QUANTUM SUPERGRAVITY & APPLICATIONS

Applications to aspects of (Heterotic string-derived) effective SUGRA theories

⋆ Hidden sector gaugino condensation

⋆ Vacuum stability

⋆ Flavor-changing neutral currents

⋆ Soft SUSY breaking

Need reliable regularization procedure that

⋆ Respects local SUSY & BRST

⋆ Retains information of effective cut-off Λ. Cf:

- $\Lambda \sim$ 2GeV $\Rightarrow$ charm quark mass
- $\Lambda \sim$ 300GeV $\Rightarrow$ W, Z masses
- $\Lambda \sim$ TeV $\Rightarrow$ Higgs + ?

$\Longrightarrow$ Pauli-Villars regularization

arXiv:1602.03916, NPB (Stora Memorial Issue)

OUTLINE

- Overview of Pauli-Villars regularization of SUSY & SUGRA
- Quadratic divergences
  - Vacuum stability
  - Flavor changing neutral currents
- Anomalies, their cancellation and applications
  - Anomaly cancellation
  - Gauge coupling unification
  - Hidden sector gaugino condensation
  - Axion physics
- Soft SUSY breaking
- Unfinished business

PV regularization of SUSY and SUGRA

PV reg of ordinary non-Abelian gauge theory not possible:

breaks gauge (BRST) invariance of (gauge-fixed) Lagrangian

SUSY $\Rightarrow$ cancellations of diagrams that require high spin PV fields

SUSY YM & matter: only chiral PV superfields needed

SUGRA + YM & matter: chiral + Abelian vector PV superfields

$\Rightarrow$ cancellation of all quadratic and log UV divergences

Provided $C_M^a = (\mathrm{Tr}T_a^2)_M \equiv C_R^a$ for some real rep R

$\surd$ MSSM & extensions & (known) hidden sectors

+ most linear divergences $\Rightarrow$ triangle anomalies canceled or reappear in PV masses

Except for linear divergences from SUGRA contributions to gravitino & gaugino connections

Form SUSY anomalies with uncanceled log divergences $\propto$ total divergence (e.g. Gauss-Bonnet) provided

$$\Lambda(Z,\bar{Z}) = \mu_0 e^{K/4}$$

Z = chiral superfield, $K(Z,\bar{Z})$ = Kähler potential

Dan Butter & MKG

QUADRATIC DIVERGENCES

$$V_Q = \epsilon\Lambda^2 \left[(N_\chi - 1)\,|M_\psi|^2 - N_G|M_\lambda|^2 - R_{i\bar m}F^i\bar F^{\bar m}\right], \quad \text{if } \langle V_{\rm tree}\rangle = \langle D_a\rangle = 0, \quad \epsilon = \frac{1}{16\pi^2}$$

in canonical Einstein basis

$$F^i = -e^{K/2}K^{i\bar m}(\overline{W}_{\bar m} + K_{\bar m}\overline{W}) = \partial^2 Z^i/\partial\theta^2\Big| = \text{chiral aux. field},$$

$W(Z)$ = superpotential, $K(Z,\bar Z)$ = Kähler potential, $_i = \partial/\partial z^i$, etc.

Typically: $N_\chi \gtrsim 300$ = # of chiral (Z^i) $\gg N_G \lesssim 65$ = # of gauge (V_a) supermultiplets

$M^2_\lambda(z,\bar z)$ = gaugino mass^2 = $\frac{1}{4}f_i\bar f^i M^2_\psi(z,\bar z) \ll M^2_\psi$ = gravitino mass^2,

$f_i = \partial f(z)/\partial z^i$, $\quad f(Z)$ = gauge kinetic function, $\quad z^i = Z^i\Big|$, $\quad \bar f^i = K^{i\bar m}\bar f_{\bar m}$

$\epsilon N_\chi \sim 1 \overset{?}{\Longrightarrow}$ Large vacuum energy (1st term)? FCNC (last term)? Choi et al. $\times 2$

$$R_{i\bar m} = \sum_{j=1}^{N_\chi} R^j_{ij\bar m} = \text{Kähler Ricci tensor}$$

Should sum $(\epsilon N_\chi)^n$

PV regulated theory: $V_Q \to \epsilon\left[|M|^2\left(N_\chi\Lambda^2_\chi - \Lambda^2_{\rm grav}\right) - N_G M^2_\lambda \Lambda^2_G - R_{i\bar m}F^i\bar F^{\bar m}\Lambda'^2_\chi\right] +$

finite terms $\propto$ PV masses^2 M^2_I $\qquad \Lambda_A = \Lambda_A(M^2_I) \qquad \Longrightarrow$

$\mathcal{L}_Q = \mathcal{L}_{\rm tree}(g^R_{\mu\nu}, K^R) - \mathcal{L}_{\rm tree}(g_{\mu\nu}, K) + O(\epsilon^2), \qquad K^R = K + \Delta K = K + \epsilon\sum_A \Lambda^2_A$

Leading order in ϵN_χ : $V_{\rm tree}(K) \to V_{\rm tree}(K^R)$. E.g: if $\Delta K \sim 1$ = constant: $V_{\rm LO} = e^{\Delta K}V_{\rm tree} \sim V_{\rm tree}$!

Also FCNC can be evaded by • isometries of Kähler geometry: $R_{i\bar m} \propto K_{i\bar m}$ (e.g. no-scale),

• judicious choice of PV masses

MKG & Brent Nelson

ANOMALIES

Effective field theory anomalous under T-duality (target space modular invariance)

& (with Wilson lines) $U(1)_X$: exact symmetries of heterotic string

Simplest case: 3 "diagonal" "Kähler moduli" T^i: T-duality $\ni$ $SL(2,\mathbf{Z})$

$$T'^i = \frac{aT^i - ib}{icT^i + d} = (aT^i - ib)e^{-F^i(T^i)}, \quad G(T',\bar{T}') = G(T,\bar{T}) + F(T) + \bar{F}(\bar{T}), \quad ad - bc = 1$$

$K = G(T,\bar{T}) + G_{\rm inv}$, $\quad F(T) = \sum_i F^i(T^i)$ generated by • axion shift ${\rm Im}T^i \to {\rm Im}T^i - 1$,

• ${\rm Re}T^i \to 1/{\rm Re}T^i$ = inversion of radius of $i^{\rm th}$ torus $t^i = T^i\big|$ in 6-d orbifold

Also: $V'_X = V_X + \Lambda_X + \bar{\Lambda}_X$, $\quad \Phi'^a = e^{-q^a_X\Lambda_X - q^a_i F^i}\Phi^a$, $\quad \Lambda_X$ chiral

At one loop: in Kähler $U(1)$ superspace: Binétruy, Girardi & Grimm

$$\Delta\mathcal{L}_{\rm anom} \ni \frac{1}{8}\int d^4\theta \frac{E}{R}\Phi H + {\rm h.c.}, \qquad \Phi = \frac{1}{3}W^{\alpha\beta\gamma}W_{\alpha\beta\gamma} + W^\alpha_a W^a_\alpha$$

E = superdet(supervielbein); $\quad R = \frac{1}{2}e^{K/2}W = \frac{1}{2}M_\psi$ = SUGRA aux. field;

$W^{\cdots}$ = spacetime curvature & YM field strengths, resp.

$H = -bF(T) + \frac{1}{2}\delta_X\Lambda_X$ with (e.g., $\mathbb{Z}_3$ and $\mathbb{Z}_7$)

$$8\pi^2 b = \frac{1}{24}\left(2\sum_p q^p_i - N + N_G - 21\right) \quad \forall \quad i \quad = -C^a - C^a_M + 2\sum_b (T^2_a)^b_b q^b_i \quad \forall \quad i, a,$$

$$2\pi^2\delta_X = -\frac{1}{24}{\rm Tr}T_X = -\frac{1}{3}{\rm Tr}T^3_X = -{\rm Tr}(T^2_a T_X) \quad \forall \quad a \neq X$$

Universality conditions allow anomaly cancellation by 4-d Green-Schwarz mechanism:

$$\mathcal{L}_{\rm tree} \ni \frac{1}{8}\int d^4\theta \frac{E}{R} S\Phi + {\rm h.c.}, \qquad S = \text{dilaton (chiral) superfield}$$

$\Delta\mathcal{L}_{\rm tree}$ cancels loop anomaly provided: $\Delta S = -H = bF(T) - \frac{1}{2}\delta_X\Lambda_X$

Require: $K_{\rm tree} \ni k(S+\bar{S}) \to k(S+\bar{S}+V)$, $V = -b[G(T,\bar{T}) + O|\Phi|^2] + \frac{1}{2}\delta_X V_X$

string/QFT matching, MKG & Tom Taylor

Orbifolds $\neq \mathbb{Z}_3$, $\mathbb{Z}_7$: $b^{a,i}_{\rm QFT\ loop} \neq b$:

Anomaly canceled by GS term V and T-dependent string loop corrections $\propto b - b^{a,i}_{\rm QFT\ loop}$

More elegant formulation: (modified) linear multiplet $L = (\ell, \chi, b_{\mu\nu})$

10-d SUGRA $\longrightarrow$ 4-d two-form $b_{\mu\nu}$ dual to axion ${\rm Im}s$, $s = S|$: $\epsilon^{\mu\nu\rho\sigma}\partial_\nu b_{\rho\sigma} \propto \partial^\mu {\rm Im}s$

Superfield duality transformation: $-\int dLk'(L)/L \leftrightarrow S + \bar{S} + V$ $\to L^{-1}$ if $k(L) = \ln L$

(classical limit)

GS term: $\mathcal{L}_{\rm GS} = -\int d^4\theta ELV$ $\qquad \Delta\mathcal{L}_{\rm GS} = -\int d^4\theta EL(H+\bar{H})$

$$= \int d^4\theta \frac{E}{8R}(\bar{\mathcal{D}}^2 - 8R)LH + {\rm h.c.} = -\frac{1}{8}\int d^4\theta \frac{E}{R}\Phi H + {\rm h.c.}$$

from partial integration and (modified) linearity condition: $(\bar{\mathcal{D}}^2 - 8R)L = -\Phi$

Applications

★ Gauge coupling unification — Regulated theory: Coefficient $g_{a\,\mathrm{eff}}^{-2}$ of $F^a_{\mu\nu}F_a^{\mu\nu}$ at string scale

$\mu_s = g_s = g_a(\ell_0)$, $\ell_0 = \langle\ell\rangle$, $g^{-2}(\ell) = -\frac{1}{2}\int d\ell k'/\ell = \mathrm{Res} + \frac{1}{2}\,V|$ in units: $m_P = (8\pi G_N)^{-\frac{1}{2}} = 1$

determined by PV masses. Here $M_{\rm PV}$ uniquely fixed by cancellation of UV divergences

$\mathbb{Z}_{\mathbf{3}}$, $\mathbb{Z}_{\mathbf{7}}$: T-dependence cancels: $p_b =$ coefficient of $|\Phi^b|^2 e^{\sum_i q^b_i g^i}$ in V

$$\frac{1}{g^2_{a\,\mathrm{eff}}} = \frac{1}{g^2(\ell_0)} - \frac{1}{16\pi^2}\left(C^a - C^a_M\right)k(\ell_0) - \frac{2}{16\pi^2}\sum_b C^a_b \ln(1 - p_b\ell_0)$$

Compare RGE invariant: Shifman & Vainshtein

$$\delta_a = \frac{1}{g^2_a(\mu)} - \frac{1}{16\pi^2}(3C^a - C^a_M)\ln\mu^2 + \frac{2C^a}{16\pi^2}\ln g^2_a(\mu) + \frac{2}{16\pi^2}\sum_b C^a_b \ln Z^a_b(\mu)$$

Set $g^{-2}_{a\mathrm{eff}} = \delta_a$ with $g(\ell_0) = g_s = g_a(\mu_s)$, $Z^a_b(\mu_s) = (1 - p_b l)^{-1}$, $k(\ell_0) \stackrel{\text{tree}}{\to} \ln(g^2_s/2) = \ln(\mu^2_s/2)$

$\Rightarrow\quad \mu^2_{\rm unif} = \frac{\mu^2_s}{2e} = \frac{g^2_s m^2_P}{2e} \approx 2\times 10^{17}\mathrm{GeV}$ for gauge coupling unification scale in $\overline{MS}$ bar scheme

V. Kaplunovski, MKG & TT, VK & Lewis, MKG & Rulin Xiu

$\mu_{\rm unif} \sim 20\times$ larger than MSSM value. Corrections from: • vector like massive states generic to orbifold compactifications, • QFT loop & string nonperturbative corrections to $k(\ell) \neq \ln(\ell)$,

• T-dependent terms in orbifolds with threshold corrections (small for weak coupling $|T| \sim 1$)

NOTE: 2-loop (all loop!) effects encoded in 1-loop QFT calculation through Z-dependence of PV masses imposed by SUSY

★ Hidden sector gaugino condensation VYT, Binétry, G & Y-Y Wu, Casas

- Introduce composite chiral superfields charged under strongly coupled gauge group $\mathcal{G}_a$:

$$U_a \simeq W^\alpha_a W^a_\alpha, \qquad \Pi^\alpha_a \simeq \prod_b (\Phi^b_a)^{n^\alpha_b}$$

$W^\alpha_a W^a_\alpha \to U_a$ in $\mathcal{L}_{\rm tree}$ & (modified) linearity condition

- Match (all) anomalies of effective theory to underlying YM theory: $\Longrightarrow$

$$\mathcal{L}_{\rm eff}(U_a, \Pi^\alpha_a) = \frac{1}{8}\int d^4\theta \frac{E}{R}\sum_a U_a\left[b'_a \ln\left(e^{-K/2}U_a\right) + \sum_\alpha b^\alpha_a \ln \Pi^\alpha_a\right] + \text{h.c.}$$

$$b'_a = \frac{1}{8\pi^2}\left(C^a - \sum_b C^a_b\right), \qquad b^\alpha_a = \sum_{b\in\alpha}\frac{C^a_b}{4\pi^2 d^\alpha_a}, \qquad d^\alpha_a = \dim\left(\Pi^\alpha_a\right)$$

- Add GS term; T-duality & $U(1)_X$ anomalies canceled as before

- Add modular & gauge invariant superpotential $W(\Pi) = \sum_{a,\alpha} C^a_\alpha(T)\Pi^\alpha_a$
  $\Rightarrow$ solution to EOM's with $\langle u_a\rangle \neq 0,\ u_a = U_a|$, SUSY broken

$\exists$ always solution $u_a = g_s = 0$, no SUSY breaking

only solution without GS term: "runaway dilaton"

GS term $\Rightarrow$ 2nd runaway direction $\rightarrow$ strong coupling

Dilaton stabilized at weak coupling with broken SUSY if string nonperturbative effects included

★ Axion physics — Banks & Dine; MKG, Dan Butter, Ben Kain

R symmetry: $\Phi(\theta) \to \Phi'(\theta') = e^{i\alpha}\Phi(\theta')$, e.g., $\Phi = W_a^\alpha W_\alpha^a,\ U_a,\ e^{K/2}W(Z$ or $\Pi)$ such that

$$\int d^4\theta' \frac{E(\theta')}{R(\theta')}\Phi(\theta') = \int d^4\theta \frac{E(\theta)}{R(\theta)}\Phi(\theta) \left(\sim \int d^2\theta \Phi(\theta) \text{ in flat SUSY}\right) \text{ is invariant}$$

Broken by quantum anomalies in presence of instantons [Cf. QCD: broken $U(1)_\chi$]

$$\Delta\mathcal{L}(U_a) = i\frac{\alpha}{8}b_a'' \int d^4\theta \frac{E}{R}U_a + \text{h.c.}, \quad b_a'' = b_a' + \sum_\alpha b_a^\alpha = \frac{1}{8\pi^2}\left[C^a - \sum_b C_b^a \left(1 - \frac{2}{d_a}\right)\right],\ d_a \equiv d_a^\alpha$$

$d_a = 3$ (SS: $W \sim (\Phi^a)^{n\geq 3} \overset{\sim}{\to}$ scale invariance) $\Rightarrow b_a'' = \frac{1}{8\pi^2}\left(C^a - \frac{1}{3}\sum_b C_b^a\right) \equiv b_a =$ $(-2/3\times)\beta$-function coefficient. Single condensate U_a: $\Delta\mathcal{L}(U_a)$ canceled by $\Delta \text{Im} s = -\alpha b_a''$

R-symmetry unbroken, massless axion = Peccei-Quinn axion $a = \langle\sqrt{2\ell/k'(\ell)}\rangle \text{Im} s$?

Not if $\exists$ two hidden condensates: $m_a \sim |b_1'' - b_2''|\langle\sqrt{|u_1 u_2|}\rangle \gtrsim \Lambda_{QCD}, \quad \langle|u_i|\rangle \sim \Lambda_i^3 \sim e^{-1/b_i'' g_s^2}\mu_s^3$

Assumed no broken global symmetry in hidden sector: $m_{u_a,\pi_a^\alpha} \gtrsim \Lambda_a$

QCD: composite (pseudo) goldstone bosons of spontaneously broken (nonanomalous) $SU(N_F)_\chi$

Simple toy model: SUSY $SU(N_c)$: $W(\Pi) = C\text{Tr}M\Pi$, $\Pi_B^A = Q^A Q_B^c$, $\dim(\Pi) = 2$, $\Rightarrow$ $b_Q'' = \dfrac{C_a}{8\pi^2} = \dfrac{N_c}{8\pi^2}$

$$\mathcal{L}(U_Q)_{\text{anom}} \ni \ln\det\Pi;\ \lim_{m_P\to\infty}\int dU_Q e^{iS[U_Q,\Pi]} \to e^{iS[\Pi]_\text{h}}$$

(h = holomorphy) Davis, Dine,& Seiberg

NOTE: If $M \to 0,\ W(\Pi) \to 0$ SUSY YM has larger R-symmetry: $U_Q \to e^{i\alpha} U_Q, \quad \Pi \to e^{i\beta}\Pi$

R-symmetry if $U_a, U_Q \neq 0$: $\Delta \mathrm{Im} s = -\alpha b_a = -\frac{1}{8\pi^2}\left[\alpha(N_c - N_F) + \beta N_F\right], \quad \beta = \alpha\left(\frac{8\pi^2 b_a - N_c}{N_F} + 1\right)$

$M \neq 0$: R-symmetry nonanomalous only at symmetric point: $\beta = \alpha, \quad N_c = 8\pi^2 b_a$

Axion mass: $m_a = \frac{F_\pi}{F_a}\frac{|8\pi^2 b_a - N_c|}{\sqrt{2n} b_a} m_\pi,$ standard result? At string scale:

$$\mathcal{L} \ni -\frac{\mathrm{Im} s}{4}\sum_a F^a \cdot \tilde{F}_a = -\frac{a}{4\langle\sqrt{2\ell/k'(\ell)}\rangle}\sum_a F^a \cdot \tilde{F}_a \equiv -\frac{a}{4F_a}\sum_a F^a \cdot \tilde{F}_a$$

Real QCD: Integrate out SUSY partners, heavy quarks... Same result! (up to $m_u \neq m_d$)

Integrating out gauginos shifts axion couplings: $\Delta \mathcal{L}_{\mathrm{QCD}} = \frac{a}{4F_a}\frac{N_c}{8\pi^2 b_a}(F\cdot\tilde{F})_Q \Rightarrow$

$$\mathcal{L}_{\mathrm{QCD}} \ni -\frac{a}{4F_a}\left(1 - \frac{N_c}{8\pi^2 b_a}\right)(F\cdot\tilde{F})_Q \equiv -\frac{na}{32\pi^2 f_a}(F\cdot\tilde{F})_Q, \qquad n = \#\text{ of light quarks}$$

$n = 2: \quad m_a = \frac{2\sqrt{z}}{1+z}\frac{F_\pi}{f_a} m_\pi, \quad z = \frac{m_u}{m_d}$ (operators $\propto U_a^{n>1}$ highly suppressed by T-duality)

At symmetry point $m_a = 0$ & no solution* to QCD CP problem: *(Need small misalignment angle)

if $Q_A(\theta) \to e^{\frac{i}{2}\beta} Q_A(\theta') \Rightarrow q_A(\theta) \to e^{\frac{i}{2}(\beta-\alpha)} q_A(\theta') = q_A(\theta')$

Cannot set $\theta_{QCD} = -\frac{n\langle a\rangle}{8\pi^2 f_a} = 0 = \arg\det m_q$

Soft SUSY breaking at one loop

Important if tree level terms

- absent: e.g., "no-scale" SUGRA with F-term mediated SUSY breaking
- suppressed: e.g., gaugino masses, A-terms $\ll$ scalar masses if $b_a \ll b$ in hidden sector condensation models

Include (super-Weyl) anomaly mediated contributions:

$$m_a \ni -\frac{3}{2}g_a^2(m_a)M_\psi = \frac{\beta(m_a)}{g_a(m_a)}M_\psi$$ Randall & Sundrum★

Giudice, Luty, Murayama & Rattazzi ★∗† MKG, Nelson & Wu∗† Bagger, Moroi & Poppitz†

$$A_{abc} \ni (\gamma_a + \gamma_g + \gamma_c)\, M_\psi, \qquad B_{ab} \ni (\gamma_a + \gamma_b)\, M_\psi$$ R&S★ GLM&R★∗†

G&R, Arkani-Hamed&GLR♯ G&N∗†

- $\mu_a^2 \ni \gamma_a |M_\psi|^2$ G&N♯∗†

†Effective Operator, ★ Chiral compensator, ♯ spurion

∗ PV reg: B-term, A-term and soft mass^2 PV insertions

- R&S: specific class of models $\Rightarrow \mu_a^2$ at 2-loop only
- ♯: holomorphic spurion previously assumed $\Rightarrow$ PV A&B-term insertions only.

Double B-insertion cancels in $\mu_a^2 \Rightarrow$ 2-loop only.

One loop contribution from PV soft mass^2 $\leftrightarrow$ real spurion

ALL gauge-charged PV fields contribute to renormalization of gauge sector.
Relevant contributions completely specified by cancellation of UV divergences.
$\Rightarrow$ m_a uniquely determined by low energy theory.

IN CONTRAST only subset of PV superfields Z^I contribute to renormalization of Kähler potential through superpotential couplings to light fields Z^i: $W_{\rm PV} \ni c_{IJk} Z^I Z^J Z^k$
Z^I gets large mass through superpotential coupling to $Z^{I'}$: $W_{\rm PV} \ni \mu_{II'} Z^I Z^{I'}$

$$m_I = m_{I'} = e^{K/2}(K_{I\bar{I}} K_{I'\bar{I}'})^{\frac{1}{2}} \mu_{II'} \equiv f_I \mu_{II'}$$

$K_{I\bar{I}}$ fixed by finiteness of ΔK; $K_{I'\bar{I}'}$ unconstrained $\Rightarrow$ f_I unconstrained

$W_{\rm soft}^{1-{\rm loop}}(z^i)$ depends on scalar derivatives of $\ln f_I$ $\Leftarrow$ Planck scale physics

Full expressions (PV): Binétruy,G&N

Unfinished business

• Effective field theory from heterotic string: we don't know

★ Twisted sector Kähler potential beyond order $|\Phi_{\rm tw}|^2$

∗ Details of Planck scale physics as encoded in PV masses

• Anomaly cancellation: terms nonlinear in parameters of anomalous transformations are model dependent. Scrucca & Serone, Butter & MKG

Full anomaly cancellation may constrain

★ Cf. constraint on gauge reps

&

∗ implications for soft SUSY breaking at one loop

• Waiting for guidance from LHC. If nothing new found????