Review of Collective Effects in Low Emittance Rings

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Introduction

Recently there has been much development in the design of storage rings with very low emittances, and projects include MAX IV, APS, ESRF, Spring 8, Sirius, … A so-called "ultimate storage ring" indicates one with emittances that are diffraction limited at 1 angstrom wavelength.

Among the collective effects that become important for such rings are: intrabeam scattering (IBS) that limits the emittances that are achievable and the Touschek effect that shortens the beam lifetime; there are impedance driven effects, like the microwave instability, the single-bunch, transverse mode coupling (TMCI) instability, and the multi-bunch transverse instability; one needs also to consider the fast ion instability and possibly space charge effects

Will review these effects, and use as example the PEP-X designs, a 4.5 GeV ring designed for the PEP tunnel:

A: 2010 (SLAC-PUB-13999), ε_x= 165 pm, ε_y= 8 pm, I= 1.5 A, lattice TME + DBA "Baseline"

B: 2012 (SLAC-PUB-14785), ε_x= ε_y= 12 pm (round beam), I= 0.2 A, lattice 7BA + damping wigglers, (following MAX IV's pioneering ideas) "Ultimate"

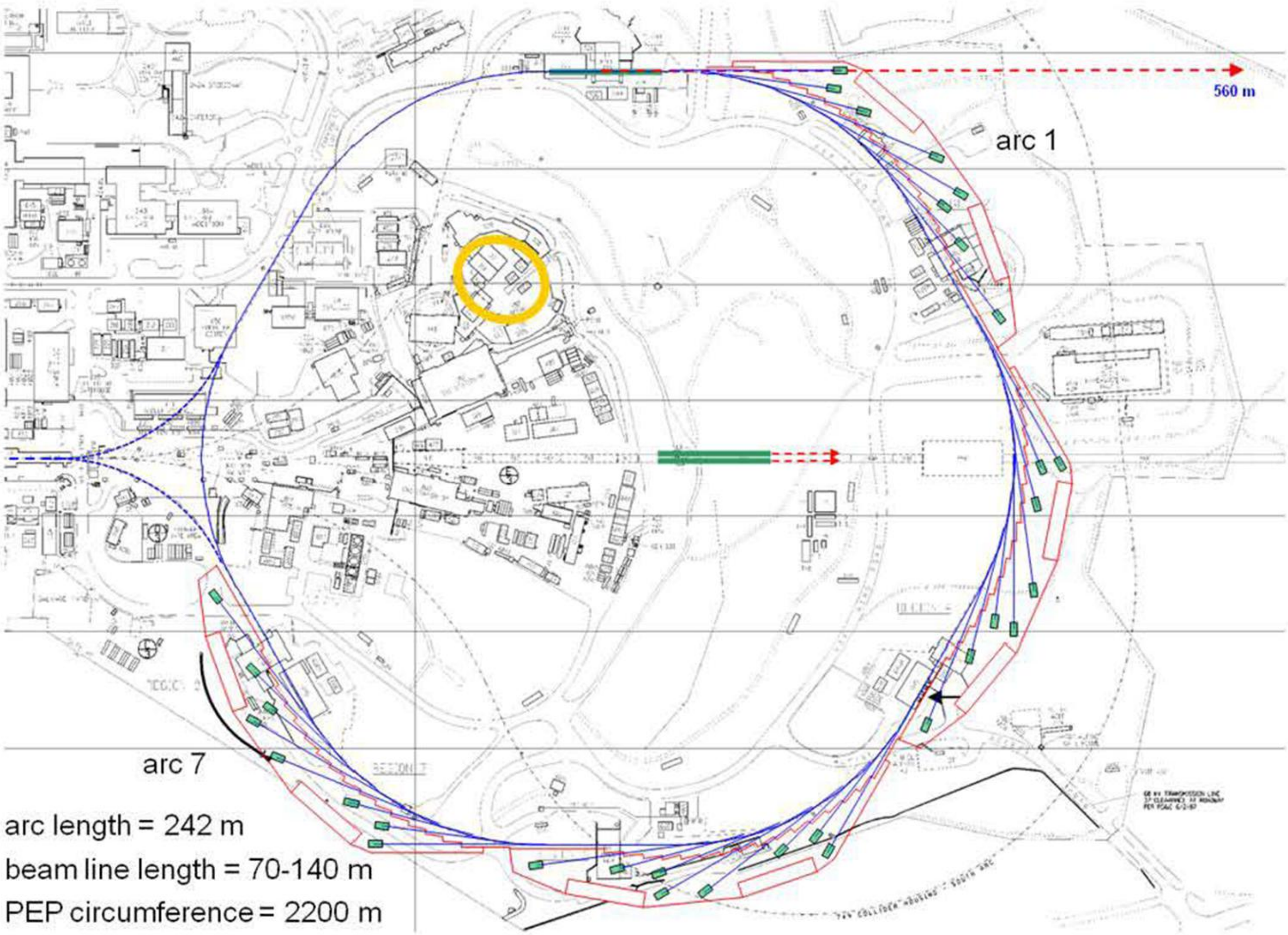
Outline

- Intra-beam scattering (IBS)
- Touschek lifetime
- Longitudinal microwave instability
    - --Impedance budget and pseudo-Green function wake
    - --Instability, simulations
    - --Coherent synchrotron radiation (CSR)
- Other instabilities
- Conclusions

As specific example illustrating the above topics I will use cases of PEP-X, a study effort of a group led by Y. Cai (SLAC-PUB-14785, Mar. 2012)

Collective effect contributors included G.Stupakov, Y. Cai, L. Wang, M. Borland, …

PEP-X: An Ultimate Storage Ring



Yunhai Cai, SLAC

Selected Parameters for PEP-X

Parameter	A	B	Units
Energy, E	4.5	4.5	GeV
Circumference, C	2199	2199	m
Average current, I	1.5	0.2	A
Bunch population, N_b	2.18	0.28	10^{10}
Number of bunches, M	3154	3300	
Rel. rms energy spread, σ_p	1.14	1.2	10^{-3}
Rms bunch length, σ_z	3.0	3.0	mm
Nominal emittance, ε_0	85.7	11.0	pm
Momentum compaction, α	5.8	5.0	10^{-5}
Synchrotron tune, ν_s	7.7	6.9	10^{-3}
Horiz. rad. damping time, τ_x	13.5	19.	ms
Long. rad. damping time, τ_p	7.2	12.	ms

A—Baseline, B—Ultimate. Note that the nominal horizontal emittance $\varepsilon_{x0} = \varepsilon_0/(1+\kappa)$, with κ the x-y coupling parameter

Intra-Beam Scattering (IBS)

IBS describes multiple scattering that leads to an increase in all bunch dimensions and in energy spread. In low emittance e^- rings IBS increases the steady-state beam dimensions

In low emittance rings, IBS is typically what limits the emittance at design current, or conversely sets the design current

Theory of IBS initially developed by Brueck and LeDuff (1965). More systematically developed by Piwinski (1974), Bjorken-Mtingwa (1983, using quantum mechanical scattering theory), Martini (1984, modification of Piwinski's formulation).

IBS theory was originally developed for proton machines. The predictions for growth rates seem to agree well for e.g. protons in the Tevatron (Lebedev 2005) and heavy ions in RHIC (Fedotov et al 2006)

Bjorken-Mtingwa (BM) Formulation

IBS (amplitude) growth rates (i= x, y, or p):

$$\boxed{\begin{aligned}\frac{1}{T_i} &= 4\pi A(\log)\left\langle \int_0^\infty \frac{d\lambda\, \lambda^{1/2}}{[\det(L+\lambda I)]^{1/2}} \left\{ \right.\right. \\ & TrL^{(i)}Tr\left(\frac{1}{L+\lambda I}\right) - 3TrL^{(i)}\left(\frac{1}{L+\lambda I}\right) \left.\left.\right\}\right\rangle\end{aligned}}$$

with

$$A = \frac{r_0^2 cN}{64\pi^2 \bar{\beta}^3 \gamma^4 \epsilon_x \epsilon_y \sigma_s \sigma_p}$$

$$L = L^{(p)} + L^{(x)} + L^{(y)} \quad ,$$

$$L^{(p)} = \frac{\gamma^2}{\sigma_p^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ,$$

$$L^{(x)} = \frac{\beta_x}{\epsilon_x}\begin{pmatrix} 1 & -\gamma\phi_x & 0 \\ -\gamma\phi_x & \gamma^2\mathcal{H}_x/\beta_x & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$L^{(y)} = \frac{\beta_y}{\epsilon_y}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma^2\mathcal{H}_y/\beta_y & -\gamma\phi_y \\ 0 & -\gamma\phi_y & 1 \end{pmatrix}$$

(log) is Coulomb log

r_0 is classical radius of electron

$\bar{\beta} = v/c$

$H_x = [\eta_x^2 + (\beta_x \eta_x' - \beta_x' \eta_x/2)^2]/\beta_x$

$\phi_x = \eta_x' - \beta_x' \eta_x / 2\beta_x$

Finding T_i^{-1} means performing a type of elliptical integral at all lattice positions

Nagaitsev algorithm for evaluating elliptical integrals speeds up the calculation a factor of ~25 (in Mathematica)

Expected accuracy ~1/(log). For low emittance electron rings (log)~ 10

Vertical emittance in a ring is usually due to either η_y, produced by orbit errors, and/or by *x*-*y* coupling. Formulas so far have been without coupling. IBS with coupling is described in A. Piwinski (1991), B. Nash et al (2002), V. Lebedev (2005).

Steady-State Emittances

In electron machines the IBS growth is counteracted by synchrotron radiation damping (with $\tau_i^{-1} >> T_i^{-1}$), leading to increased stead-state emittances

Steady-state IBS emittance and energy spread with coupling (A. Xiao):

$$\epsilon_x = \frac{\epsilon_{x0}}{1 - \tau_x^*/T_x}, \qquad \sigma_p^2 = \frac{\sigma_{p0}^2}{1 - \tau_p/T_p}$$

with $\tau_x^* = \tau_x/(1+\kappa\tau_x/\tau_y)$ and $\varepsilon_y = \kappa\varepsilon_x$.

Solution involves (i) integration at every lattice element to obtain T_i^{-1}, (ii) averaging around the ring, (iii) solving the above equations simultaneously (e.g. using Newton’s method)

Programs that solve IBS (mostly BM formulation) are ZAP, SAD, MAD-X, Elegant, …

SAD treats the three axes equally and includes coupling (e.g. *x*-*y*, *x*-*p*) in a general way by diagonalizing to normal modes (K. Oide)

Simplified Model of IBS

(K. Bane, EPAC02)

Longitudinal growth rate:

$$\frac{1}{T_p} \approx \frac{r_e^2 c N_b (\log)}{16\gamma^3 \epsilon_x^{3/4} \epsilon_y^{3/4} \sigma_z \sigma_p^3} \left\langle \sigma_H \, g(a/b) \; (\beta_x \beta_y)^{-1/4} \right\rangle$$

$= <\delta(1/T_p)>$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{\mathcal{H}_x}{\epsilon_x}\,, \quad a = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_x}{\epsilon_x}}\,, \quad b = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_y}{\epsilon_y}}$$

$$g(\alpha) = \alpha^{(0.021 - 0.044 \ln \alpha)}$$

Transverse growth rate:

$$\frac{1}{T_x} = \frac{\sigma_p^2}{\epsilon_x} \langle \mathcal{H}_x \delta(1/T_p) \rangle$$

Valid for a, b<< 1, “high energy approximation”

Solution for PEP-X

- For PEP-X consider round beam, κ= 1

I [mA]	ε_x [pm]	ε_y [pm]	σ_p [10^{-3}]	σ_z [mm]
0	5.5	5.5	1.20	3.0
200	11.5	11.5	1.25	3.1

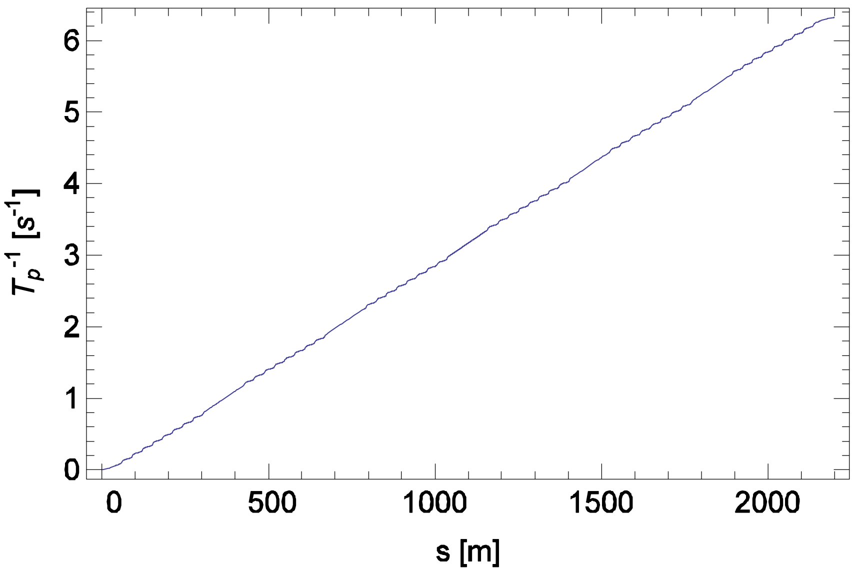
Table. Steady-state beam properties in PEP-X at zero current and nominal current. Results were obtained using the Bjorken-Mtingwa (B-M) formalism.

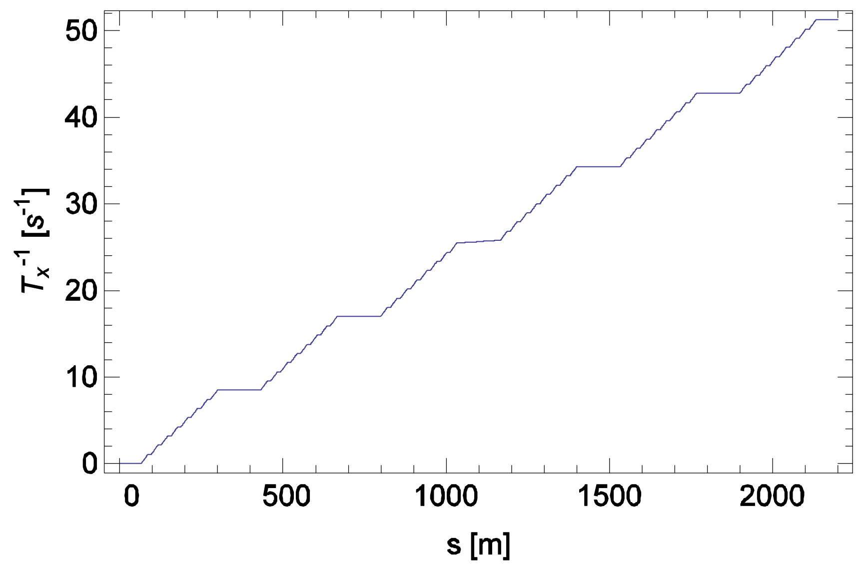
- Note: almost no growth in p or z

- In nominal configuration T_x^{-1}= 52. s^{-1}, T_p^{-1}= 7.4 s^{-1} (simplified model gets T_x^{-1}= 53.7 s^{-1}, T_p^{-1}= 8.9 s^{-1})

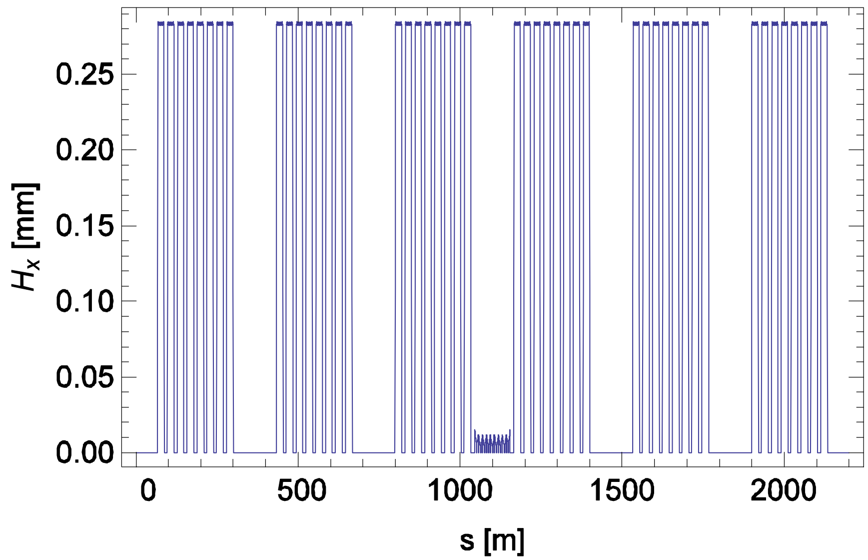
- Checked with SAD, an optics program that treats coupling without simplifying assumptions. In dispersion-free regions quad strengths were adjusted to bring tunes close together; then 800 of these quads were rotated by small random amount and scaled to give $\varepsilon_{x0} \approx \varepsilon_{y0}$; repeated for 10 seeds; results $\varepsilon_x \approx \varepsilon_y \approx$ 11 pm (K. Kubo)

Accumulated IBS Growth Rates

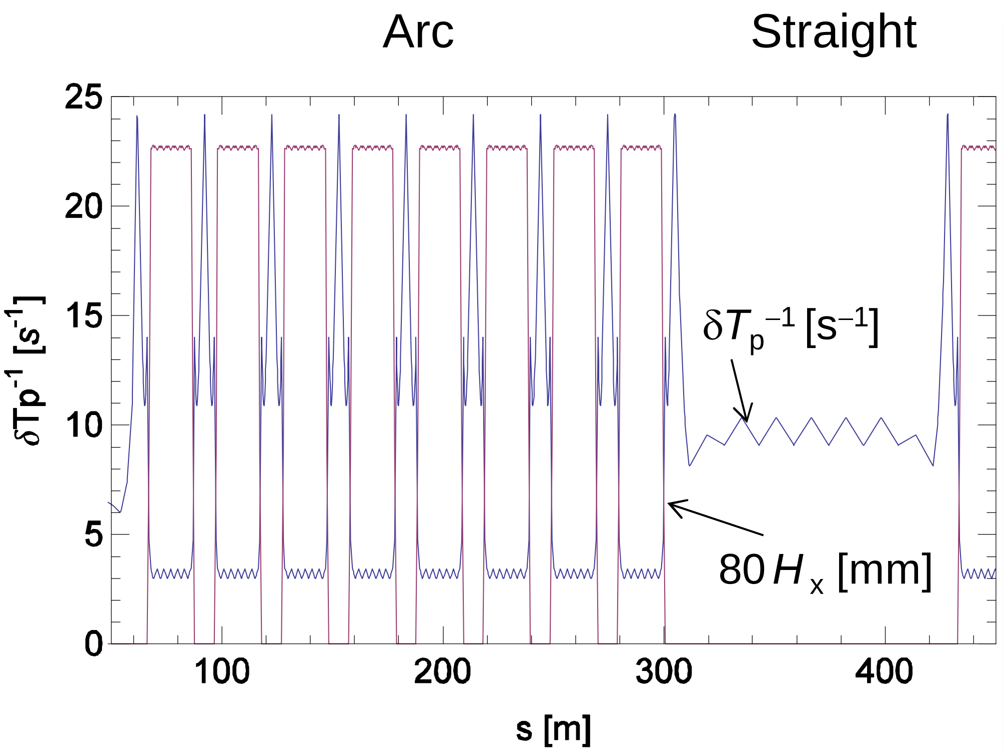




Accumulated growth rates in p, x;
H_x optics function



Correlation between H_x and $\delta(1/T_p)$ in PEP-X



H_x and $\delta(1/T_p)$ over one arc and one straight of PEP-X

Note the anti-correlation of the two functions in arc

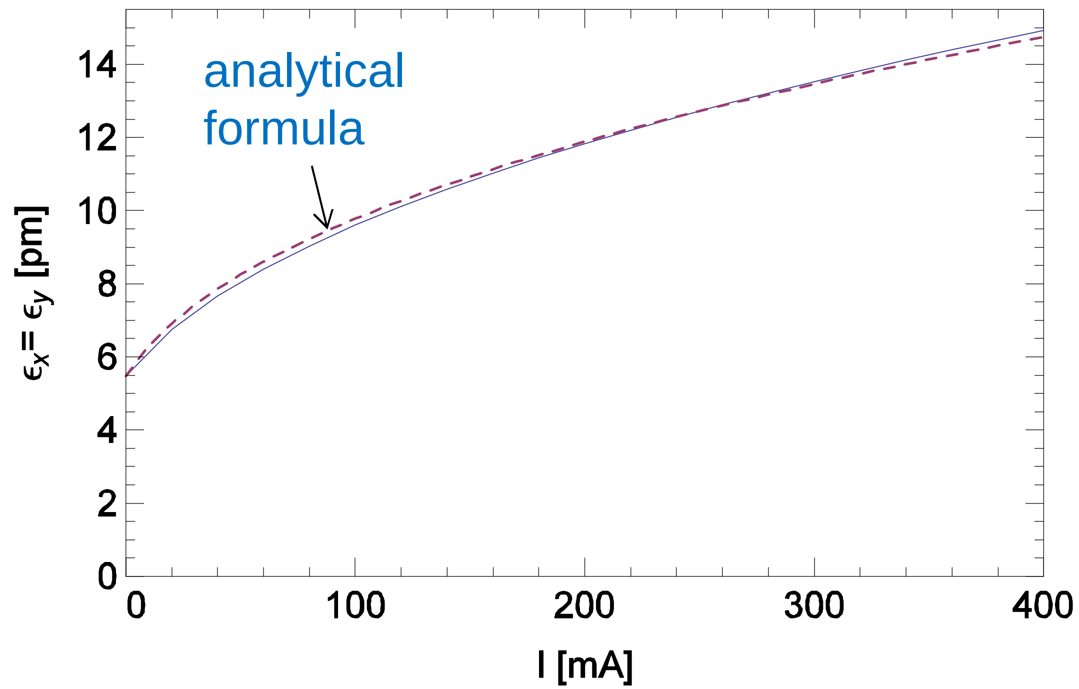
With no correlation but "same" lattice parameters, $1/T_x$ would be twice as large

Emittance Dependence on Current

- With T_p^{-1} small, from simplified model can show that steady-state emittances can be approximated by

$$\left(\frac{\epsilon_x}{\epsilon_{x0}}\right)^{5/2} - \left(\frac{\epsilon_x}{\epsilon_{x0}}\right)^{3/2} = \alpha\left(\frac{I}{I_A}\right)$$

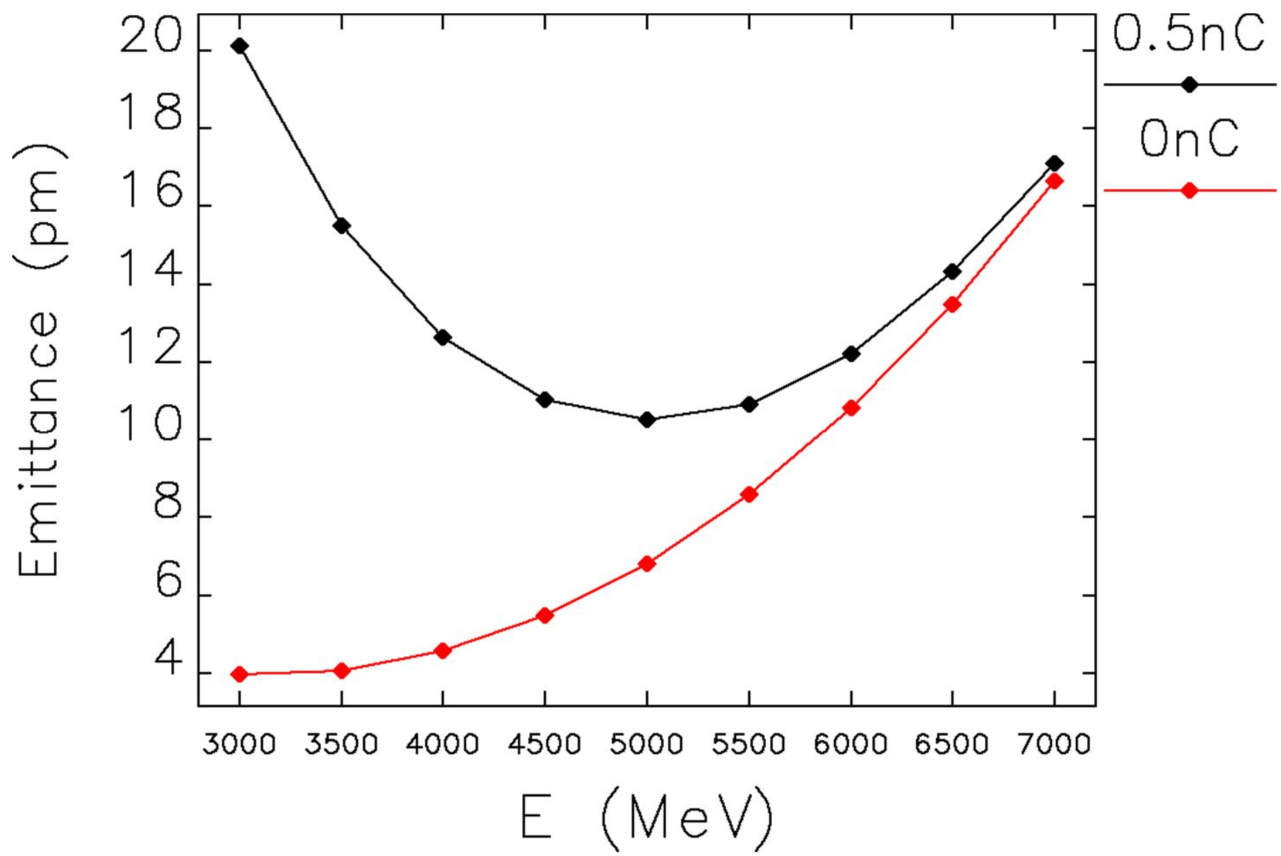
with I_A= 17 kA and α a constant



Steady-state emittances as function of bunch current in PEP-X. The dashed curve gives the analytical approximation.

Dependence on Energy

(M. Borland)



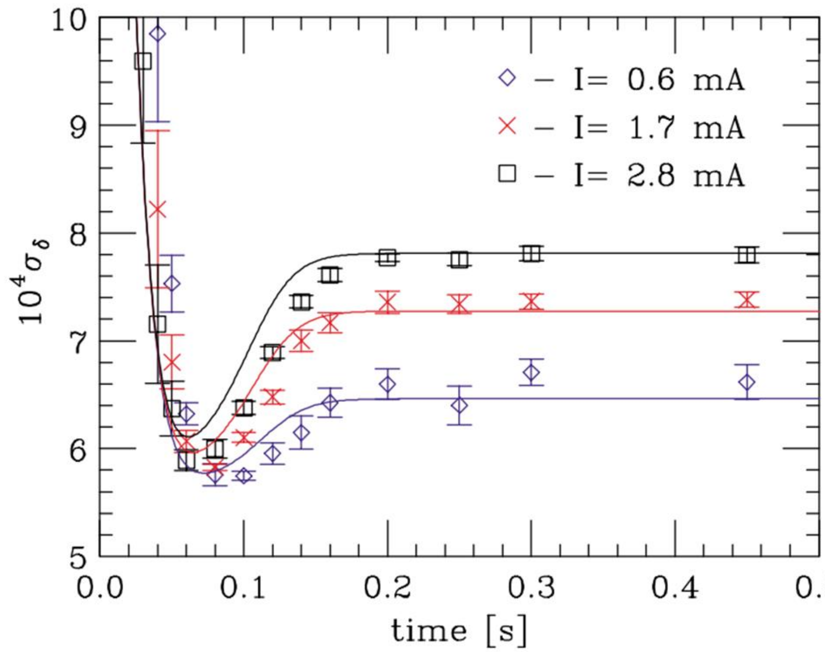
Emittance $\varepsilon_x = \varepsilon_y$ vs. energy for a round beam at nominal current (black) and at zero current (red).

KEK's Accelerator Test Facility (ATF)

ATF is an electron storage ring with *C*= 138 m, *E*= 1.28 GeV, ε_{x0}~ 1 nm, ε_{y0}~ 10 pm, maximum *N*~ 10^{10}. Complete IBS measurements performed in short time in April 2000

At ATF all bunch dimensions can be measured. Unique is that σ_p can be measured to a few percent (at high dispersion point after extraction). λ_z measured by streak camera.

Measured energy spread as function of time after injection, for three different currents (the plotting symbols). The curves give BM simulations.

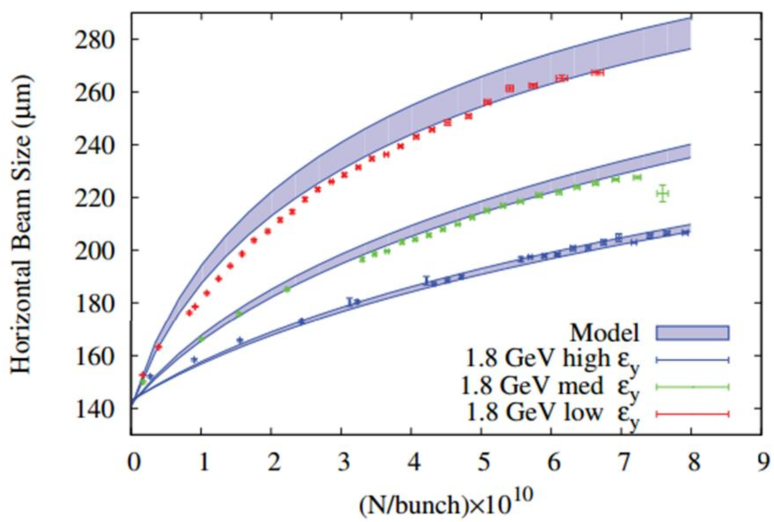


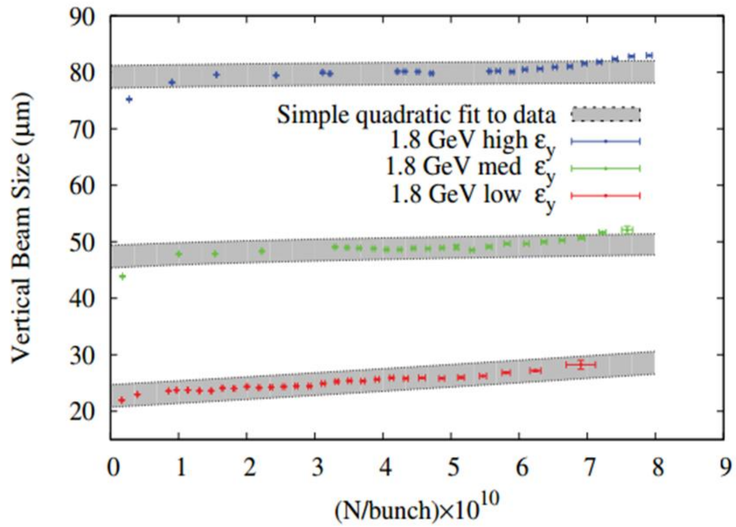
K. Kubo

Other Measurements in Electron Machines

A more complete set of measurements was also performed at the ATF (simultaneous measurement of σ_p, σ_z, ε_x, ε_y) [K. Bane et al, PRST-AB 2002]. Agreement with theory was good but not perfect. Now it is believed that discrepancy was largely due to ε_y measurement errors

IBS measurements were performed at CESR-TA and seemed to agree reasonably well with theory, except for unaccounted growth in *y* [Ehrlichman et al, IPAC 12, Blaser et al, IPAC 14]





Blaser et al, IPAC 14

More on IBS

F. Antoniou and Y. Pappaphilippou pointed out that the IBS models, BM, Piwinski, Bane, disagree (~10—20%), particularly when IBS is strong (TWIICE14)

Y. Pappaphilippou et al have managed to optimize the lattice of low emittance rings to significantly reduce IBS (IPAC13)

Touschek Lifetime

• Touschek effect concerns large, single Coulomb scattering events where energy transfer from transverse to longitudinal leads to immediate particle loss

• Number of particles in bunch decays as:

$$N_b = \frac{N_{b0}}{1 + t/\mathcal{T}}$$

• Normally, for flat beams, use formula of Brueck

• Otherwise use general formula due to Piwinski. Inverse of Touschek lifetime:

$$\frac{1}{\mathcal{T}} = \frac{r_e^2 c N_b}{8\sqrt{\pi}\beta^2\gamma^4\sigma_z\sigma_p\epsilon_x\epsilon_y}\langle\sigma_H\mathcal{F}(\delta_m)\rangle$$

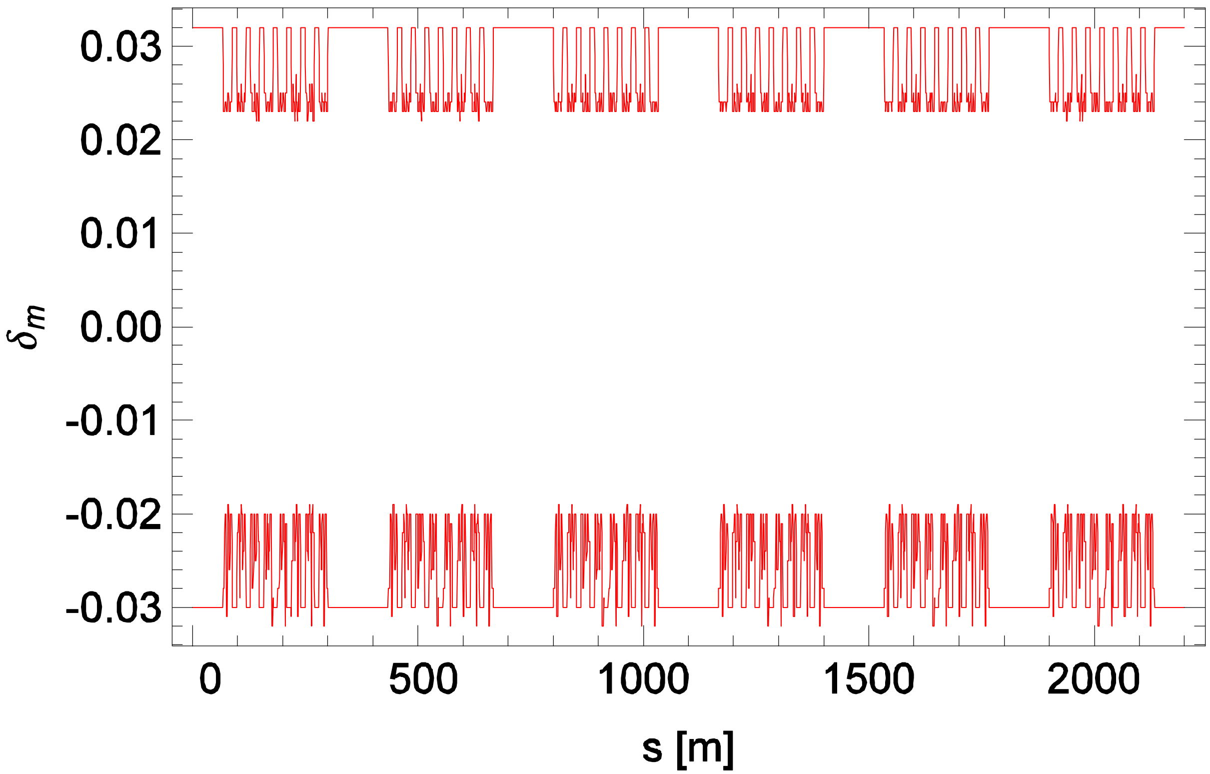
$$\mathcal{F}(\delta_m) = \int_{\delta_m^2}^{\infty}\frac{d\tau}{\tau^{3/2}}\boxed{e^{-\tau B_1}}I_0(\tau B_2)\left[\frac{\tau}{\delta_m^2} - 1 - \frac{1}{2}\ln\left(\frac{\tau}{\delta_m^2}\right)\right]$$

$$B_{1,2} = \frac{1}{2\beta^2\gamma^2}\left|\frac{\beta_x\sigma_x^2}{\epsilon_x\tilde{\sigma}_x^2} \pm \frac{\beta_y}{\epsilon_y}\right|$$

• $B_1 \sim \beta_{x,y}/\varepsilon_{x,y}$ => where $\sigma_{x,y}$ is large, $1/\mathcal{T}$ is small because of $\exp(-\tau B_1)$ factor in integral. This factor is also reason $1/\mathcal{T}$ becomes small at very small $\varepsilon_{x,y}$.

Momentum Acceptance in PEP-X

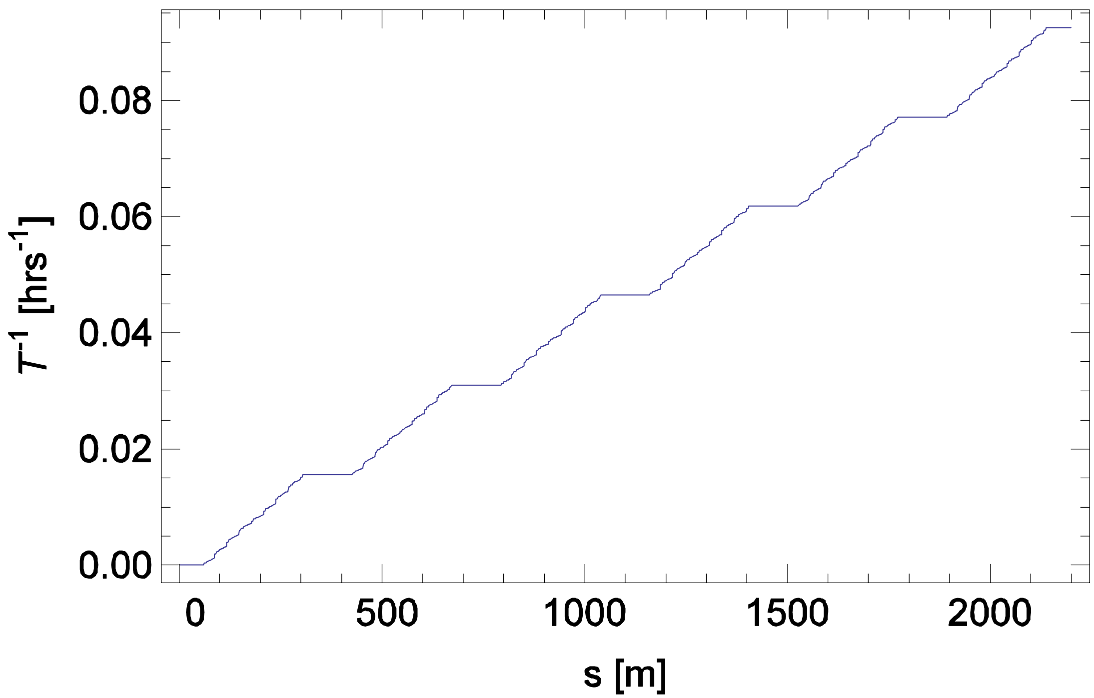
(Min-Huey Wang)



Momentum acceptance due to linear optics for PEP-X. The average value is $\delta_m = \pm 2.8\%$.

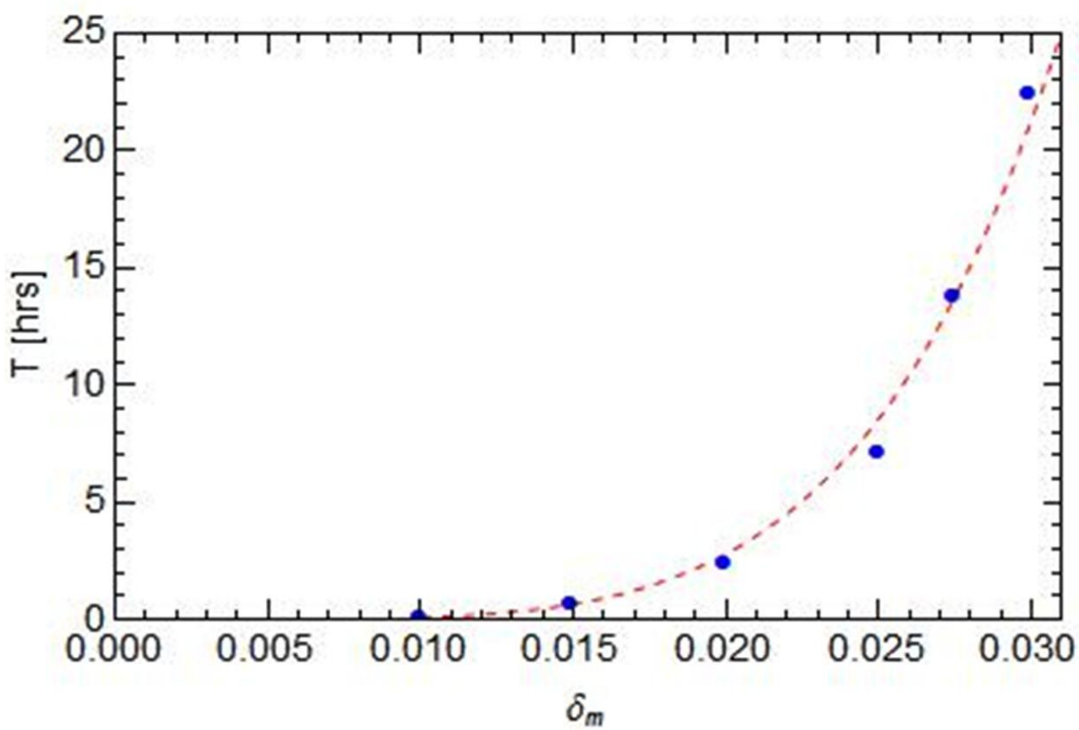
Touschek Lifetime Results

- Result for the IBS-determined steady-state beam sizes is: T= 11 hrs



Accumulation around the ring of the Touschek growth rate in PEP-X. The growth is significant only in the arcs, where $\sigma_{x,y}$ are small.

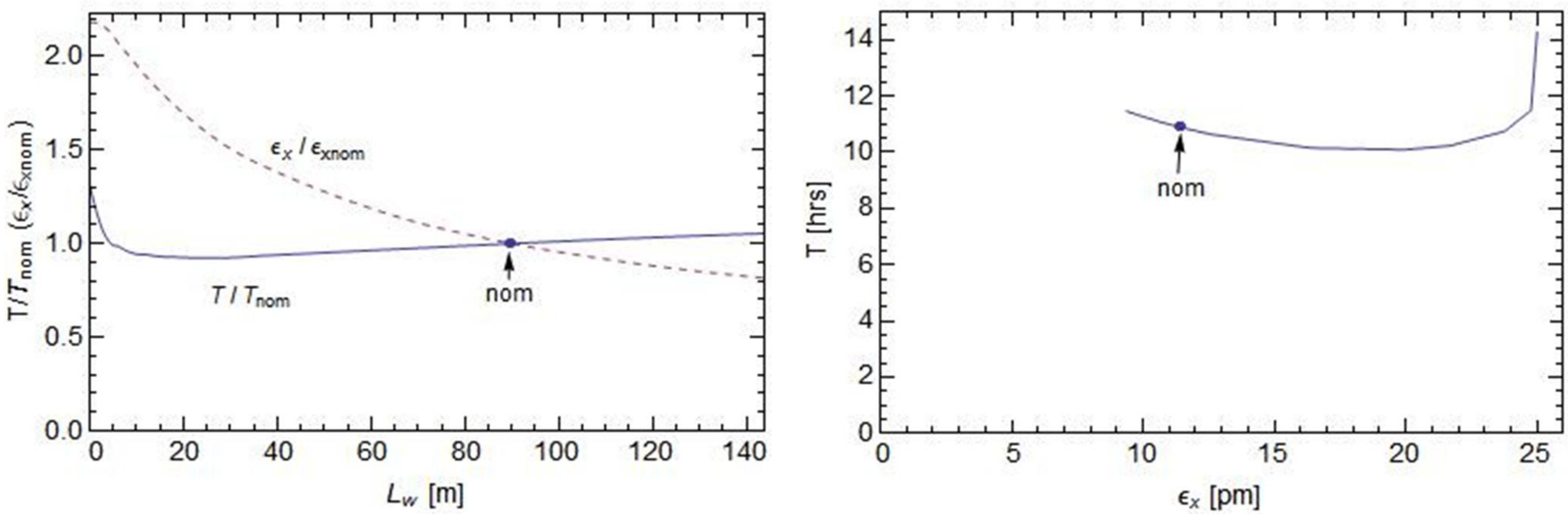
Dependence on Momentum Acceptance



Touschek lifetime T *vs. (global) momentum acceptance parameter, δ_m (blue symbols). The dashed curve gives the fit:* T*= 0.088(δ_m/0.01)^5 hrs.*

Touschek Lifetime vs Emittance

As the length of wiggler L_w increases, the emittance decreases. In PEP-X design L_w= 90 m, ε_{xnom}= 11 pm (including IBS), T_{nom}= 11 hrs



Emittance ε_x (= ε_y) and Touschek lifetime T *vs wiggler length* L_w *(left plot), and* T *vs* ε_x *(right). These are results of self-consistent calculations including IBS.*

Longitudinal Impedance Calculations for PEP-X

- For PEP-X, without an actual vacuum chamber design available, we developed a straw man design, inspired by objects in other machines, such as PEP-II

Sources include: RF cavities, BPM's, wiggler transitions, undulator transitions, resistive wall, coherent synchrotron radiation (CSR)

For the microwave instability, generate:

(i) a pseudo-Green function wake representing the ring—to be used in simulations (σ_z= .5 mm; nominal is 3 mm)

(ii) an impedance budget—to assess relative importance of contributors

People involved in 3D code development and impedance calculation include L.-Q. Lee, C.-K. Ng, L. Wang, L. Xiao

This approach was successfully applied many years ago to the SLC damping rings and DaΦne, where drawings of the vacuum chamber components were available [see e.g. Bane et al, HHH-2004 and references therein]

- Another approach is one or more Q= 1 resonators to represent ring impedance (e.g. used for LEP); each resonator has two parameters ω_r, R_s

SLC Damping Ring Measurements (Old Ring)

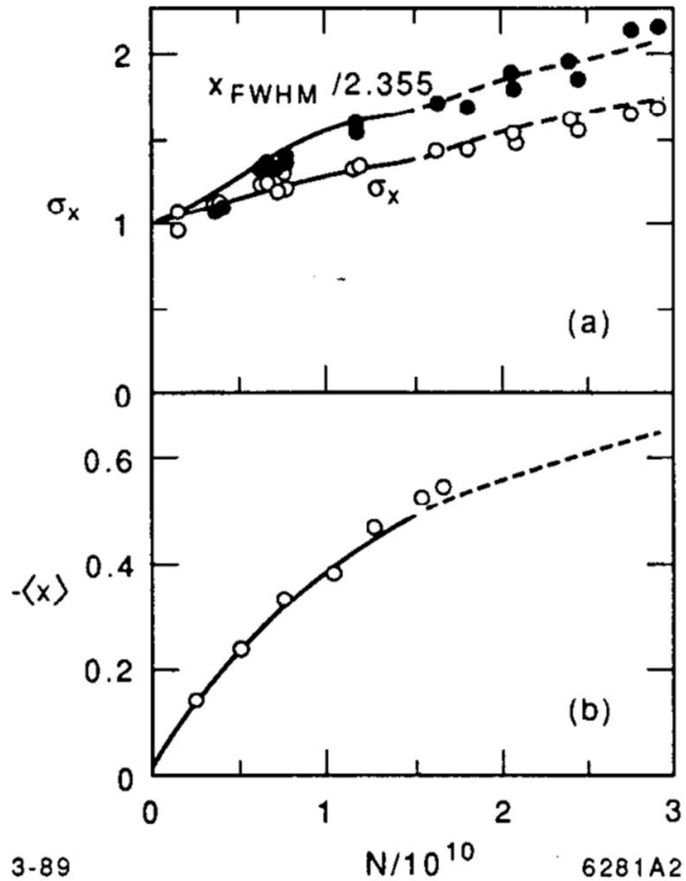
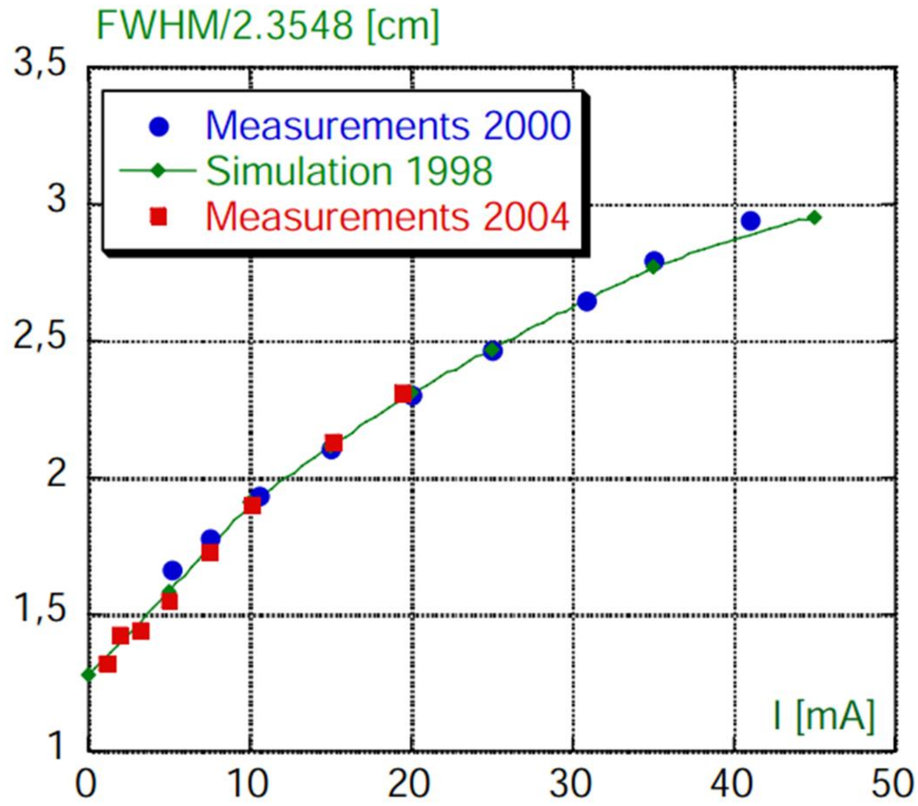


Fig. 5. (a) Bunch lengthening, and (b) the centroid shift calculated for the SLC damping rings at $V_{rf} = 0.8$ MV. The symbols indicate the measurement results.

K. Bane, R. Ruth, PAC89

Wake used in the simulations was obtained from drawings of vacuum chamber components

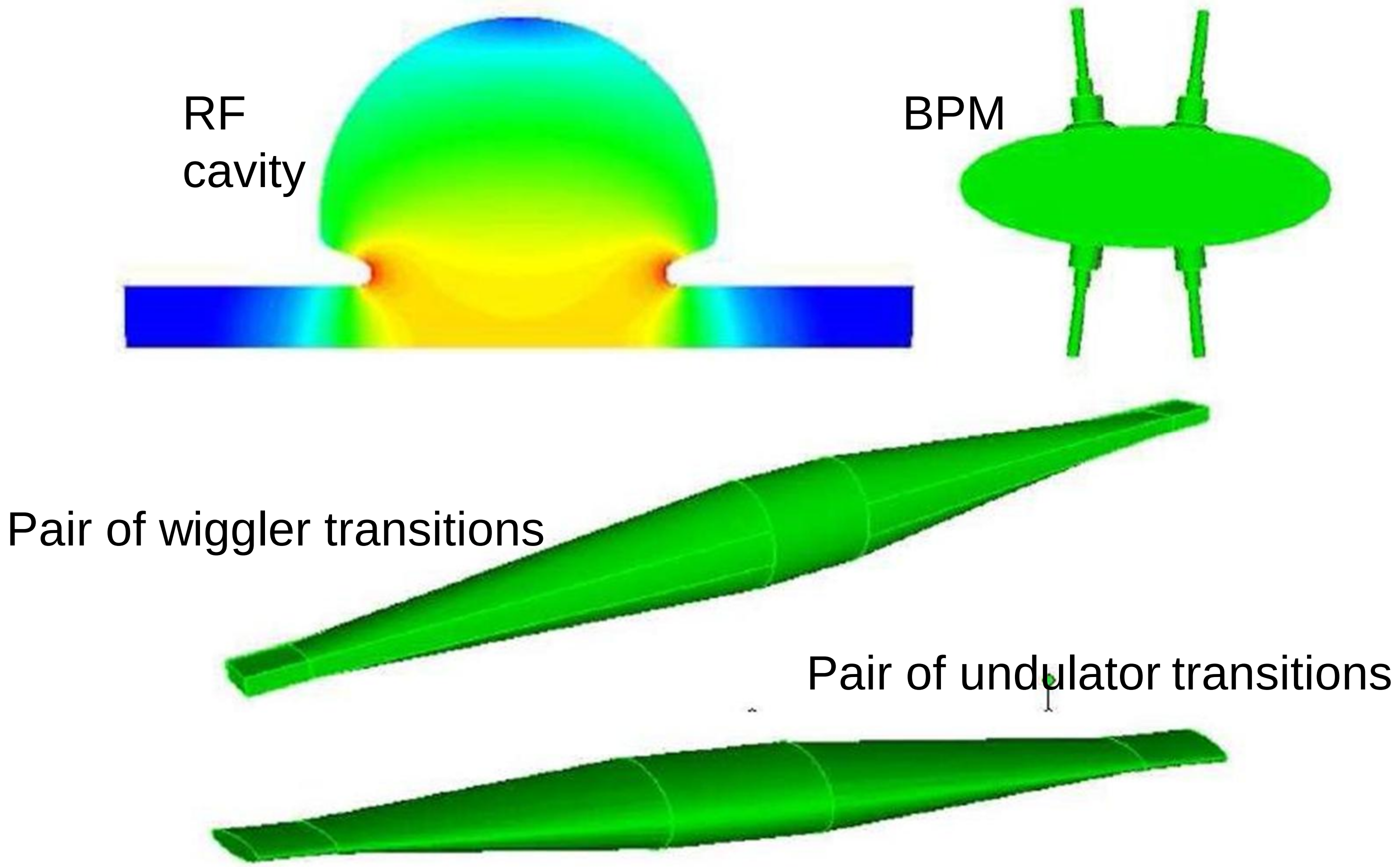
DaΦne Bunch Length Measurements



M. Zobov

Comparison of bunch lengthening simulations (green line) with measurements (symbols) for DaΦne. The wake used in the simulations was obtained from the drawings of vacuum chamber components

Selected PEP-X Impedance Sources



Selected impedance objects included in our straw man PEP-X design.
Note: the fundamental mode fields are shown in the RF cavity.

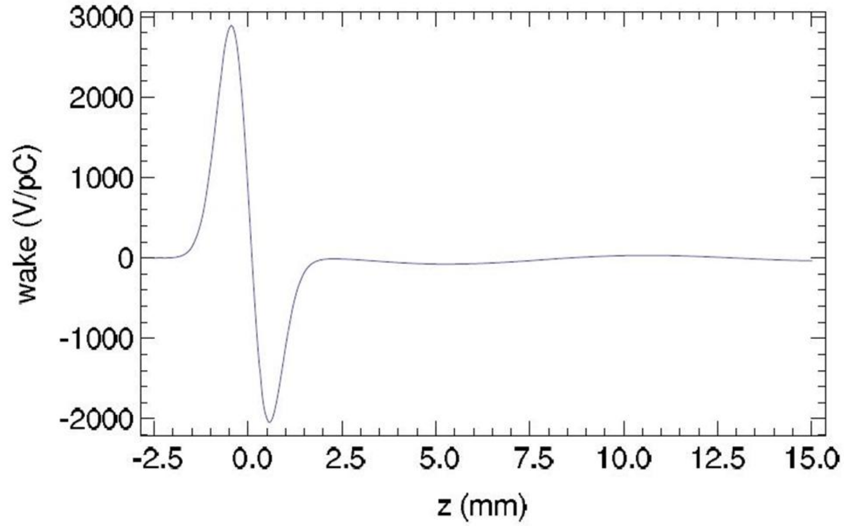
Impedance Budget

Object	Single Contribution			Total Contribution			
	k_{loss}[V/pC]	R [Ω]	L [nH]	N_{obj}	k_{loss}[V/pC]	R [Ω]	L [nH]
RF cavity	.92	30.4	–	16	14.7	487	–
Undulator taper (pair)	.06	3.2	.32	30	1.9	95	9.6
Wiggler taper (pair)	.43	21.4	.72	16	6.8	340	11.5
BPMs	.013	.6	.005	839	11.3	465	4.1
Bellows slots	.00	.0	4e-4	720	.0	.0	.3
Bellows masks	.005	.2	.004	720	3.7	142	2.7
Resistive wall wake					21.3	880	11.3
Total					59.7	2409	39.5

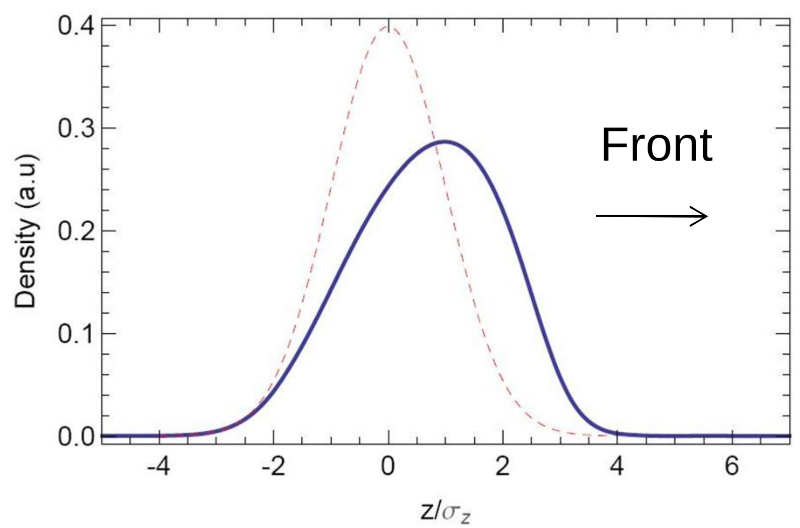
Impedance budget for PEP-X, giving the loss factor, and the effective resistance and inductance of the various objects in the ring. The results are at nominal bunch length σ_z= 3 mm.

Pseudo-Green Function Wake

Pseudo-Green function wake representing the PEP-X ring: wake of a σ_z= .5 mm bunch

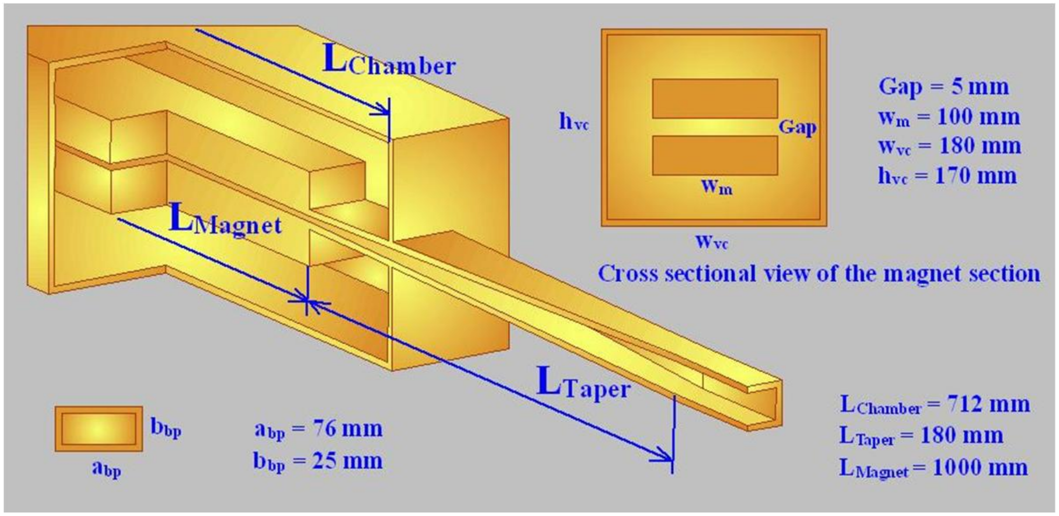


Haissinski solution, giving the steady-state bunch shape. Bunch length is 25% above nominal length.



(G. Stupakov)

A Difficult Example: an Insertion Device



From "Impedance calculation for the NSLSII storage ring," A. Blednykh

With the insertion gap becoming ever smaller, the insertion region becomes a dominating part of the ring impedance*

Insertion transitions tend to be long, gradually tapered, and 3D => it is very challenging to obtain the wakefield for a short bunch

*(especially true for vertical impedance)

Microwave Instability: Two Types

- Coasting beam analysis gives a (conservative) estimate of the threshold to the microwave instability (Boussard, 1975):

$$\frac{e\hat{I}_{th}|Z/n|}{2\pi\alpha E\sigma_{p0}^2} = 1$$

with $n = ck/\omega_0$ and $|Z/n|$ taken at a representative frequency, *e.g*. at $k = 1/\sigma_z$

- In 1994 the SLC damping ring vacuum chambers were replaced with smoother chambers. Surprisingly the threshold dropped, from $N_{th} = 3\times10^{10}$ to 1.5--2.0×10^{10}

Analysis showed that a new type of instability was found, which we call the weak instability. Unlike the normal (strong) microwave instability, it is sensitive to the damping time ($N_{th} \sim \tau_d^{-1/2}$) and can be suppressed by Landau damping (see *e.g*. Oide, 1994)

Macro-particle Tracking (A. Renieri, 1976)

- The development of longitudinal phase space can be followed using macroparticle tracking

- Track $i = M$ particles, with energy p_i (normalized to $\sigma_{\delta 0}$) and longitudinal position q_i (normalized to σ_{z0}), over $n = N_T$ time steps:

$$\begin{pmatrix} p_i \\ q_i \end{pmatrix}_{n+1} = \begin{bmatrix} 1 - c_d & -\theta \\ (1 - c_d)\theta & 1 - \theta^2 \end{bmatrix} \begin{pmatrix} p_i \\ q_i \end{pmatrix}_n + \begin{bmatrix} 1 \\ \theta \end{bmatrix} (\theta I \mathcal{C} W_\lambda(q_i \sigma_{z0}) + r_e)$$

with time step $\theta = 2\pi \Delta t / T_s$, and c_d and r_e represent effects of radiation damping and quantum excitation

- After each time step the bunch distribution $\lambda(s)$ and then $W_\lambda(s)$ are recomputed

- Algorithm is easy to implement (see *e.g. Elegant*), though results tend to be noisier than the Vlasov-Fokker-Planck solutions

Vlasov-Fokker-Planck (VFP) Equation Solver

- Beam longitudinal density distribution $\psi(\theta, q, p)$ follows

$$\frac{\partial\psi}{\partial\theta} - \{H, \psi\} = 2\beta\frac{\partial}{\partial p}(p\psi + \frac{\partial\psi}{\partial p}) \ .$$

The independent variable $\theta = \omega_{s0}t$, with ω_{s0} synchrotron frequency and t time. Here phase space variables $q = z/\sigma_{z0}$ and $p = -\delta/\sigma_{\delta 0}$; $\{f, g\}$ is a Poisson bracket; $\beta = 1/\omega_s\tau_p$

- The Hamiltonian

$$H(\theta, q, p) = \frac{1}{2}(q^2 + p^2) - I\int_{-\infty}^{q} dq''\int_{-\infty}^{\infty} dq'\lambda(\theta, q')\, w(q'' - q') \ ,$$

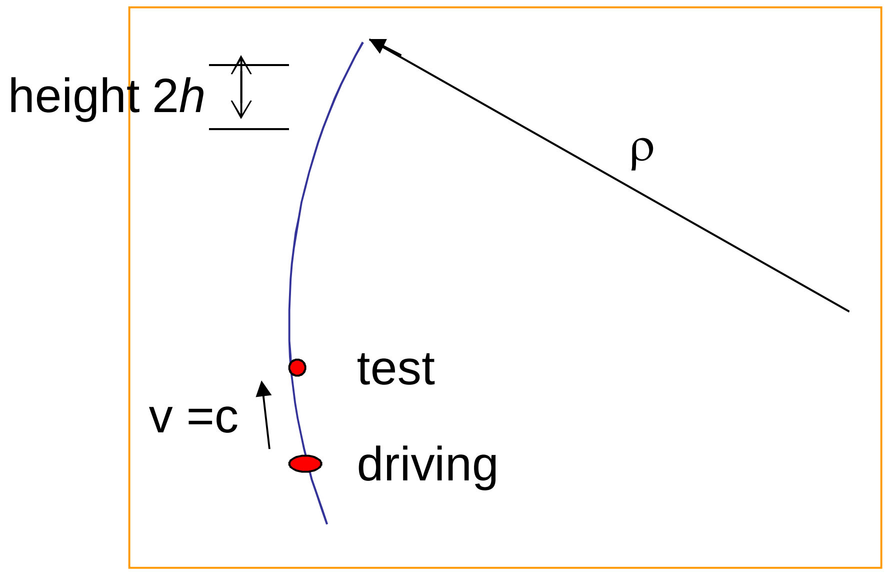
with longitudinal distribution $\lambda(\theta, q) = \int_{-\infty}^{\infty}\psi(\theta, q, p)dp$, and $w(q) \equiv \mathcal{C}W(q\sigma_{z0})$ is point charge wake (per turn).

- Warnock and Ellison in 2000 developed a robust algorithm for solving the VFP equation. It follows the development of the distribution function ψ on a rectangular mesh in phase space

- Using macro-particle tracking with the pseudo-Green function, we find that for PEP-X, the microwave threshold is very high (> 8 A)

Microwave Instability Due to Shielded CSR

- For short bunches (*e.g.* in low α mode) CSR can dominate the ring impedance, since strength $\sim\sigma_z^{-4/3}$



K. Bane, Y. Cai, G. Stupakov, PRST-AB, 2010

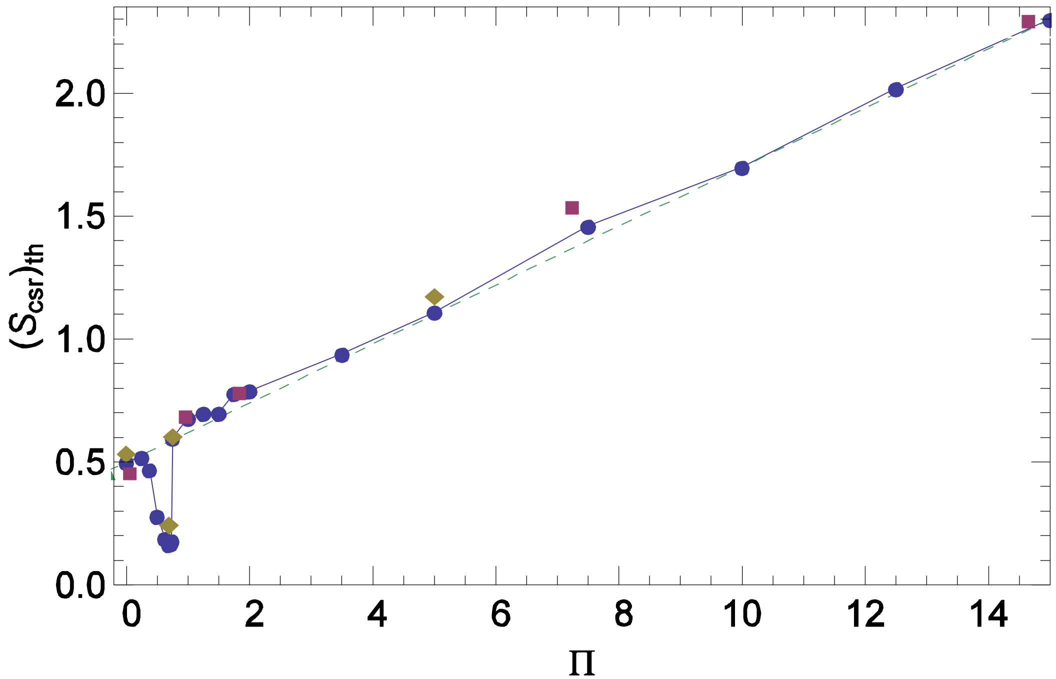
- Wake of particles moving at speed *c*, on circle of radius ρ, between two metallic plates located at $y = \pm h$ was found by J. Murphy, et al.

- For a bunch, the normalized threshold current *S* is a function of normalized shielding parameter Π, with

$$S = \frac{eN_b\rho^{1/3}}{2\pi\nu_s\gamma\sigma_\delta\sigma_z^{4/3}}\,, \qquad \Pi = \frac{\sigma_z\rho^{1/2}}{h^{3/2}}$$

- Used Vlasov-Fokker-Planck solver (a la Warnock) to find threshold current

Threshold for Shielded CSR Impedance



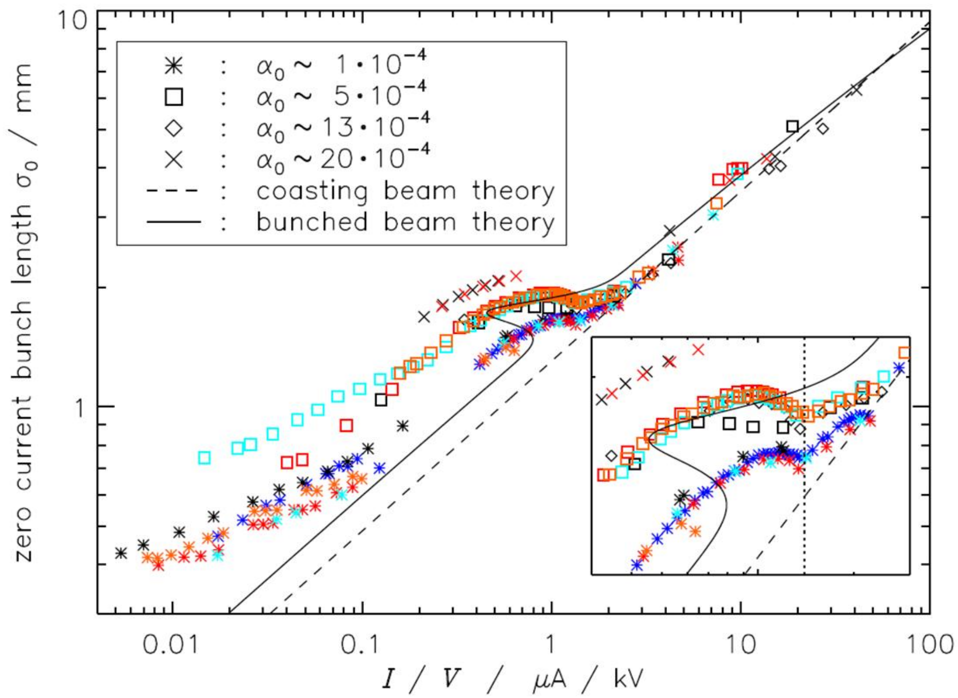
For the CSR wake, threshold values of current S vs. shielding parameter Π. Symbols give simulation results.

- The dashed curve is S_{th}= 0.50 + 0.12 Π. Result also agrees well with coasting beam theory (for Π not small), for which $S_{th}\sim \Pi^{2/3}$ (Y. Cai, IPAC2011)

- We think dip at $\Pi\sim$ 0.7 indicates a change from a strong to a weak instability

• For PEP-X, vacuum chamber is elliptical with axes (20.0, 12.5) mm and bending radius ρ= 100.8 m => Π= 22.7 and I_{th}= 3.6 A, far above design *I*.

Comparison with Measurement



M. Ries, et al, IPAC 12

Scaled values of measured bursting threshold at Metrology Light Source. Colors indicate measurement series within one α_0 set.

- Reasonably good agreement at relatively longer bunch lengths and for location of dip. Note that simulations assumed simple phase space (not low α)

- Similarly good agreement for BESSY-II (P. Kuske), ANKA (M. Klein). Shows that for short bunches CSR can dominate a ring impedance, and can be simply modeled

Other Instabilities

Transverse Single Bunch (TMCI):

In light sources with regions of small aperture vacuum chambers, the resistive wall impedance is typically the dominant contributor. For a Gaussian bunch the average kick in a round chamber of radius *b*

$$\kappa_y = (0.723)\frac{c}{\pi^{3/2}b^3}\sqrt{\frac{Z_0}{\sigma_z\sigma_c}}$$

Single-bunch threshold can be approximated by [S. Krinsky]

$$I_b^{th} \approx 0.7\frac{4\pi c\,\nu_s(E/e)}{\mathcal{C}}\frac{1}{\sum_i \ell_i\beta_{y,i}\kappa_{y,i}}$$

For PEP-X, multi-bunch threshold $I^{th} = MI_b^{th} = 1.8$ A

PEP-X beam chamber types. The first three entries are Al, the last two Cu

Type	Length [m]	Shape	(b_x, b_y) [mm]	$\langle\beta_y\rangle$ [m]
Arcs	1318	E	(30.0, 12.5)	7.0
Straights r	510	C	(48.0, 48.0)	15.6
Straights i	123	C	(48.0, 48.0)	60.0
Undulators	158	E	(25.0, 3.0)	2.8
Wigglers	90	R	(22.5, 4.0)	12.0

At times the small aperture insertion devices dominate the transverse impedance, with the geometric and resistive wall components being about equal in strength. Optimizing the 3D geometry of the transitions and then estimating the instability threshold can be challenging (see e.g. Y.-C. Chae's talk on APS insertion devices at TWIICE14)

Multi-bunch Transverse Instability:

Including only the resistive wall wake, which often dominates the multibunch transverse instability, the growth rate is given by [A. Wolski et al, 2006]

$$\Gamma = \frac{c\,(I/I_A)}{4\gamma\sqrt{\mathcal{C}(1-[\nu_y])}}\langle\beta A\rangle$$

with

$$\langle\beta A\rangle = \frac{4}{\sqrt{\pi Z_0}}\sum_i \frac{\ell_i \beta_{y,i}}{b_i^3\sqrt{\sigma_{c,i}}}$$

For PEP-X (ultimate) the growth rate is 1.4 ms^{-1} or 99 turns

Fast Ion Instability:

Places requirements on vacuum pressure, gap in bunch train. Multi-particle tracking shows that the instability is manageable (L. Wang)

Conclusions

- In low emittance rings such as PEP-X “ultimate”, impedance effects tend not to be important since the current is quite low (200 mA)

- Our impedance calculations used a simplified, “straw dog” vacuum chamber model. More precise calculations, when based on engineering drawings for a real machine, can change this conclusion, particularly when low aperture insertion devices are present

- IBS sets the limit of current that can be stored in an ultimate ring. In PEP-X with round beams, IBS doubles the emittance to 11 pm at the design current of 200 mA.

- The Touschek lifetime in ultimate PEP-X is quite large, 11 hrs, but it is a very sensitive function of the momentum acceptance

- How to run a machine with a round beam needs serious study. The choice will affect the IBS and Touschek effect