Efficiency of Feedbacks and Landau Damping for Low-Emittance Bunches

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TWIICE-2, Feb 8, 2016, Abingdon, Oxfordshire, UK

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- Efficiency of Feedbacks
- NHT: Examples for APS
- Broken Landau damping for e-beams
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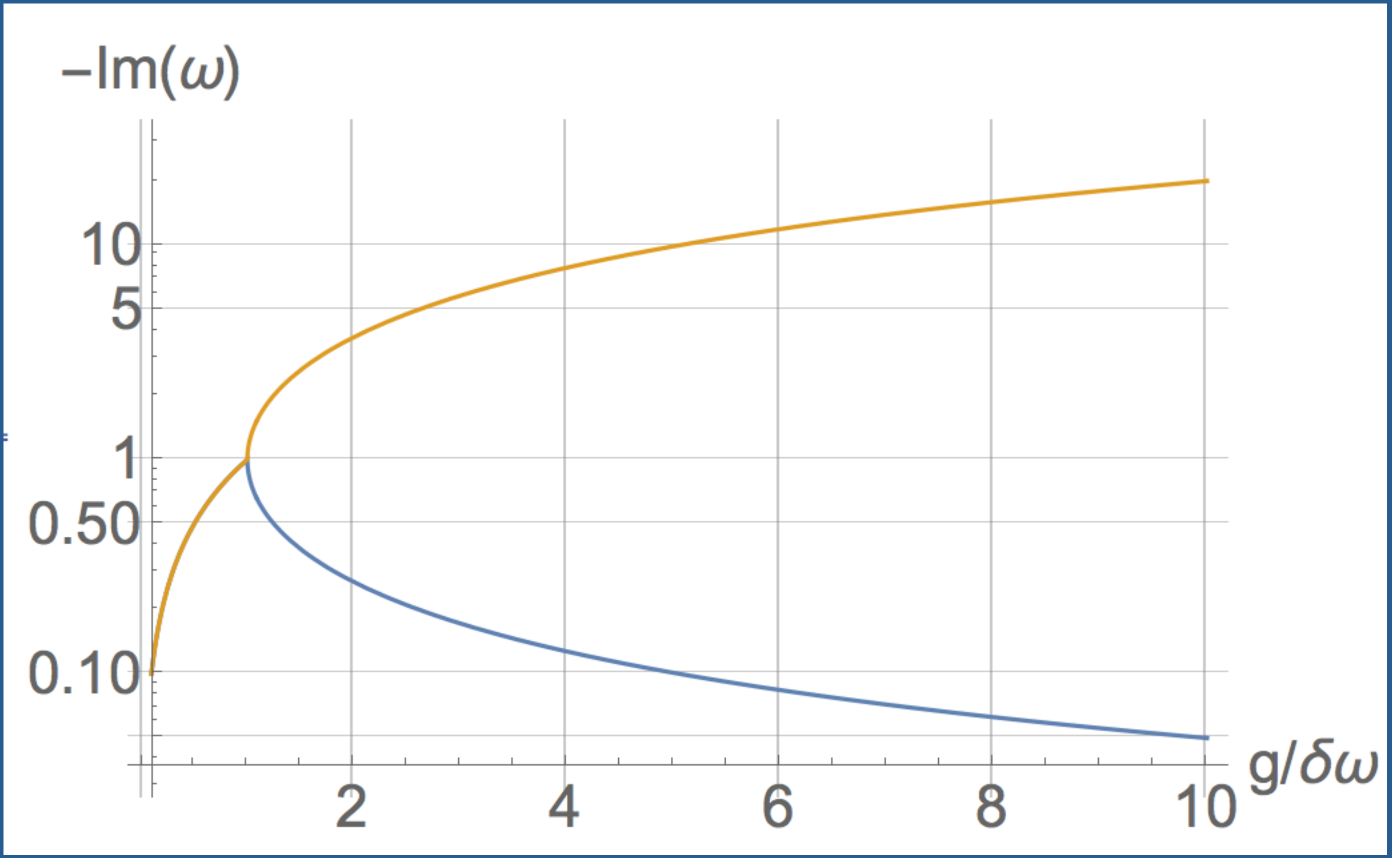
Splendors and Miseries of Feedbacks

$$\dot{A}_1 + i\omega_1 A_1 = -g(A_1 + A_2);$$
$$\dot{A}_2 + i\omega_2 A_2 = -g(A_1 + A_2);$$



$$A \propto \exp(-i\omega t);$$
$$\delta\omega = |\,\omega_1 - \omega_2\,|.$$

With a higher gain, one of the modes is damped less!

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **17,** 021007 (2014)

Nested head-tail Vlasov solver

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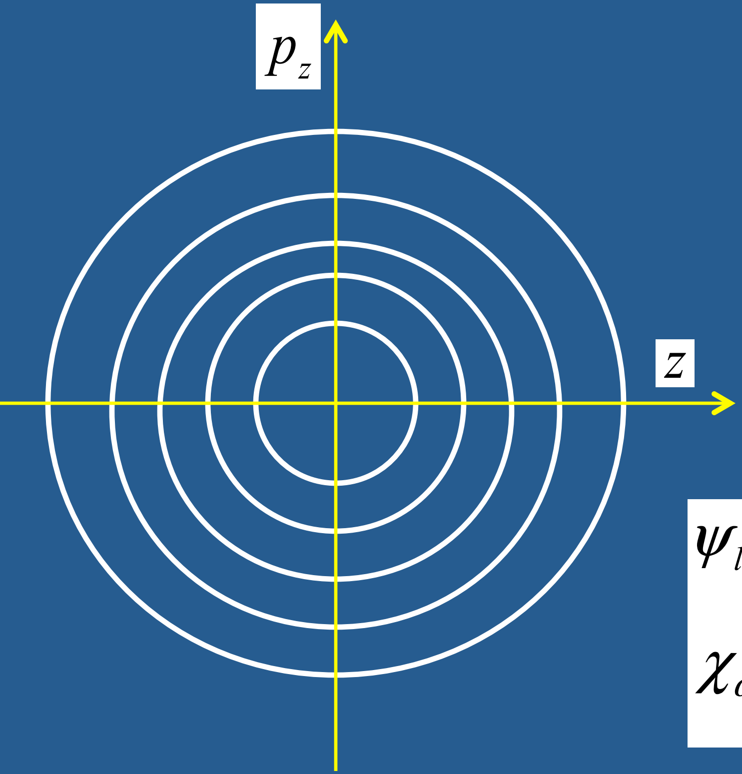
(Received 4 September 2013; published 25 February 2014)

Nested head-tail is a Vlasov solver for transverse oscillations in multibunch beams. It takes into account azimuthal, radial, coupled-bunch, and beam-beam degrees of freedom affected by arbitrary dipole wakes, feedback damper,beam-beam effects and Landau damping.



$$\psi_{l\alpha} \propto \exp(il\phi + i\chi_\alpha \cos\phi - i\omega_b t)\,;$$

$$\chi_\alpha = \frac{Q' \omega_0 r_\alpha}{c\eta}\;;$$

Starting Equation, single bunch

- In the air-bag single bunch approximation, beam equations of motion can be presented as in Ref [A. Chao, Eq. 6.183]:

$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X$$

where X is a vector of the HT mode amplitudes,

$$(\hat{S} + \hat{Z})_{lm\alpha\beta} = -il\delta_{lm}\delta_{\alpha\beta} - i^{l-m}\frac{\kappa}{n_r}\int_{-\infty}^{\infty} d\omega Z(\omega) J_l(\omega\tau_\alpha - \chi_\alpha) J_m(\omega\tau_\beta - \chi_\beta)$$

$$\kappa = \frac{N_b r_0 R_0}{8\pi^2 \gamma Q_b Q_s}$$

Time is in units of the angular synchrotron frequency.

Damper

- Damper is assumed to see a bunch center of mass and then to kick this bunch as a whole, i.e. it is a space-wise flat response.
- The damper matrix is added to the RHS of the dynamic equation as a special impedance:

$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X$$

$$(\hat{S} + \hat{Z})_{lm\alpha\beta} = -il\delta_{lm}\delta_{\alpha\beta} - i^{l-m} \frac{\kappa}{n_r} \int_{-\infty}^{\infty} d\omega Z(\omega) J_l(\omega\tau_\alpha - \chi_\alpha) J_m(\omega\tau_\beta - \chi_\beta)$$

$$Z_{\text{damper}}(\omega) \propto \delta(\omega) \;\Rightarrow\; \hat{D}_{lm\alpha\beta} = -i^{m-l} \frac{g}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta)$$

g is the damper gain in units of the damping rate.

Damper changes modes structure, blocking their centroids.

Coupled Equidistant Bunches

Unless high-frequency high-Q HOM, wake field of preceding bunches can be taken as flat within the bunch length.

The only difference between the bunches is CB mode phase advance, otherwise they are all identical.

Thus, the CB kick felt by any bunch is proportional to its own offset, so the CB matrix $\hat{C}$ has the same structure as the damper matrix $\hat{D}$:

$$\dot{X} = \hat{S}\cdot X + \hat{Z}\cdot X + \hat{D}\cdot X + \hat{C}\cdot X;$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l}\frac{d_\mu}{n_r}J_l(\chi_\alpha)J_m(\chi_\beta); \qquad \hat{C} = 2\pi i\kappa W(\varphi_\mu)\hat{D}/g_\mu;$$

$$W(\varphi_\mu) = \sum_{k=1}^{\infty} W(-ks_0)\exp(-ik\varphi_\mu); \quad \varphi_\mu = \frac{2\pi(\mu+\{Q_x\})}{M_b}; \quad 0 \le \mu \le M_b - 1\,.$$

Wake and impedance are determined according to A. Chao book.

$$E_0 = 7 \text{ Gev}$$
$$C_0 = 1.1 \text{ km}$$
$$Q_s = 0.008$$
$$N_b = 4 \cdot 10^{11}$$
$$\varepsilon_x = 2.5 \text{ nm}$$
$$\varepsilon_y = 0.04 \text{ nm}$$
$$\sigma_z = 1.5 \text{ cm}$$
$$\delta E / E_0 = 1 \cdot 10^{-3}$$

Example: Argonne APS

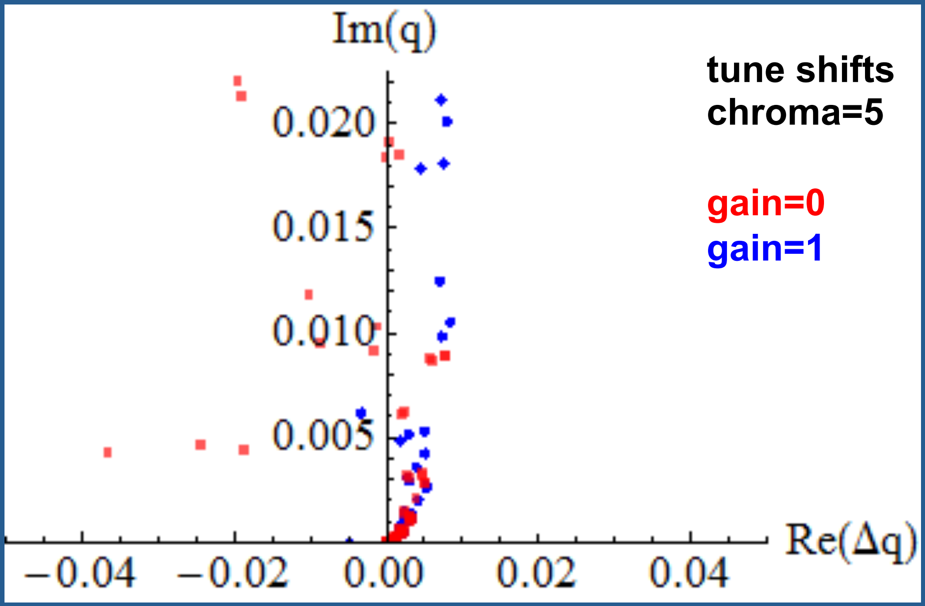
Toy impedance model: broadband resonance with

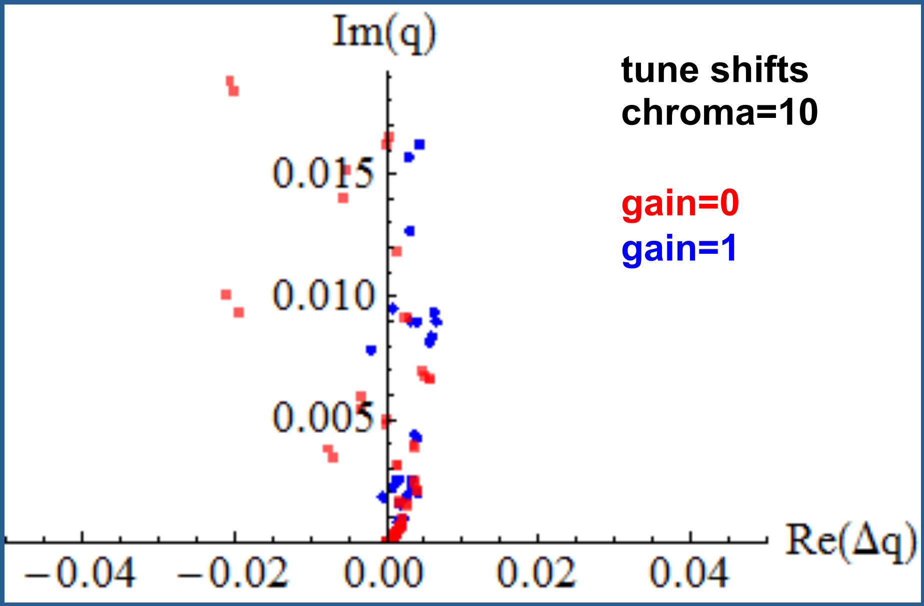
$$f_r = 3\,\text{GHz}; \quad Q_r = 1; \quad R_r \beta = 1\,\text{M}\Omega.$$
$$2\pi f_r \sigma_z / c \approx 1.$$

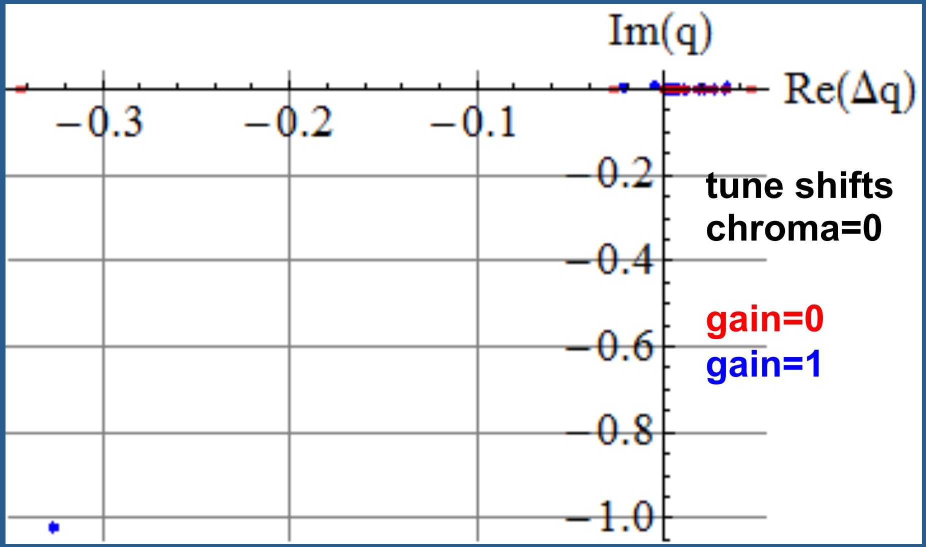
$$Z_\perp(\omega) = \frac{\omega_r}{\omega} \frac{R_r}{1 + iQ_r\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$$

Tune shifts at various gain and chroma

$$q = \frac{\Omega}{\Omega_s}; \quad \Delta q = \mathrm{frac}(q)$$







Note the opposite asymmetries at gain=0 and gain=1.

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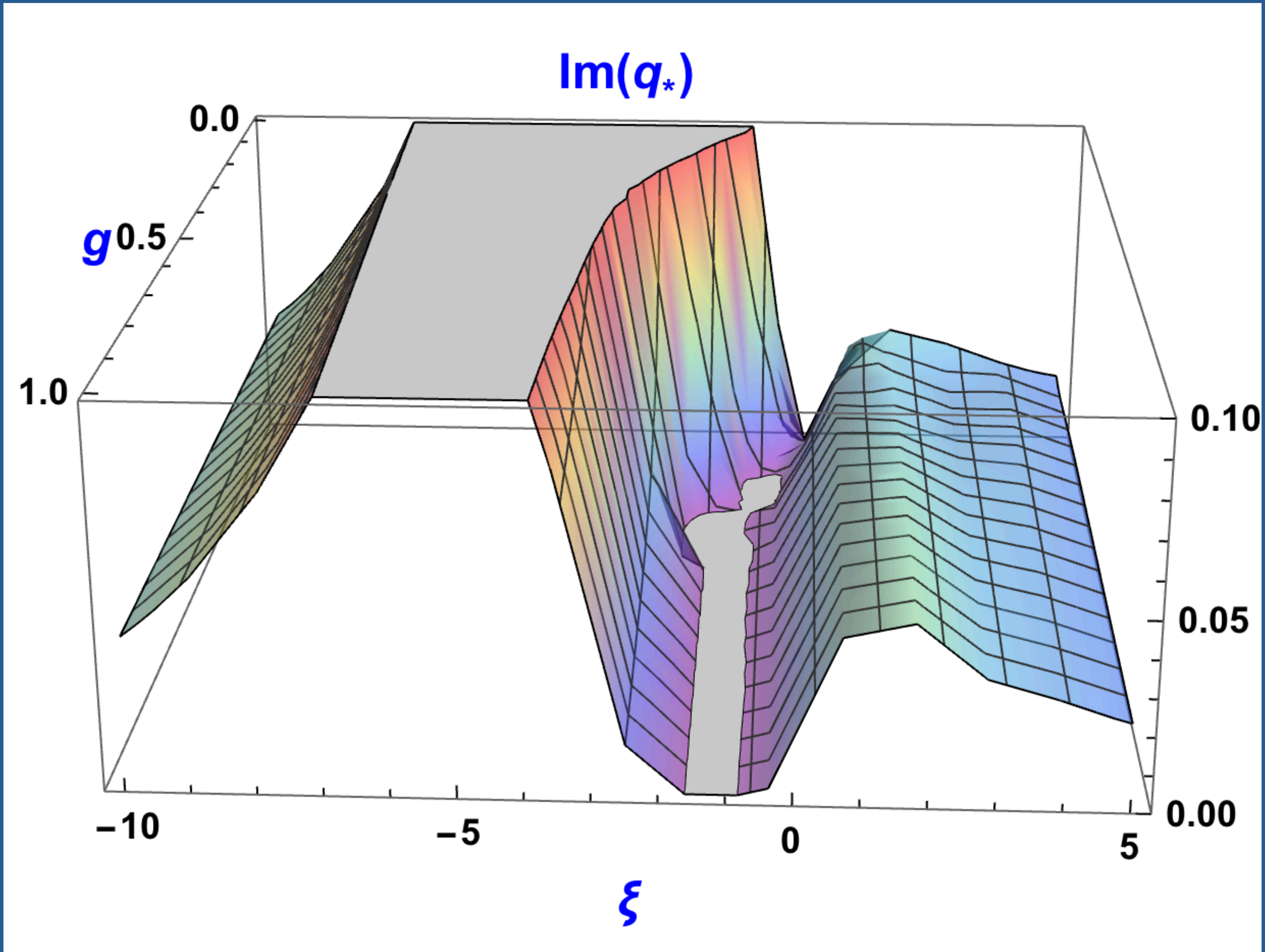
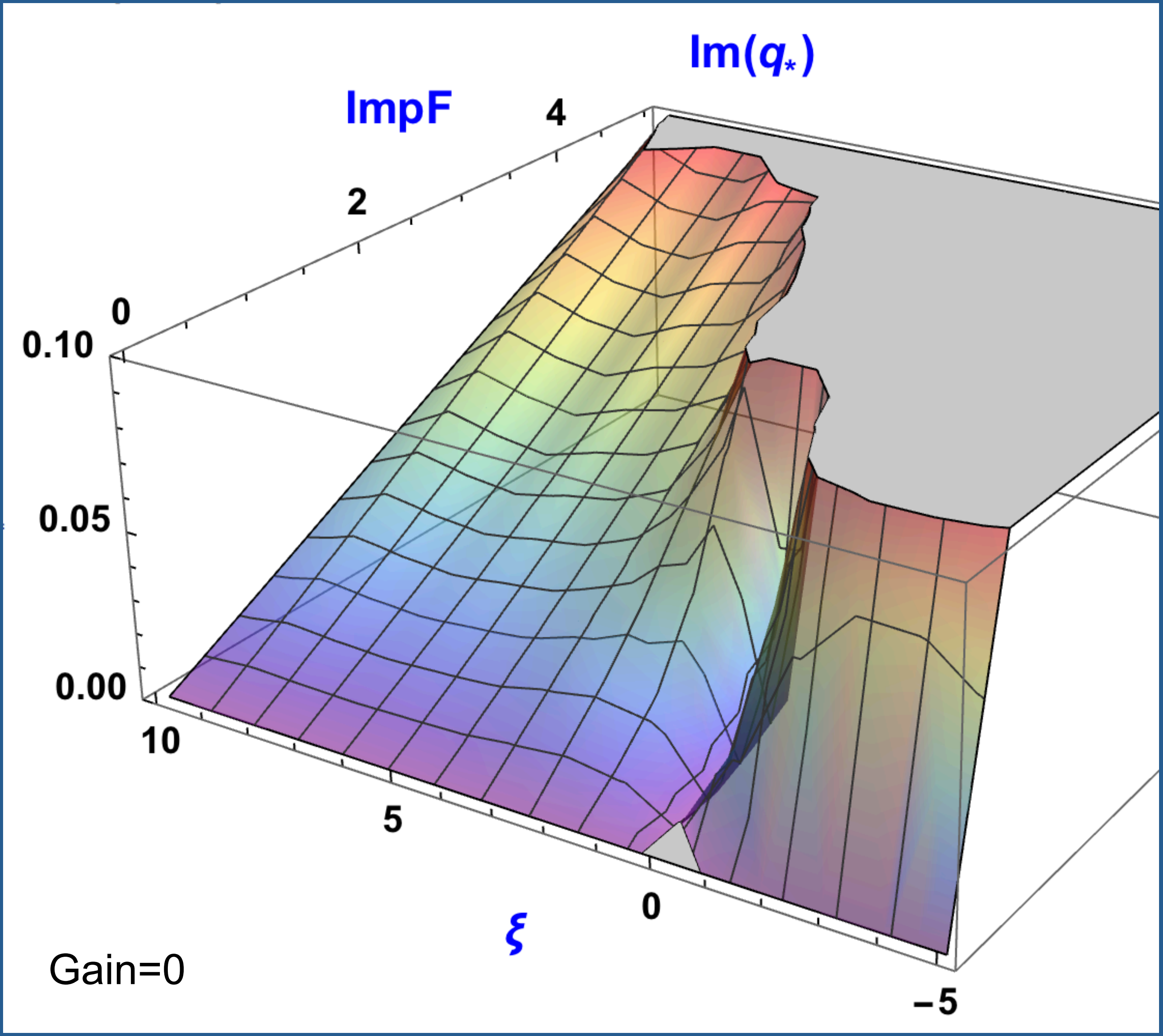
Most unstable mode (MUM) growth rate vs gain and chroma



Gain dependence is very weak except some special regions.

Growth rate vs intensity and chroma, no gain



$\text{ImpF} \equiv N_b / 4E11$

Note TMCI

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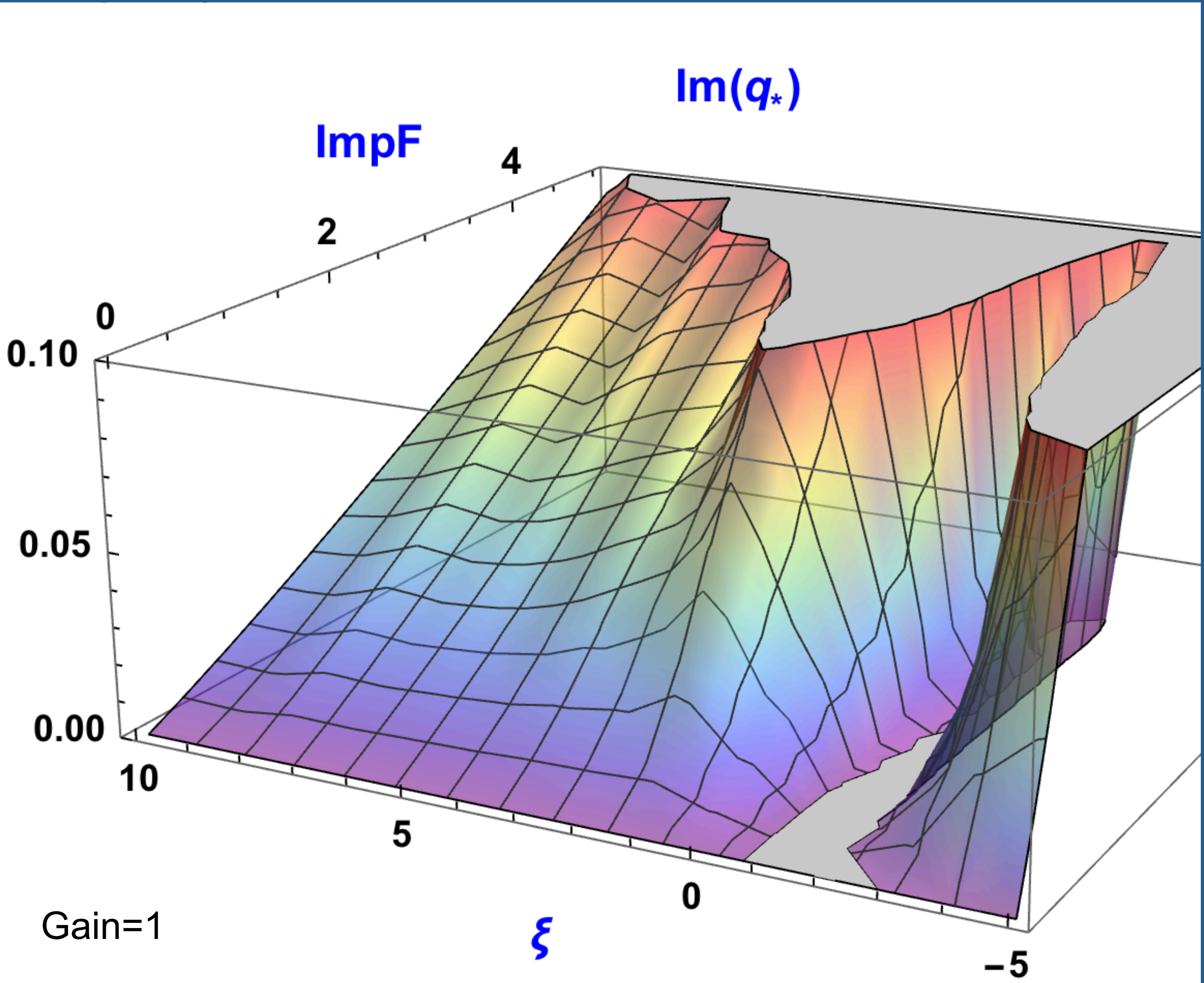
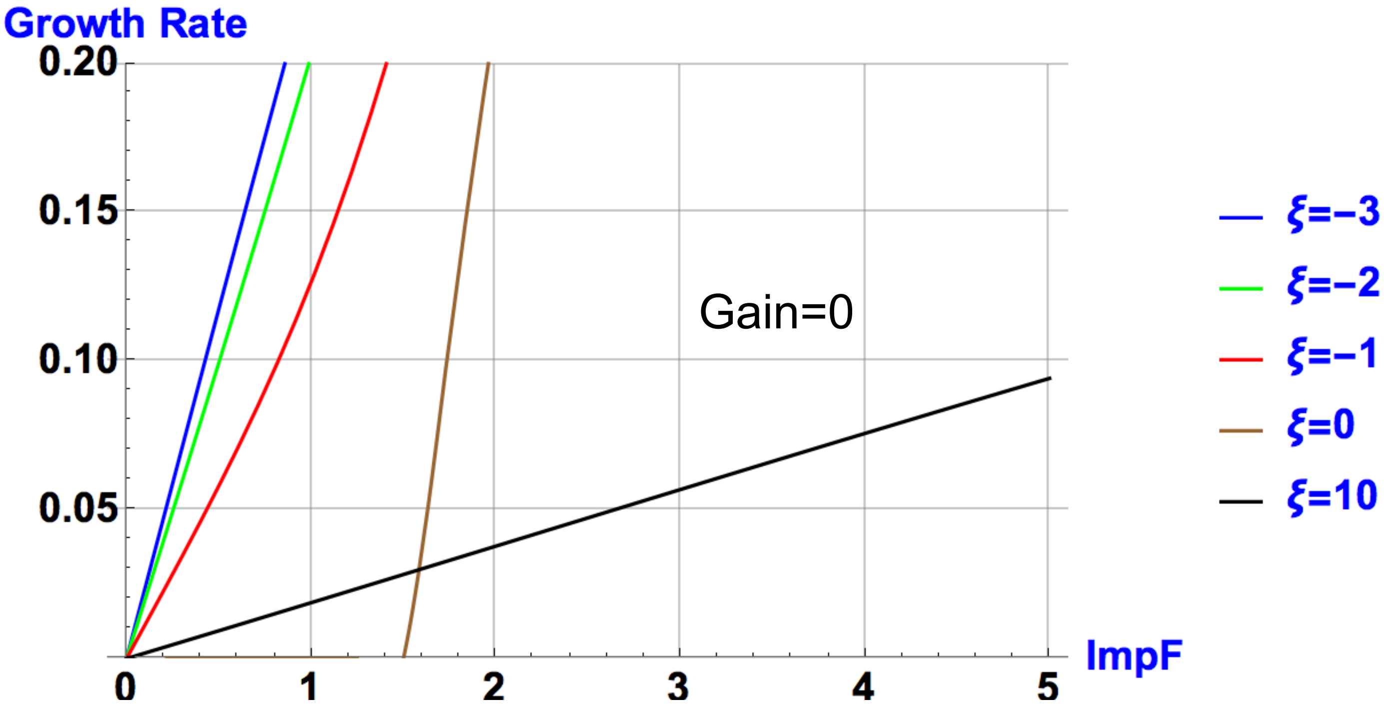
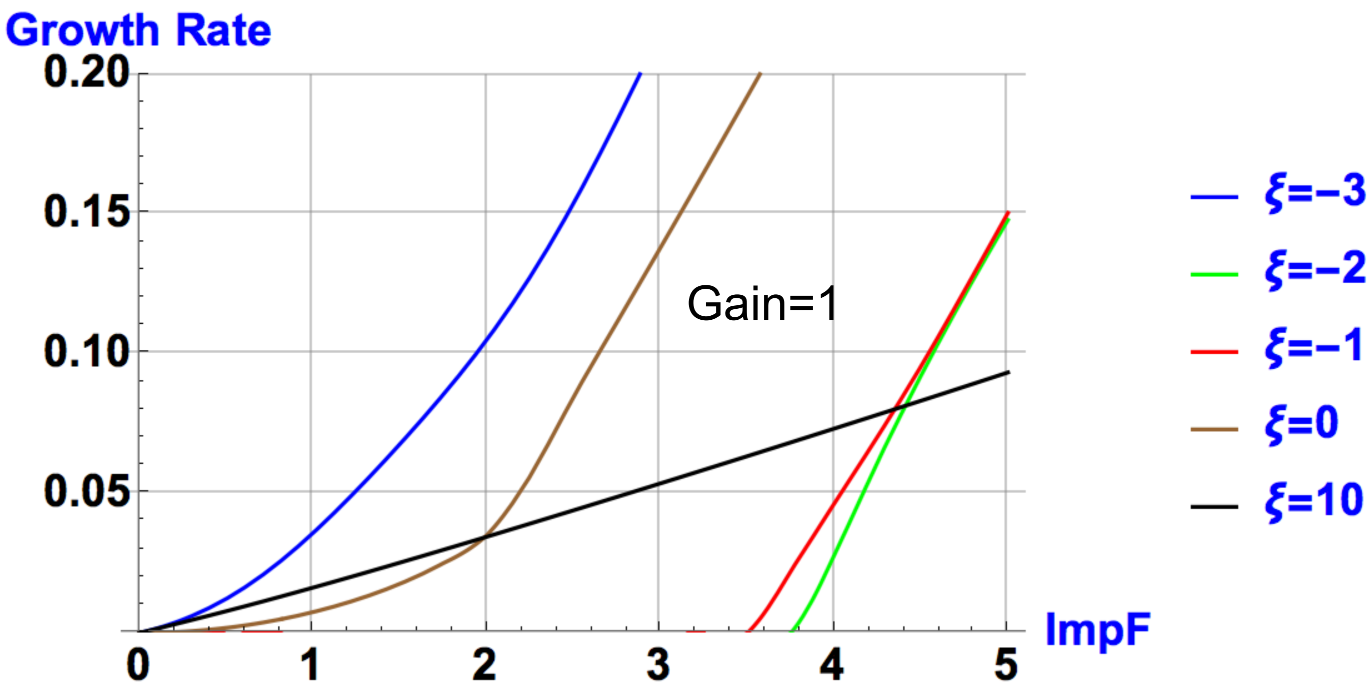
Same, with gain=1



Note a stable area at slightly negative chromaticity





Growth rate vs intensity for various chromaticity, without gain (top) and with it (bottom).

Note increase of the TMCI threshold at chroma = -1, -2.

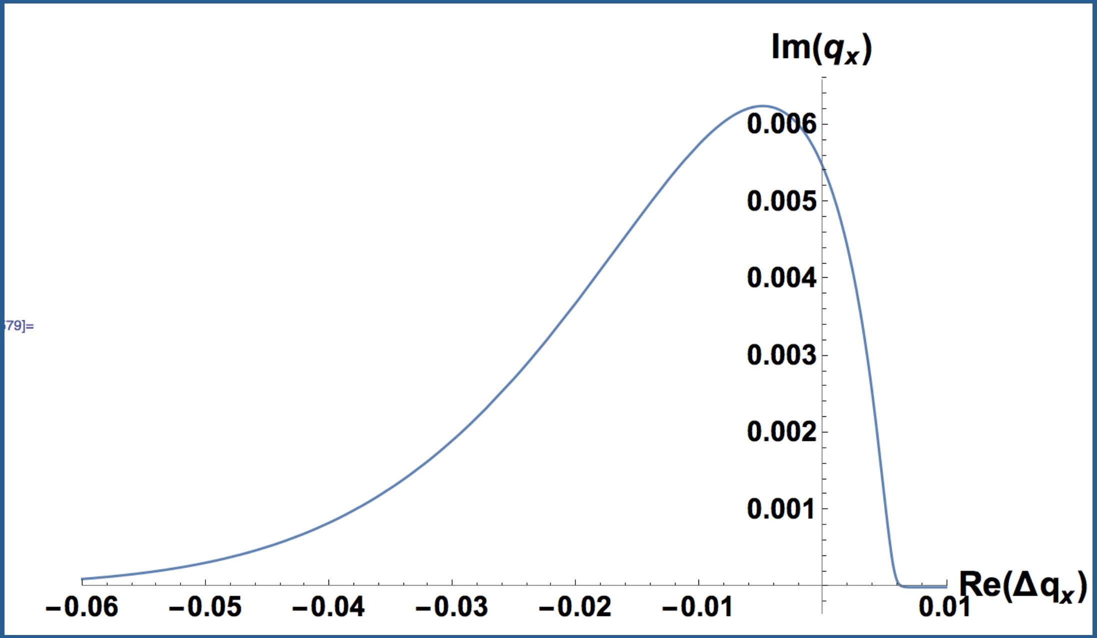
Transverse-to-Transverse (TT) Stability Diagrams

Stability diagram (SD) is defined as a map of real axes ν on the complex plane:

$$D_l(\nu) = \left(-\int \frac{J_x \partial F / \partial J_x}{\nu - l - \delta q_x + io} d\Gamma \right)^{-1}; \quad \delta q_x = \frac{\delta \omega_x}{\omega_s}; \quad \int F d\Gamma = 1; \quad d\Gamma = dJ_x \cdot dJ_y \cdot dJ_z$$

$$\delta q_x = A_{xx} J_x / \varepsilon_x + A_{xy} J_y / \varepsilon_y$$
$$\langle J_i \rangle = \varepsilon_i$$

To be stable, all the coherent tune shifts q have to be below the SD.



Example matrix A_{ik} was provided by Vadim Sajaev.

Since normally $\varepsilon_x \gg \varepsilon_y$, the TT diagrams are very asymmetric. The same happens when the signs of A_{xk} are identical.

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Longitudinal-to-Transverse (LT) Landau Damping

Stability diagram can be estimated as modified maps

$$\mathrm{D}_l(\nu) = \left(-\int \frac{J_x \partial F / \partial J_x}{\nu - l - l\delta q_s - \delta q_x + io} d\Gamma \right)^{-1}$$

In the RF bucket, $\delta q_s < 0$, so the diagram is shifted to the left for positive l and inversely.

With potential well distortion, the effect could be stronger and of the opposite sign, but still asymmetric and proportional to l.

Thus, bucket nonlinearity does not seem to help much: some modes could be even destabilized with or without the bucket distortion.

How to repair Landau damping?

With that asymmetry of SD, what can be suggested for the stabilization?

- Octupoles at high β_y (sign matters!)
- With the damper ON, make the nonlinear tune shifts > 0
- High second-order chromaticity $Q''_{x,y}$ with the proper sign and sufficient spread $Q''_{x,y}(\delta p / p)^2 / 2$ (octupoles and skew-octupoles at high-dispersion area).

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Summary

- Dampers can increase TMCI threshold , factor of 2 or so, with a slightly negative chromaticity (above transition).

- At positive chromaticity (i.e. conventional one), damper enhances presence of unstable modes with positive coherent tune shifts, reducing those with negative tune shift.

- Due to relatively low vertical emittance, the TT stability diagram is normally very asymmetric, which may dramatically reduce LD

- To repair LD, several measures can be applied:
  - High beta_y octupole;
  - Positive-side SD with the damper ON;
  - Q'' of the proper sign.

Many thanks!

Backup slides

E-Cloud in Positron Rings

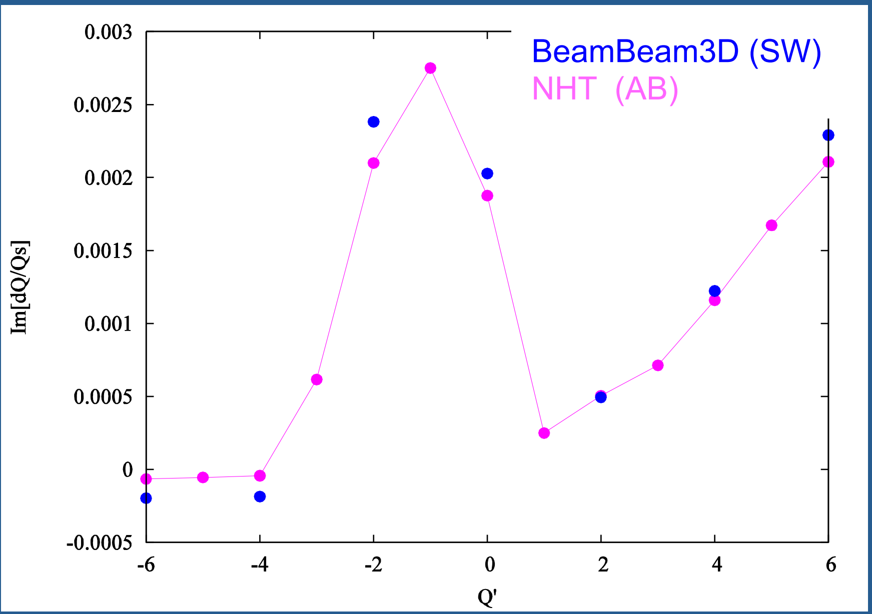
- E-cloud introduces both impedance and frequency spread, so by itself it both stabilizes and destabilizes, fighting with itself, see my estimations on the both factors at "Three-beam instability in the LHC" (2012).

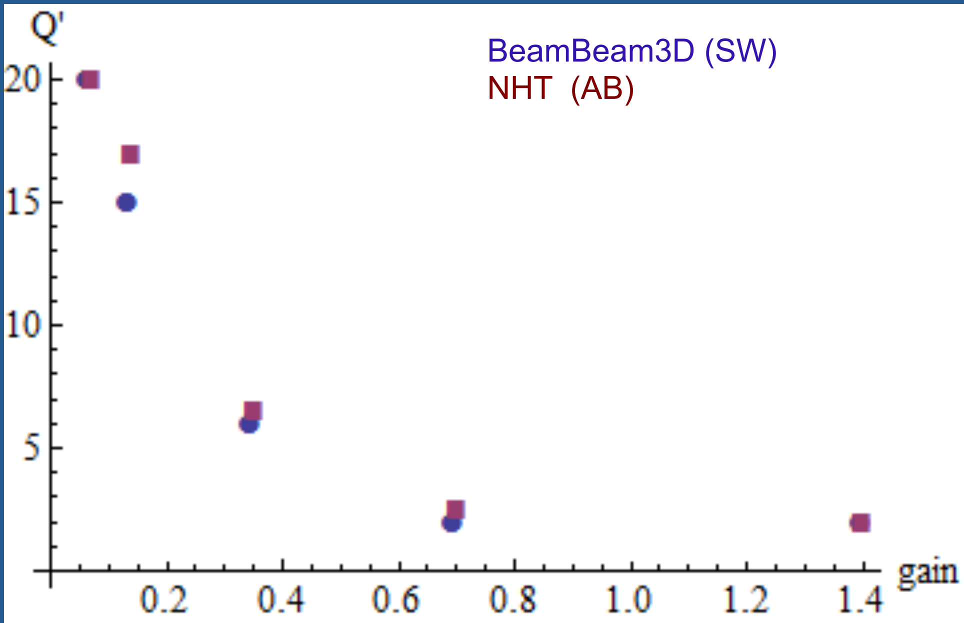
- Those not very reliable estimations tell that by itself e-cloud cannot drive weak head-tail instability: its frequency spread wins over its impedance.

- Independently of that, e-cloud can lead to the weak HT when its nonlinearity cancels one of the octupoles. Then the stability diagram collapses, and even a low wake could drive the instability, see "Three beam…"

- Thus, simulation with e-cloud must take into account machine nonlinearity. With e-cloud, it can surprisingly destabilize the beam.

Benchmarking: NHT vs BeamBeam3D (S.White)





Highest growth rates for

single beam, single bunch,
maximal gain and nominal impedance

Threshold chromaticity vs gain for

two single-bunch LR-colliding beams,
end of the squeeze parameters,
no octupoles.

BeamBeam3D data – ICE mtg, 07/11/2012.

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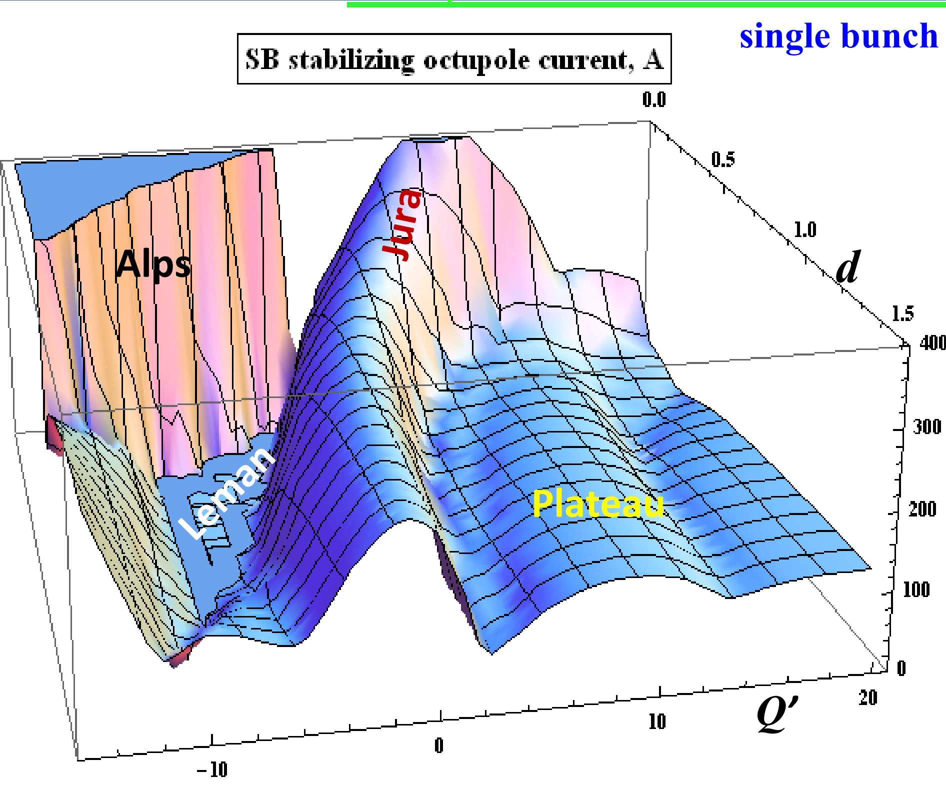
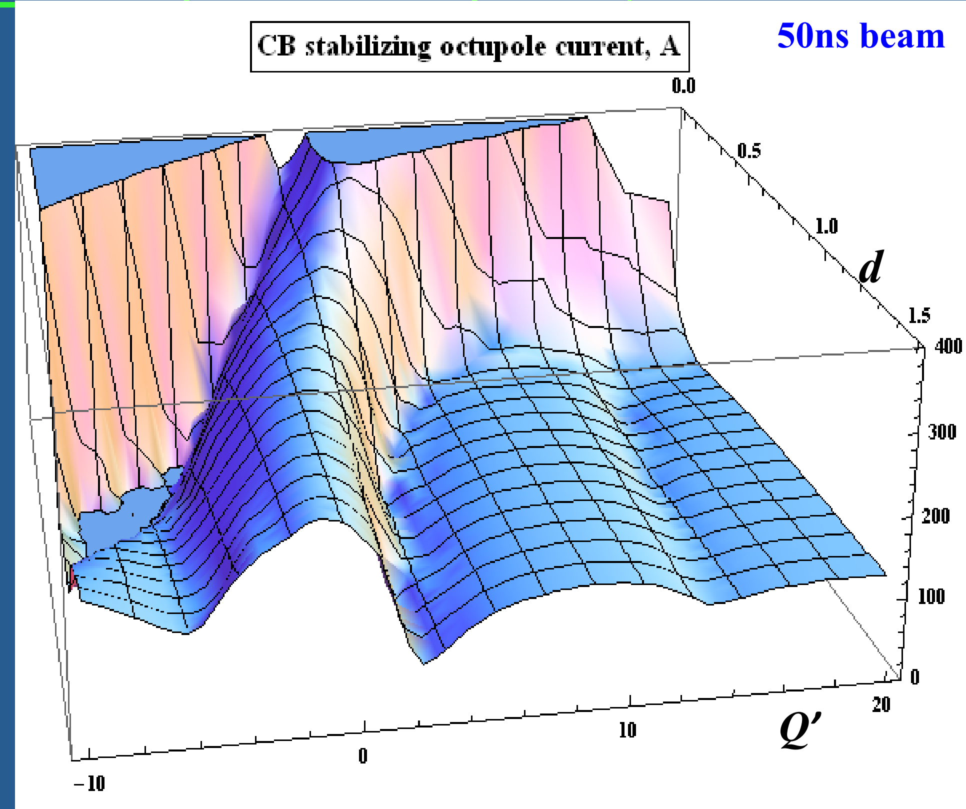
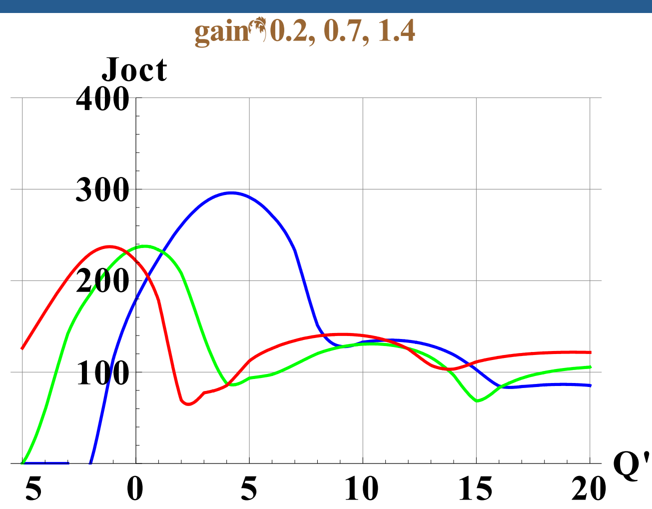
Couple Bunch Factor: LO+, bbb ADT, 2Imp

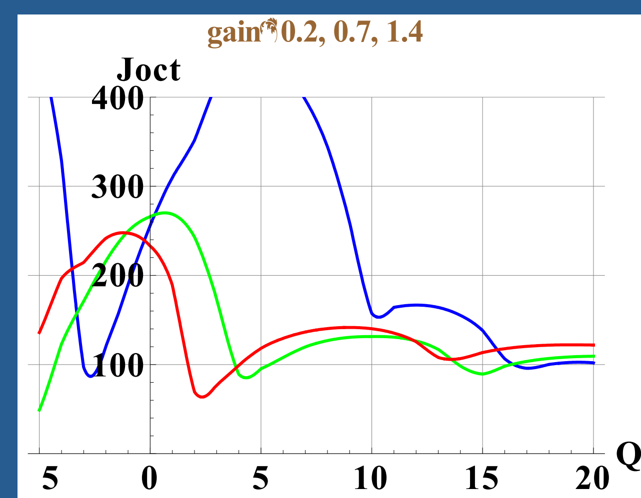




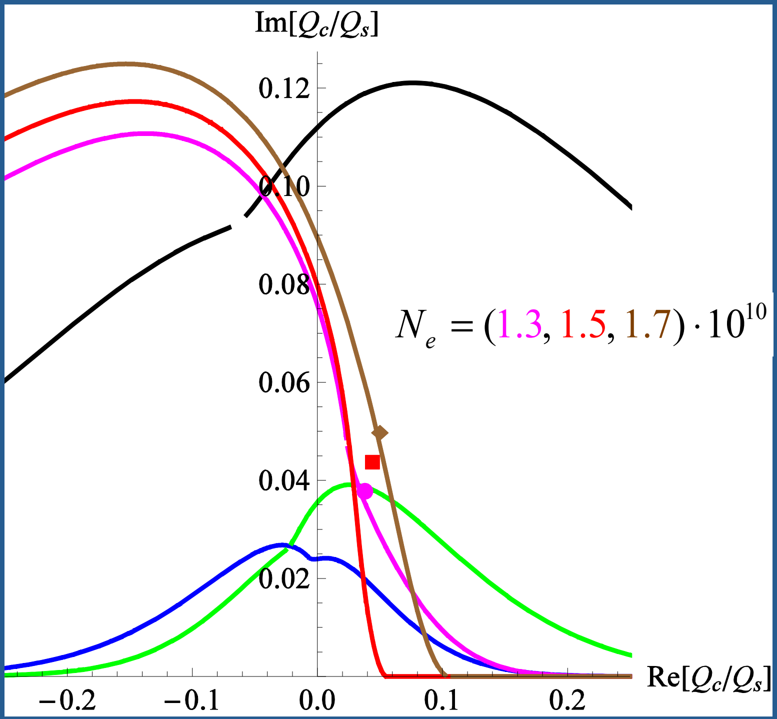


Almost no difference at Plateau.

All the plots – for 1.5E11 p/b, emittances of 2μm, 50ns beam.



Beam-Beam-Beam Effect in LHC



LO=200A – computed threshold

(Pacman) BB only, LO=0

BB and LO=500A

BB, LO=500A, dQe0=6.0E-4

BB, LO=500A, dQe0=8.0E-4

BB, LO=500A, dQe0=1.0E-3

Markers – MUMs, colors correspond

Instability is driven by e-cloud attracted by 2 beams in the high-beta area of IR1&5.

It happens due to a right-collapse of the SD + low-frequency e-wake with positive coherent tune shifts.

Electron wake:

$$W(\tau)\ ;\ W_0 \sin(\omega_e \tau)\exp\left(\omega_e \tau / 2Q\right);$$

$$W_0 = \frac{N_e r_e c}{4\sigma_\perp^4 \omega_e}, \qquad Q \sim 3-5,\ \ \tau < 0$$

$$\psi_e \equiv \omega_e \sigma_z / c = \begin{cases} 6.5 \text{ rad} & \text{for } \beta\text{=300m} \\ 1.4 \text{ rad} & \text{for } \beta\text{=4km} \end{cases}$$

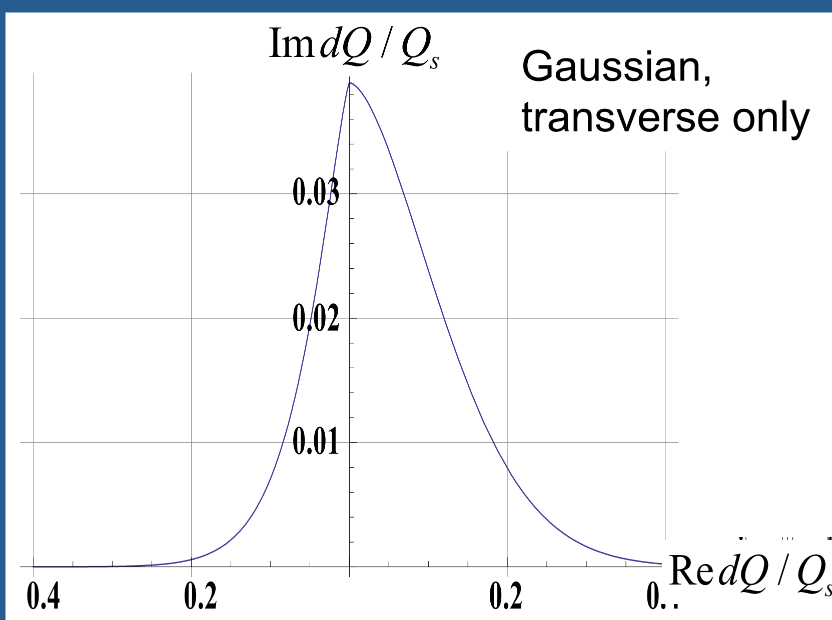
Analysis of solutions

1. For every given gain and chromaticity, the eigensystem is found for the provided impedance tables or functions.

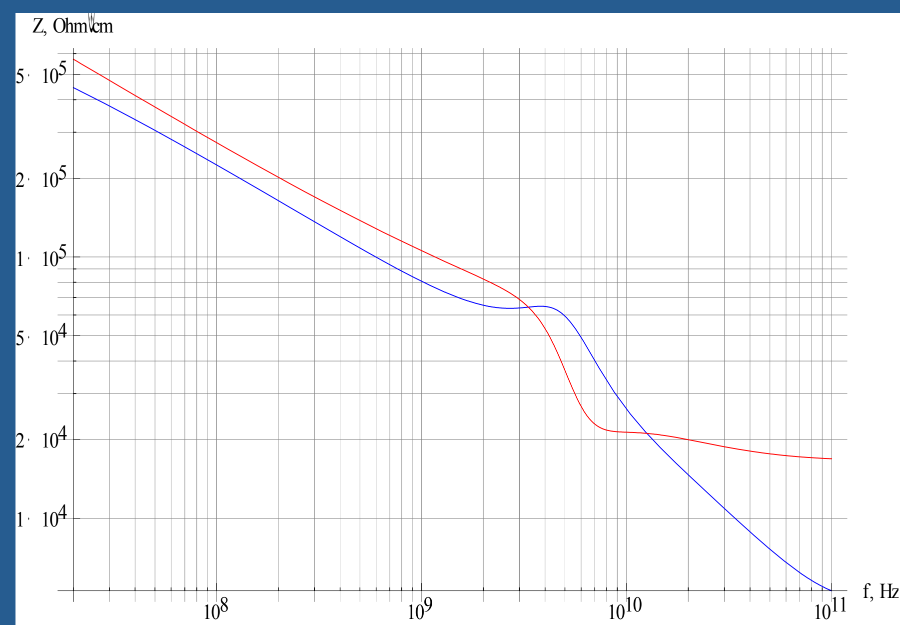
2. The complex tune shifts are found from the eigenvalues $\Delta\Omega_{l\alpha} = \Omega_{l\alpha} - l$

3. The stabilizing octupole current is found from the stability diagram for every mode, then max is taken.

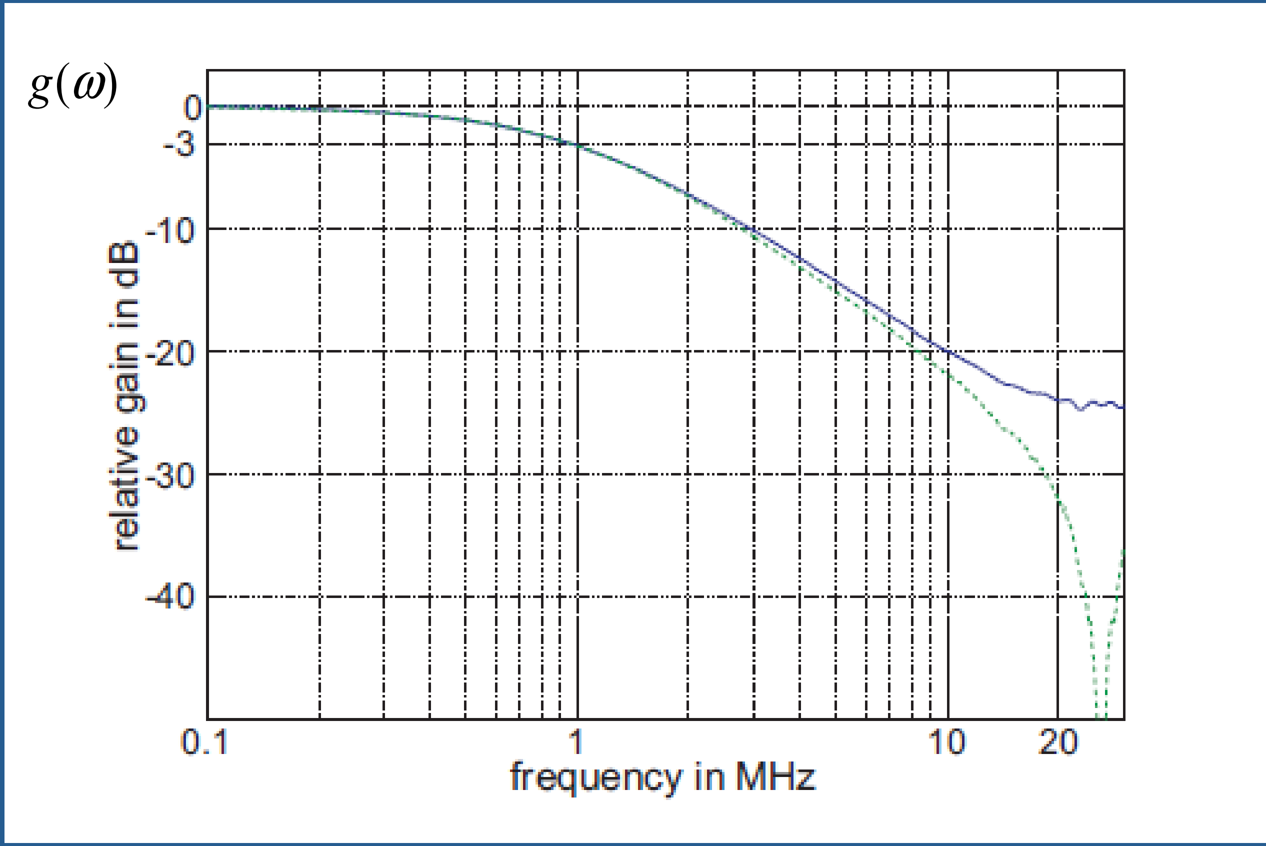
Stability diagram at +200 A of octupoles



LHC Horizontal Impedances (N. Mounet)



Old damper gain



Old narrow-band ADT gain profile (W. Hofle, D. Valuch) .
At 10 MHz it drops 10 times. The new damper is bbb for 50ns beam.

Below gain is measured in omega_s units, max gain=1.4 is equivalent to 50 turns of the damping time.

CB Mode Damping Rate

With $g(\omega)$ as the frequency response function of the previous plot, the time-domain damper's "wake" is

$$G(\tau) = \int_0^\infty g(\omega)\cos\left(\omega\tau\right)d\omega/\pi\ ,$$

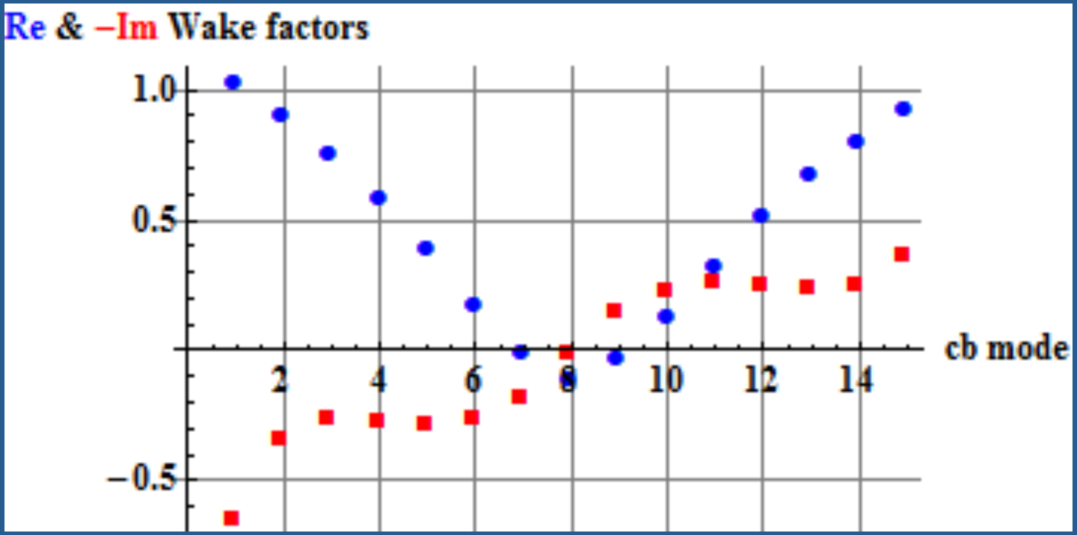
assuming this response to be even function of time (no causality for the damper!).

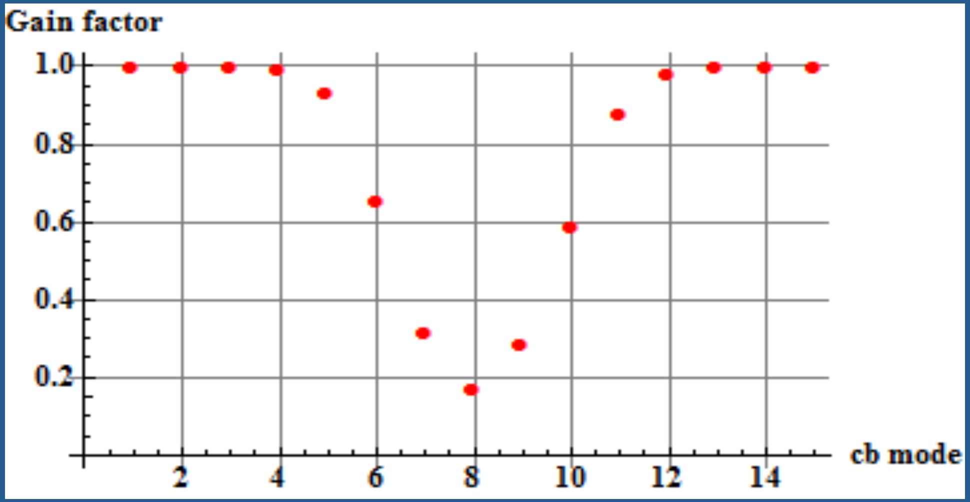
From here (equidistant bunches!):

$$d_\mu = d\,\frac{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)\cos(k\varphi_\mu)}{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)}\ ;$$

where d is the rate provided for low-frequency CB zero-head-tail modes at zero chromaticity.

CB Wake and Gain Factors for the Old ADT





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