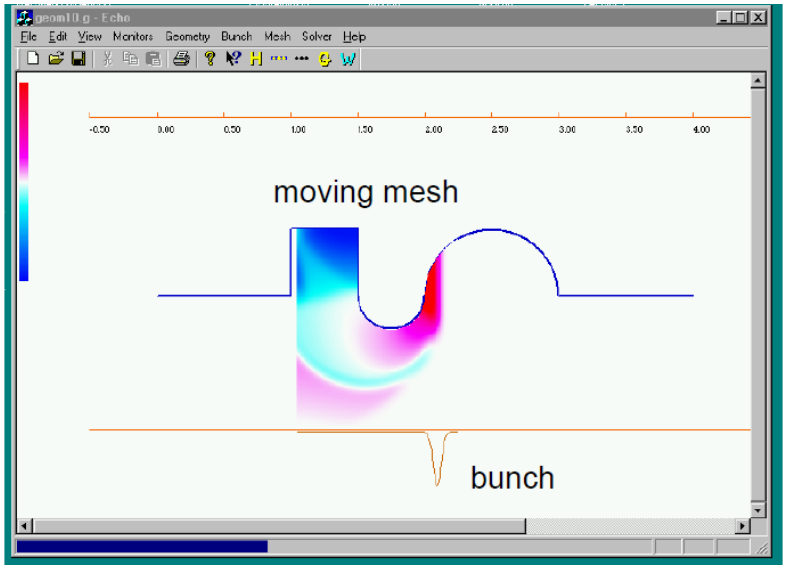
Wakefield Code ECHO

Algorithms and Applications



Igor Zagorodnov

Second Topical Workshop on Instabilities, Impedance and Collective Effects

Abingdon, Oxfordshire
8. February 2016

HELMHOLTZ GEMEINSCHAFT

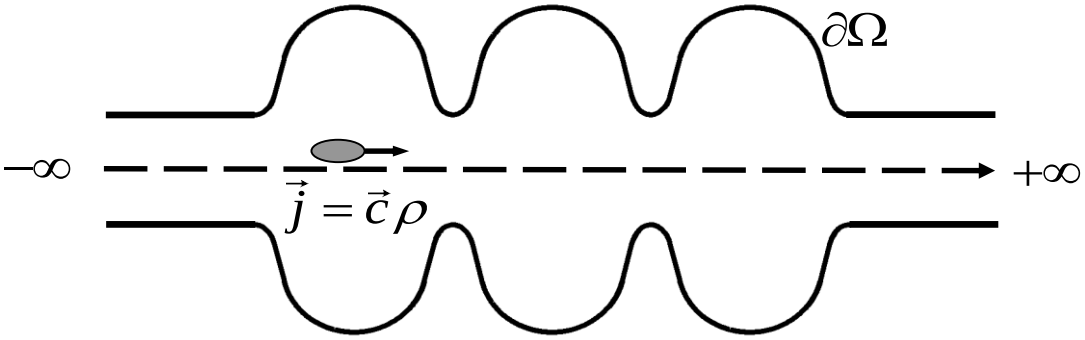
DESY

Overview

- Motivation
- Numerical Methods
  - low-dispersive schemes
  - boundary approximation
  - indirect integration algorithm
  - modelling of conductive walls
- Code Status and Applications
  - geometries of revolution (ECHOz1, ECHOz2)
  - rectangular structures (ECHO2D)
  - fully 3D geometries (ECHO3D)
  - Particle-In-Cell code

Motivation

Wake field calculation – estimation of the effect of the geometry variations on the bunch



First codes in **time domain** ~ 1980

A. Novokhatski (BINP),

T. Weiland (CERN)

Wake potential

$$W_{\parallel}(\mathbf{r}_0, \mathbf{r}, s) = \frac{1}{Q}\int_{-\infty}^{\infty} E_z\left(\mathbf{r}_0, \mathbf{r}, z, \frac{z-s}{c}\right) dz$$

$$\frac{\partial}{\partial s}\mathbf{W}_{\perp}(\mathbf{r}_0, \mathbf{r}, s) = \nabla_{\perp} W_{\parallel}(\mathbf{r}_0, \mathbf{r}, s)$$

Motivation

MAFIA*

- rotationally symmetric and 3D
- longitudinal and transverse wakes
- triangular geometry approximation;
- arbitrary materials
- moving window
- dispersion error

NOVO**

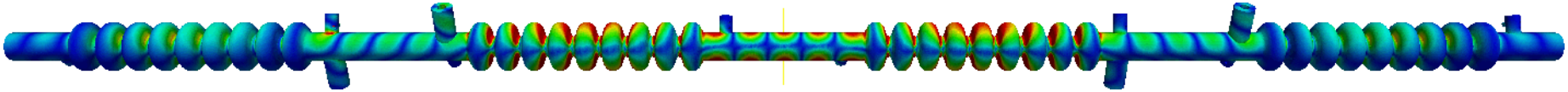
- rotationally symmetric
- only longitudinal wake
- “staircase” geometry approximation;
- only PEC
- moving mesh
- low dispersion error

* MAFIA Collaboration, MAFIA Manual, CST GmbH, Darmstadt, 1997.

** A. Novokhatski, M. Timm, T.Weiland, *Transition Dynamics of the Wake Fields of Ultra Short Bunches*. Proc. of the ICAP 1998, Monterey, California, USA.

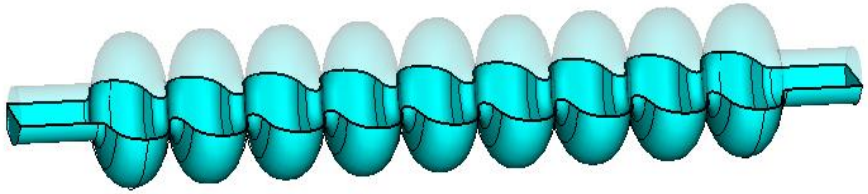
Motivation

(Picture from W.Ackermann, TU Darmstadt, 2012)



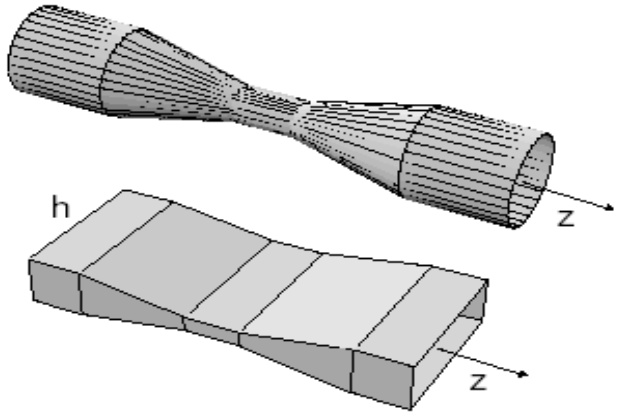
New projects with

- short bunches;
- long structures;
- tapered collimators

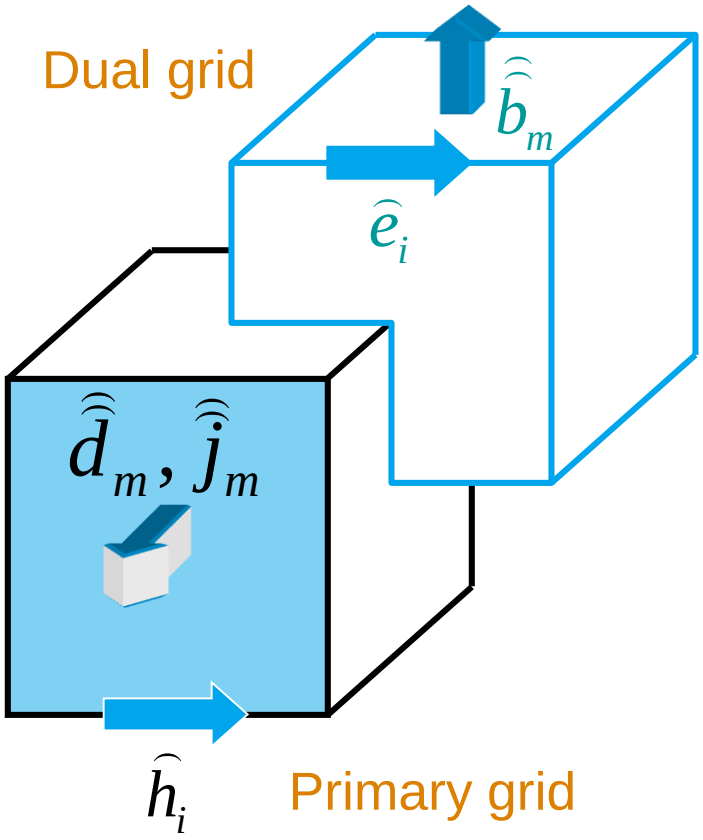


Solutions

- zero dispersion in longitudinal direction;
- “conformal” meshing;
- moving mesh and “explicit” or “split” methods



Low-dispersive schemes



Maxwell Grid Equations*

$$\mathbf{C}\hat{\mathbf{e}} = -\frac{d}{dt}\hat{\tilde{\mathbf{b}}}, \qquad \tilde{\mathbf{C}}\hat{\mathbf{h}} = \frac{d}{dt}\hat{\tilde{\mathbf{d}}} + \hat{\tilde{\mathbf{j}}},$$

$$\mathbf{S}\hat{\tilde{\mathbf{b}}} = \mathbf{0}, \qquad \tilde{\mathbf{S}}\hat{\tilde{\mathbf{d}}} = \mathbf{q},$$

$$\hat{\mathbf{e}} = \mathbf{M}_{\varepsilon^{-1}}\hat{\tilde{\mathbf{d}}}, \qquad \hat{\mathbf{h}} = \mathbf{M}_{\mu^{-1}}\hat{\tilde{\mathbf{b}}}$$

$$curl\ \mathbf{C} = \begin{pmatrix} \mathbf{0} & -\mathbf{P}_z & \mathbf{P}_y \\ \mathbf{P}_z & \mathbf{0} & -\mathbf{P}_x \\ -\mathbf{P}_y & \mathbf{P}_x & \mathbf{0} \end{pmatrix}$$

$$div\ \mathbf{S} = \begin{pmatrix} \mathbf{P}_x^* & \mathbf{P}_y^* & \mathbf{P}_z^* \end{pmatrix}$$

* T. Weiland, *A discretization method for the solution of Maxwell's equations for six-component fields*, Electronics and Communication (AEÜ) **31**, p. 116 (1977)

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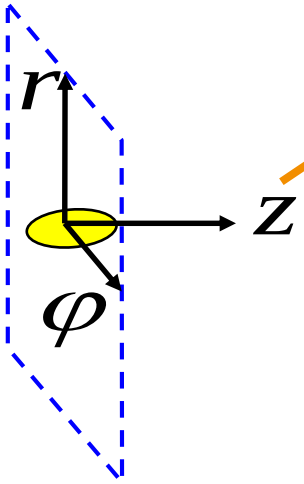
Low-dispersive schemes

$$\mathbf{C} = \begin{pmatrix} \mathbf{0} & -\mathbf{P}_z & \mathbf{P}_\varphi \\ \mathbf{P}_z & \mathbf{0} & -\mathbf{P}_r \\ -\mathbf{P}_\varphi & \mathbf{P}_r & \mathbf{0} \end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{P}_\varphi \\ \mathbf{0} & \mathbf{0} & -\mathbf{P}_r \\ -\mathbf{P}_\varphi & \mathbf{P}_r & \mathbf{0} \end{pmatrix}$$

$$\mathbf{L} = \begin{pmatrix} \mathbf{0} & -\mathbf{P}_z & \mathbf{0} \\ \mathbf{P}_z & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

transversal plane



longitudinal direction

Low-dispersive schemes

FDTD (1966)

$$\frac{\mathbf{e}^{n+1/2}-\mathbf{e}^{n-1/2}}{\Delta t}=\mathbf{M}_{\varepsilon^{-1}}(\mathbf{C}^{*}\mathbf{h}^{n}-\mathbf{j}^{n})$$

$$\frac{\mathbf{h}^{n+1}-\mathbf{h}^{n}}{\Delta t}\mathbf{h}^{n}=\mathbf{M}_{\mu^{-1}}\mathbf{C}\mathbf{e}^{n+1/2}$$

$$\mathbf{h}^{n}=\begin{pmatrix}\mathbf{h}_{x}^{n}\\ \mathbf{h}_{y}^{n}\\ \mathbf{h}_{z}^{n}\end{pmatrix} \qquad \mathbf{e}^{n+0.5}=\begin{pmatrix}\mathbf{e}_{x}^{n+0.5}\\ \mathbf{e}_{y}^{n+0.5}\\ \mathbf{e}_{z}^{n+0.5}\end{pmatrix}$$

Implicit Scheme* (2002)

$$\frac{\mathbf{e}^{n+1/2}-\mathbf{e}^{n-1/2}}{\Delta t}=\mathbf{M}_{\varepsilon^{-1}}(\mathbf{T}^{*}\bar{\mathbf{h}}^{n}+\mathbf{L}^{*}\mathbf{h}^{n}-\mathbf{j}^{n})$$

$$\frac{\mathbf{h}^{n+1}-\mathbf{h}^{n}}{\Delta t}\mathbf{h}^{n}=\mathbf{M}_{\mu^{-1}}\mathbf{C}\mathbf{e}^{n+1/2}$$

$$\bar{\mathbf{h}}^{n}\equiv\theta\mathbf{h}^{n+1}+(1-2\theta)\mathbf{h}^{n}+\theta\mathbf{h}^{n-1}$$

* I. Zagorodnov, R. Schuhmann, T. Weiland, *Long-Time Numerical Computation of Electromagnetic Fields in the Vicinity of a Relativistic Source*, Journal of Computational Physics **191**, No.2 , pp. 525-541 (2003)

DESY

Low-dispersive schemes

Implicit Scheme (2002)

$$\mathbf{a}^n = \int_{-\infty}^{t_n} \widehat{\mathbf{h}}_s \, d\tau$$

$$\left(\mathbf{I} + \theta \mathbf{T}_1\right)\mathbf{a}^{n+1} = 2\mathbf{a}^n - \mathbf{a}^{n-1} - \mathbf{T}_1\left((1-2\theta)\mathbf{a}^n + \theta \mathbf{a}^{n-1}\right) - \mathbf{L}_1 \mathbf{a}^n + \mathbf{F}^n$$

$$\mathbf{T}_1 = \Delta t^2 \mathbf{M}_{\mu^{-1}} \mathbf{C} \mathbf{M}_{\varepsilon^{-1}} \mathbf{T}^*, \quad \mathbf{L}_1 = \Delta t^2 \mathbf{M}_{\mu^{-1}} \mathbf{C} \mathbf{M}_{\varepsilon^{-1}} \mathbf{L}^*$$

For fully rotationally symmetric problems (**monopole** mode m=0) our scheme in staircase approximation with θ=0.5 coincides with scheme realized in code NOVO**.

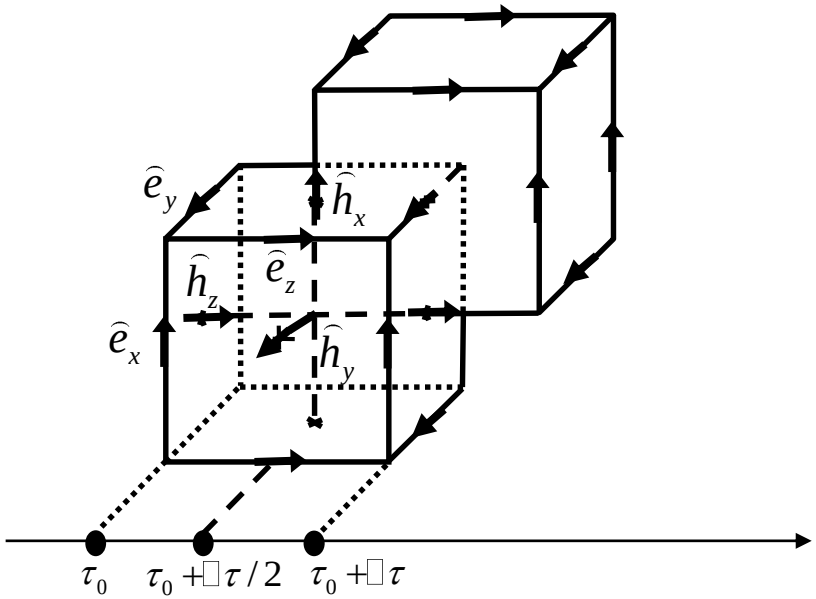
$$\mathbf{M}^{-1}_{\mu_\varphi^{-1}} \Delta t^{-2}\left(\mathbf{a}_\varphi^{n+1} - 2\mathbf{a}_\varphi^n + \mathbf{a}_\varphi^{n-1}\right) = \mathbf{P}_z \mathbf{M}_{\varepsilon_r^{-1}} \mathbf{P}_z^T \mathbf{a}_\varphi^n + \mathbf{P}_r \mathbf{M}_{\varepsilon_z^{-1}} \mathbf{P}_r^T \frac{\mathbf{a}_\varphi^{n+1} + \mathbf{a}_\varphi^{n-1}}{2} + \mathbf{F}_\varphi^n,$$

** A. Novokhatski, M. Timm, T. Weiland, *Transition Dynamics of the Wake Fields of Ultra Short Bunches*. Proc. of the ICAP 1998, Monterey, California, USA.

DESY

Low-dispersive schemes

E/M and TE/TM* splitting (2004)



Subdue the updating procedure to the bunch motion

* Zagorodnov I.A, Weiland T., *TE/TM Field Solver for Particle Beam Simulations without Numerical Cherenkov Radiation,* Phys. Rev. ST Accel. Beams **8**, 042001 (2005)

E/M splitting (Yee's Scheme)

$$\mathbf{h}^n = \begin{pmatrix} \mathbf{h}_x^n \\ \mathbf{h}_y^n \\ \mathbf{h}_z^n \end{pmatrix} \qquad \mathbf{e}^{n+0.5} = \begin{pmatrix} \mathbf{e}_x^{n+0.5} \\ \mathbf{e}_y^{n+0.5} \\ \mathbf{e}_z^{n+0.5} \end{pmatrix}$$

TE/TM splitting

$$\mathbf{v}^n = \begin{pmatrix} \mathbf{e}_x^n \\ \mathbf{e}_y^n \\ \mathbf{h}_z^n \end{pmatrix} \qquad \mathbf{u}^{n+0.5} = \begin{pmatrix} \mathbf{h}_x^{n+0.5} \\ \mathbf{h}_y^{n+0.5} \\ \mathbf{e}_z^{n+0.5} \end{pmatrix}$$

Low-dispersive schemes

$$\mathbf{C}_0 = c^{-1}\mathbf{M}^{1/2}_{\mu^{-1}}\mathbf{C}\mathbf{M}^{1/2}_{\varepsilon^{-1}} = \begin{pmatrix} \mathbf{0} & -\mathbf{P}^0_z & \mathbf{P}^0_y \\ \mathbf{P}^1_z & \mathbf{0} & -\mathbf{P}^0_x \\ -\mathbf{P}^1_y & \mathbf{P}^1_x & \mathbf{0} \end{pmatrix}$$

transversal plane

longitudinal direction

$$\mathbf{T}_i = \begin{pmatrix} \mathbf{0} & \mathbf{0} & -\mathbf{P}^i_y \\ \mathbf{0} & \mathbf{0} & \mathbf{P}^i_x \\ \left(\mathbf{P}^i_y\right)^* & -\left(\mathbf{P}^i_x\right)^* & \mathbf{0} \end{pmatrix}, i = 0,1$$

$$\mathbf{L} = \begin{pmatrix} \mathbf{0} & \mathbf{P}^0_z & \mathbf{0} \\ -\mathbf{P}^1_z & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

DESY

Low-dispersive schemes

E/M splitting (1966)

$$\frac{\mathbf{e}^{n+0.5}-\mathbf{e}^{n-0.5}}{\Delta\tau}=\mathbf{C}_0{}^{*}\mathbf{h}^{n}+\mathbf{j}^{n},$$

$$\frac{\mathbf{h}^{n+1}-\mathbf{h}^{n}}{\Delta\tau}=-\mathbf{C}_0\mathbf{e}^{n+0.5}$$

$$\mathbf{h}^{n}=\begin{pmatrix}\mathbf{h}_x^{n}\\ \mathbf{h}_y^{n}\\ \mathbf{h}_z^{n}\end{pmatrix}\qquad \mathbf{e}^{n+0.5}=\begin{pmatrix}\mathbf{e}_x^{n+0.5}\\ \mathbf{e}_y^{n+0.5}\\ \mathbf{e}_z^{n+0.5}\end{pmatrix}$$

dispersion error

TE/TM splitting (2004)

$$\frac{\mathbf{u}^{n+0.5}-\mathbf{u}^{n-0.5}}{\Delta\tau}=\mathbf{T}_0\frac{\mathbf{u}^{n+0.5}+\mathbf{u}^{n-0.5}}{2}+\mathbf{L}\mathbf{v}^{n}+\mathbf{j}_u^{n},$$

$$\frac{\mathbf{v}^{n+1}-\mathbf{v}^{n}}{\Delta\tau}=\mathbf{T}_1\frac{\mathbf{v}^{n+1}+\mathbf{v}^{n}}{2}-\mathbf{L}^{*}\mathbf{u}^{n+0.5}+\mathbf{j}_v^{n+0.5}$$

$$\mathbf{v}^{n}=\begin{pmatrix}\mathbf{e}_x^{n}\\ \mathbf{e}_y^{n}\\ \mathbf{h}_z^{n}\end{pmatrix}\qquad \mathbf{u}^{n+0.5}=\begin{pmatrix}\mathbf{h}_x^{n+0.5}\\ \mathbf{h}_y^{n+0.5}\\ \mathbf{e}_z^{n+0.5}\end{pmatrix}$$

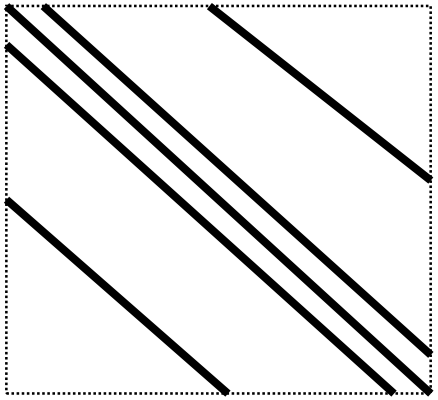
dispersion error suppressed for $\Delta\tau=\Delta z$

Low-dispersive schemes

Implicit TE/TM formulation

$$\mathbf{W}^e \frac{\widehat{\mathbf{e}}_z^{n+0.5} - \widehat{\mathbf{e}}_z^{n-0.5}}{\Delta\tau} = \mathbf{M}_{\varepsilon_z^{-1}} \left[\mathbf{P}_y^* \widehat{\mathbf{h}}_x^{\#} - \mathbf{P}_x^* \widehat{\mathbf{h}}_y^{\#} + \widehat{\widehat{\mathbf{j}}}_z^n \right]$$

$$\mathbf{W}^h \frac{\mathbf{h}_z^{n+1} - \mathbf{h}_z^n}{\Delta\tau} = \mathbf{M}_{\mu_z^{-1}} \left[\mathbf{P}_y \widehat{\mathbf{e}}_x^{\#} - \mathbf{P}_x \widehat{\mathbf{e}}_y^{\#} \right]$$



$$\mathbf{W}^e = \mathbf{W}_{CN}^e = \mathbf{I} + \frac{\Delta\tau^2}{4} \mathbf{M}_{\varepsilon_z^{-1}} \mathbf{P}_y^* \mathbf{M}_{\mu_x^{-1}} \mathbf{P}_y + \frac{\Delta\tau^2}{4} \mathbf{M}_{\varepsilon_z^{-1}} \mathbf{P}_x^* \mathbf{M}_{\mu_y^{-1}} \mathbf{P}_x$$

Low-dispersive schemes

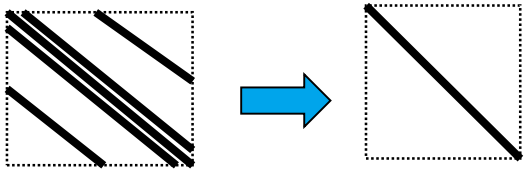
ADI and Explicit TE/TM formulations

TE/TM-ADI scheme

$$\mathbf{W}^e_{CN} = \mathbf{W}^e_{ADI\,2} + O(\Delta\tau^4)$$ - splitting error

Explicit TE/TM scheme*

$$\mathbf{W}^e = \mathbf{W}^h = \mathbf{I}$$



$$\mathbf{W}^e_{CN} = \mathbf{I} + O(\Delta\tau^2)$$ - splitting error

* Dohlus M., Zagorodnov I., *Explicit TE/TM Scheme for Particle Beam Simulations*// Journal of Computational Physics **225**, No. 8, pp. 2822-2833 (2009)

Low-dispersive schemes

Stability, energy and charge conservation

$$\mathbf{B}\frac{\mathbf{y}^{n+1}-\mathbf{y}^{n}}{\Delta\tau}+\mathbf{A}\mathbf{y}^{n}=\mathbf{f}^{n}$$

The stability condition $\mathbf{Q}\equiv\mathbf{B}-0.5\Delta\tau\mathbf{A}\geq 0$

E/M splitting

(FDTD scheme)

$$\Delta\tau\leq\frac{1}{\sqrt{\Delta x^{-2}+\Delta y^{-2}+\Delta z^{-2}}}$$

TE/TM implicit

$$\Delta\tau\leq\Delta z$$

TE/TM explicit

$$\Delta\tau\leq\min\left(\frac{1}{\sqrt{\Delta x^{-2}+\Delta y^{-2}}},\Delta z\right)$$

dispersion error suppressed for $\Delta\tau=\Delta z$

Low-dispersive schemes

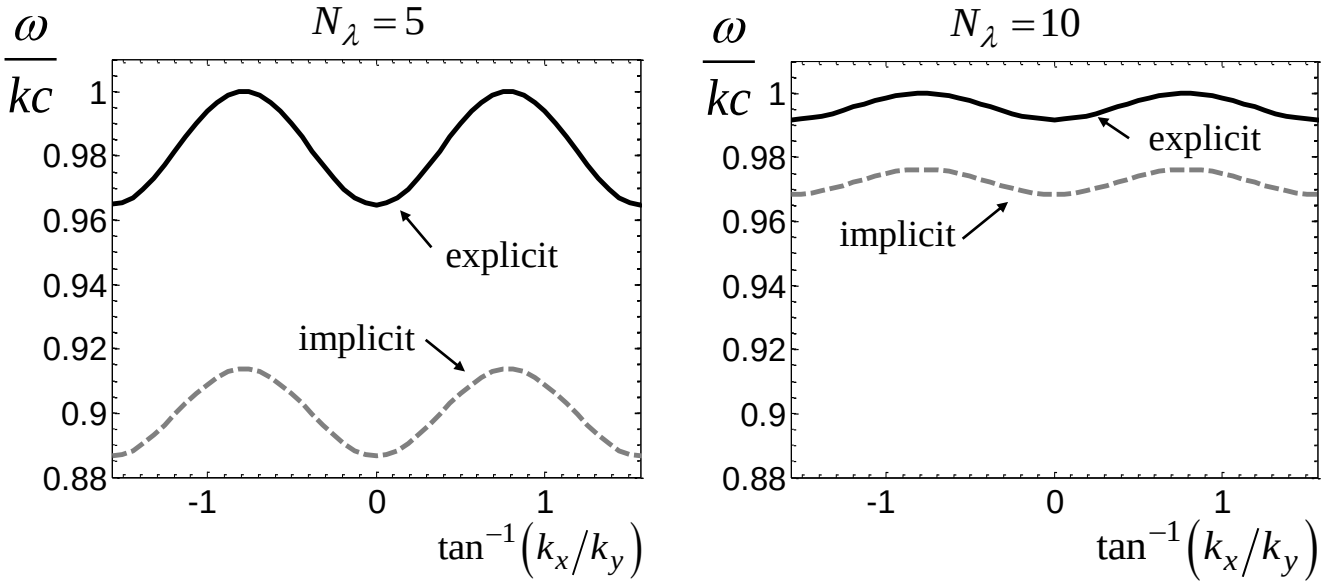
Dispersion relation in the transverse plane

$$\frac{\sin^2\Omega}{\Delta\tau^2}=\frac{\sin^2 K_z}{\Delta z^2}+\left(\frac{\sin^2 K_x}{\Delta x^2}+\frac{\sin^2 K_y}{\Delta y^2}\right)\cos^2\Omega$$

Implicit TE/TM scheme

$$\frac{\sin^2\Omega}{\Delta\tau^2}=\frac{\sin^2 K_z}{\Delta z^2}+\left(\frac{\sin^2 K_x}{\Delta x^2}+\frac{\sin^2 K_y}{\Delta y^2}\right)\left(1-\frac{\Delta\tau^2}{\Delta z^2}\sin^2 K_z\right)$$

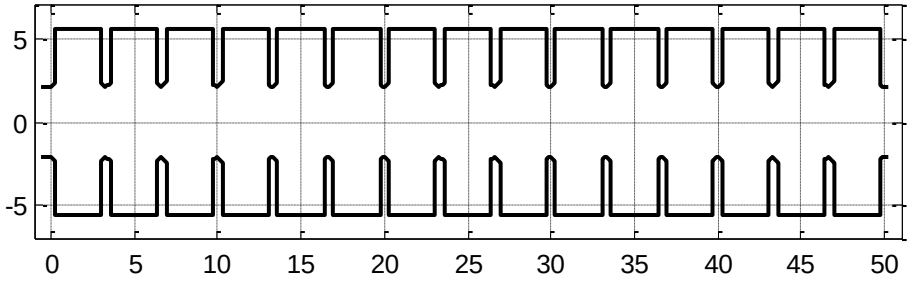
Explicit TE/TM scheme



No dispersion in z-direction + no dispersion along XY diagonals

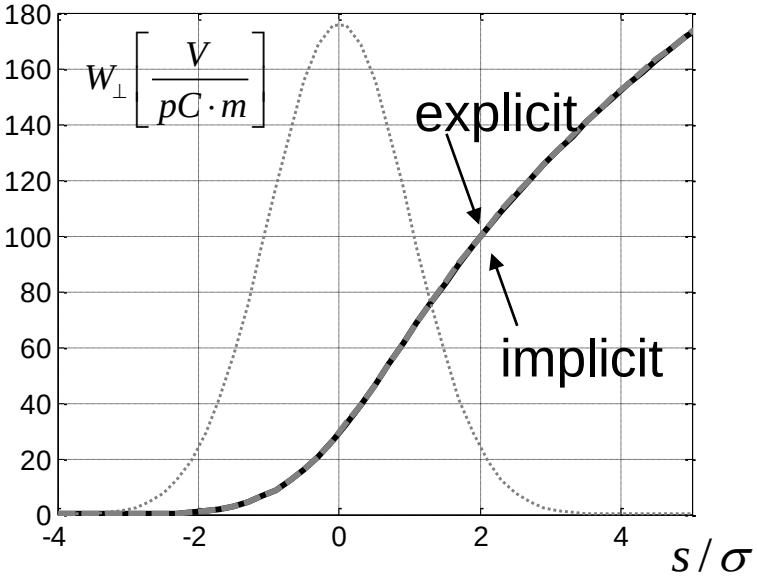
Low-dispersive schemes

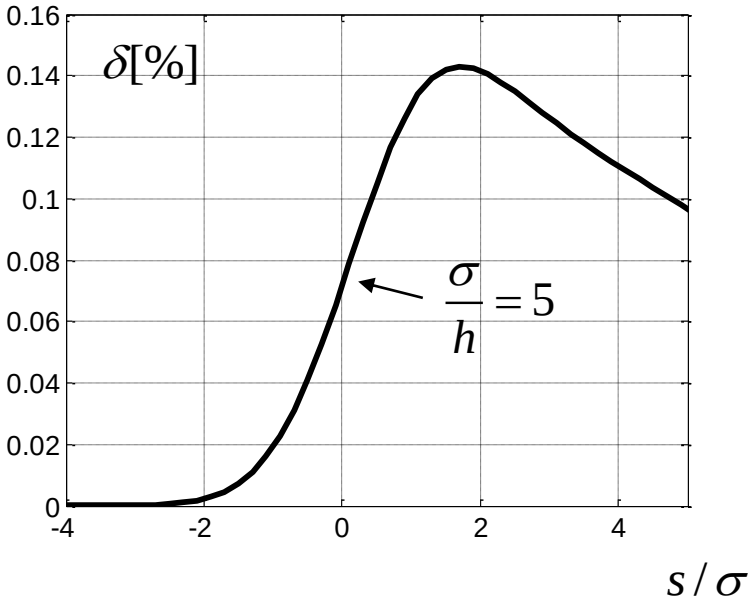
Transverse Deflecting Structure



$$\delta = \frac{\left| W_{\perp}^{\exp} - W_{\perp}^{imp} \right|}{\max W_{\perp}^{\exp} - \min W_{\perp}^{\exp}} * 100\%$$

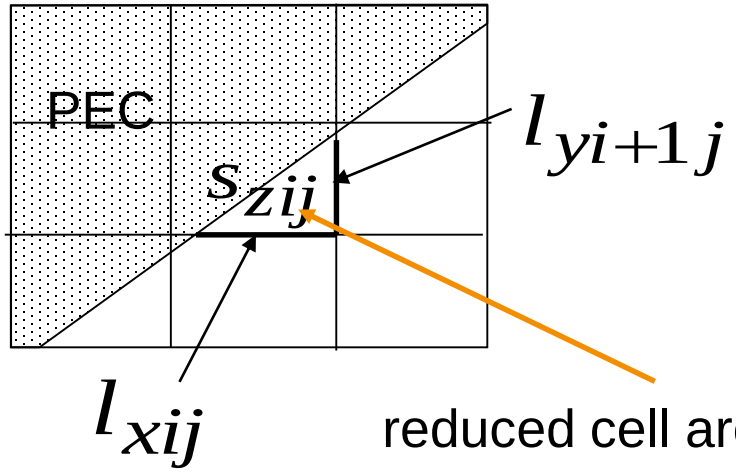
Gaussian bunch with sigma=300μm





Boundary approximation

Standard Conformal Scheme



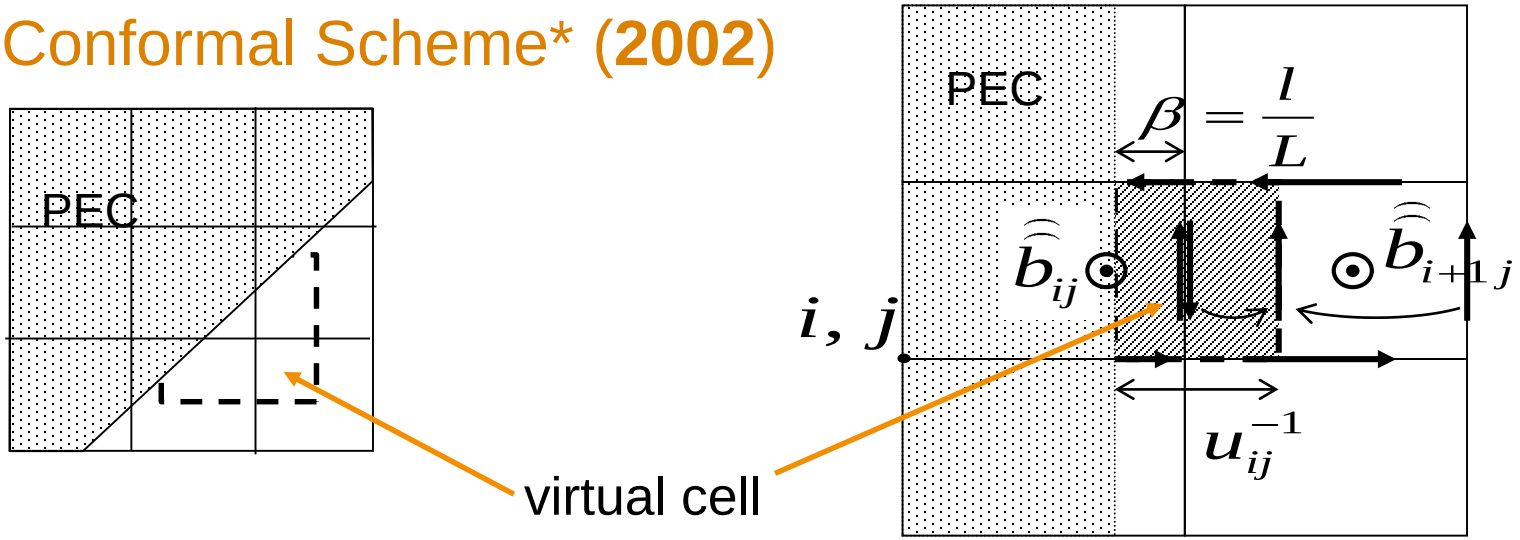
reduced cell area → **time step must be reduced**

Dey S, Mittra R. A locally conformal finite-difference time-domain (FDTD) algorithm for modeling threedimensional perfectly conducting objects. IEEE Microwave and Guided Wave Letters 7(9):273–275 (1997)

Thoma P. Zur numerischen Lösung der Maxwellschen Gleichungen im Zeitbereich. Dissertation Dl7: TH Darmstadt,1997.

Boundary approximation

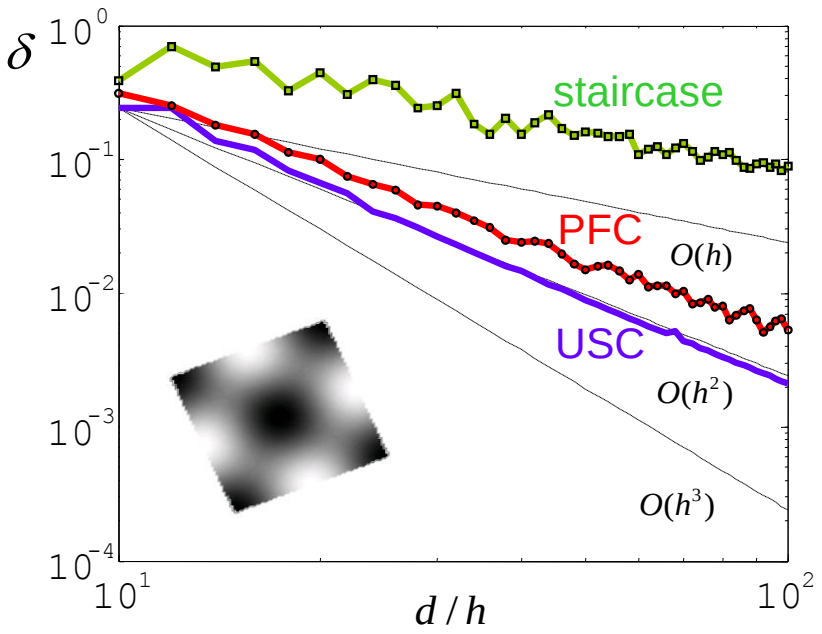
New Conformal Scheme* (**2002**)

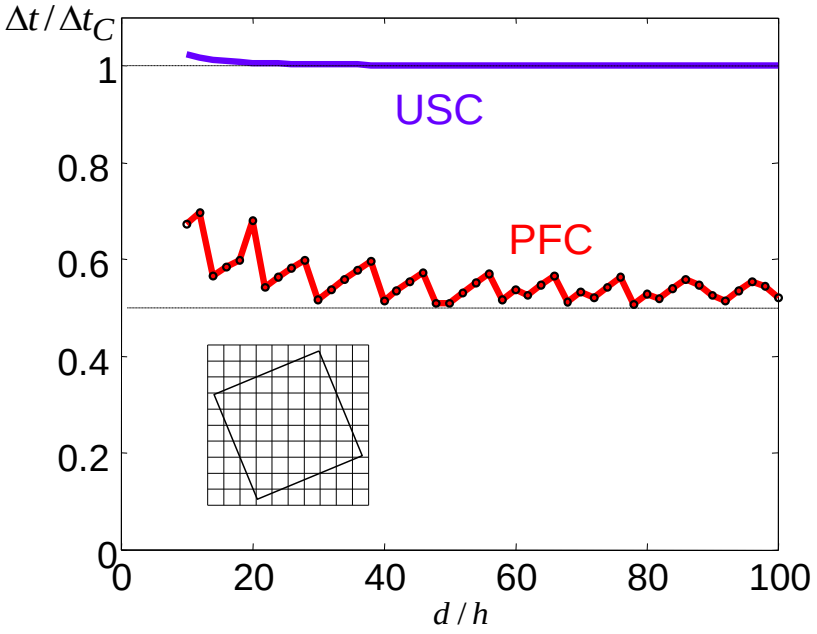


Time step is not reduced!

* Zagorodnov I., Schuhmann R.,Weiland T., *A Uniformly Stable Conformal FDTD-Method on Cartesian Grids*, International Journal on Numerical Modeling, vol. 16, No.2, pp. 127-141 (2003)

Boundary approximation



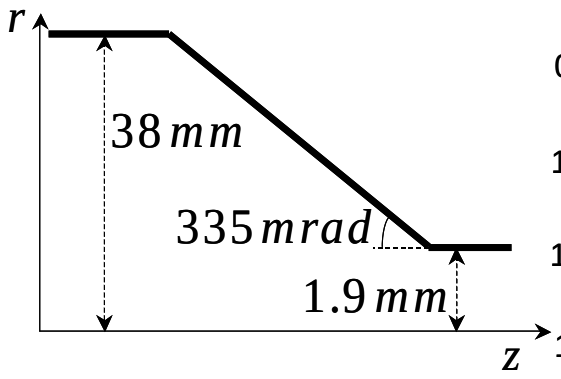


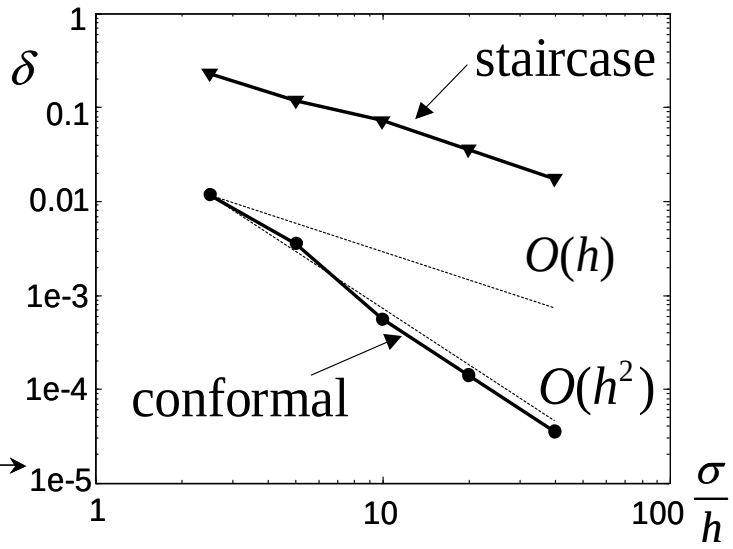
Square rotate by angle $\pi/8$.

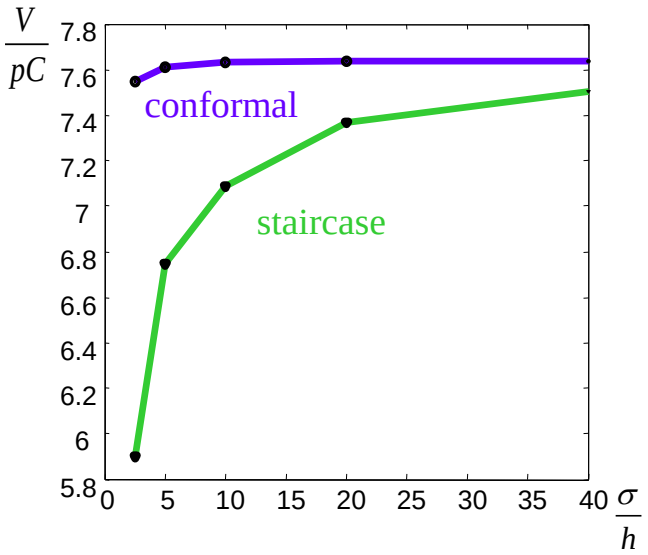
$$\delta = \frac{\left\| H_z - \tilde{H}_z \right\|_{L_2^h}}{\left\| H_z \right\|_{L_2^h}}$$

PFC (Partially Filled Cells) ~ Dey-Mittra
USC (Uniformly Stable Conformal)

Boundary approximation



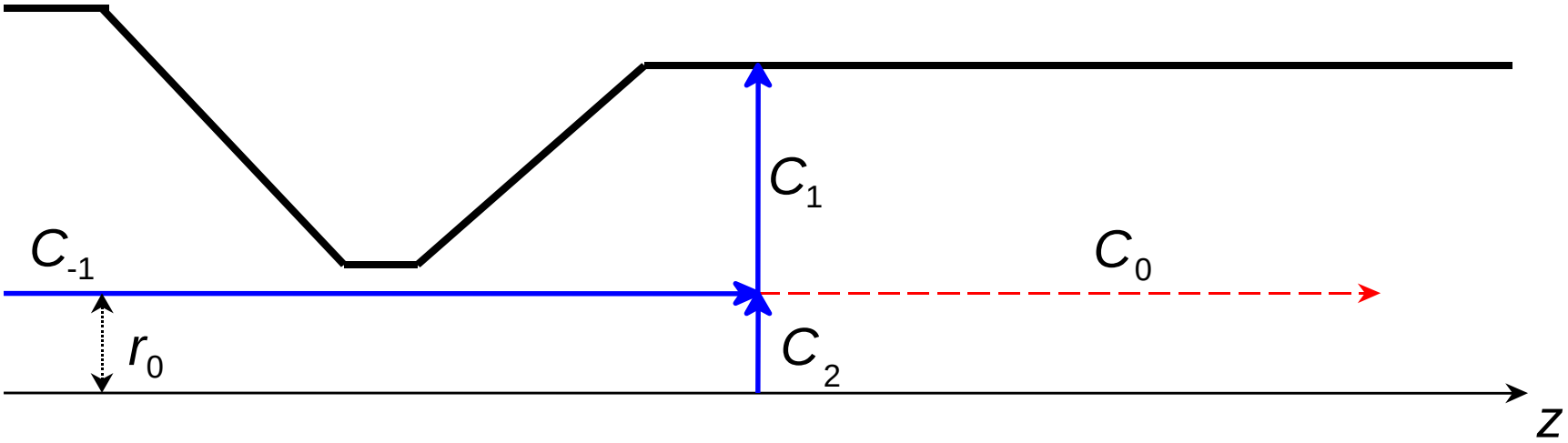




Error in loss factor for a taper $\delta = \left| L_{calc} - L \right| L^{-1}$

The error δ relative to the extrapolated loss factor L=-7.63777 Vp/C for bunch with σ=1 mm is shown.

Indirect Integration Algorithm



$$z = z_0$$

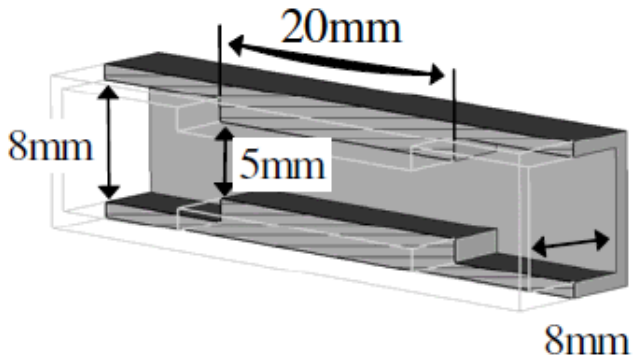
$$QW_{\square}^{m} = \int_{C_{-1}} e_z^s dz + \int_{C_0} e_z^s dz$$

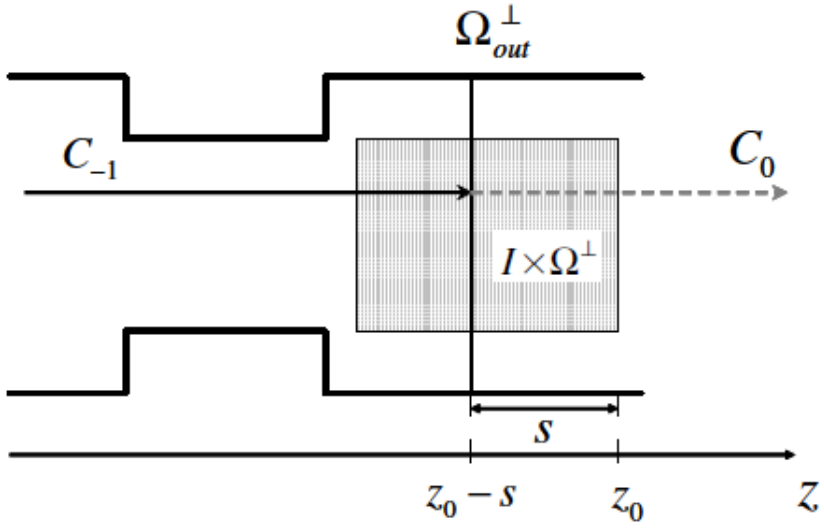
$$\int_{C_0} e_z^s dz = -\frac{1}{2}\left(\int_{C_1} \left(r_0^m \omega_D + r_0^{-m} \omega_S - \frac{\beta}{a^m} \omega_S \right) - \frac{\beta}{a^m} \int_{C_2} \omega_S \right)$$

I. Zagorodnov, R. Schuhmann, T. Weiland, Journal of Computational Physics **191**, No.2 , pp. 525-541 (2003)

O. Napoly, Y. Chin, and B. Zotter, Nucl. Instrum. Methods Phys. Res., Sect. A **334**, 255 (1993)

Indirect Integration Algorithm





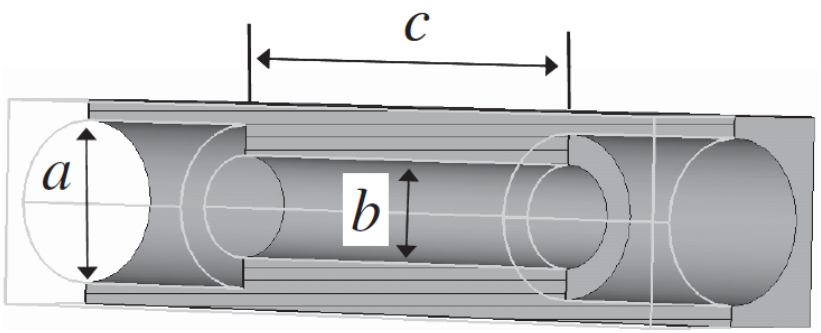
$$QW_{\parallel}(\vec{r}_0, s) = -\int_{C_{-1}(\vec{r}_0, z_0 - s)} E_z^{sc}[\vec{r}_0, z, t(z, s)]dz - u(\vec{r}_0, s), \qquad t(z, s) = \frac{z + s}{c}$$

$$\Delta u(\vec{r}, s) = -\left[\frac{\partial}{\partial s} + \frac{\partial}{c\,\partial t}\right] E_z^{sc}(\vec{r}, z_0 - s, t_0), \qquad \vec{r} \in \Omega^{\perp}_{out}, \qquad u(\vec{r}, s) = 0, \qquad \vec{r} \in \partial\Omega^{\perp}_{out}$$

Zagorodnov I., *Indirect Methods for Wake Potential Integration,* Phys. Rev. STAB **9**, 102002 (2006)

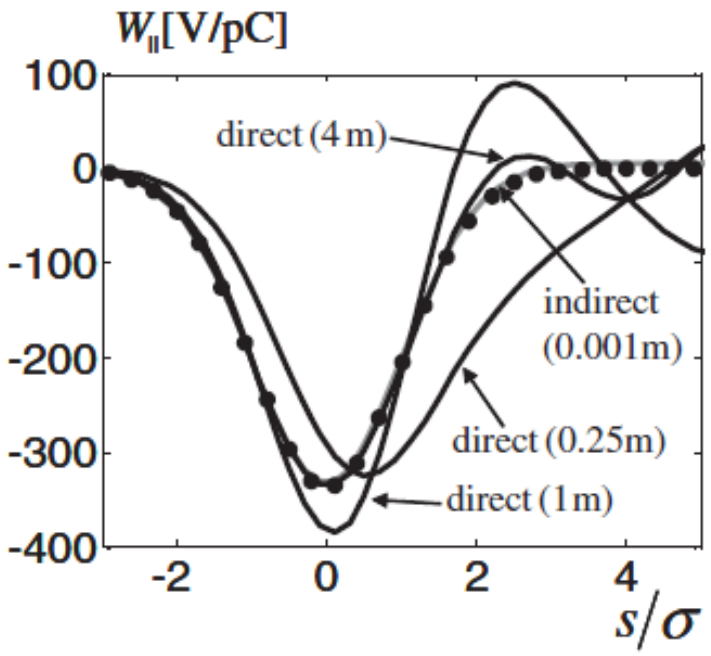
H. Henke and W. Bruns, in Proceedings of EPAC 2006, Edinburgh, Scotland (WEPCH110, 2006)

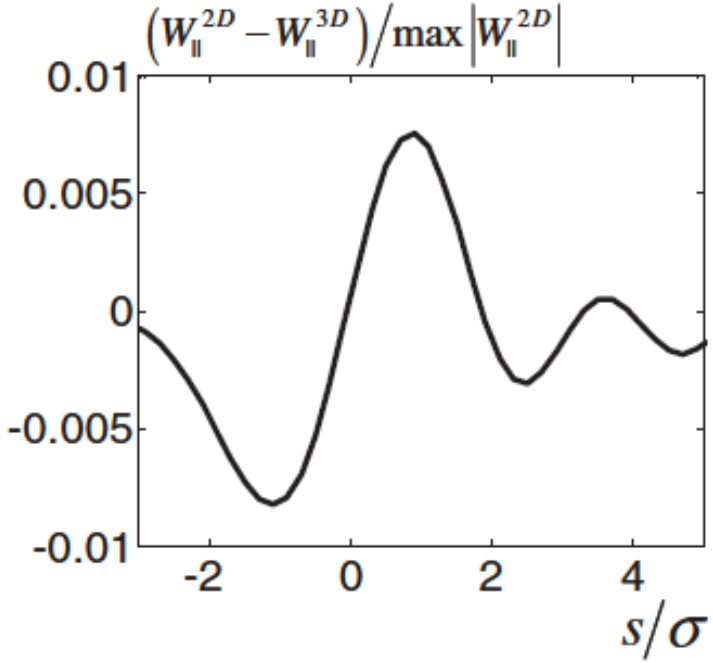
Indirect Integration Algorithm



$a = 8$ mm, $b = 5$ mm, and $c = 20$ mm

Gaussian bunch with rms length 25 µm





Modelling of Conductive Walls

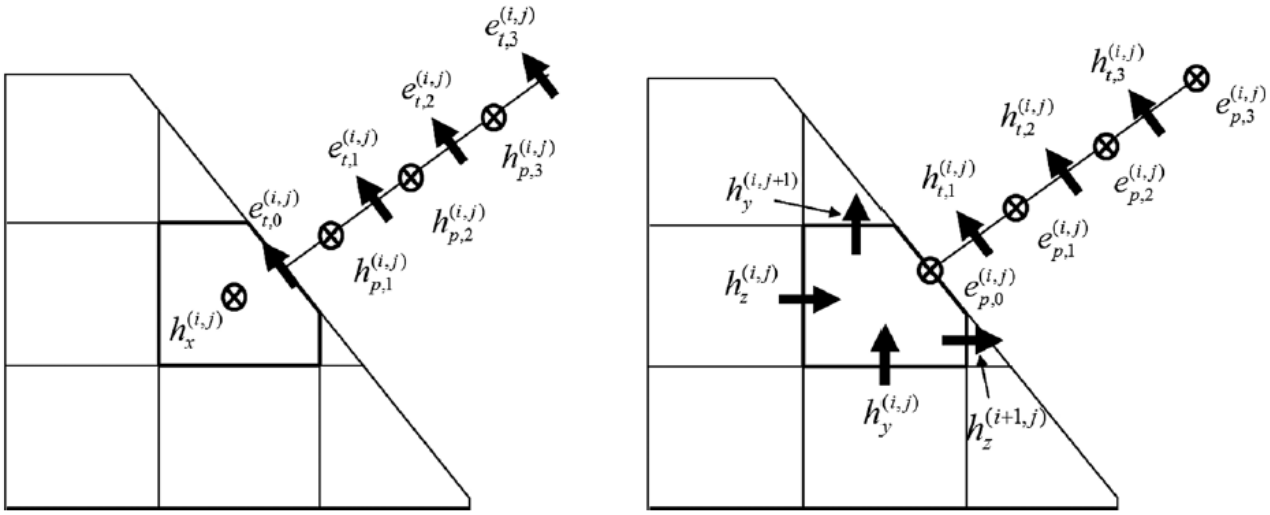
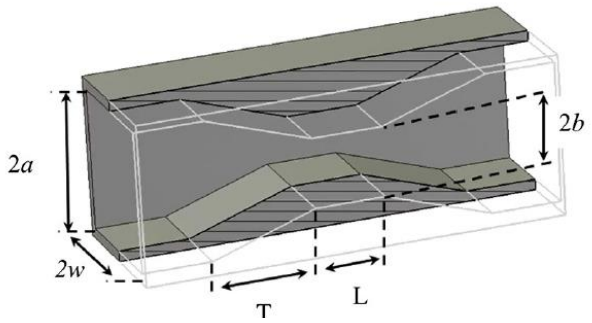


FIG. 6. A boundary cell in the vacuum and 1D conductive line in the metal.

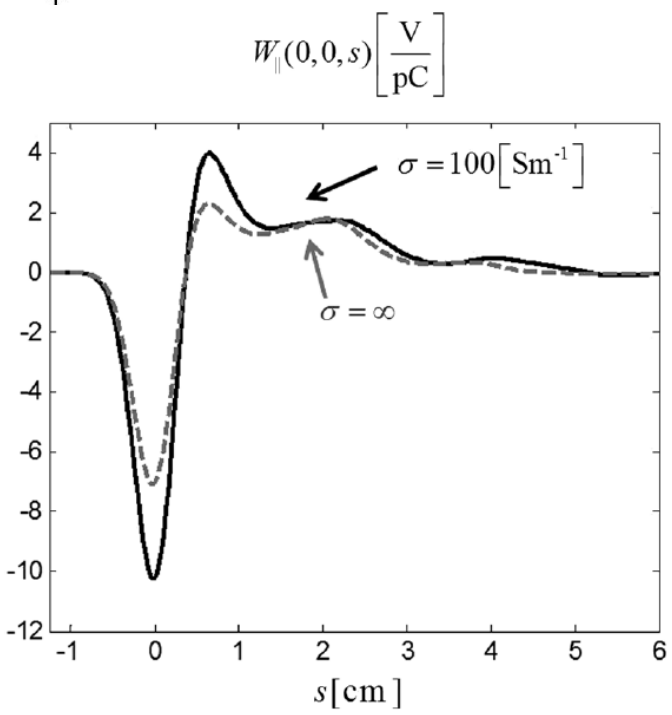
Tsakanian A., Dohlus M., Zagorodnov I., *Hybrid TE-TM scheme for time domain numerical calculations of wakefields in structures with walls of finite conductivity,* Phys. Rev. STAB **15**, 054401 (2012)

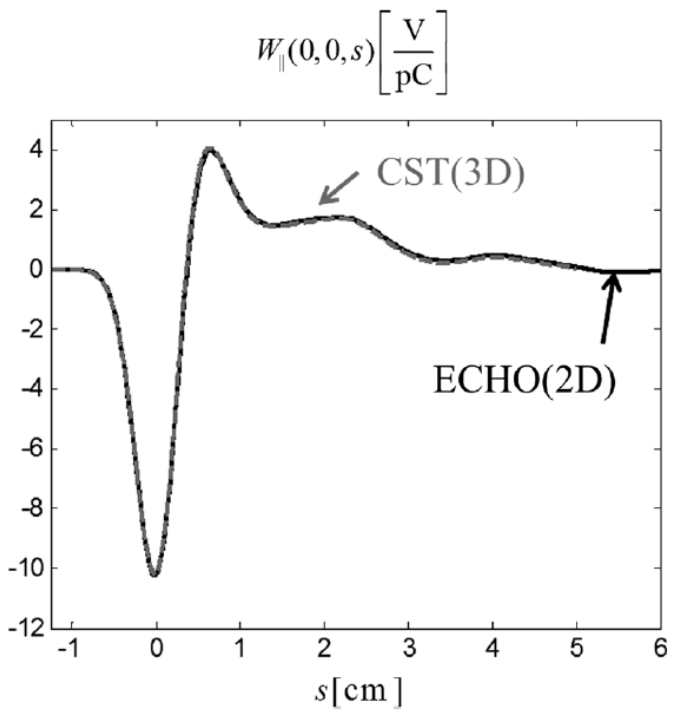
Zagorodnov I., Bane K., Stupakov G. Calculations of wakefields in 2D rectangular structures, Phys. Rev. STAB **18**, 104401 (2015)

Modelling of Conductive Walls



2*w*= 10 cm, *T*=5 cm, 2*b*=2cm, *L*=12 cm.
Gaussian bunch with rms length 25mm





Longitudinal wake potential of tapered collimator

Code Status

- ECHOz1, ECHOz2 (rotationally symetric)
- ECHO2D (rectangular and rotationally symetric)

https://www.desy.de/~zagor/WakefieldCode_ECHOz/

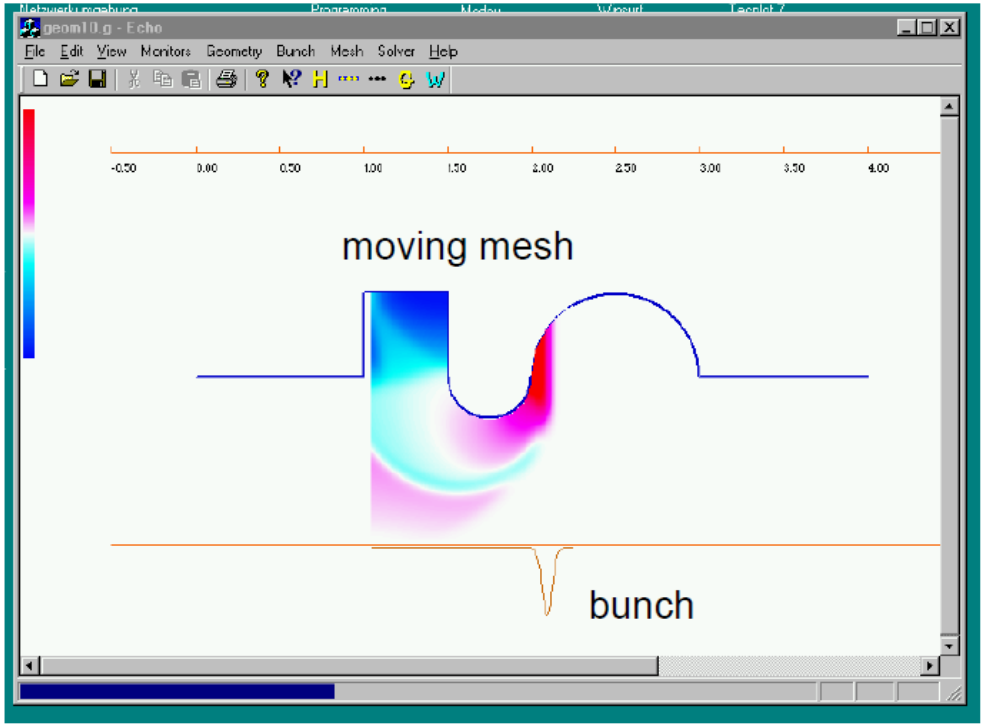
- ECHO3D (fully 3D)
- ECHO-PIC (Particle-In-Cell for rotationally symmetric and rectangular)

Rotationally Symmetric Geometries

$$\begin{pmatrix} E_r \\ H_\theta \\ E_z \end{pmatrix} = \sum_{m=0}^{\infty} \begin{pmatrix} E_{r,m} \\ H_{\theta,m} \\ E_{z,m} \end{pmatrix} \cos(m\theta) \qquad \begin{pmatrix} H_r \\ E_\theta \\ H_z \end{pmatrix} = \sum_{m=0}^{\infty} \begin{pmatrix} H_{r,m} \\ E_{\theta,m} \\ H_{z,m} \end{pmatrix} \sin(m\theta)$$

$$W_{\parallel}(r_0, \theta_0, r, \theta, s) = \sum_{m=0}^{\infty} W_m(s) r_0^m r^m \cos\left(m(\theta - \theta_0)\right)$$

ECHOz1



ECHO

Electromagnetic
Code for
Handling
Of
Harmful
Collective
Effects

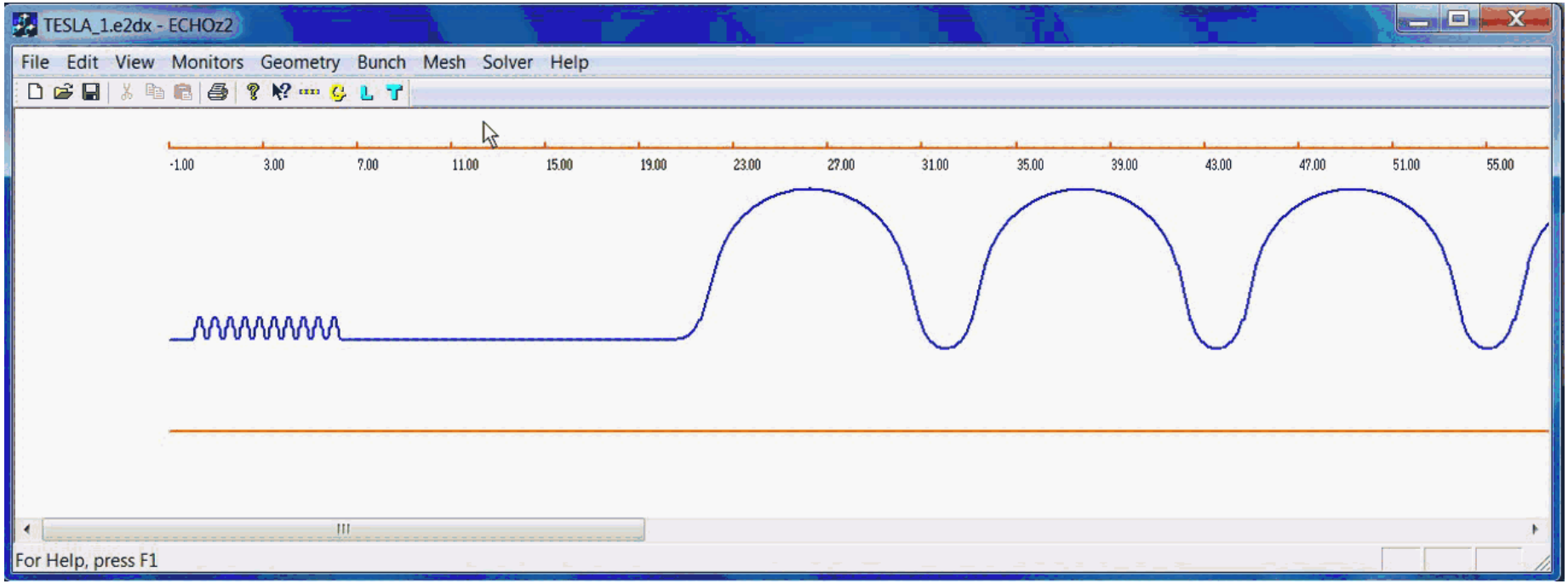
Only fully rotationally symmetric problems (m=0 mode)

- vector potential wave equation (slide 9)
- only PEC
- stand-alone Windows GUI application

ECHOz2

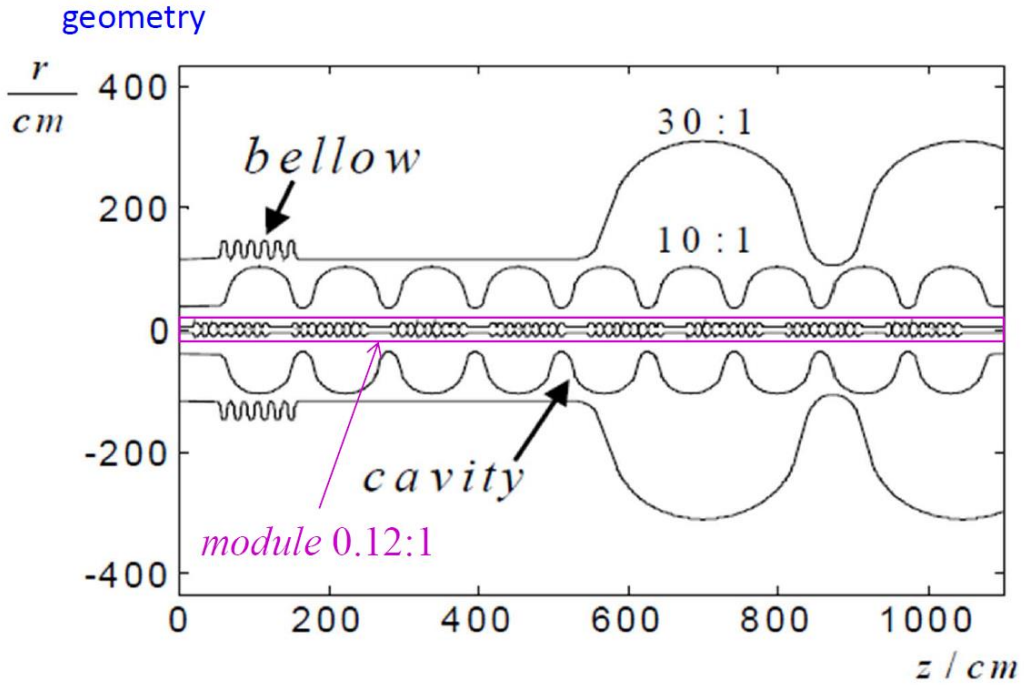
Rotationally symmetric problems (all modes)

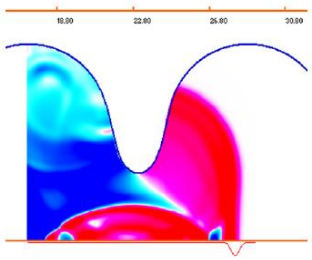
- TE/TM implicit (slide 10)
- surface conductivity
- stand-alone Windows GUI application



ECHOz2

Short-Range Wake Functions for TESLA Cryomodule





σ = 700 ... **50 μm**

Longitudinal wake functions was found earlier with code NOVO

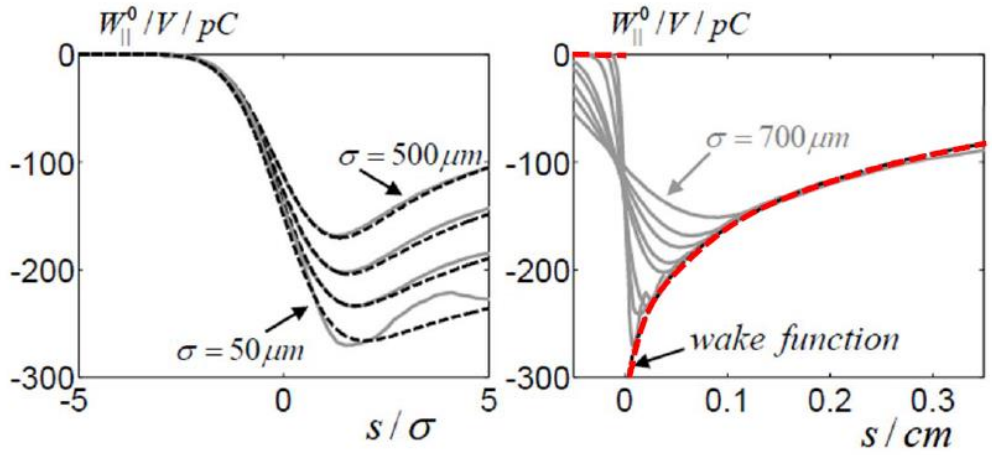
Novokhatski A., Timm M., Weiland T., Single Bunch Enetgy Spread in the TESLA Cryomodule, DESY TESLA-99-16, 1999

Transverse wake functions is found with code ECHO

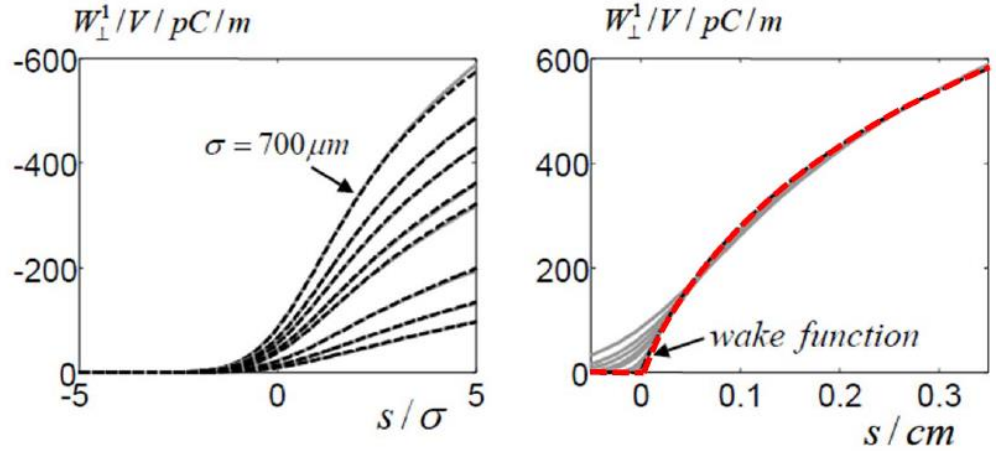
T. Weiland, I. Zagorodnov, The short-range transverse wake function for TESLA accelerating structure, TESLA Report 2003-19

ECHOz2

longitudinal wake (monopole)



transverse wake (dipole)



Novokhatski A., Timm M., Weiland T. (1999, NOVO)

$$W_{\parallel}(s) = A\left[(1+\beta_0)e^{-\sqrt{\frac{s}{s_0}}} - \beta_0\right]$$

Weiland T., Zagorodnov I. (2003, ECHO)*

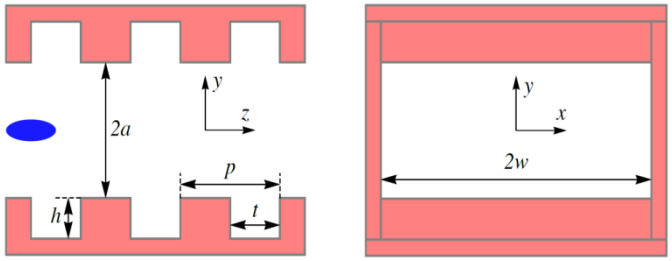
$$W_{\parallel}(s) = Ae^{-\sqrt{\frac{s}{s_0}}}$$

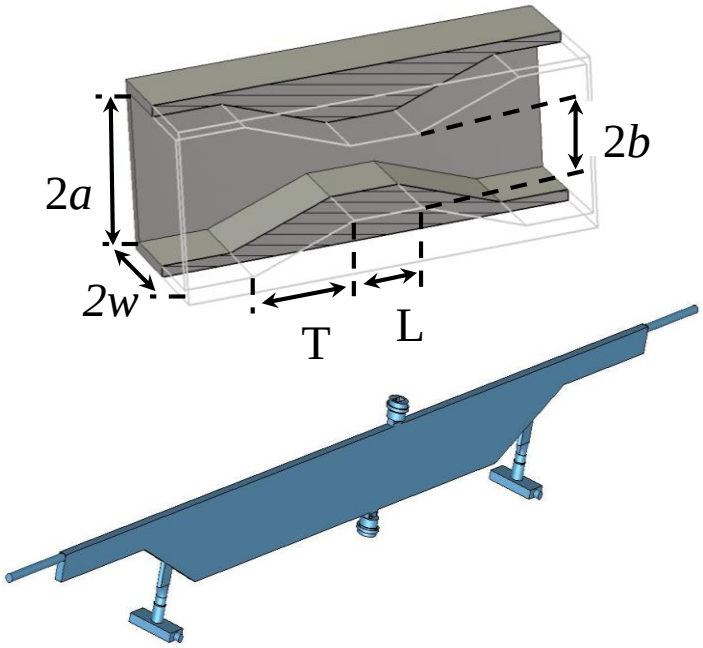
$$W_{\perp}(s) = A_1\left[1-\left(1+\sqrt{\frac{s}{s_1}}\right)e^{-\sqrt{\frac{s}{s_1}}}\right]$$

Bane K.L.F., *Short-Range Dipole Wakefields in Accelerating Structures for the NLC,* SLAC-PUB-9663, LCC-0116, 2003

ECHO2D

Rectangular Geometries with Constant Width





$$\begin{pmatrix} E_x \\ H_y \\ H_z \end{pmatrix} = \frac{1}{w}\sum_{m=1}^{\infty} \begin{pmatrix} E_{x,m} \\ H_{y,m} \\ E_{z,m} \end{pmatrix} \cos\left(\frac{\pi}{2w} mx\right)$$

$$\begin{pmatrix} H_x \\ E_y \\ E_z \end{pmatrix} = \frac{1}{w}\sum_{m=1}^{\infty} \begin{pmatrix} H_{x,m} \\ E_{y,m} \\ E_{z,m} \end{pmatrix} \sin\left(\frac{\pi}{2w} mx\right)$$

Zagorodnov I., Bane K., Stupakov G. Calculations of wakefields in 2D rectangular structures, Phys. Rev. STAB **18**, 104401 (2015)

The idea can be found, for example, in

A. Tremaine, J. Rosenzweig, and P. Schoessow, Phys. Rev. E **56**, 7204 (1997)

DESY

ECHO2D

Wake field expansion is new (to our knowledge)

$$W_{\parallel}(x_0, y_0, x, y, s) = \frac{1}{w}\sum_{m=0}^{\infty} W_m(y_0, y, s)\sin\left(k_{x,m}x_0\right)\sin\left(k_{x,m}x\right)$$

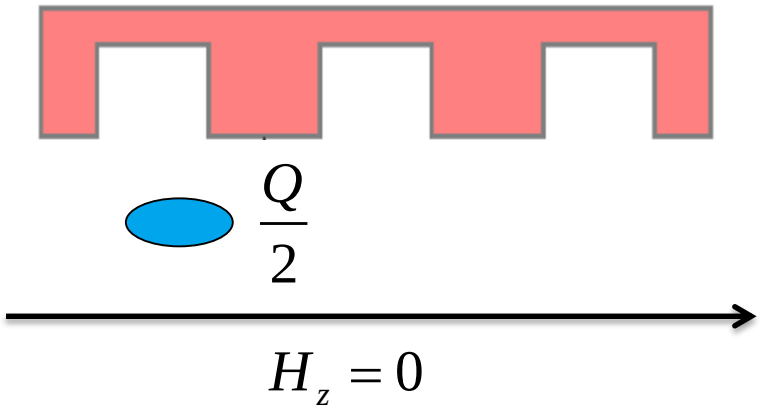
General case

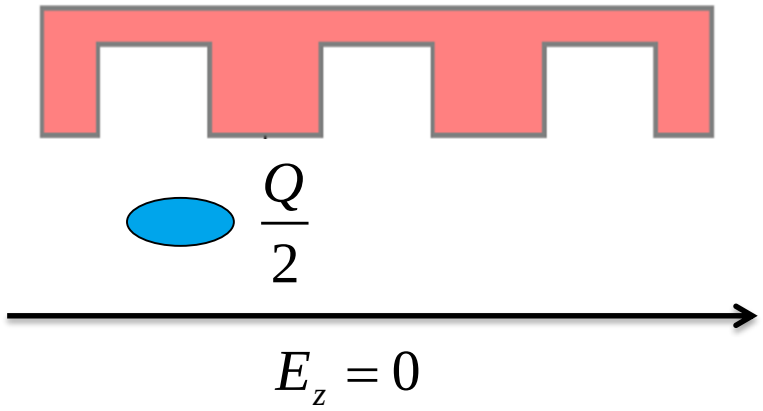
$$W_m(y_0, y, s) = \begin{pmatrix}\cosh(k_{x,m}y_0) & \sinh(k_{x,m}y_0)\end{pmatrix} \begin{pmatrix} W_m^{cc}(s) & W_m^{cs}(s) \\ W_m^{sc}(s) & W_m^{ss}(s)\end{pmatrix} \begin{pmatrix}\cosh(k_{x,m}y) \\ \sinh(k_{x,m}y)\end{pmatrix}$$

With symmetry in y

$$W_m(y_0, y, s) = W_m^{cc}(s)\cosh(k_{x,m}y_0)\cosh(k_{x,m}y) + \\ + W_m^{ss}(s)\sinh(k_{x,m}y_0)\sinh(k_{x,m}y)$$

ECHO2D





$$W_m(y_0, y, s) = W_m^H(y_0, y, s) + W_m^E(y_0, y, s)$$

$$W_m^{cc}(s) = \frac{W_m^H(y_0, y_0, s)}{\cosh(k_{x,m} y_0)^2}$$

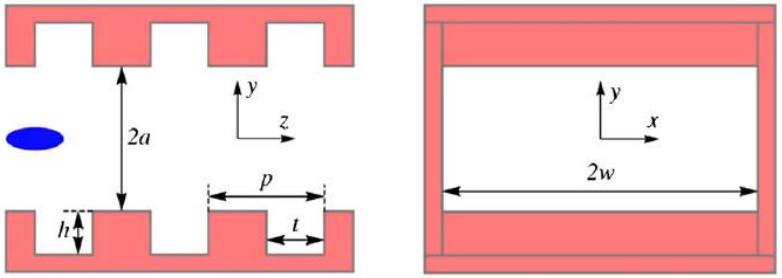
$$W_m^{ss}(s) = \frac{W_m^E(y_0, y_0, s)}{\sinh(k_{x,m} y_0)^2}$$

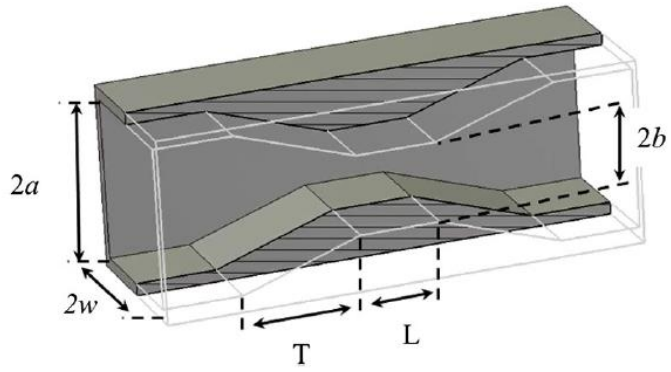
$$W_m(y_0, y, s) = W_m^{cc}(s)\cosh(k_{x,m} y_0)\cosh(k_{x,m} y) + W_m^{ss}(s)\sinh(k_{x,m} y_0)\sinh(k_{x,m} y)$$

ECHO2D

Rectangular and rotationally symmetric problems (all modes)

- TE/TM implicit (slide 10)
- surface conductivity
- stand-alone console application (Windows, Mac OS, Linux)
- Parallelized (MPI and threads)

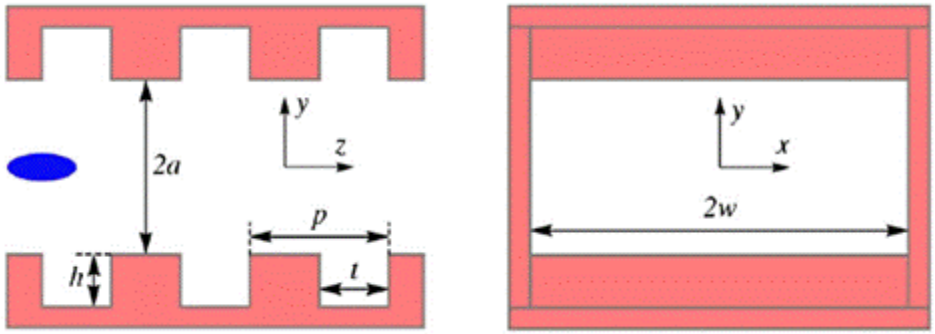




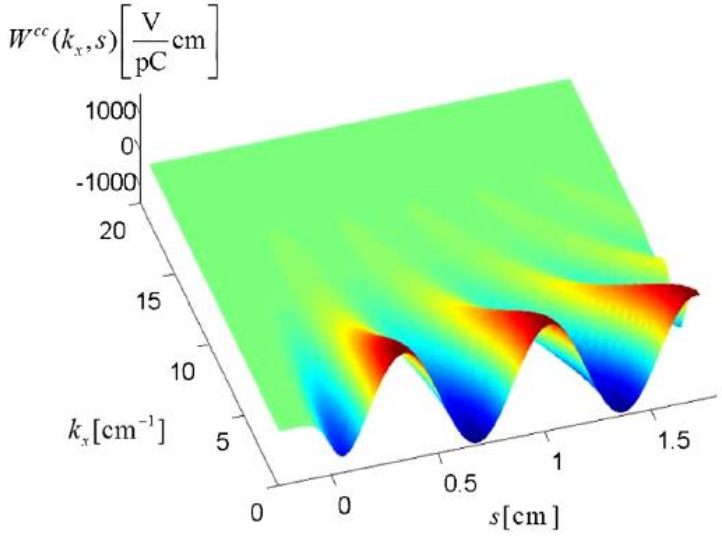
Zagorodnov I., Bane K., Stupakov G. Calculations of wakefields in 2D rectangular structures, Phys. Rev. STAB **18**, 104401 (2015)

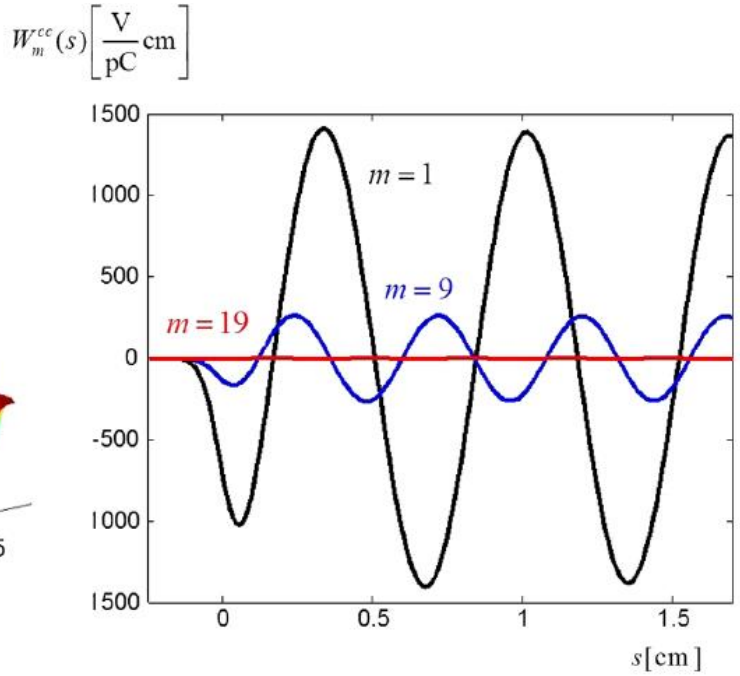
ECHO2D

Pohang Dechirper Experiment



2*w*= 5 cm, *p*=0.5 mm,
h=0.6mm,
t=p/2, *L*=1 m, 2a= 6mm.
Gaussian bunch with rms length 0.5mm

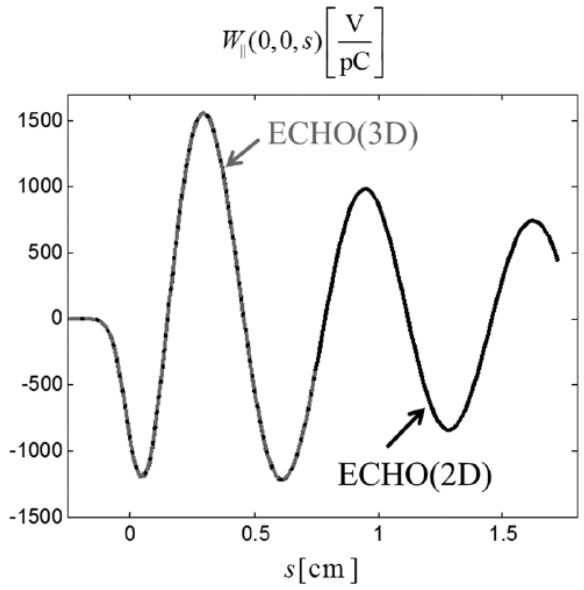


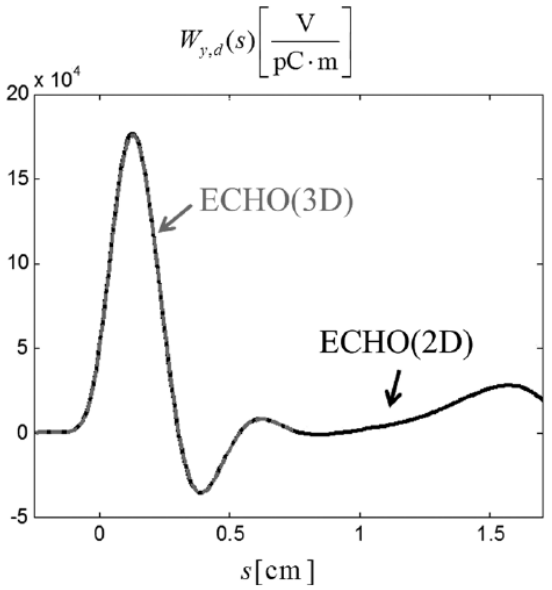


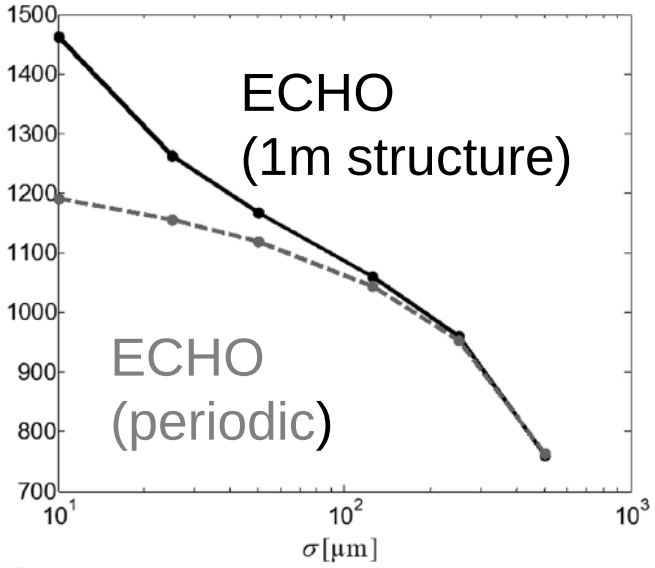
ECHO2D

Pohang Dechirper Experiment*

$$\frac{\langle W_{\parallel}(\mathbf{0},\mathbf{0},s)\rangle}{\mathrm{V/pC}}$$







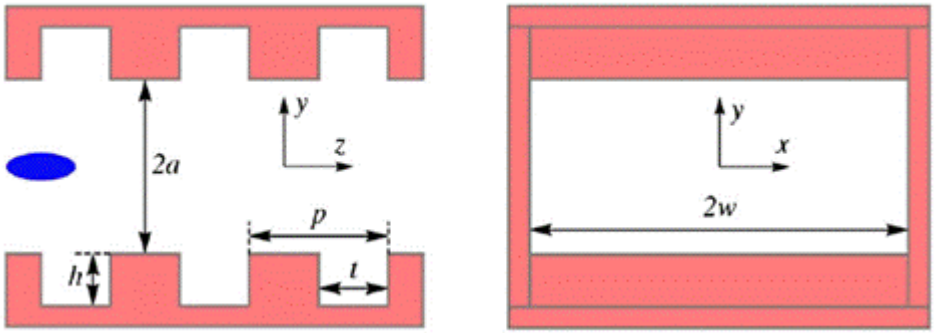
Wake	r[T3P]	r[ECHO]
Longitudinal, loss factor	0.84	0.83
Dipole, kick factor	1.08	0.79
Quad, kick factor	0.73	0.73

$$r = \frac{\text{measured}}{\text{analytical}} = 0.75$$

*P. Emma et al., Phys. Rev. Lett. **112**, 034801 (2014)

ECHO2D

SLAC Dechirper



2*w*= 1.2 cm, *p*=0.5 mm, h=0.5mm, t=p/2, *L*=2 m, **2a= 1.4 mm**.

$$W_{\parallel}(\mathbf{0},\mathbf{0},s) = Ae^{-\sqrt{\frac{s}{s_0}}}$$

	Analytical*	Periodic (ECHO)	Length 2m (ECHO)	Length 2m (NOVO**)
A [V/pC]	9.05	9.08	**9.29**	9.80
s_0 [mm]		0.25	**0.21**	0.22

*Bane K., Stupakov G., Dechirper Wakefields for Short Bunches, Report LCLS-II TN-16-01(2016)

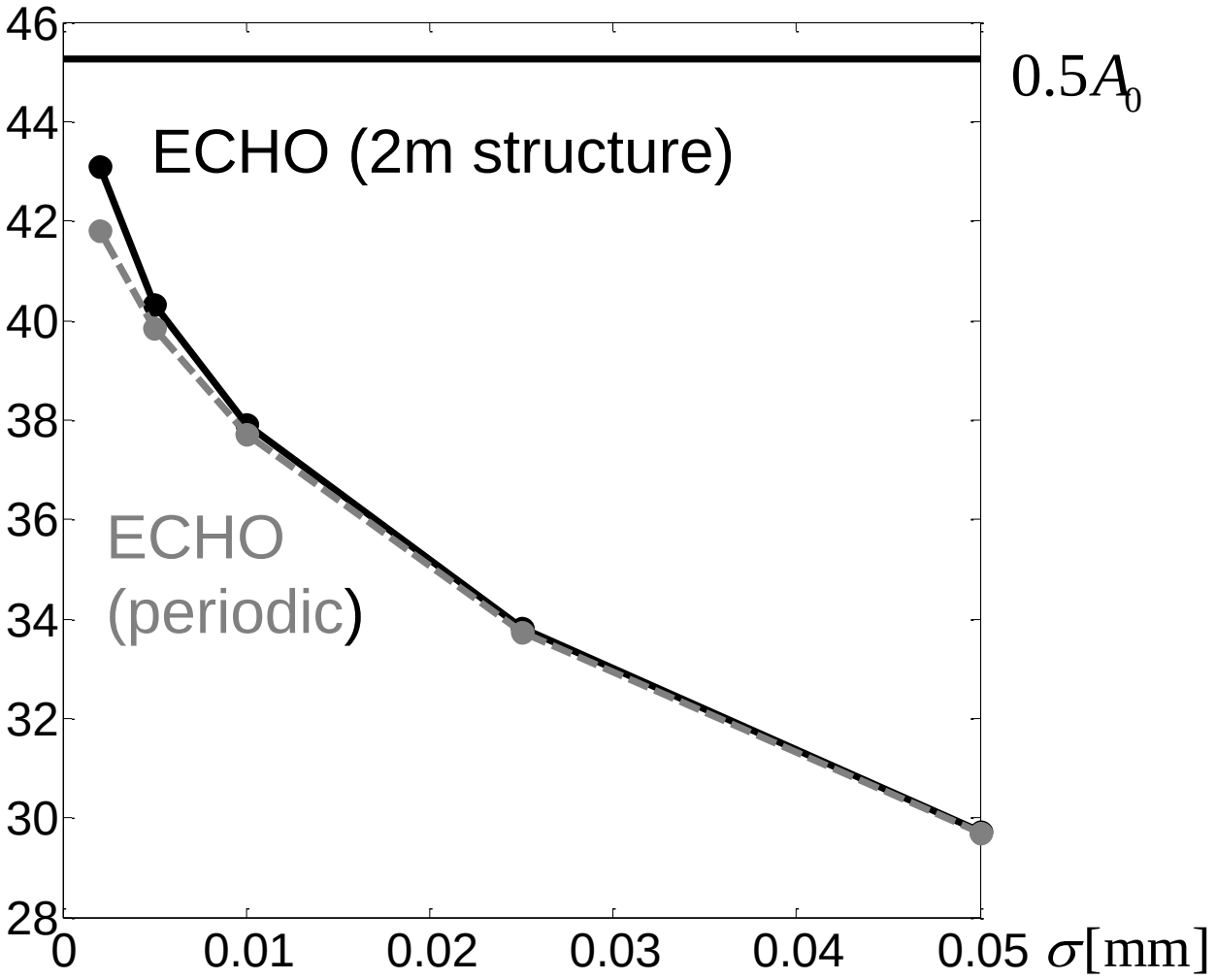
$$*A_0 = \frac{Z_0 c\pi}{16a^2} L$$

Novokhatski A., Wakefield potentials of corrugated structures, PR-STAB **18, 104402 (2015)

ECHO2D

SLAC Dechirper

$$\frac{\langle W_{\parallel}(\mathbf{0},\mathbf{0},s)\rangle}{\mathrm{kV/pC}}$$



ECHO2D

SLAC Dechirper

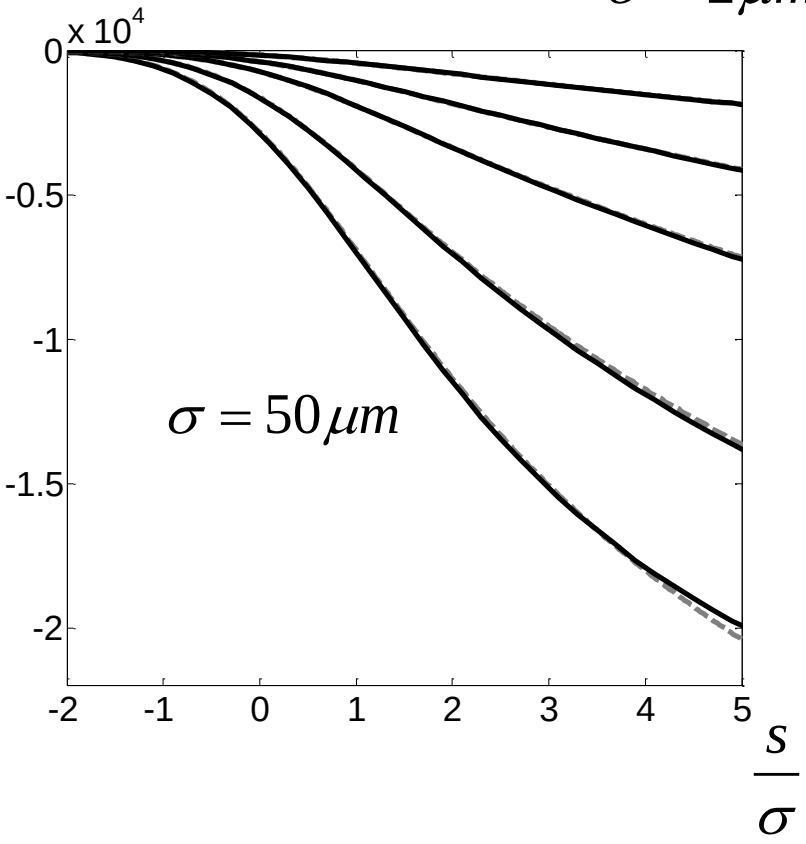
$$W_y(y_0, y, s) \approx y_0 \frac{\partial}{\partial y_0} W_y(\mathbf{0}, \mathbf{0}, s) + y \frac{\partial}{\partial y} W_y(\mathbf{0}, \mathbf{0}, s)$$

$$\frac{-\frac{\partial}{\partial y} W_y(\mathbf{0}, \mathbf{0}, s)}{\text{kV/pC/mm}}$$

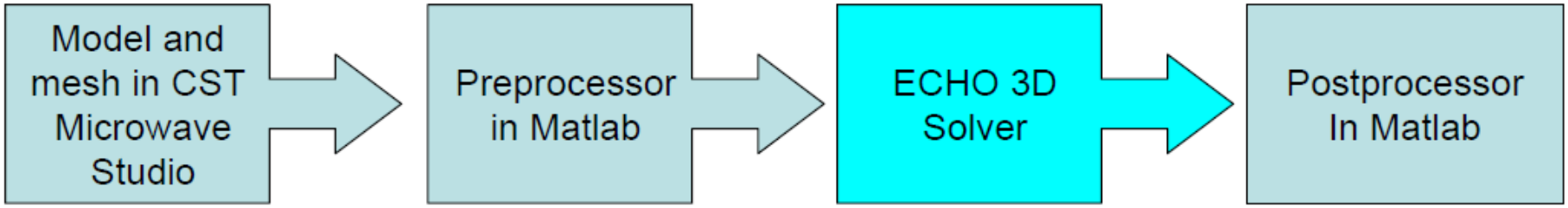
$$\begin{pmatrix} \frac{\partial}{\partial y} W_y(\mathbf{0}, \mathbf{0}, s) \\ \frac{\partial}{\partial y_0} W_y(\mathbf{0}, \mathbf{0}, s) \end{pmatrix} \sim \frac{A}{2}\left[1 - \left(1 + \sqrt{\frac{s}{s_0}}\right) e^{-\sqrt{\frac{s}{s_0}}}\right]$$

	Quadrupole	Dipole
A [V/pC]	8.08	6.94
s_0 [mm]	0.086	0.066

$\sigma = 2\,\mu m$



ECHO3D

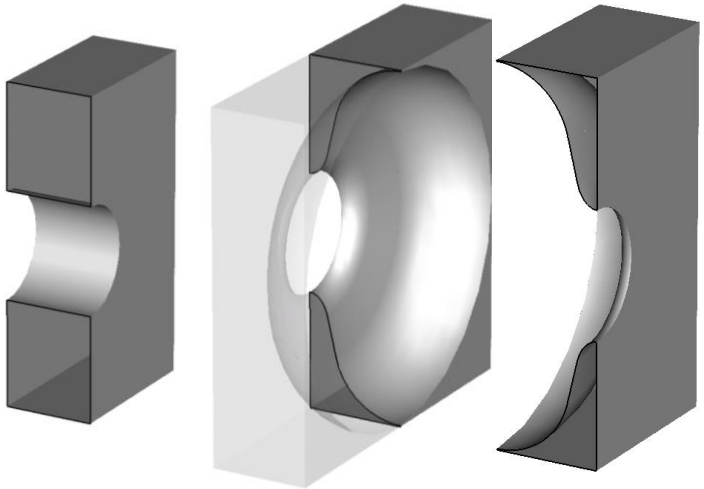


- in **3D** only solver, modelling and meshing in CST Microwave Studio
- allows for accurate calculations on conventional single-processor PC
- To be parallelized ...

3D simulation. Cavity

Moving mesh

20 TESLA cells structure

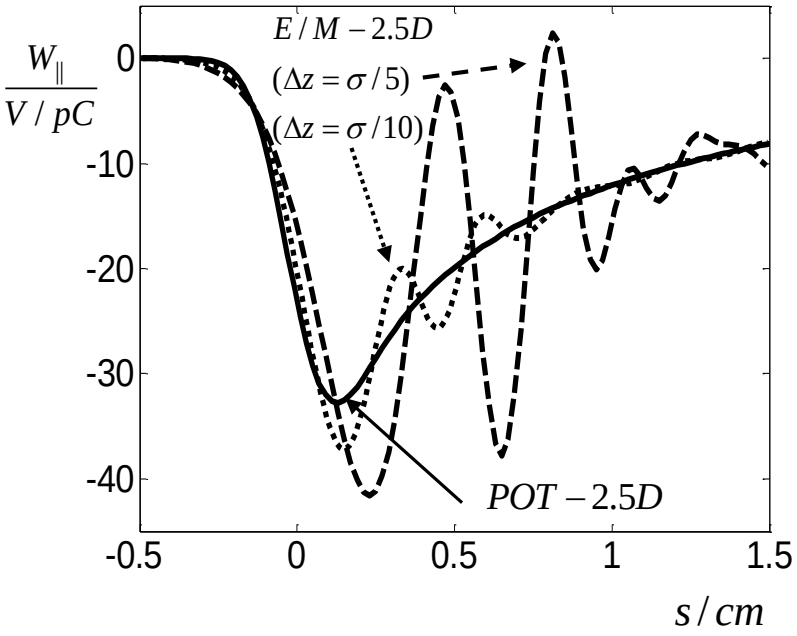


3 geometry elements

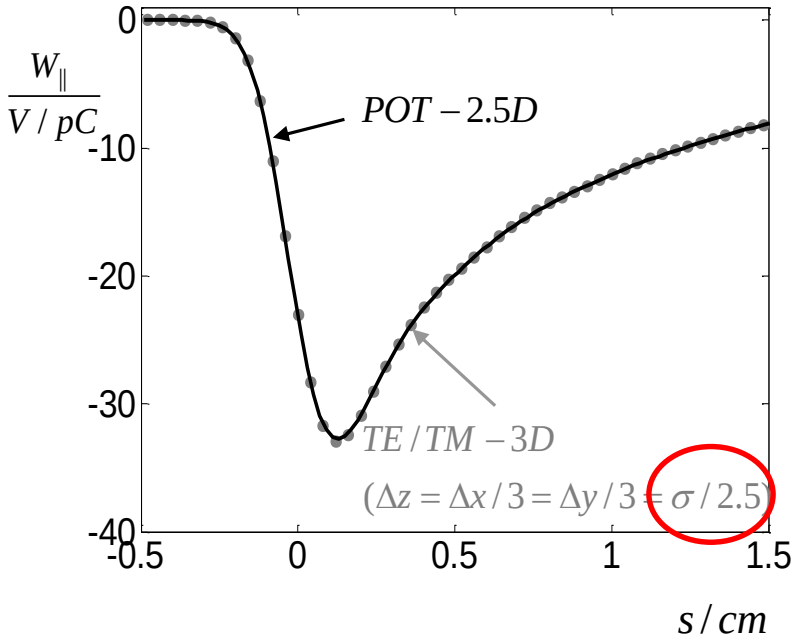
The geometric elements are loaded at the instant when the moving mesh reach them. During the calculation only 2 geometric elements are in memory.

ECHO3D

E/M



ECHO-3D (TE/TM)



Comparison of the wake potentials obtained by different methods for structure consisting of 20 TESLA cells excited by Gaussian bunch $\sigma = 1mm$

E/M splitting $\Delta z \sim \sqrt{\frac{\sigma^3}{L}}$

TE/TM splitting $\Delta z \sim \sigma$

ECHO 3D

Coupler Kick

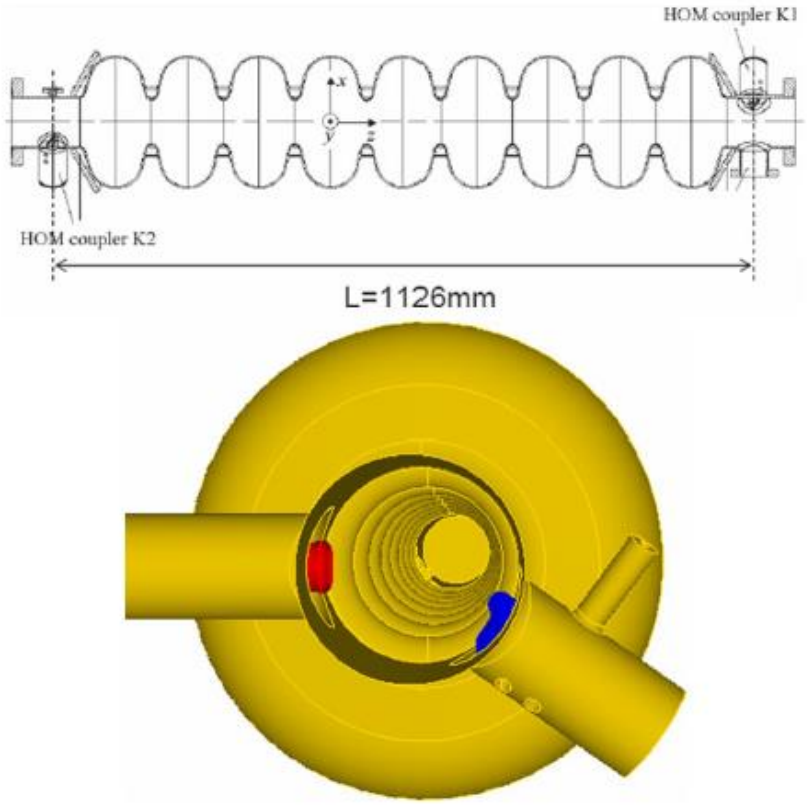


Figure 1: Sketch of an ILC cavity with the HM couplers (top). View from the downstream end of a cavity showing the FM coupler (red) and one HM coupler (blue).

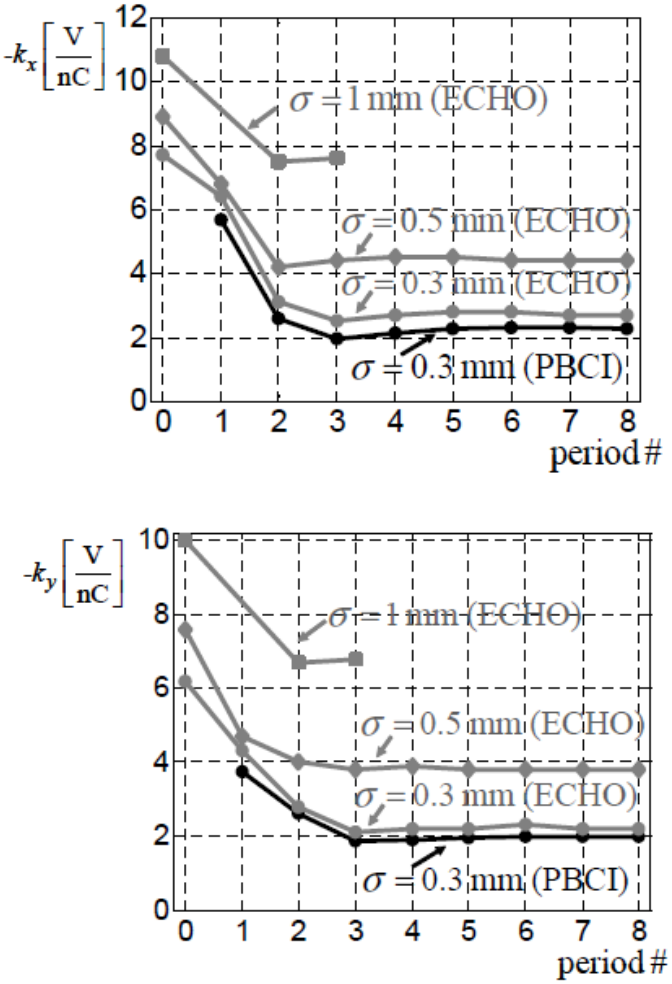


Figure 3: Horizontal and vertical kick vs. number of periods.

ECHO-PIC

Stupakov G., Using pipe with corrugated walls for a subterahertz free electron laser, PR-STAB **18**, 030709 (2015)

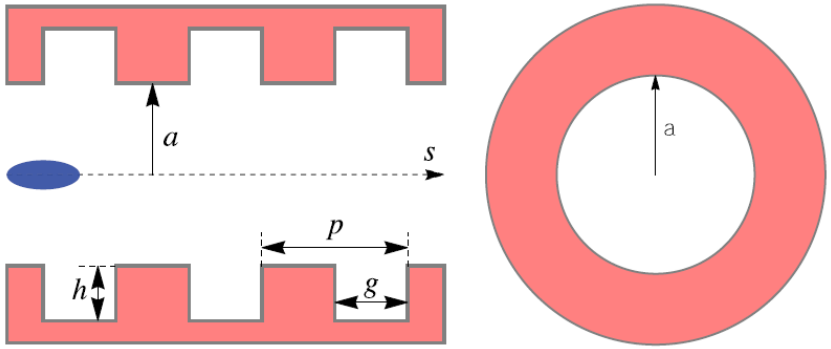
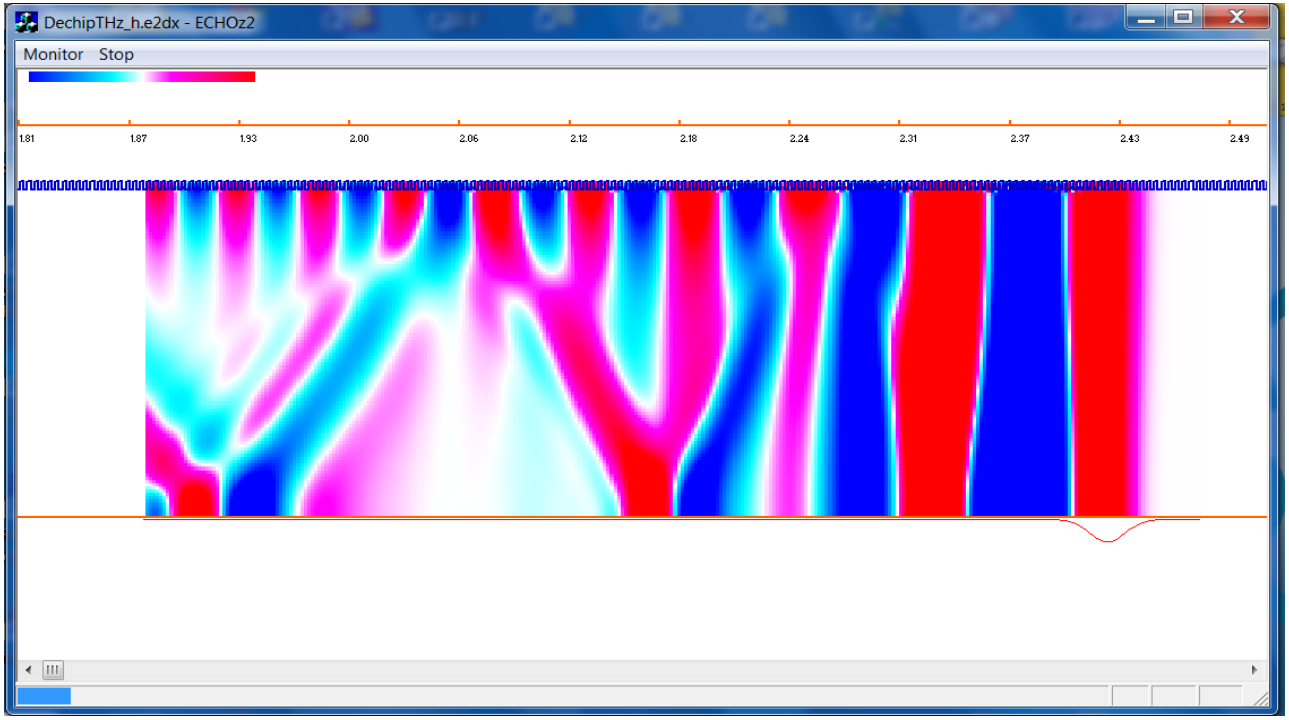


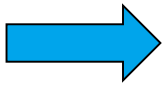
TABLE I. Corrugation and beam parameters.

Pipe radius (mm)	2
Depth h (μm)	50
Period p (μm)	40
Gap g (μm)	10
Bunch charge (nC)	1
Energy (MeV)	5
Bunch length (ps)	10

ECHO-PIC



Wakefield code ECHO
(with resistivity)

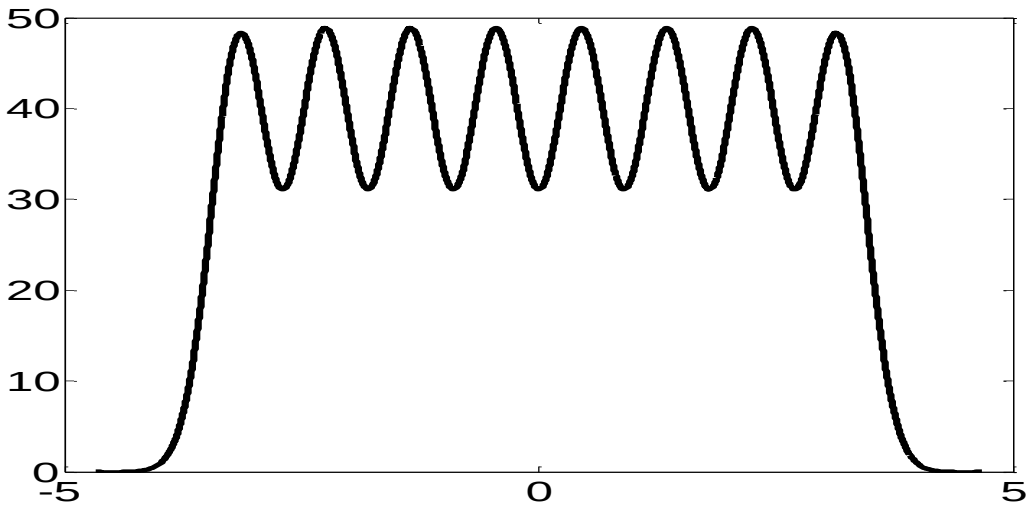


Particle-in-Cell code ECHO-PIC
(only longitudinal dynamics)

ECHO-PIC

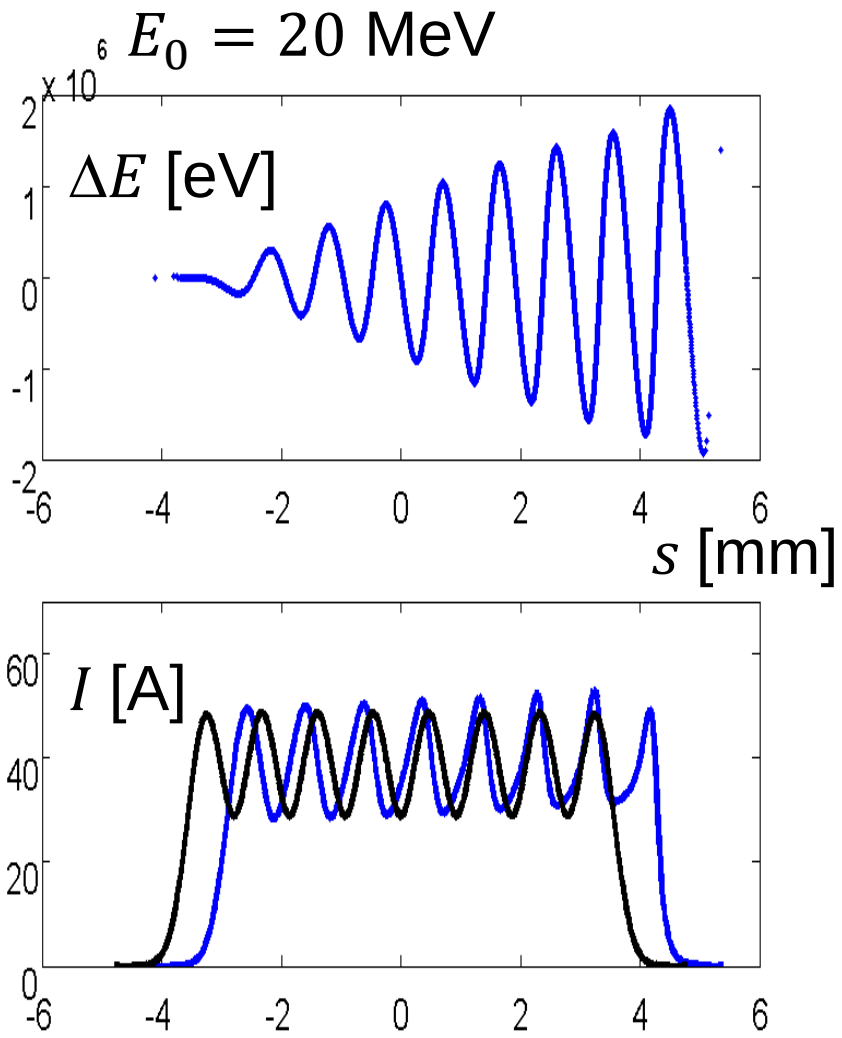
Pipe length L, m	1.5-2
Bunch enegry E_0, MeV	5-20
Gausian bunch rms σ, mm	0.3*8
Charge Q, nC	0.96

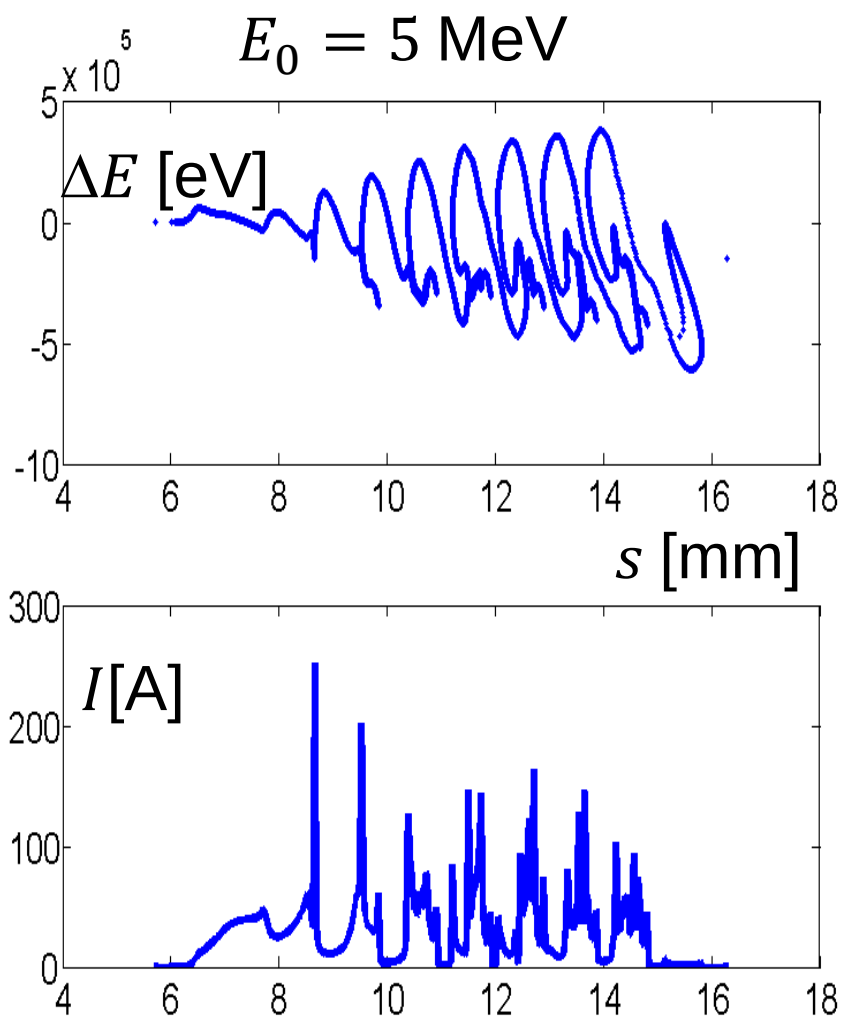
I [A]



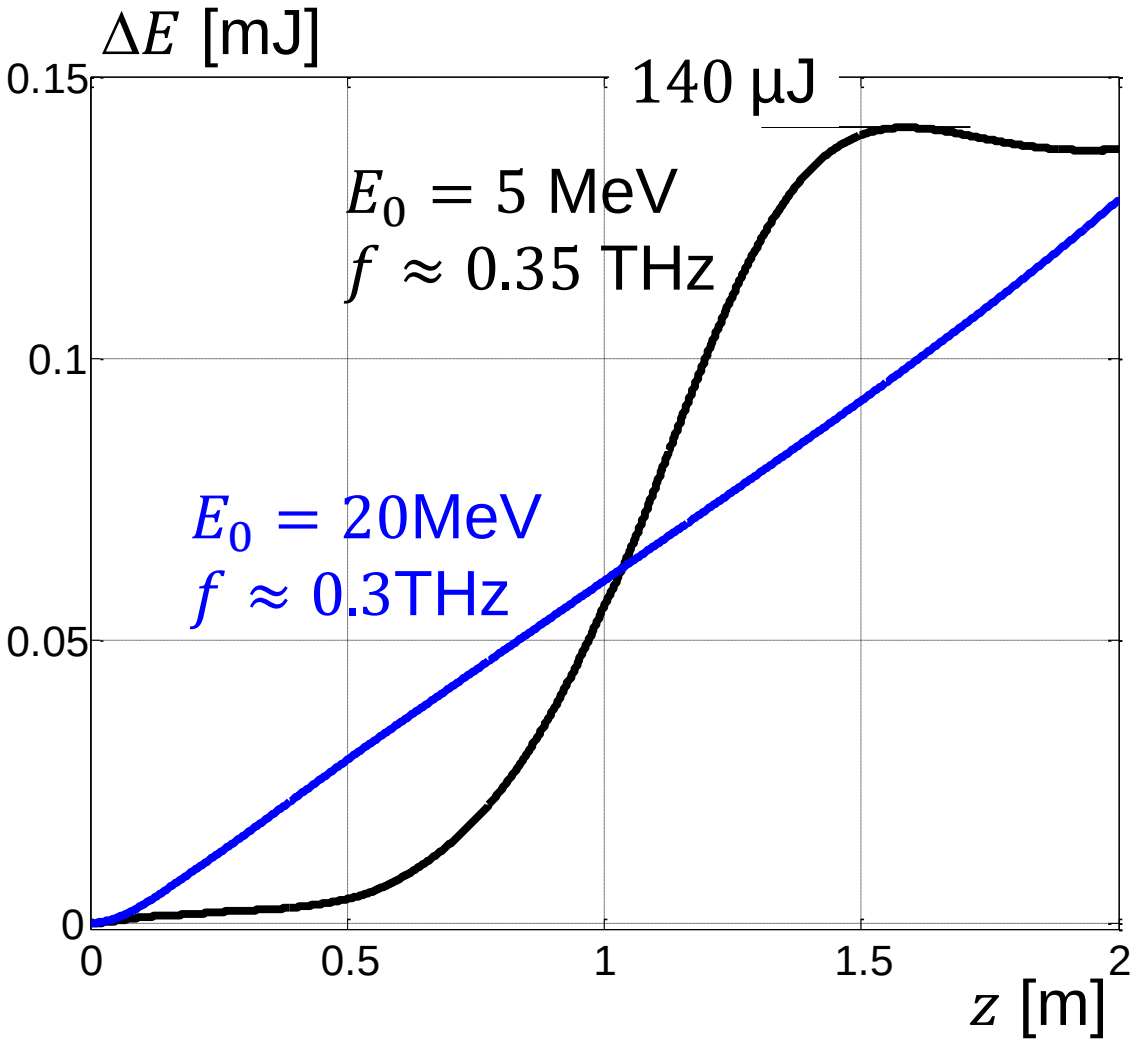
s [mm]

ECHO-PIC

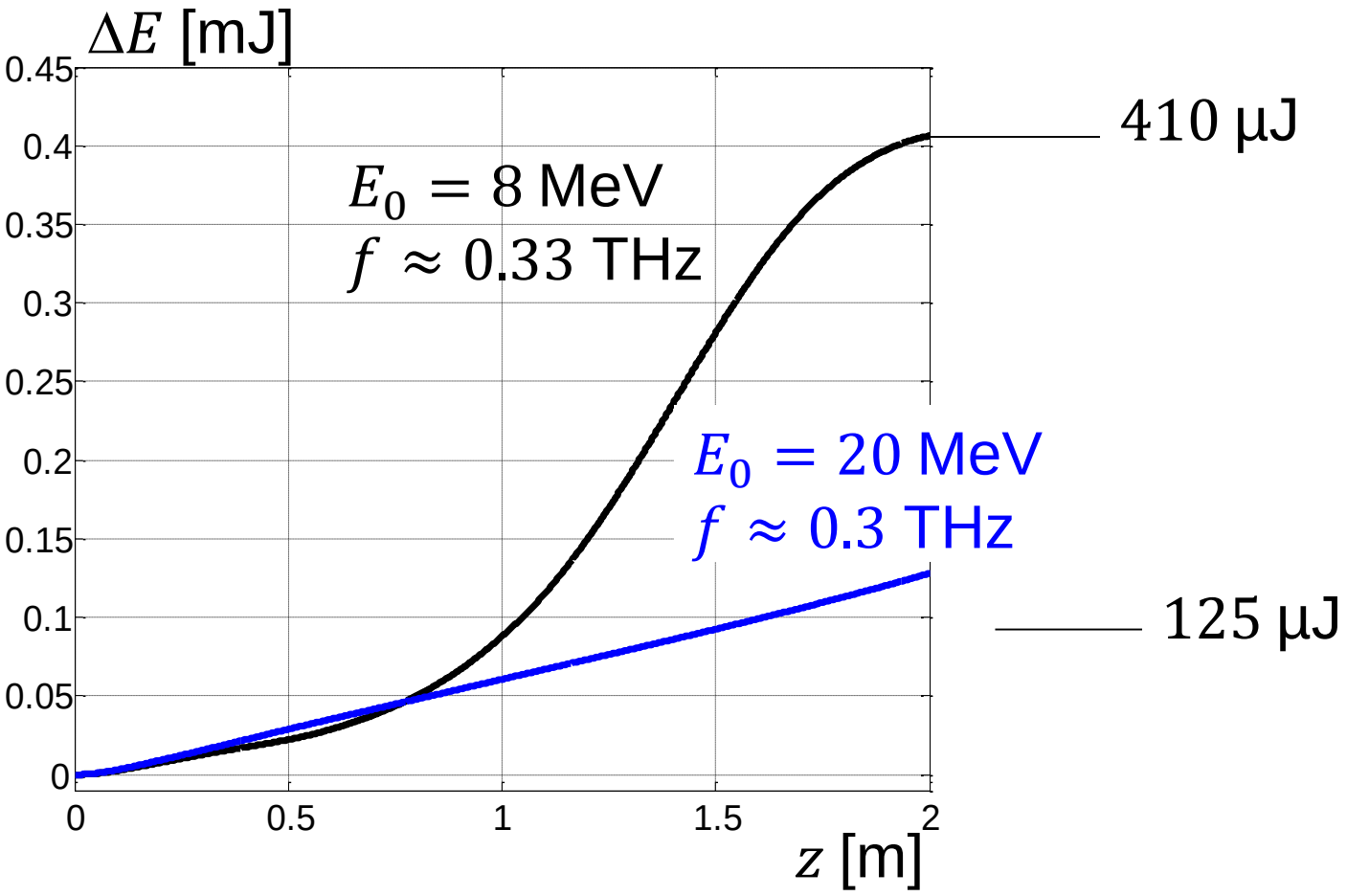




ECHO-PIC


ΔE [mJ]
140 μJ
$E_0 = 5$ MeV
$f \approx 0.35$ THz
$E_0 = 20$MeV
$f \approx 0.3$THz
0.15
0.1
0.05
0
0
0.5
1
1.5
2
z [m]

ECHO-PIC


ΔE [mJ]
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
z [m]
$E_0 = 8$ MeV
$f \approx 0.33$ THz
$E_0 = 20$ MeV
$f \approx 0.3$ THz
410 µJ
125 µJ