

# APFEL

## Recent Developments

[arXiv:1310.1394](https://arxiv.org/abs/1310.1394)

Valerio Bertone

University of Oxford



**xFitter external meeting**

February 18 - 20, 2016, Dubna

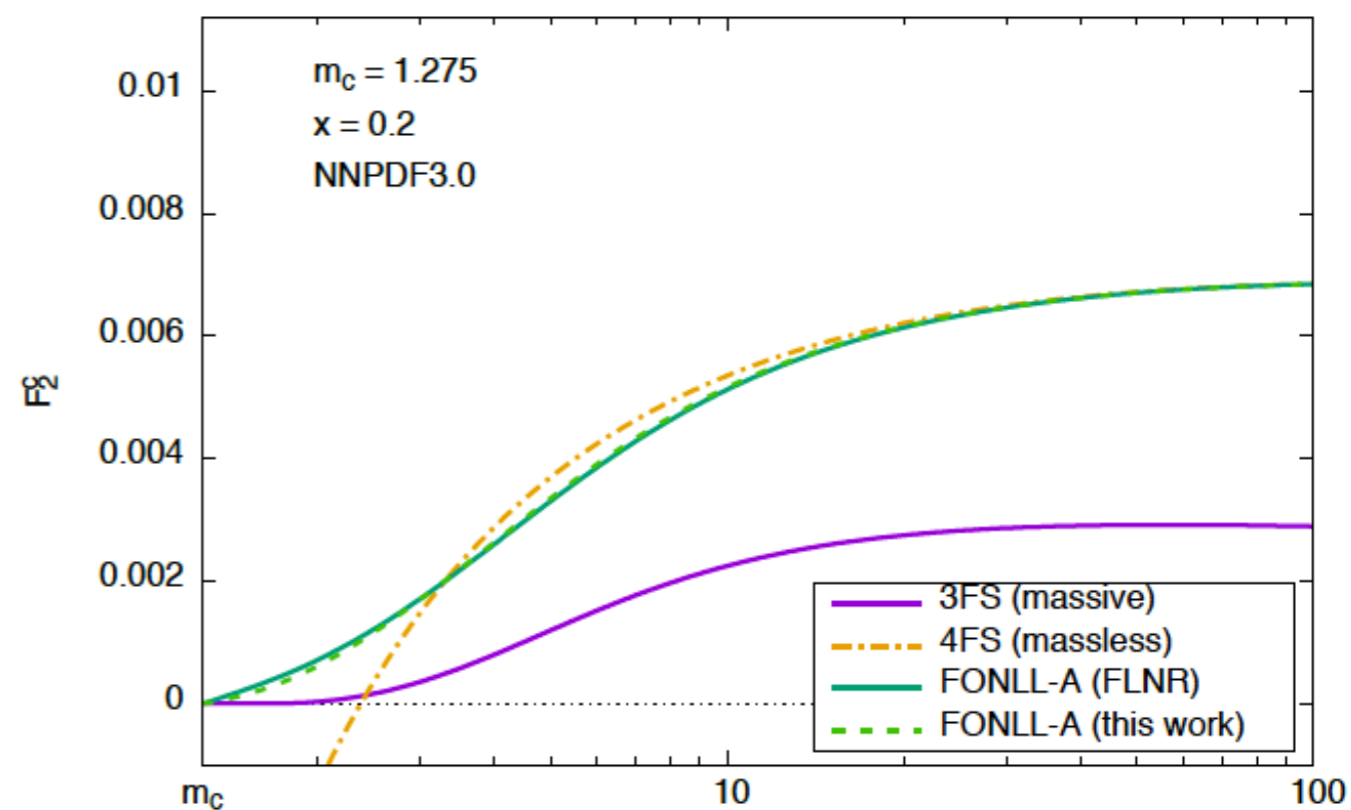
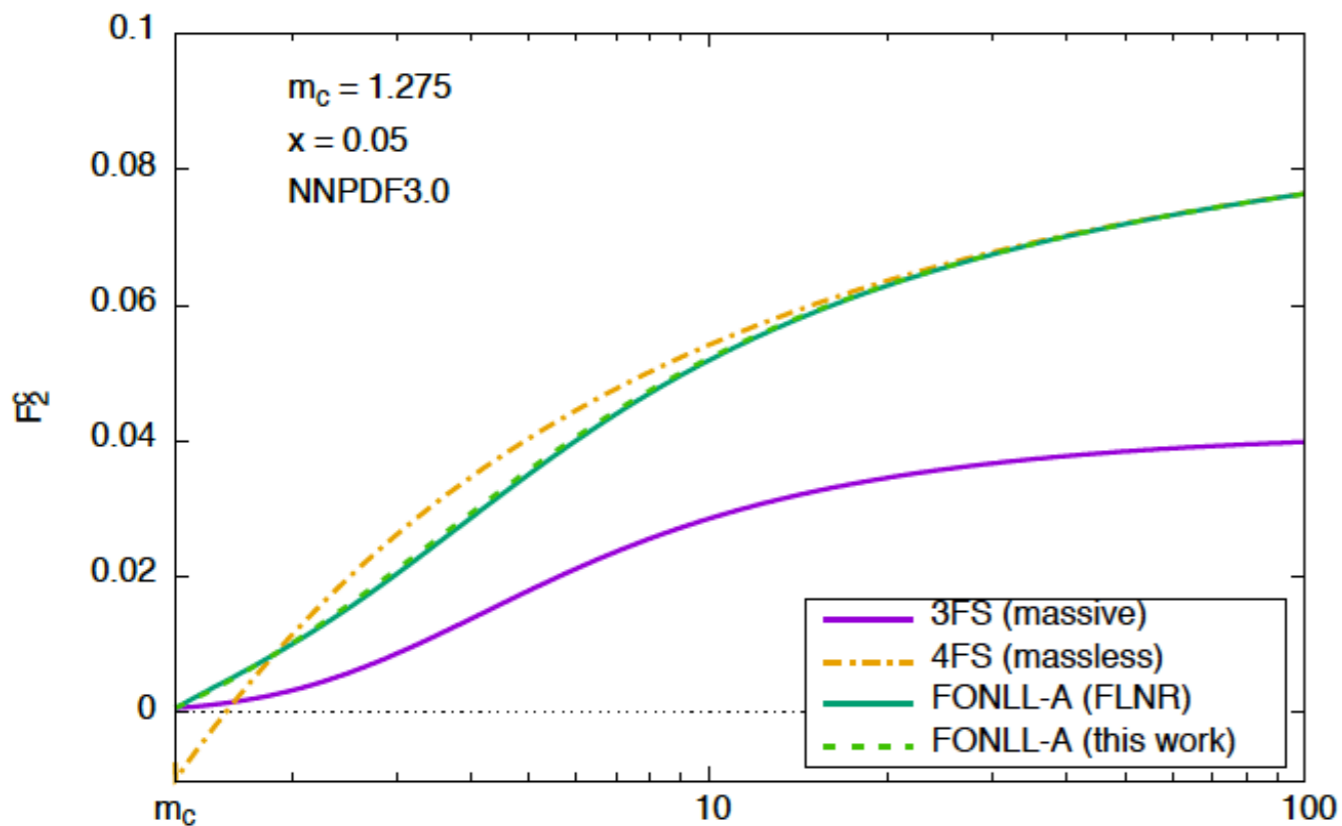
# Recap on APFEL

- 🍏 APFEL is a **public** library for PDF evolution:
  - 🍏 up to NNLO in QCD combined to LO QED corrections.
  - 🍏 FFNS and VFNS.
  - 🍏 Pole and  $\overline{\text{MS}}$  heavy quark masses.
  - 🍏 Module for the fast computation of DIS NC and CC observables up to NNLO in different mass schemes (ZM-VFNS, FFNS and FONLL).
  - 🍏 interfaces to FORTRAN, C/C++ and Python.
  - 🍏 Amazing web interface available on <http://apfel.mi.infn.it>.
  - 🍏 APFEL is available from <http://apfel.hepforge.org/>.
- 🍏 Interfaced to xFitter and Alpos.
- 🍏 Used for the next generation of the NNPDF fits.

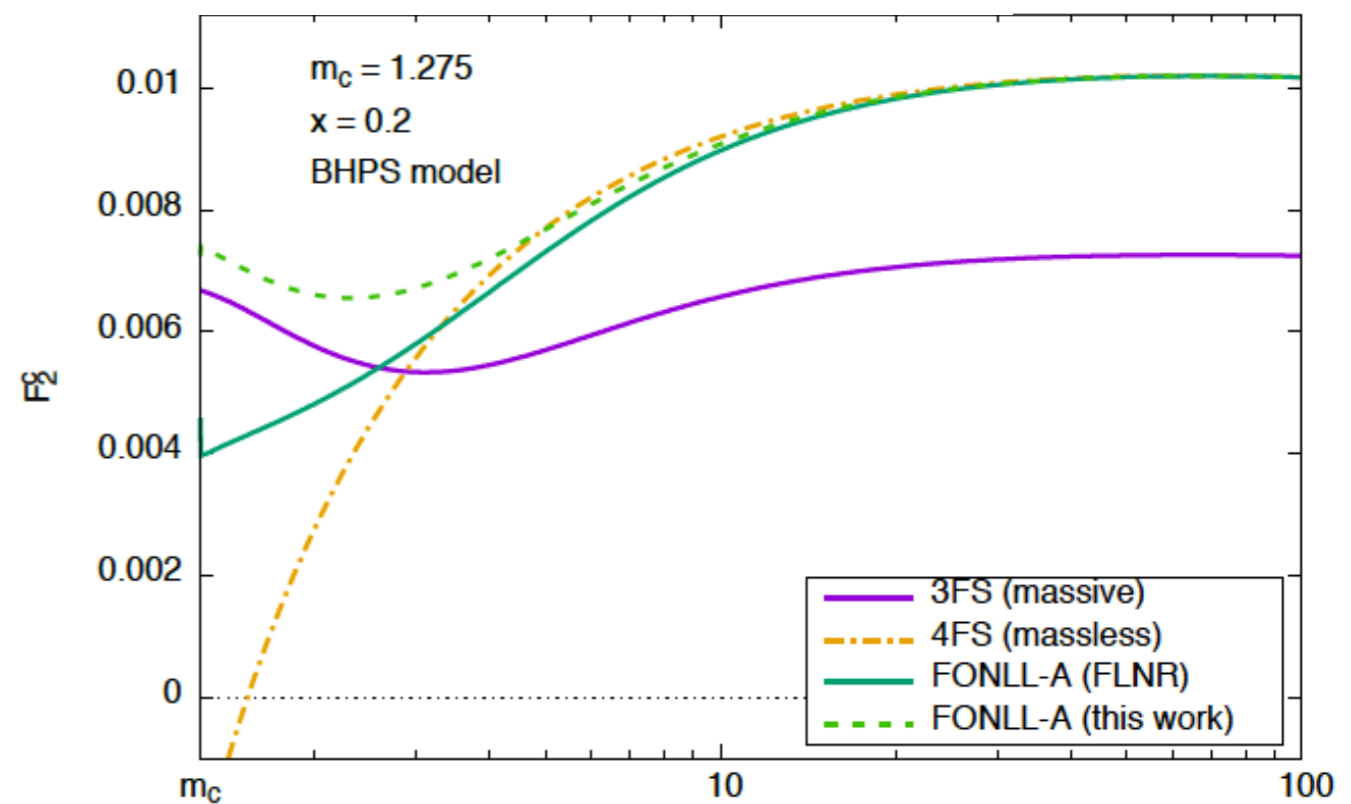
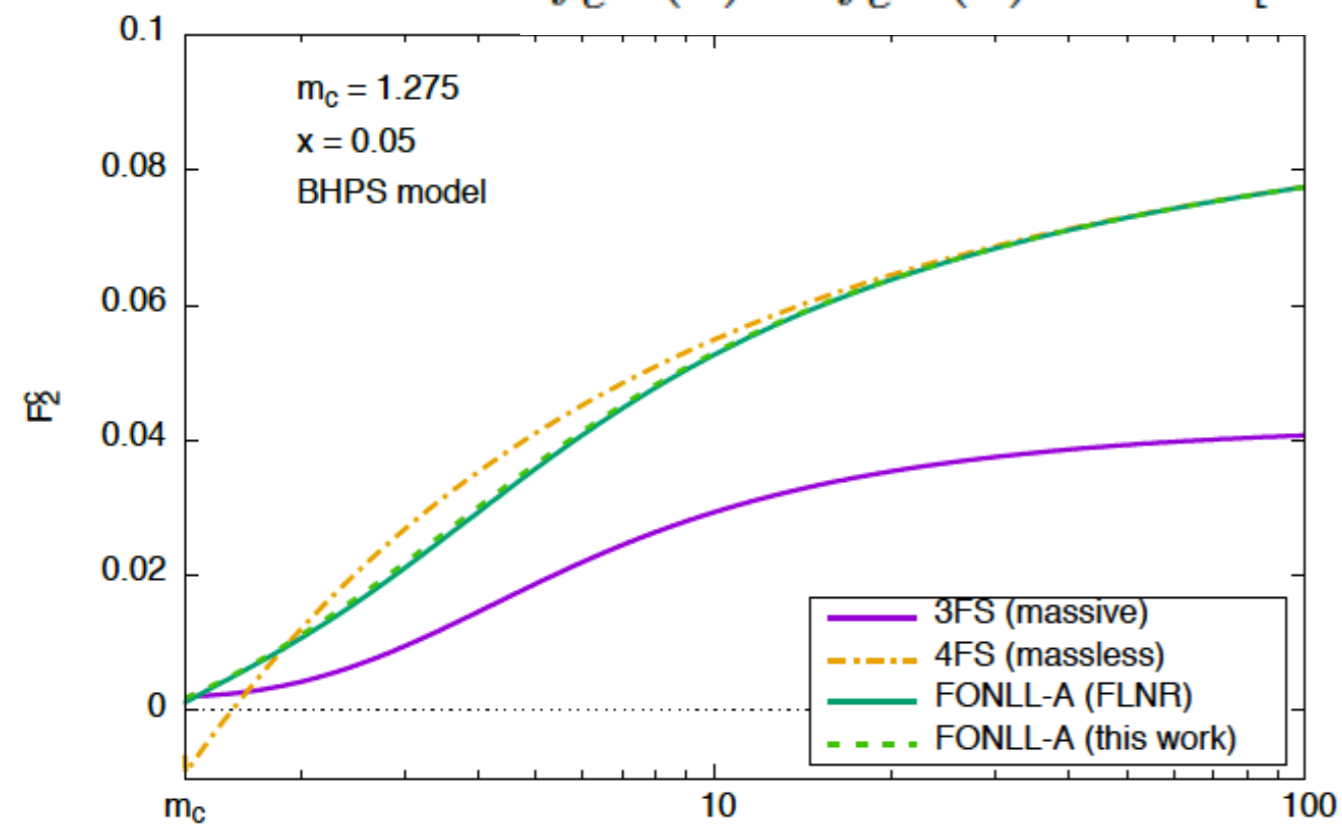
# Intrinsic Charm

- 🍏 Introducing an intrinsic charm (IC) component in the context of a GM-VFNS like FONLL (or ACOT, or TR) requires some care:
  - 🍏 relax the assumption of pure perturbative generation of heavy quarks at the thresholds,
  - 🍏 take into account charm-initiated diagrams both in the massive and in the massless sectors [[arXiv:1510.00009](https://arxiv.org/abs/1510.00009)].
- 🍏 A full formulation of the FONLL scheme in the presence of IC has recently been achieved [[arXiv:1510.02491](https://arxiv.org/abs/1510.02491)]:
  - 🍏 implemented in APFEL up to NLO both in the NC and CC sector and benchmarked against the public massiveDISsFunction code (<https://www.ge.infn.it/~bonvini/massivedis/>).
  - 🍏 Interestingly, it has been found that FONLL with IC is equivalent to full ACOT to all orders, while the standard FONLL (w/o IC) is instead equivalent to S-ACOT.

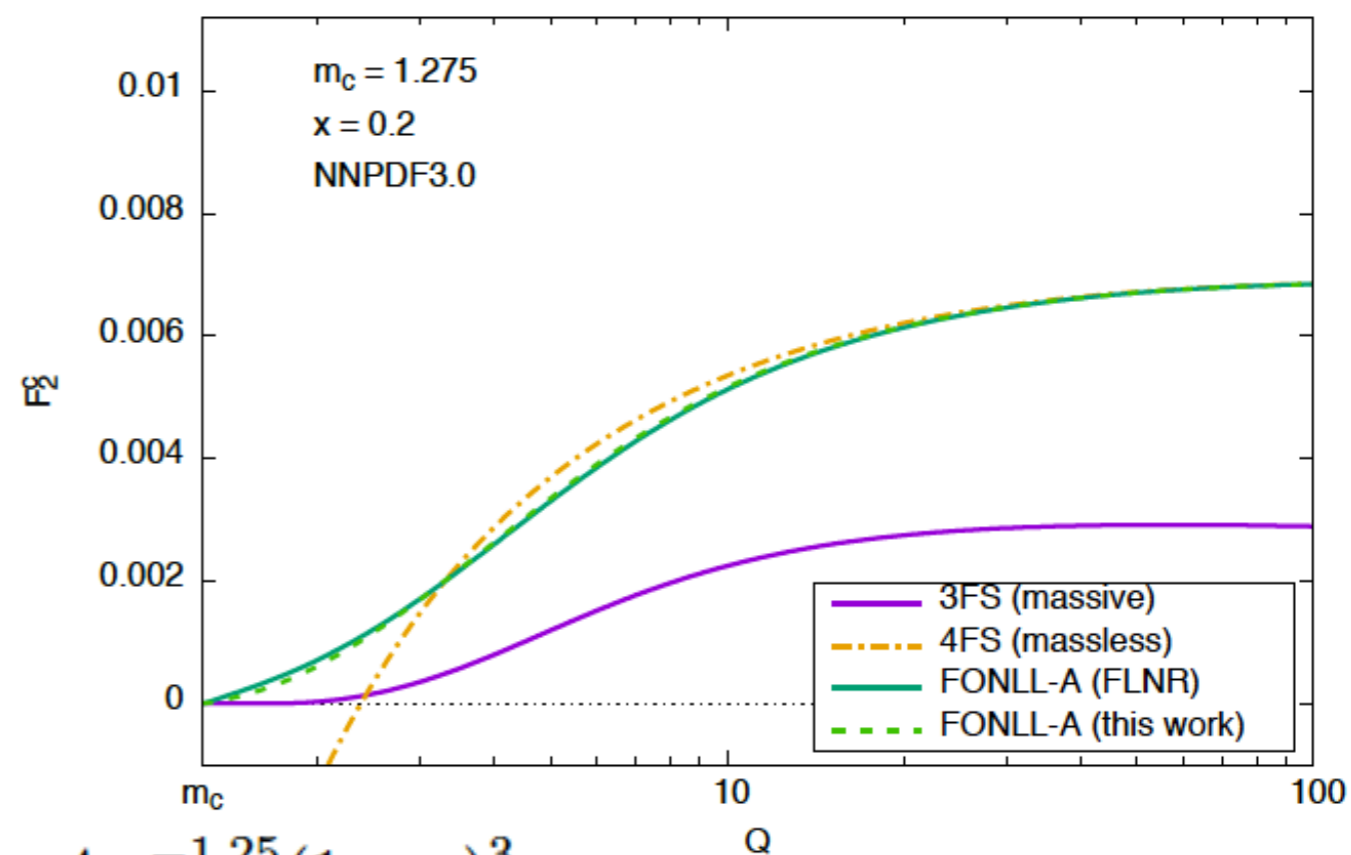
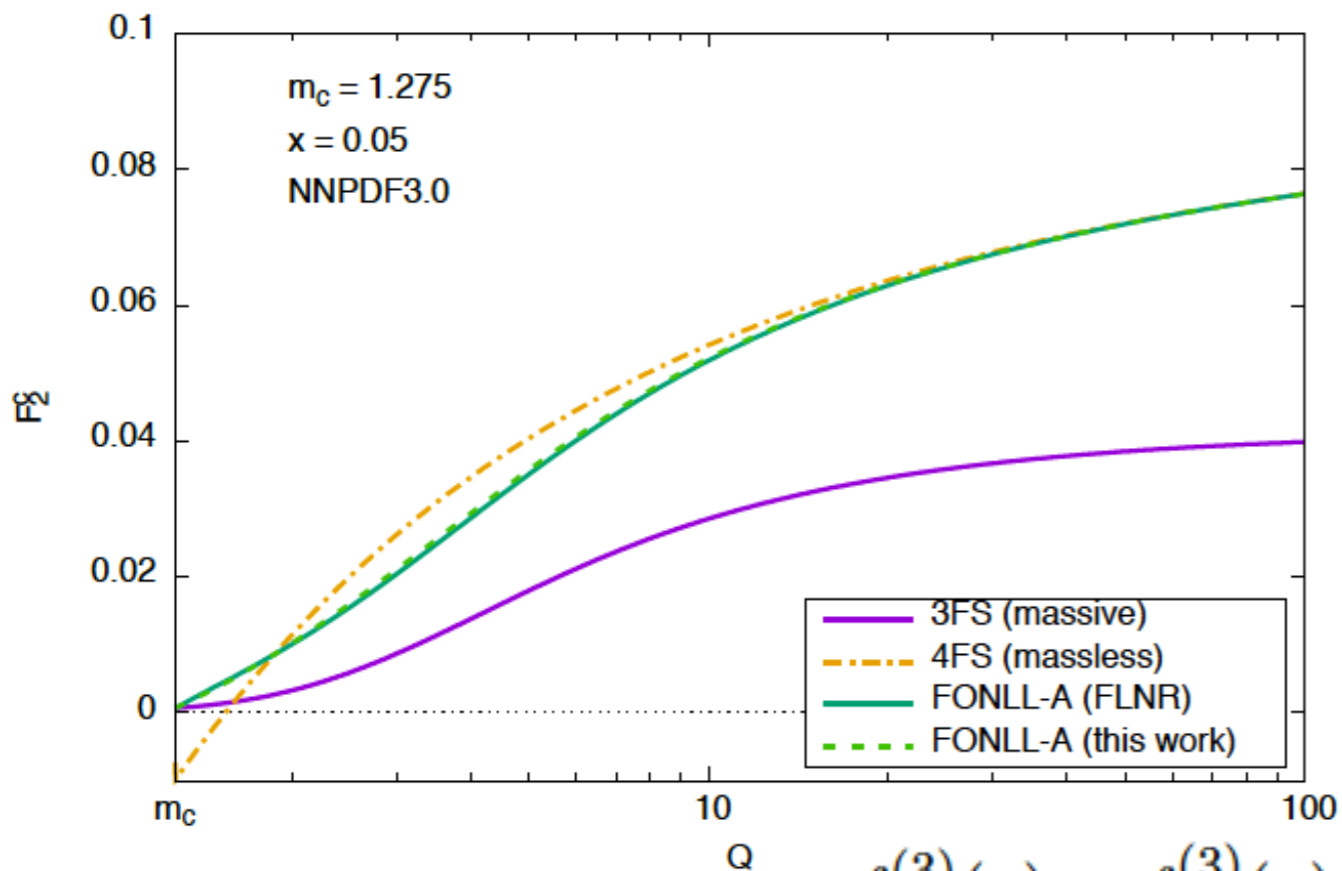
# Intrinsic Charm



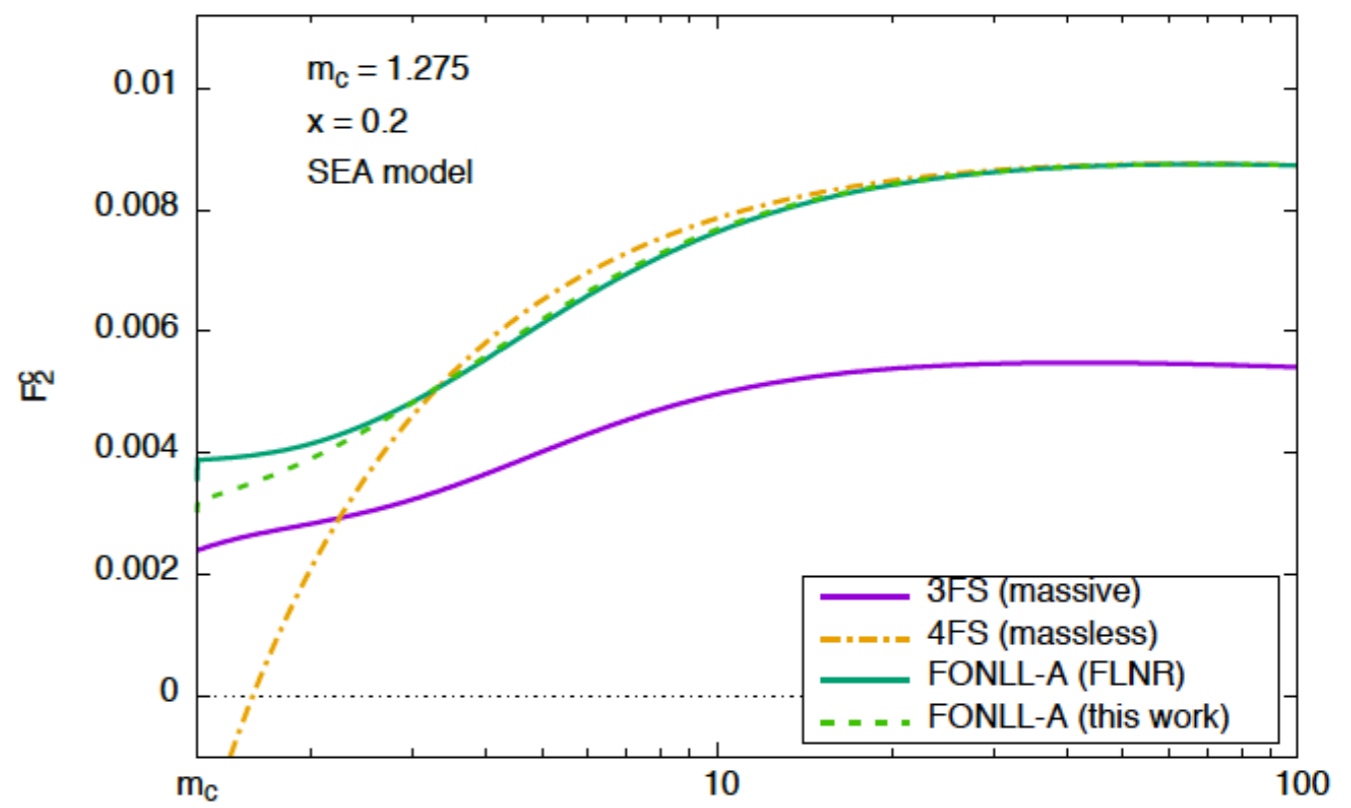
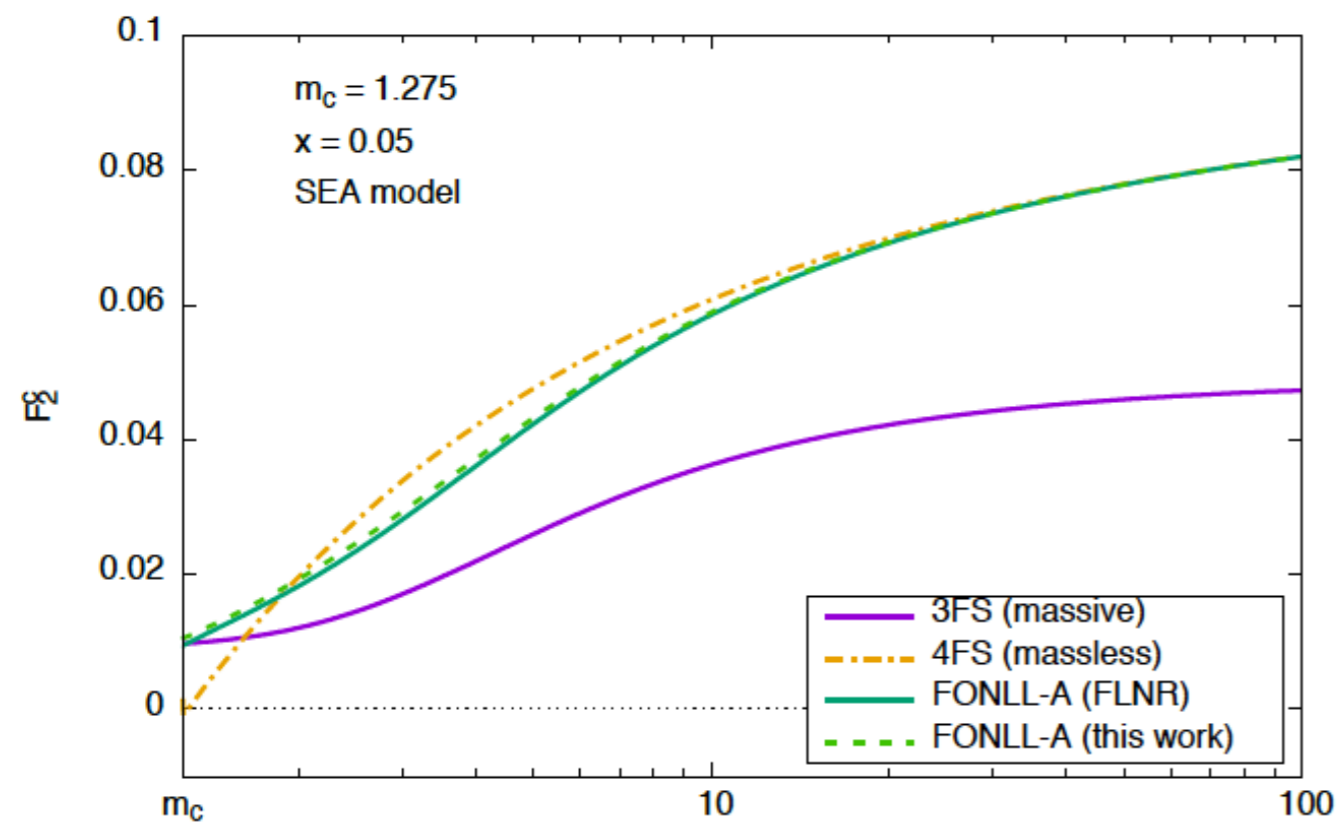
$$f_c^{(3)}(x) = f_{\bar{c}}^{(3)}(x) = A x^2 [6x(1+x) \ln x + (1-x)(1+10x+x^2)]$$



# Intrinsic Charm

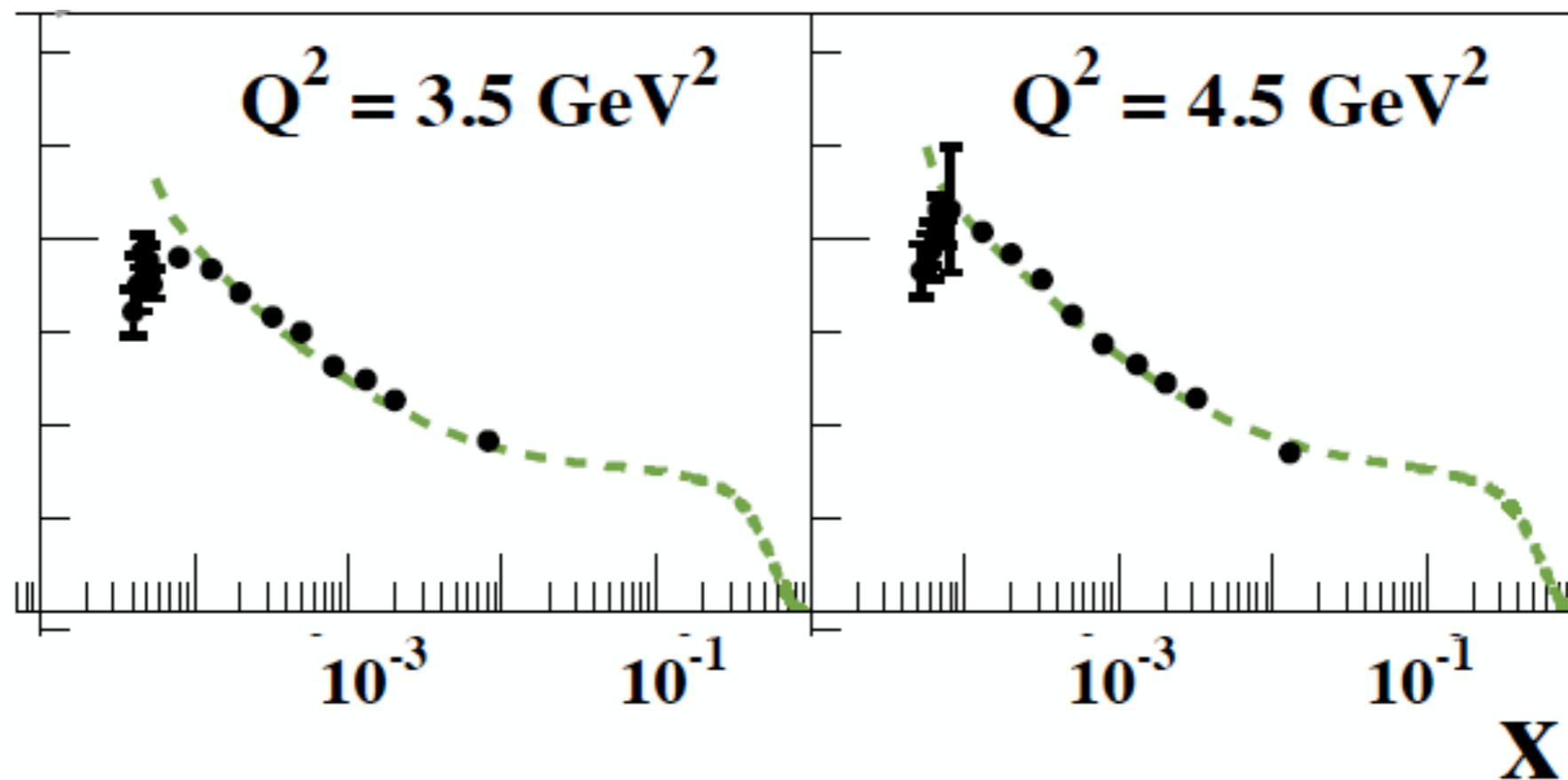


$$f_c^{(3)}(x) = f_{\bar{c}}^{(3)}(x) = A x^{-1.25} (1-x)^3$$



# Small- $x$ Resummation

- Some **tension** between fixed-order predictions and data in the low- $x$  region reached by HERA:



From Eram Rizvi talk  
at QCD@LHC14

- A similar effect was observed some time ago in the NNPDF framework by F. Caola *et al.* [[arXiv:1007.5405](https://arxiv.org/abs/1007.5405)].
- Strong suggestion of the need for **small- $x$  resummation**.

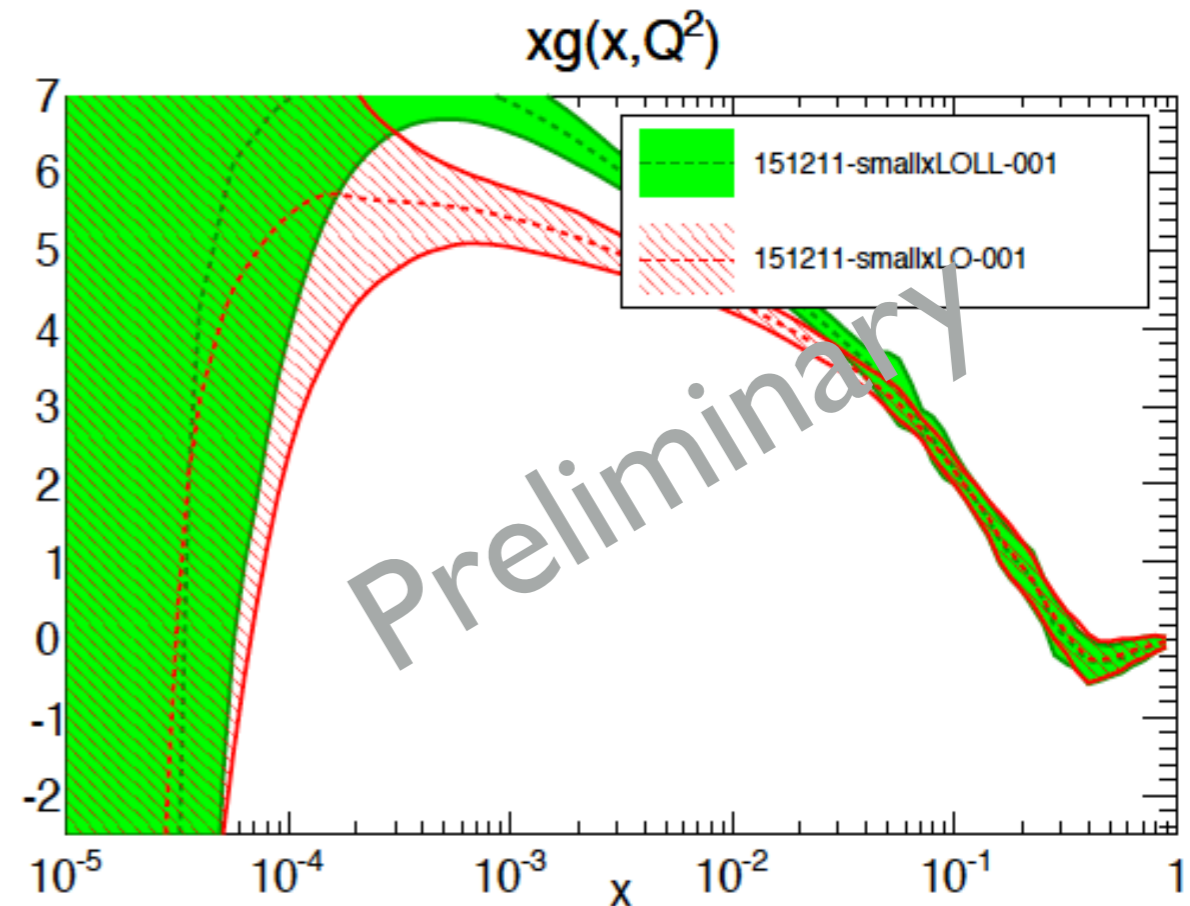
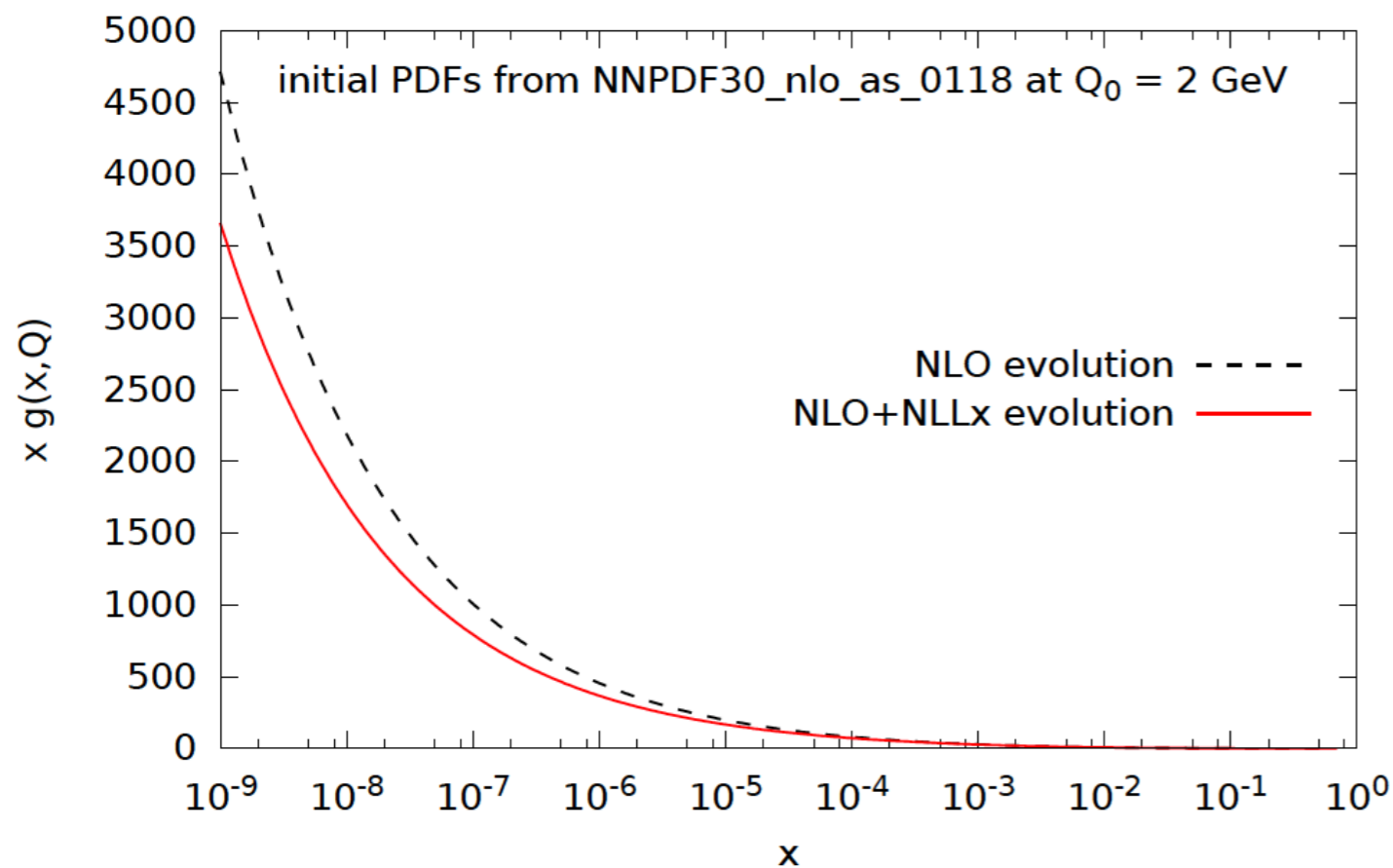
# Small- $x$ Resummation

- 🍏 In collaboration with Marco Bonvini, quite some work has been done to interface to interface the **HELL** code to APFEL:
  - 🍏 HELL implements small- $x$  resummed splitting functions up to **NLL** accuracy based on the ABF approach [[arXiv:0802.0032](#)].
  - 🍏 it will soon implement also small- $x$  resummed DIS coefficient functions (Marco Bonvini, Luca Rottoli and Tiziano Peraro are presently working on that).
- 🍏 The actual interface is **already in place** and fully operative.
- 🍏 As a proof of concept, we have already run PDF fits with small- $x$  resummed evolution obtaining encouraging results.
- 🍏 A fully consistent PDF fit would require resummed coefficient functions which should be available in HELL within a few weeks.

# Small- $x$ Resummation

- 🍏 Enhancement of the fitted gluon PDF at small values of  $x$  due to the relative suppression of the resummed evolution.
- 🍏 Compensation expected when also resummed coefficient functions will be introduced.

gluon PDF at  $Q = 100$  GeV



- 🍏 Other PDFs mostly unchanged.

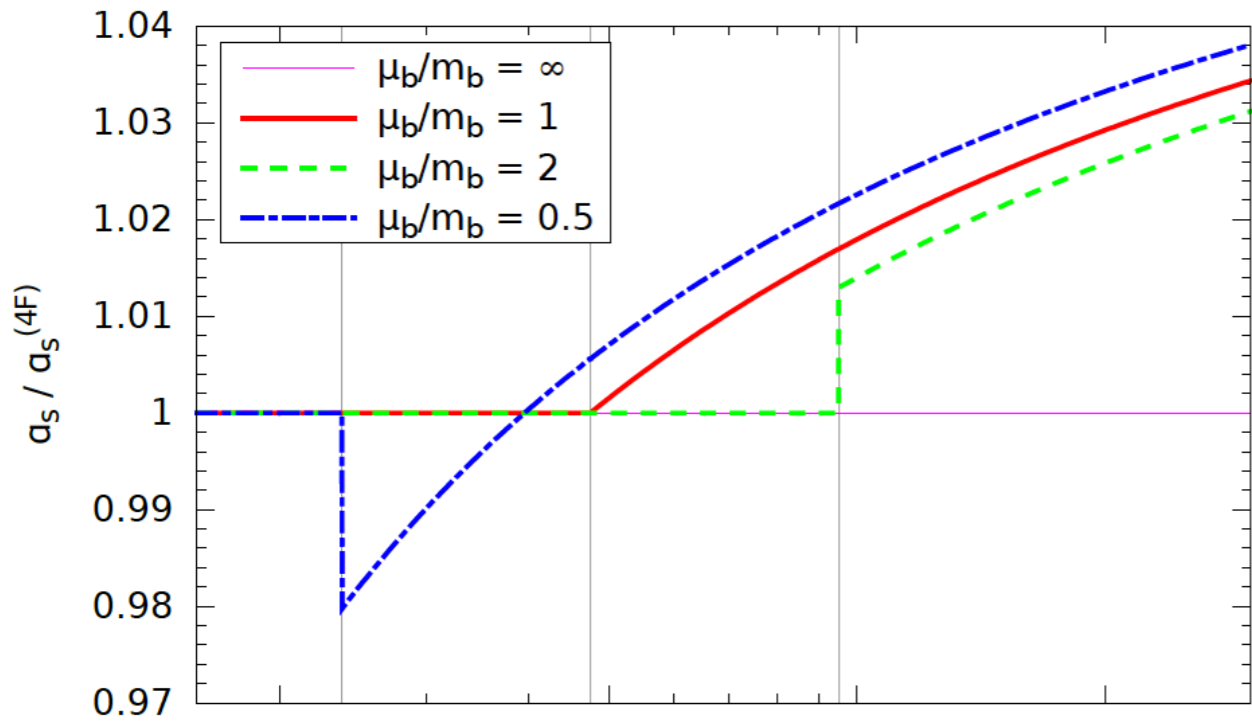


# Displaced Heavy-Quark Thresholds

- 🍏 The implementation of the VFNS evolution both for PDFs and  $\alpha_s$  requires **matching** factorization schemes that differ in the number of active (light) flavours:
  - 🍏 the scale at which two consecutive factorization schemes are matched are usually referred to as **heavy-quark thresholds**.
  - 🍏 Heavy-quark thresholds are usually (and for convenience) identified with the heavy quark masses by means of the so-called **matching conditions** presently known up to  $\mathcal{O}(\alpha_s^2)$  [[hep-ph/9612398](#)].
  - 🍏 However, heavy-quark thresholds are actually free parameters and can be chosen **arbitrarily**.
  - 🍏 If masses and thresholds are taken to be different, the matching conditions need to be “generalized” including **logarithmic terms** that would vanish otherwise.
- 🍏 APFEL now implements the possibility to set masses and thresholds to different values in a consistent way both in the pole mass and in the  $\overline{\text{MS}}$  renormalization scheme.

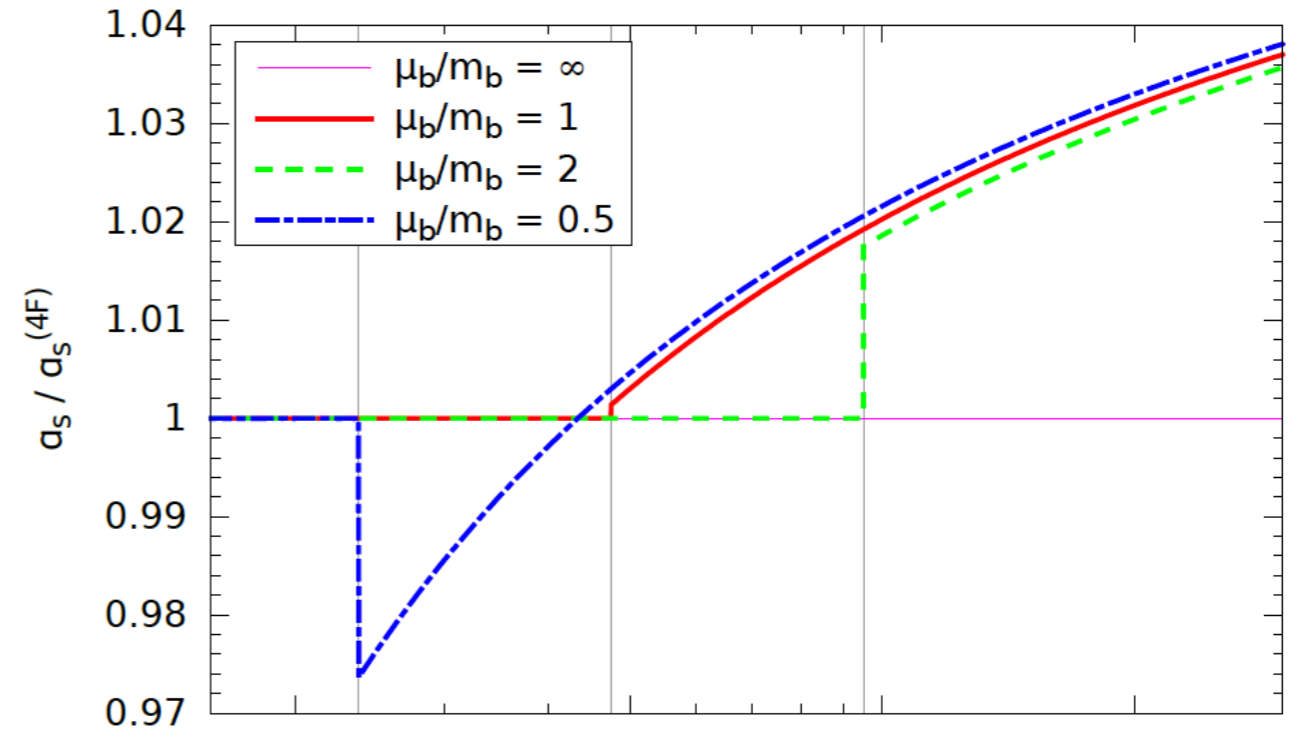
# Displaced Heavy-Quark Thresholds

NLO

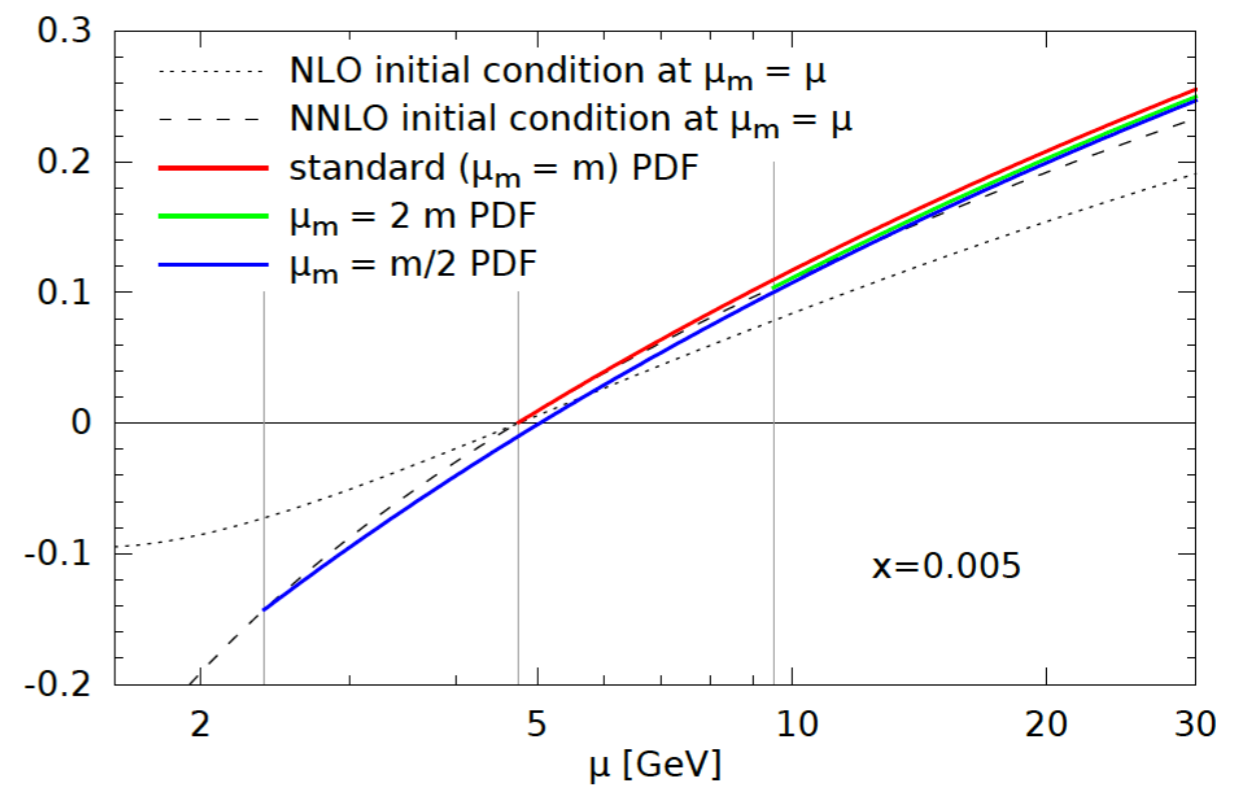
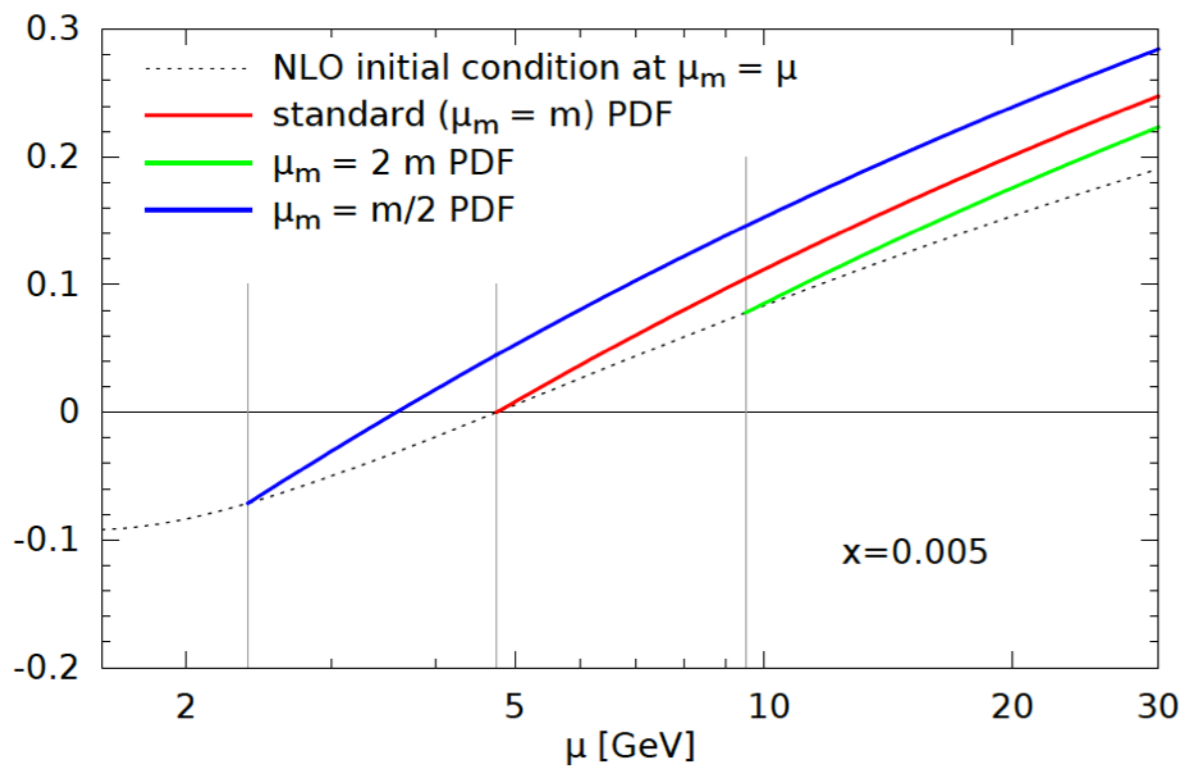


bottom PDF at NLO

NNLO



bottom PDF at NNLO



# APFELgrid

*A fast(er) interface to hadron-collider observables*

- While being an extremely useful tool, APPLgrid might not be appropriate to be directly employed in a global PDF fit where several thousands of iterations can be needed:
  - many tables need to be loaded with the concrete risk of exceeding the memory limit (pretty common on clusters).
  - Need to calculate PDF and  $\alpha_s$  evolution in real time.
  - Not particularly fast convolution.
- In order to overcome these problems, in the NNPDF collaboration we have developed a tool (that we named APFELgrid) which, starting from an existing APPLgrid file, combines PDF evolution to the hard cross sections producing a derived interpolation tables (that we call FK tables):

Observable	APPLGRID	FK	optimized FK
$W^+$ production	1.03 ms	0.41 ms (2.5x)	0.32 ms (3.2x)
Inclusive jet production	2.45 ms	20.1 $\mu$ s (120x)	6.57 $\mu$ s (370x)

- APFELgrid will soon be made public in APFEL.

# Other Recent Developments

- 🍏 Polarized DGLAP evolution up to NNLO in QCD [[arXiv:1409.5131](#)].
- 🍏 Time-like evolution + computation of SIA structure functions up to NNLO in QCD (getting ready to fit fragmentation functions).
- 🍏 Independent factorization and renormalization scale variations both in the DIS structure functions and in the evolution.

## In the Pipeline

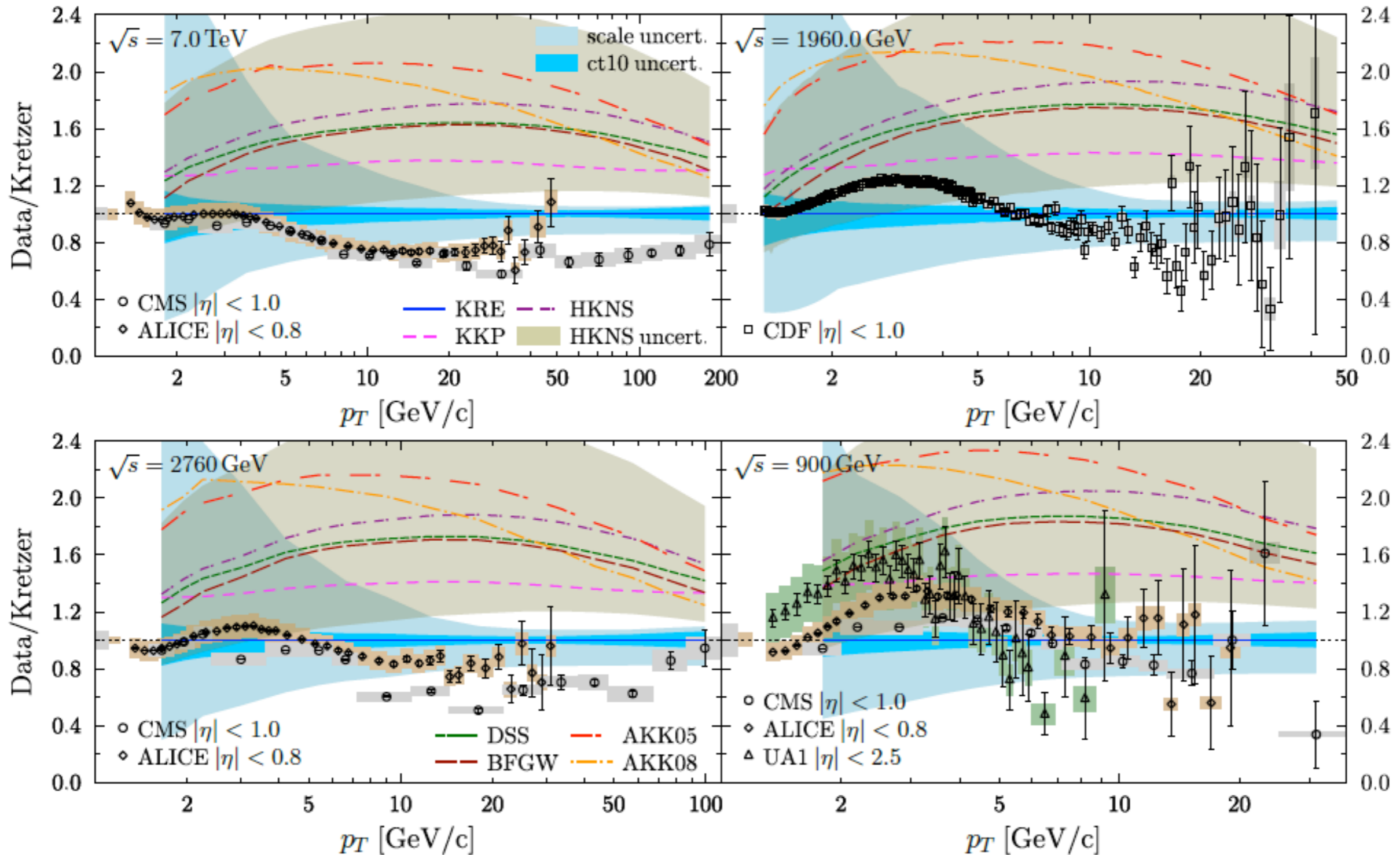
- 🍏 Full NLO corrections to the PDF and  $\alpha_s$  evolution (including the mixed QCD-QED corrections) [[arXiv:1512.00612](#)].
- 🍏 Inclusion of the photon-initiated channels in DIS.
- 🍏 Implementation of the polarized structure functions.

# Tools for Determining FFs

- 🍏 A faithful determination of fragmentation functions (FFs) is extremely important to study the universality of the QCD factorization theorem.
- 🍏 The inclusive hadron measurements at the LHC, sensibly extending the previous kinematical coverage, are particularly useful for studying the FFs.
- 🍏 Moreover, a good knowledge of FFs is functional to the determination of the **polarized PDFs**.
- 🍏 The **spread between the different FFs** present on the market is currently very large.
- 🍏 In addition, none of the existing FF sets can reproduce the experimental results optimally.

# Tools for Determining FFs

🍏 Inclusive charge-hadron spectrum:



d'Enterria et al. [arXiv:1311.1415]

# Tools for Determining FFs

- 🍏 APFEL implements the time-like evolution:
  - 🍏 up to NLO in the VFNS,
  - 🍏 up to NNLO in the FFNS (NNLO matching conditions missing).
- 🍏 In collaboration with E. Nocera and S. Carrazza, we have performed a careful **benchmark** of the time-like evolution:
  - 🍏 we are in contact with the people who calculated the time-like splitting functions: A. Mitov, S.O. Moch, A. Vogt.
- 🍏 Single-inclusive  $e^+e^-$  annihilation (SIA) structure functions also implemented in APFEL up to NNLO:
  - 🍏 partial benchmark against DSS implementation.
- 🍏 APFEL can now effectively be used to fit FFs.

# Improvements

## *A New Fast Evolution*

- 🍏 In the previous versions of APFEL the DGLAP evolution equations were written in terms of the **evolution operator**:

$$\mu^2 \frac{\partial}{\partial \mu^2} M_{ij}(\mu, \mu_0) = P_{ik}(\mu) \otimes M_{kj}(\mu, \mu_0) \quad \text{with} \quad f_i(\mu) = M_{ij}(\mu, \mu_0) \otimes f_j(\mu_0)$$

- 🍏 This may be convenient because the evolution operator can be evaluated once and for all and convoluted with any initial PDF set.
- 🍏 On the other hand, this requires solving numerically a big coupled system of ODEs, therefore it can be slow.
- 🍏 Alternatively, one can directly solve the DGLAP equations in terms of **PDFs**:

$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(\mu) = P_{ij}(\mu) \otimes f_j(\mu)$$

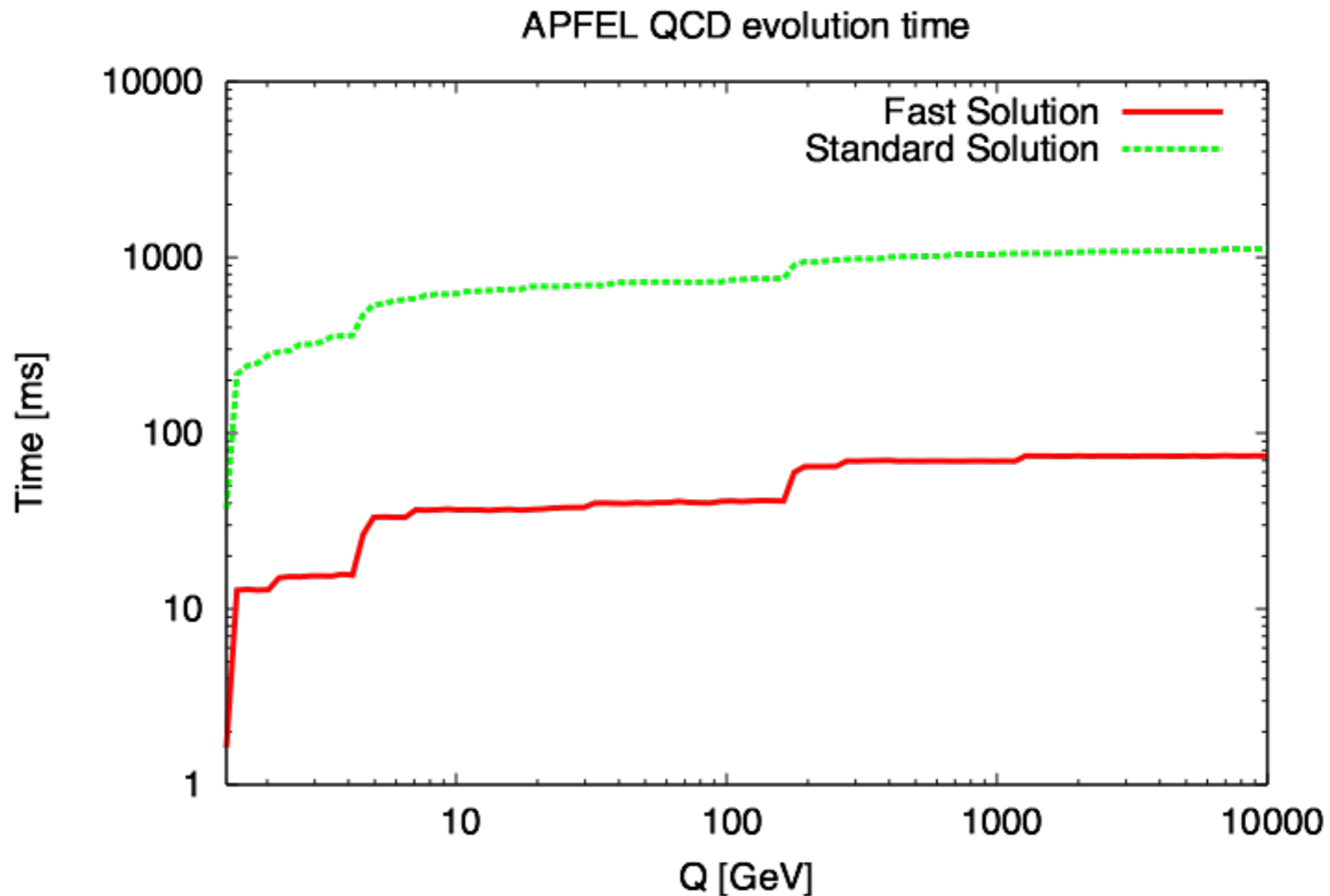
- 🍏 This requires the solution of a much smaller system of equations and is consequently much faster.



# Improvements

## *A New Fast Evolution*

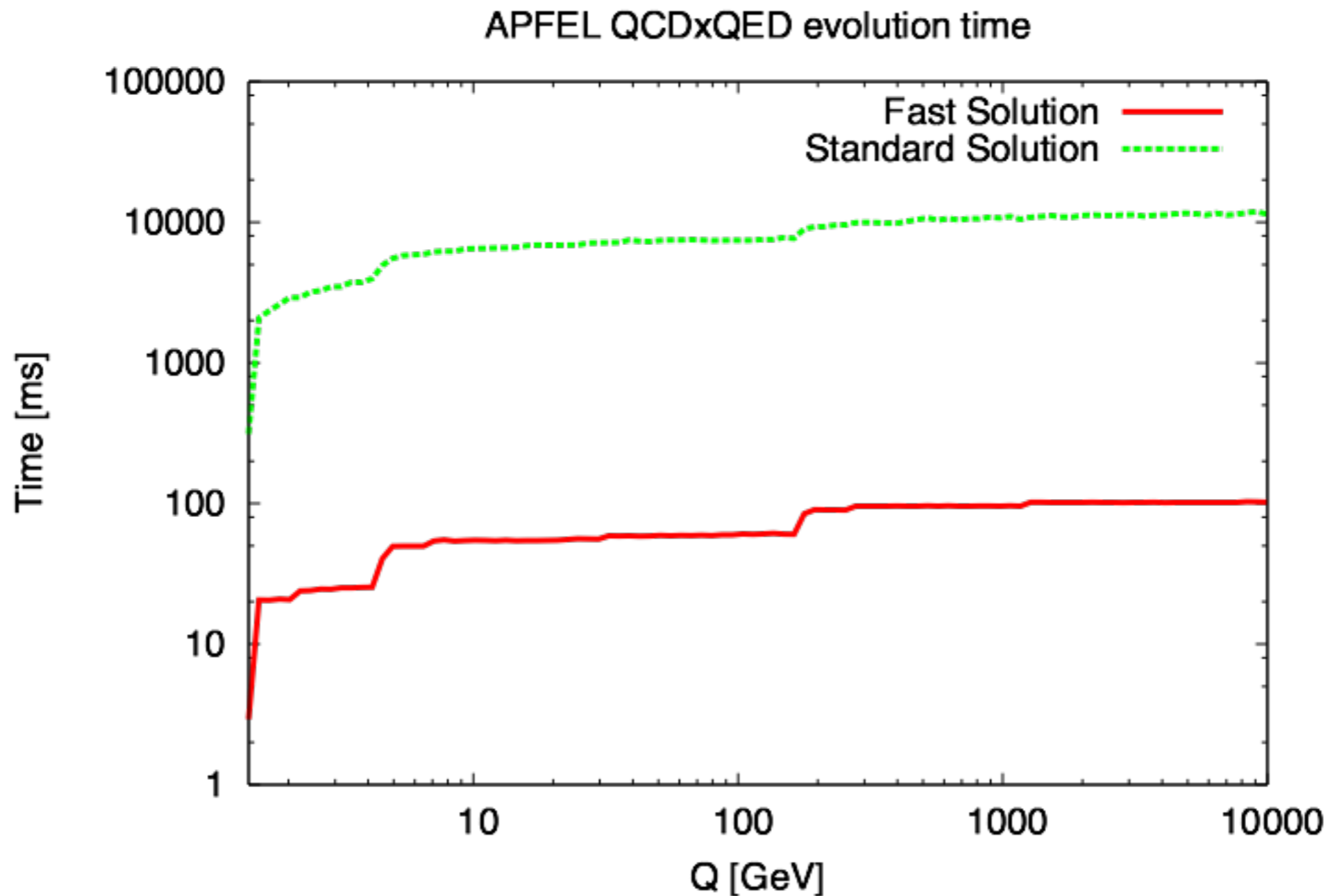
- 🍏 Comparison between old (operatorial) and new (in terms of PDFs) solution for the QCD evolution:



# Improvements

## *A New Fast Evolution*

- 🍏 Comparison between old (operatorial) and new (in terms of PDFs) solution for the QCD+QED evolution:



# Improvements

## *A New QCD+QED Evolution*

🍏 In the previous versions of APFEL the QCD+QED evolution was performed by combining the **separate** QCD and QED evolution:

🍏 we showed that the differences, of a few % at most, with the standard implementations which evolve contemporaneously in QCD and QED were due to **subleading terms in  $\alpha$** .

🍏 We have now implemented a new evolution basis which allows a **simultaneous diagonalization** of the QCD+QED evolution matrix:

1)  $g$

2)  $\gamma$

3)  $\Sigma = \Sigma_u + \Sigma_d$

4)  $\Delta_\Sigma = \Sigma_u - \Sigma_d$

5)  $T_1^u = u^+ - c^+$

6)  $T_2^u = u^+ + c^+ - 2t^+$

7)  $T_1^d = d^+ - s^+$

8)  $T_2^d = d^+ + s^+ - 2b^+$

9)  $V = V_u + V_d$

10)  $\Delta_V = V_u - V_d$

11)  $V_1^u = u^- - c^-$

12)  $V_2^u = u^- + c^- - 2t^-$

13)  $V_1^d = d^- - s^-$

14)  $V_2^d = d^- + s^- - 2b^-$

# Improvements

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14)  $V_2^d = d^- + s^- - 2b^-$

*Coupled*

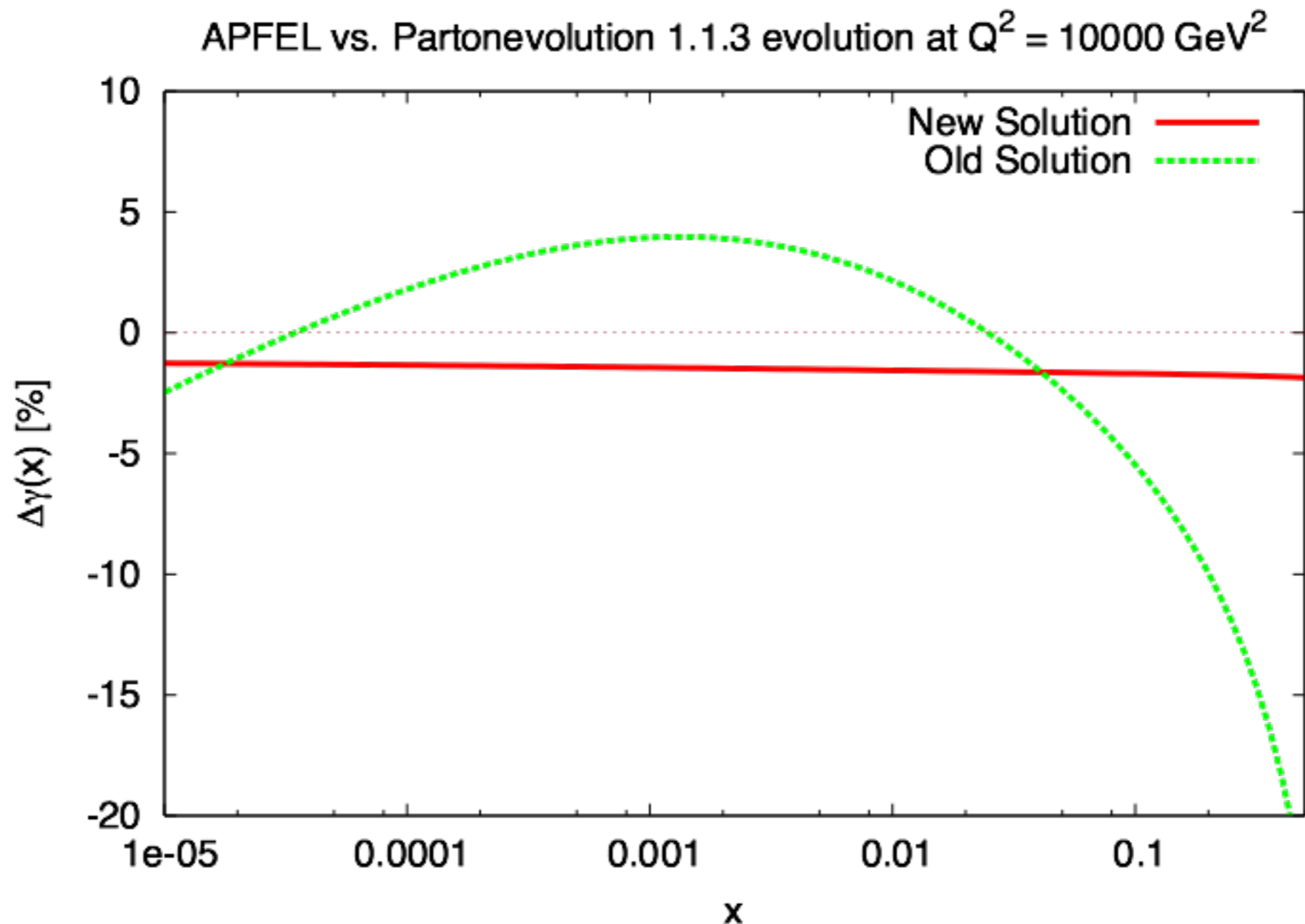
*Decoupled*

🍏 This new basis is also suitable for an easy implementation of the mixed **higher order corrections** to the evolution.

# Improvements

## *A New QCD+QED Evolution*

- 🍏 Comparison of the photon against **Partonevolution 1.1.3** in the FFNS:

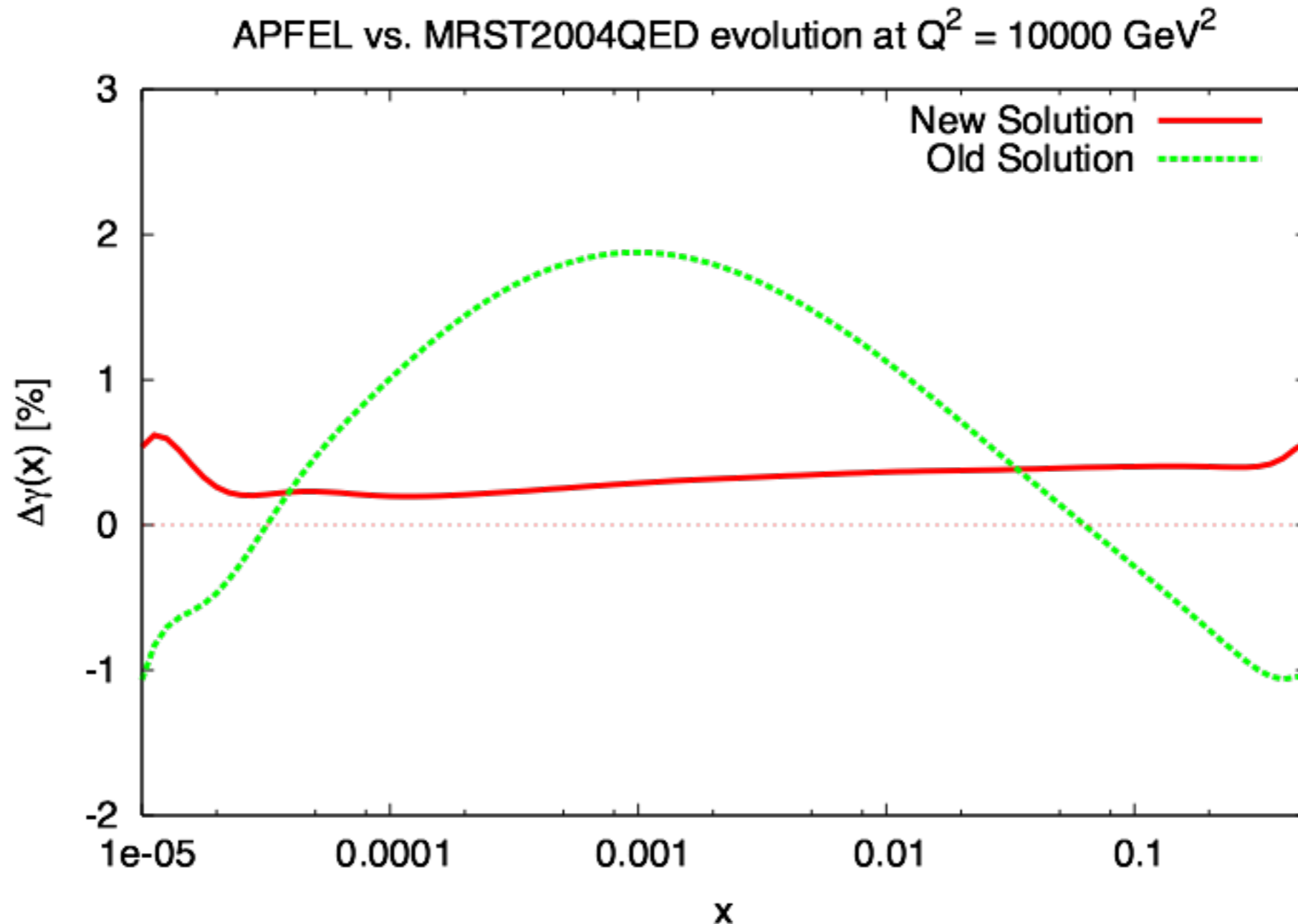


- 🍏 All the other PDFs are in excellent agreement.

# Improvements

## *A New QCD+QED Evolution*

- 🍏 Comparison of the photon against **MRST2004QED** in the VFNS:



- 🍏 All the other PDFs are in excellent agreement.