TMDs from MC evolution

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xFitter external meeting in Dubna

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In this presentation: MC results for gluons, valence and sea quarks for:

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- 1-loop-α_s.

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ccfm evolution for gluon and valence quarks.

In this presentation: MC results for gluons, valence and sea quarks for:

- DGLAP evolution,
- $xf(x,t)$,
- LO in $P(z)$,
- 1-loop-α_s.

We show:

- integrated PDFs and compare them with semi analytical results from QCDNum17,
- first results for TMDs.

Introduction to the method

Sudakov formalism

Evolution equation for parton density

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)f(\frac{x}{z},t) - \frac{\alpha_s}{2\pi}f(x,t)\int dzP(z). \qquad (1)$$

Introducing *Sudakov form factor*

$$\Delta_s(t,t_0) \equiv \Delta_s(t) = exp\Big(- \int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}P(z)\Big) \qquad (2)$$

we can rewrite (1)

$$t\frac{\partial f(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)f(\frac{x}{z},t) + f(x,t)\frac{t}{\Delta_s(t)}\frac{\partial \Delta_s(t)}{\partial t}. \qquad (3)$$

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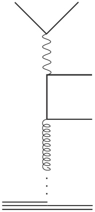

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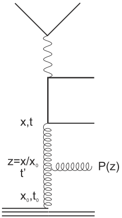
After integration

$$f(x,t) = f(x,t_0)\Delta_s(t) + \frac{\alpha_s}{2\pi}\int \frac{dt'}{t'}\frac{\Delta_s(t)}{\Delta_s(t')}\int \frac{dz}{z}P(z)f(\frac{x}{z},t'). \qquad (4)$$



Sudakov: probability of evolving from t_0 to t without any resolvable branching.
iterative solution:

$$f(x,t) = \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty}\sum_n \frac{1}{n!}log^n\Big(\frac{t}{t_0}\Big)A^n \otimes \Delta_s(t)f(\frac{x}{z},t_0), \qquad (5)$$

where $A = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P(z)$.

MC solution of the evolution equation

MC solution:

First branching: evolve from t_0 to t' obtained from $\Delta_s(t')$:

$$R_1 = \Delta_s(t'), \tag{6}$$

where R_1 is a random number in the interval $(0, 1)$.
If $t' > t$ evolution is stopped without any branching. If $t' < t$ branching is generated according to $P(z)$

$$\int_{z_{min}}^{z} dz' P(z') = R_2 \int_{z_{min}}^{z_{max}} dz' P(z') \tag{7}$$

and the evolution continues.
Second branching: evolve from t' to t'' generated according to $\Delta_s(t'', t')$. If $t'' > t$ evolution is stopped only with one branching. If $t'' < t$ branching is generated according to $P(z)$ and the evolution continues...etc.

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Observation:

$$\frac{\partial}{\partial t'} \frac{\Delta_s(t)}{\Delta_s(t')} = \frac{\alpha_s}{2\pi} \frac{\Delta_s(t)}{\Delta_s(t')} \frac{1}{t'} \int_x^{z_{max}} dz P(z) \tag{8}$$

rewrite (4)

$$f(x,t) = f_0(x,t)\Delta_s(t) + \int_x^1 \frac{dz'}{z'} \int_{t_0}^t d\Delta_s(t,t') P(z') f_0\left(\frac{x}{z'}, t'\right)\left(\int_x^{z_{max}} dz P(z)\right)^{-1} \tag{9}$$

Momentum weighted parton densities & momentum sum rule

To include all flavours in the evolution & use the Sudakov formalism we need to switch from $f(x, t)$ to $xf(x, t)$ & use momentum sum rule:

$$\sum_a \int_0^1 zP_{ab}(\alpha_s, z)dz = 0. \qquad (10)$$

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example for gluon:
P - regularized splitting function (with *plus prescription*), $\widehat{P}$ - unregularized splitting function (without *plus prescription*)

$$t\frac{\partial xg(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[zP_{gq}(z)\frac{x}{z}q\left(\frac{x}{z},t\right) + zP_{gg}(z)\frac{x}{z}g\left(\frac{x}{z},t\right)\right] \tag{11}$$

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$$\rightarrow \frac{\alpha_s}{2\pi}\int_x^{z_{max}} \frac{dz}{z}\left(z\widehat{P}_{gq}(z)\frac{x}{z}q\left(\frac{x}{z},t\right) + z\widehat{P}_{gg}(z)\frac{x}{z}g\left(\frac{x}{z},t\right)\right) - \frac{\alpha_s}{2\pi}xg(x,t)\int_x^{z_{max}} dzz\left(2n_f\widehat{P}_{qg} + z\widehat{P}_{gg}\right) \tag{11}$$

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To include all flavours in the evolution & use the Sudakov formalism we need to switch from $f(x,t)$ to $xf(x,t)$ & use momentum sum rule:

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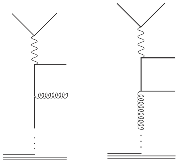
If we define now Sudakov form factor as:

$$\Delta_s(t,t_0)_i \equiv \Delta_s(t)_i = \exp\left(-\int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'} z\sum_j \widehat{P}(z)_{ji}\right) \tag{12}$$

where i, j are q or g,
we obtain equation of the same form as eq. (3), just multiplied from both sides by x and with $zP(z)$ in $\Delta_s(t)_i$ → it can be solved in analogical way.

Evolution in the code

We consider ep collisions in which we can measure different pdfs:



Forward evolution: final parton is not specified when the evolution begins.

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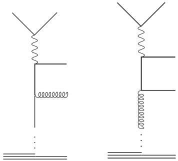


Forward evolution: final parton is not specified when the evolution begins.

Four different situations:

- gluon at the beginning and at the end,
- quark (valence or sea) at the beginning and gluon at the end
- quark (valence or sea) at the beginning and quark (valence or sea) at the end and
- gluon at the beginning and sea quark at the end.

Valence quark at the end can come only from valence quark at the beginning.

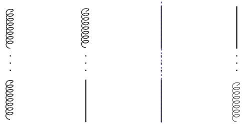
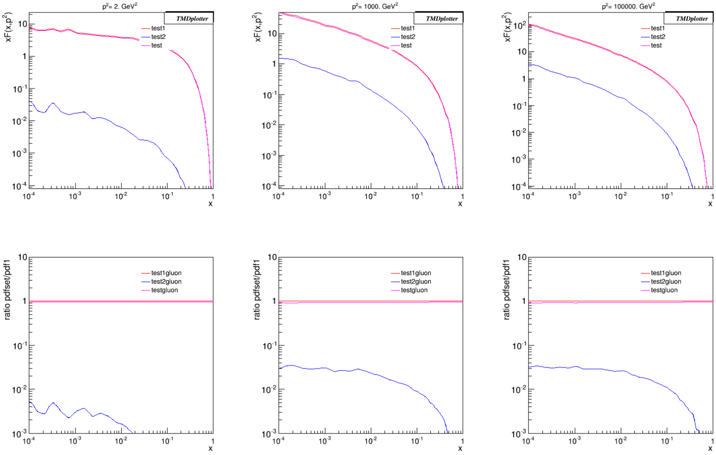
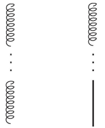


Figure : K_{gg}, K_{gq}, K_{qq},K_{qg}

Contribution from quark and gluon evolution

GLUON density: test1: merged quark and gluon evolution chain test2: quark evolution chain test: gluon evolution chain

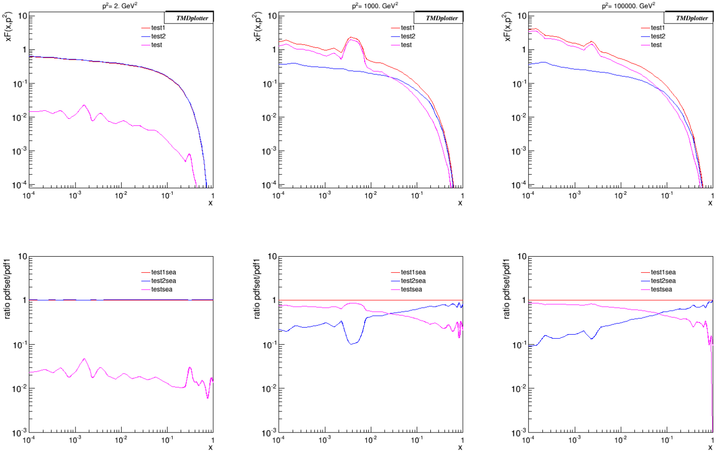


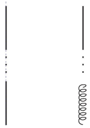


Contribution from gluon kernel dominates, contribution from quark kernel (10^{-2}) times smaller.

Contribution from quark and gluon evolution

SEA density: test1: merged quark and gluon evolution chain test2: quark evolution chain test: gluon evolution chain





At small x main contribution to sea quark density from gluon kernel, for large x quark kernel dominates.

Integrated PDFs from MC solution

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\frac{\partial xf(x,t)}{\partial t} = \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx$$
$$\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z). \tag{13}$$

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it can be shown that terms $\int_{z_{max}}^1$ skipped in the integral in eq. (13) are of order $\mathcal{O}(1 - z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x,t)}{dt}$

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Different choices of z_{max}:

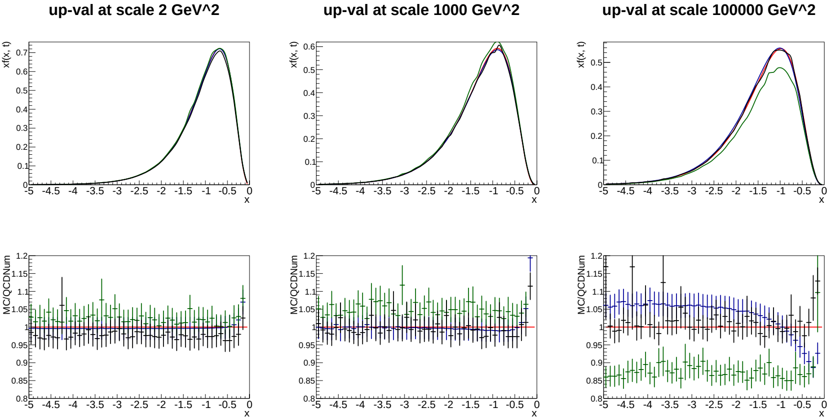
- z_{max} - fixed
- z_{max} - can change dynamically with the scale, for example:
  angular ordering: $z_{max} = \left(\frac{Q_0}{Q}\right)^2$

up-val quarks

$2GeV^2$:QCDNum, angular ordering, $z_{max} = 0.99$ $z_{max} = 0.97$

$1000GeV^2$:QCDNum, angular ordering, $z_{max} = 0.999$ $z_{max} = 0.997$

$100000GeV^2$:QCDNum, angular ordering, $z_{max} = 0.999$ $z_{max} = 0.9999$



MC results close to the QCDNum results. Effect on z_{max} observed. z_{max} value giving the results the closest to QCDNum depends on scale and shape of the distribution (terms of order $\mathcal{O}(1 - z_{max})$ multiplied by $xf(x, t)$ or $x\frac{df(x,t)}{dt}$ skipped).

sea quarks

$2 GeV^2$:QCDNum, angular ordering, $z_{max} = 0.99$ $z_{max} = 0.995$

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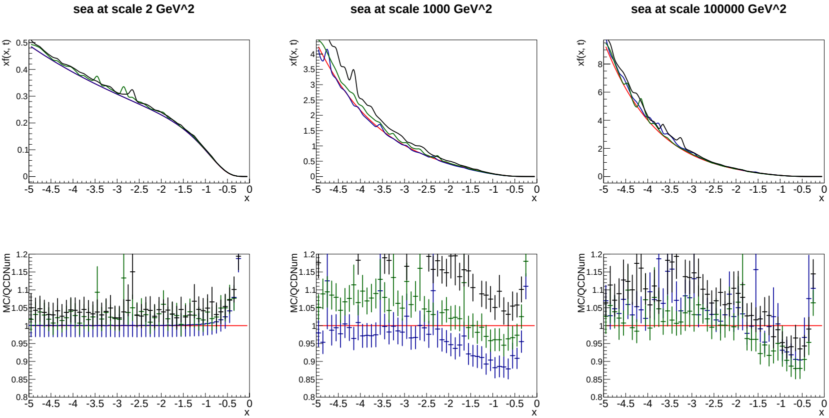
$100000 GeV^2$:QCDNum, $z_{max} = 0.996$, $z_{max} = 0.995$ $z_{max} = 0.997$



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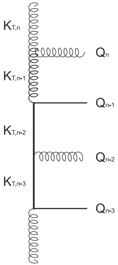
Effect on z_{max} observed. z_{max} value giving the results the closest to QCDNum depends on scale and shape of the distribution.

Results for TMDs

k_T dependence

MC solution: for every branching Q is generated and Q_x and Q_y are calculated.



k_T contains the whole history of the evolution:

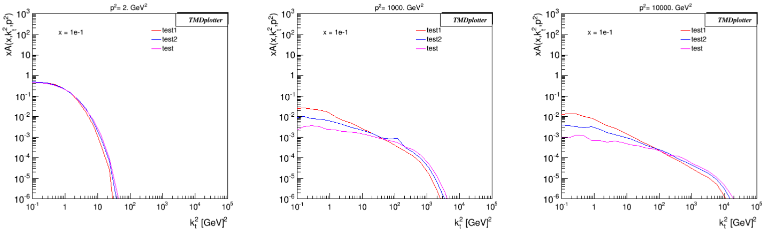
$$\overrightarrow{k}_{T,n} = \overrightarrow{k}_{T,n-1} + \overrightarrow{Q}_{T,n-1}.$$

The information about k_T is available for every branching.

up-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

x = 0.1

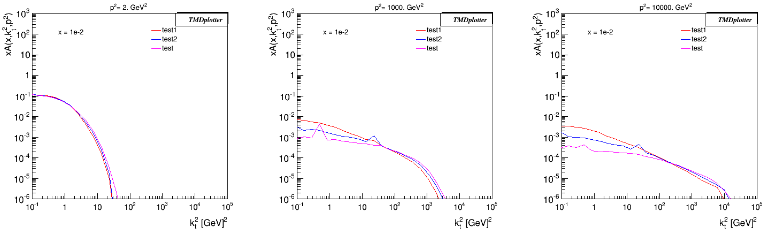


- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

up-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

x = 0.01



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up-val

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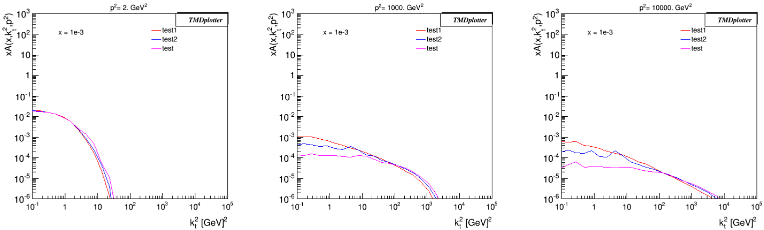
x = 0.001



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- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

Fitting method

Evolution kernels

Two different evolution kernels are defined:

- initial quark (valence or sea)$\rightarrow$ quark grid,
- initial gluon $\rightarrow$ gluon grid.

Kernels for evolution initiated by gluons and quarks are calculated separately only once per run of the code and combined at the end $\rightarrow$ fitting procedure is fast.

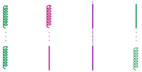

Figure : K_{gg}, K_{gq}, K_{qq},K_{qg}: gluon grid, quark grid

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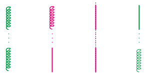


Figure : K_{gg}, K_{gq}, K_{qq},K_{qg}: gluon grid, quark grid

To get the final pdf: evolution kernel is folded with starting distribution

$$xf(x,t) = x \int dx_0 \int dz f_0(x_0) K(z,t) \delta(zx_0 - x) \quad (14)$$

For example for gluon:

$$xf(x,t)_g = x \int dx_0 \int dz \left(f_{0g}(x_0) K_{gg} + f_{0q}(x_0) K_{gq} \right) \delta(zx_0 - x), \quad (15)$$

and for up-valence quark:

$$xf(x,t)_{q,up} = x \int dx_0 \int dz \left(f_{0,up}(x_0) K_{qq} \right) \delta(zx_0 - x). \quad (16)$$

Summary

Summary

Solution of DGLAP evolution using MC method was shown.

Advantages:

- reproduce analytical solution (results consistent with QCDNum17),
- applicable for TMDs evolution,
- direct usage in PS matched calculation.

Prospects:

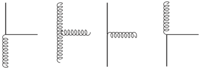
- implementation into xFitter: first full TMD distributions,
- include NLO in $P(z)$,
- development of full TMD MC - CASCADE.

Thank you!

Back up

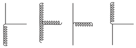
evolution in the code

Different kind of splittings can happen during the evolution process:

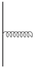


Commonly used flavour decomposition:

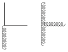
- see
  $u + \overline{u} + d + \overline{d} + s + \overline{s} = 2(\overline{u} + \overline{d} + \overline{s}) + u_v + d_v$

- valence
  $\overline{u} - u$
  $\overline{d} - d$

- gluon

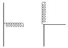
  

Flavour decomposition used in the method:

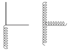
- see
  $2(\overline{u} + \overline{d} + \overline{s})$

- valence
  $\overline{u} - u$
  $\overline{d} - d$

- gluon

Ordering dependence- z_{max} origin

Some of the splitting functions are divergent for $z \to 1$.

To avoid divergences:

$$\begin{aligned}\frac{\partial xf(x,t)}{\partial t} &= \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^1 dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^1 dzP(z) \approx \\ &\approx \frac{\alpha_s}{2\pi}\frac{1}{t}\int_x^{z_{max}} dzP(z)\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_x^{z_{max}} dzP(z).\end{aligned} \tag{17}$$

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Using the form of the splitting functions:

$$P_{ab}(\alpha_s, z) = A_{ab}(\alpha_s)\delta(1-z) + K_{ab}(\alpha_s)\frac{1}{(1-z)_+} + R_{ab}(\alpha_s) \quad (18)$$

and the expansion of $xf(\frac{x}{z},t)$

$$xf\left(\frac{x}{z},t\right) = xf(x,t) + x^2 f'(x,t)(1-z) + \mathcal{O}(1-z_{max}) \quad (19)$$

it can be shown that terms $\int_{z_{max}}^1$ skipped in the integral in eq. (17) are of order $\mathcal{O}(1-z_{max})$ multiplied by $xf(x,t)$ or $x\frac{df(x' t)}{dt}$

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example:

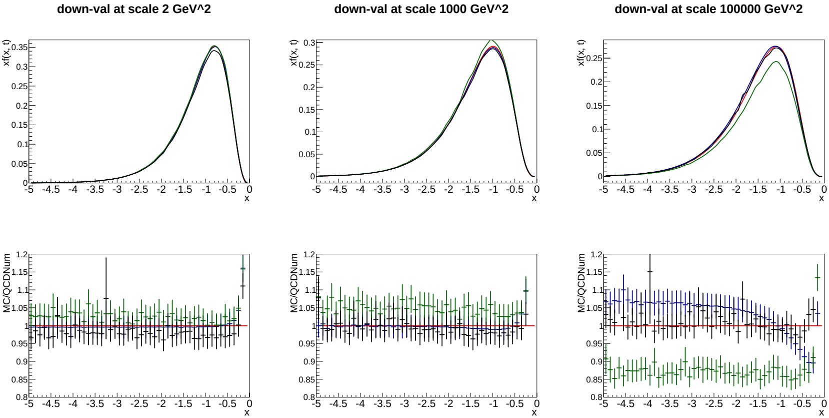
$$\frac{\alpha_s}{2\pi}\frac{1}{t}\int_{z_{max}}^1 dzK_{ab}(\alpha_s)\frac{1}{1-z}\frac{x}{z}f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi}xf(x,t)\frac{1}{t}\int_{z_{max}}^1 dzK_{ab}(\alpha_s)\frac{1}{1-z} =$$
$$= \frac{\alpha_s}{2\pi}\frac{1}{t}K_{ab}(\alpha_s)\left[x^2 f'(x,t)(1-z_{max}) + \mathcal{O}(1-z_{max})\right] \quad (20)$$

down-val quarks

$2GeV^2$:QCDNum, angular ordering, $z_{max} = 0.99$ $z_{max} = 0.97$

$1000GeV^2$:QCDNum, angular ordering, $z_{max} = 0.999$ $z_{max} = 0.997$

$100000GeV^2$:QCDNum, angular ordering, $z_{max} = 0.999$ $z_{max} = 0.9999$



MC results close to the QCDNum results.

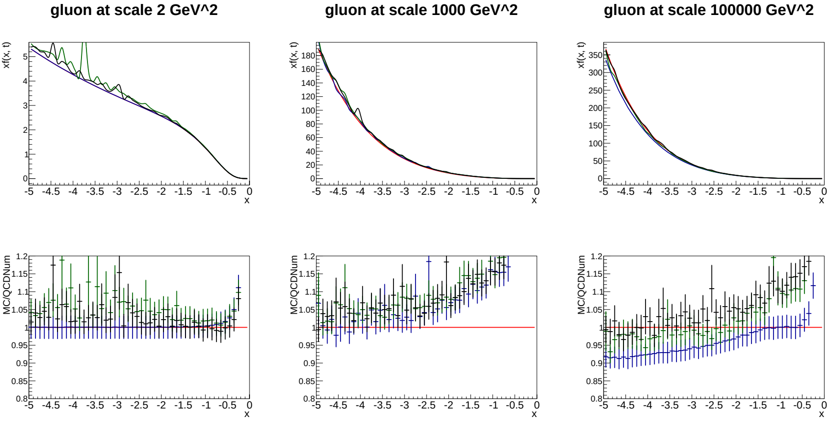
Effect on z_{max} observed. z_{max} value giving the results the closest to QCDNum depends on scale and shape of the distribution.

gluon

$2 GeV^2$:QCDNum, angular ordering, $z_{max} = 0.9$ $z_{max} = 0.87$

$1000 GeV^2$:QCDNum, $z_{max} = 0.99$, $z_{max} = 0.995$ $z_{max} = 0.993$

$100000 GeV^2$:QCDNum, angular ordering, $z_{max} = 0.995$ $z_{max} = 0.997$



MC results close to the QCDNum results.
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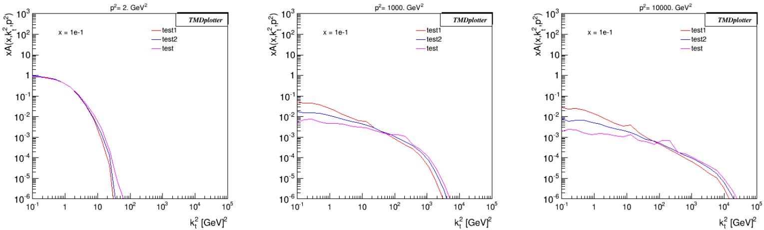
down-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

x = 0.1

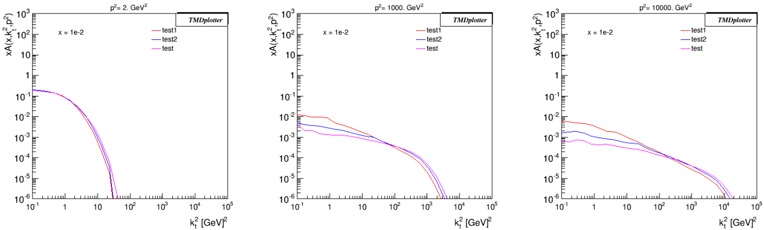


- Initial scale: intrinsic k_t distribution.
- Different choices of z_{max} lead to different uTMDs, especially different large k_T tails.

down-val

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

x = 0.01



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down-val

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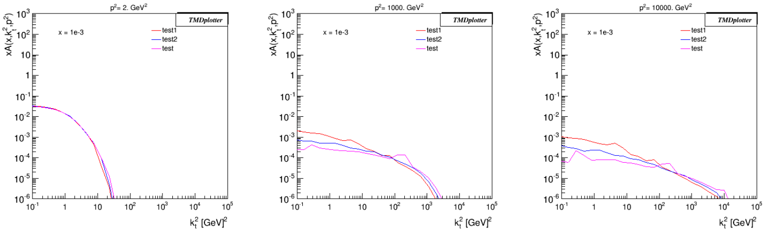

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sea

$z_{max} = 0.99$, $z_{max} = 0.999$, $z_{max} = 0.9999$

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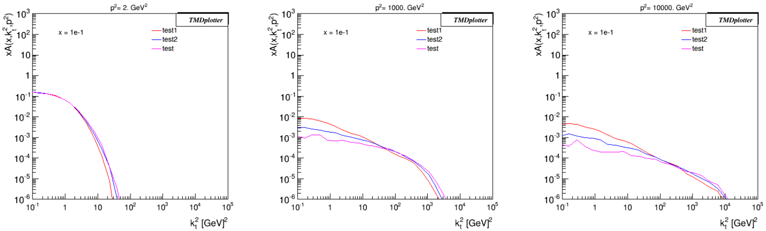

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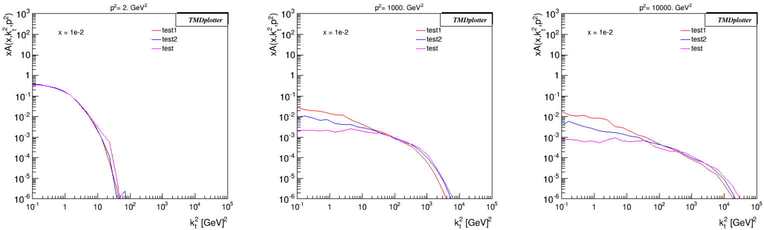

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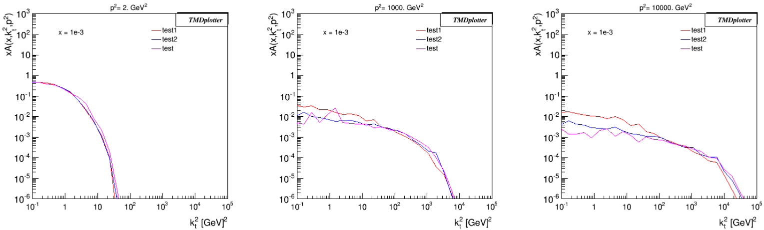

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gluon

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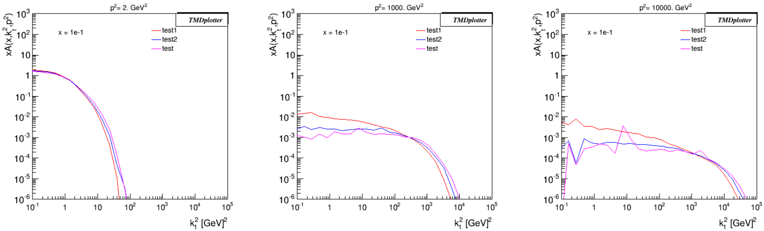

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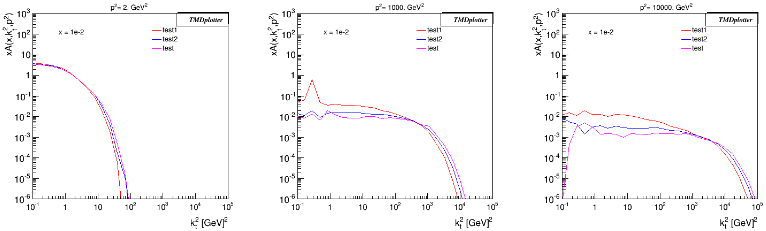

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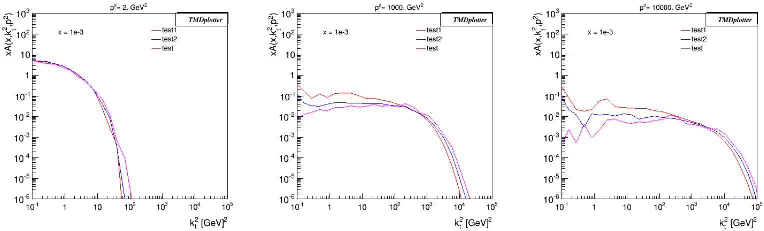

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