **N − 1**:
  - a misaligned quadrupole gives a dipolar kick,
  - a misaligned sextupole gives a quadrupolar kick, etc. etc.

- Effect of a misaligned quadrupole (the most frequently used type of magnet!)



  - deflects the beam trajectory, excites betatron oscillations, introduces unwanted dispersion

- **Unwanted dispersion** is **bad**!
  Because it adds a position-momentum correlation, and increases the beam size:

$$x(s) \to x(s) + D(s)\frac{\Delta P}{P_0}$$

$$\sigma_x^2 \to \sigma_x^2 + D^2 \underbrace{\sigma^2_{\frac{\Delta P}{P_0}}}_{\text{(energy spread)}^2}$$

Effect of accelerator structure misalignments

Particles traversing a structure with an offset excite Wakefields:

- transverse effect: z-dependent deflection

$$\Delta x' = \frac{W_{\perp}(z)\, N e^2 L_S}{P_0} \Delta x$$

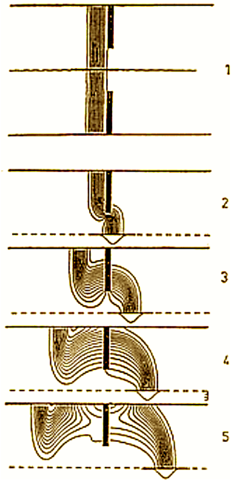
  - $W_{\perp}(z)$, transverse wakefield function, [V/pC/m/mm]
  - N, number of particles per bunch
  - L_S, structure length
  - z, particle distance from bunch head
  - Δx, transverse offset beam$\longleftrightarrow$structure

- longitudinal effect: z-dependendent energy loss

$$\Delta P_z = W_{\parallel}(z)\, N e^2 L_S$$

  - $W_{\parallel}(z)$, longitudinal wakefield function, [V/pC/m]
  - independent from misalignment

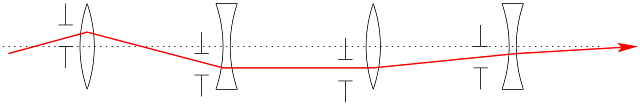
- Their effect can be felt by particles at
  - short-range (same bunch) $\rightarrow$ emittance growth
  - long-range (bunch to bunch) $\rightarrow$ beam breakup



Effect of BPM misalignments

Misaligned BPMs affect the beam, indirectly:

- they do not deflect the beam, but can compromise the effectiveness of optimisation techniques



- (and actually they can also deflect the beam, e.g. high-resolution "cavity BPMs", which can create Wakefields)

Besides, off-centred BPMs can display:

- loss of resolution, or scaling errors: $x_{\text{read}} = \alpha_{\text{scaling}}\, x_{\text{real}}$

$\Rightarrow$ **BBA, Beam-Based Alignment,** can cure this problem (e.g. **DFS, Dispersion-Free Steering)**.

Other errors that affect all elements are:

- angle errors: offsets in $\Delta x'$ and $\Delta y'$
- roll errors: rotations around the beam axis $\Delta\phi$

Beam-based alignment techniques

1. Simplest:
   - Quad-shunting: each quadrupole is moved until the magnetic centre is determined
   - **One-to-one**: transverse kickers are used to steer the beam through the centre of the BPMs

2. **Dispersion-Free Steering** (DFS) / Wakefield-Free Steering (WFS)
   - DFS: the presence of dispersion is detected and measured, and transverse kickers are used to counteract it
   - WFS: the presence of wakefields is detected and measured, and transverse kickers are used to counteract it

3. **RF Alignment** (specific to CLIC)
   - Relative beam offset in the accelerating structures is measured, and structures are moved to reduce it

- Others:
  - Emittance tunng bumps, coupling correction
  - MICADO: similar to one-to-one, but picks the best correctors
  - Kick-Minimisation: useful when DFS cannot easily be applied (e.g. ILC turnaround loops, ...)

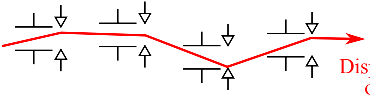
Example of emittance growth in the CLIC main linac

Misalignment of all components, $\sigma_{\text{RMS}} = 10\ \mu\text{m}$

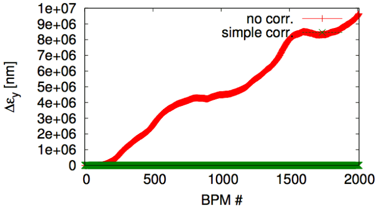
- Initial emittance is 10 nm
- Emittance growth is enormous...

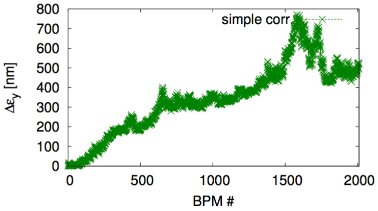
One simple mitigation technique is used: "One-to-one" steering, but it's far from being sufficient...

"One-to-one" steering:



- Each corrector is used to steer the beam through the centre of the following BPM
- Orbit makes its way through the accelerator, but dispersion is still present...





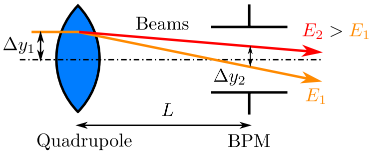
(PLACET Simulation, courtesy of D.Schulte)

Beam-based Alignment: Dispersion-Free Steering (DFS)

Principle of DFS:

1. Measure the dispersion by altering the beam energy (or scaling the magnets)
2. Compute the correction which minimise the dispersion

Graphically:



That is, minimise:

$$\chi^2 = \sum_{\text{bpms}} y_i^2 + \omega^2 \sum_{\text{bpms}} \left(y_{\Delta E,i} - y_i\right)^2 + \beta^2 \sum_{\text{corrs}} \theta_m^2$$

In practice, one needs to solve the system of equations:

$$\underbrace{\begin{pmatrix} b - b_0 \\ \omega(\eta - \eta_0) \\ 0 \end{pmatrix}}_{\text{measured observables}} = \underbrace{\begin{pmatrix} \mathbf{R} \\ \omega\mathbf{D} \\ \beta\mathbf{I} \end{pmatrix}}_{\text{response matrices}} \underbrace{\begin{pmatrix} \theta_1 \\ \vdots \\ \theta_m \end{pmatrix}}_{\text{unknowns}}$$

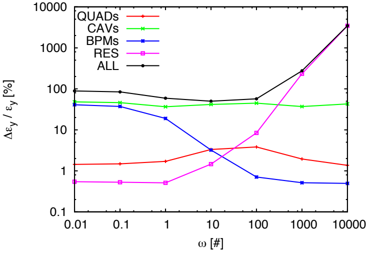
Example of DFS in the CLIC main linac

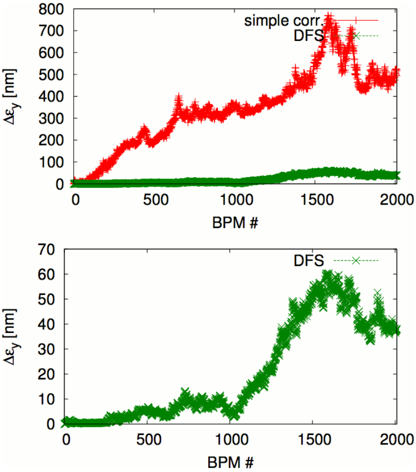
Minimise:

$$\chi^2 = \sum_{\text{bpms}} y_i^2 + \omega^2 \sum_{\text{bpms}} \left(y_{\Delta E,i} - y_i\right)^2 + \beta^2 \sum_{\text{corrs}} \theta_m^2$$

or, solve the system of equations:

$$\underbrace{\begin{pmatrix} b - b_0 \\ \omega(\eta - \eta_0) \\ 0 \end{pmatrix}}_{\text{measured observables}} = \underbrace{\begin{pmatrix} R \\ \omega D \\ \beta I \end{pmatrix}}_{\text{response matrices}} \underbrace{\begin{pmatrix} \theta_1 \\ \vdots \\ \theta_m \end{pmatrix}}_{\text{unknowns}}$$





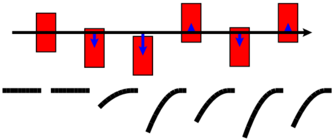
(PLACET Simulation, courtesy of D.Schulte)

Example of RF Alignment in the CLIC main linac

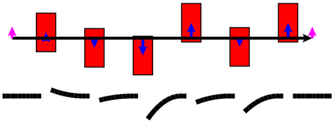
RF Alignment:

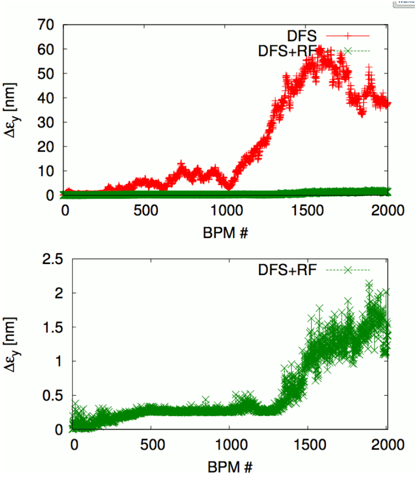
- Each structure is equipped with a wakefield monitor
- Up to eight structures on one movable girders

Before correction:



After correction:



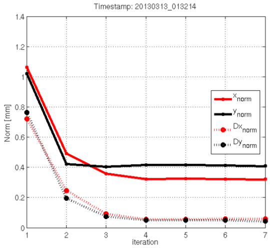


(PLACET Simulation, courtesy of D.Schulte)

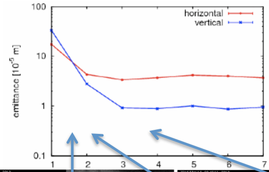
DFS Tests at FACET / SLAC

(A.Latina, E.Adli, J.Pfingstner, D.Schulte)

Orbit/Dispersion



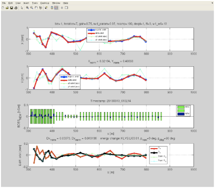
Emittance

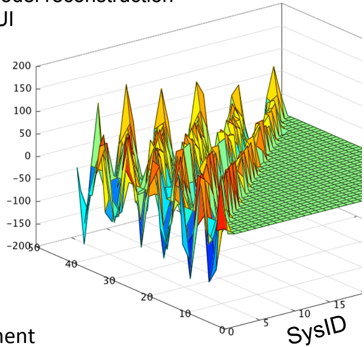


DFS correction applied to 500 meters of the SLC linac

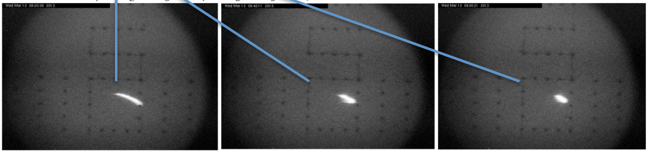
- SysID algorithms for model reconstruction
- DFS correction with GUI
- Emittance growth is measured

Graphic User Interface:





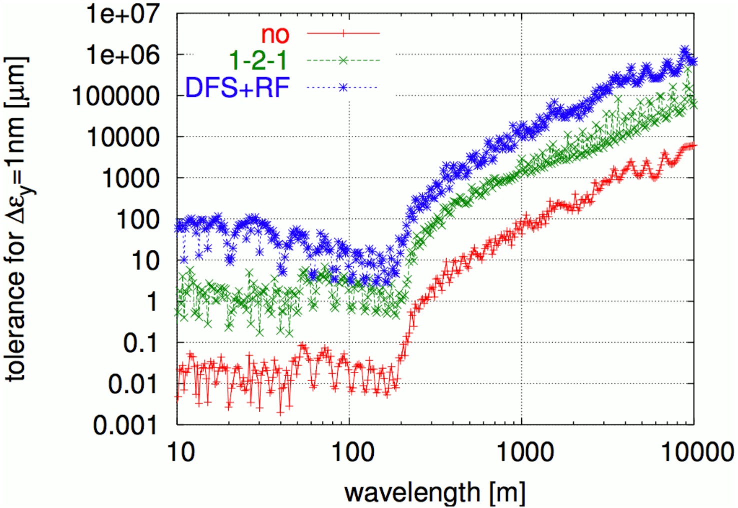
Beam profile measurement



Before correction | After 1 iteration | After 3 iterations

Incoming oscillation/ dispersion is taken out and flattened; emittance in LI11 and emittance growth significantly reduced.

Tolerance to long wavelengths



(courtesy of D.Schulte)

Tolerance Table

Element	error	with respect to	alignment	
			ILC	CLIC
Structure	offset	girder	$300\ \mu m$	$10\ \mu m$
Structure	tilts	girder	$300\ \mu radian$	$200(*)\ \mu m$
Girder	offset	survey line	$200\ \mu m$	$9.4\ \mu m$
Girder	tilt	survey line	$20\ \mu radian$	$9.4\ \mu radian$
Quadrupole	offset	girder/survey line	$300\ \mu m$	$17\ \mu m$
Quadrupole	roll	survey line	$300\ \mu radian$	$\leq 100\ \mu radian$
BPM	offset	girder/survey line	$300\ \mu m$	$14\ \mu m$
BPM	resolution	BPM center	$\approx 1\ \mu m$	$0.1\ \mu m$
Wakefield mon.	offset	wake center	—	$5\ \mu m$

(courtesy of D.Schulte)

Conclusions

- Element misalignments can greatly damage the beam, inducing emittance growth and beam breakup
  - CLIC budgets are very tight: $\Delta\epsilon_x = 60$ nm and $\Delta\epsilon_y = 10$ nm in the main linac
- Standard Pre-Alignment techniques are not sufficient to meet the CLIC requirements:
  - tightest tolerance is 14 μm over a window of 200 m
- Powerful beam-based techniques have been invented and tested experimentally
- **PACMAN** + Beam line design + Beam-based Alignment:

wonderful technical achievements!