Turnaround radius in an accelerated universe in Einstein and in modified gravity

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work with A. Prain, M. Lapierre-Léonard, V. Vitagliano *JCAP* 10, 013 (2015); *CQG* 33, 145008 (2016); *Phys. Dark Univ.* 11, 11 (2016)

Outline

Turnaround radius with Hawking mass

Part of a larger program aiming at applying quasilocal mass in cosmology. Already used to test whether Newtonian *N*-Body simulations of large scale structures are reliable (VF, Prain & Lapierre-Léonard, PRD 2015).

Consider present accelerated era of the universe and the largest bound objects in the sky. The turnaround radius was suggested as a possible way to test dark energy (Roupas *et al.* 2014, PRD 89, 083002; Pavlidou & Tomaras 2014, JCAP 09, 020; Pavlidou, Tetradis, Tomaras 2014, JCAP 05, 017)

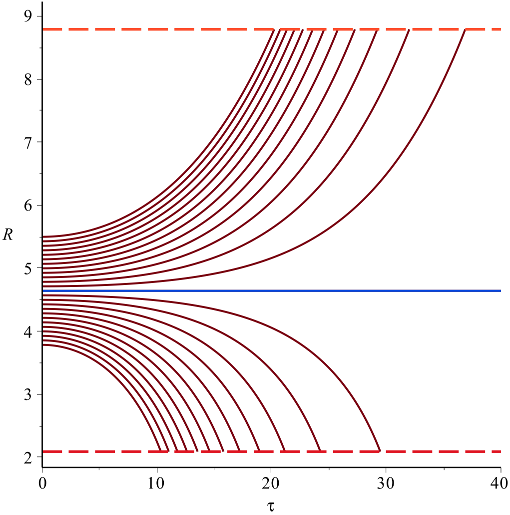
but the concept of TR is older (Souriau 1981; Stuchlik 1983; Stuchlik *et al.* 1989-2005; Mizony & Lachiéze-Rey 2005; Blau & Rollier 2008, ...)

Consider an accelerated FLRW universe with one spherical inhomogeneity; massive test particles with zero radial initial velocity cannot collapse if they are outside a critical radius R_c (*turnaround radius*), but can only expand.

For $R < R_c$, outer layers of dust reach zero radial acceleration and collapse under self-gravity. If you cross outside R_c in geodesic motion, you will never fall back.
TR studied in Schwarzschild-de Sitter, Lemaître-Tolman-Bondi, and McVittie spacetimes.
SdS (heuristic)**:**

$$ds^2 = -\left(1 - \frac{2M}{R} - H^2 R^2\right) dt^2 + \frac{dR^2}{1 - \frac{2M}{R} - H^2 R^2} + R^2 d\Omega^2_{(2)}$$

$$H = \sqrt{\Lambda/3}\,, \qquad R_c = \left(\frac{3GM}{\Lambda}\right)^{1/3}$$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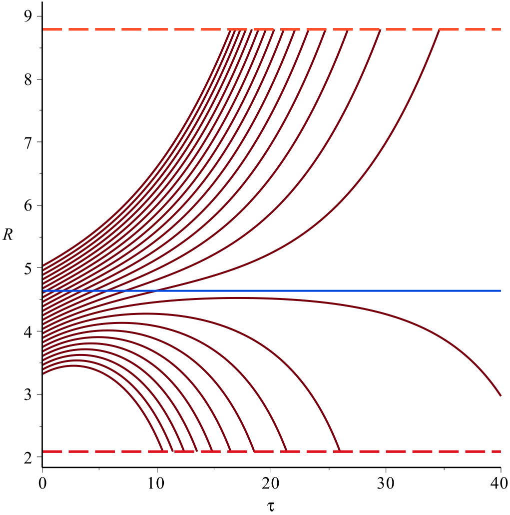
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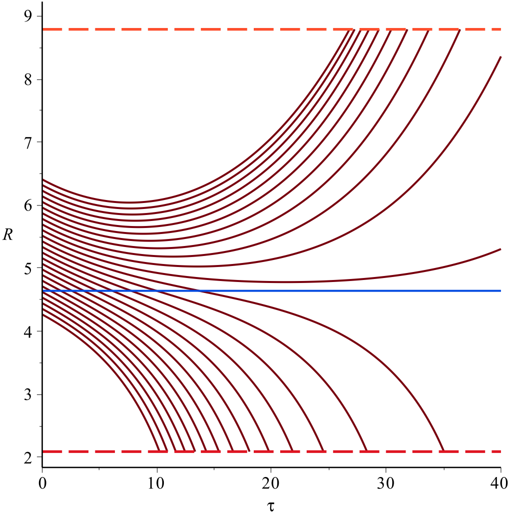
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Radial timelike geodesics obey $\ddot{R}(\tau) = \left(R^3 - R_c^3\right) H^2/R^2$
LTB models (dust) Pavlidou, Tetradis & Tomaras 2014 have

$$ds^2 = -dt^2 + \frac{R'(t,r)}{1+f(r)} dr^2 + R^2(t,r) d\Omega_{(2)}^2$$

with $' \equiv d/dr$, $f(r)$ related to initial density profile. Radial timelike geodesics obey

$$\ddot{R} = -\frac{G\mathcal{M}(r)}{R^2} + \frac{\Lambda R}{3}$$

and the turnaround radius is $R_c = \left(\frac{3G\mathcal{M}(r_c)}{\Lambda}\right)^{1/3}$ where
$\mathcal{M}(r) = \int_0^R dR\, R^2 \rho$ Lemaître mass.

More realistic: post-FLRW space (1st order)

$$ds^2 = a^2(\eta)\left[-(1+2\phi)\,d\eta^2 + (1-2\phi)\left(dr^2 + r^2 d\Omega_{(2)}^2\right)\right]$$

Pavlidou, Tetradis & Tomaras find timelike radial geodesics obey

$$\ddot{R} = -\frac{4\pi}{3}\left(\rho_{\text{DE}} + 3P_{\text{DE}}\right)R - \frac{G\mathcal{M}(r)}{R^2} = \frac{\ddot{a}}{a} - \frac{G\mathcal{M}(r)}{R^2}$$

where it is suggested (but not written down)

$$\mathcal{M}(r) = \int_0^R dR\,R^2 \rho_{\text{total}}$$

$$\rightarrow \quad R_c = \left(\frac{3\mathcal{M}}{4(3w+1)\pi\rho_{\text{DE}}}\right)^{1/3}$$

(reduces to SdS expression for $w = -1$).

Questions (not answered, nor posed):

- gauge-invariance;
- what is the "mass in a sphere of radius R"? Should it include ρ_{DE}? If not, why? Should it include only $\rho_{\mathrm{perturbation}}$? Why?

Use Hawking-Hayward quasilocal energy (reduces to Misner-Sharp-Hernandez mass in spherical symmetry) and a new splitting of it. Assumptions:

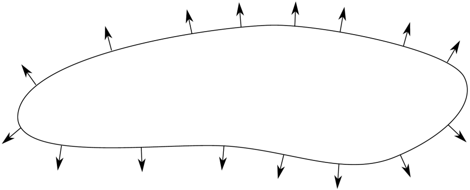
- GR is valid
- 1st order in metric perturbations; spherical symmetry $\phi = \phi(r)$ (consequences of $\phi \neq \phi(r)$ discussed in Barrow & Saich 1993, MNRAS 262, 717)
- FLRW background, spatially flat, accelerated by DE with $\rho_{\text{DE}}, P_{\text{DE}} = w\rho_{\text{DE}}$

Physical mass is the Hawking quasilocal energy[1]
Idea: total mass in a region bounded by a surface S is measured by behaviour of null geodesics at S

$S =$ closed spacelike orientable 2-surface

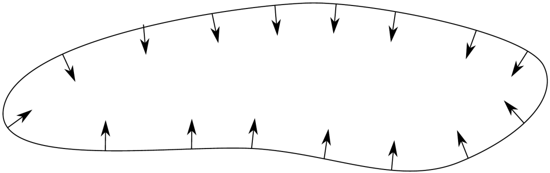
$\mathcal{R} =$ induced Ricci scalar on S

$\theta_{(\pm)} =$ expansions of outgoing/ingoing null geodesic congruences from S



[1] S.W. Hawking 1968, *J. Math. Phys.* 9, 568; S.A. Hayward 1994, *Phys. Rev. D* 49, 831

General perturbations of FLRW



$\sigma_{ab}^{(\pm)}$ = shear tensors of null congruences

ω^a = projection on S of the commutator of null normal vectors to S (anholonomicity)

μ = volume 2-form on S

A = area of S

$$M_{\rm HH} \equiv \frac{1}{8\pi}\sqrt{\frac{A}{16\pi}}\int_S \mu\left(\mathcal{R} + \theta_{(+)}\theta_{(-)} - \frac{1}{2}\sigma_{ab}^{(+)}\,\sigma_{(-)}^{ab} - 2\omega_a\,\omega^a\right)$$

Compute for

$$d\tilde{s}^2 = a^2(\eta) \underbrace{\left[-\left(1 + 2\phi_N\right) d\eta^2 + \left(1 - 2\phi_N\right) \left(dr^2 + r^2 d\Omega_{(2)}^2 \right) \right]}_{\text{post-Newtonian}}$$

and attempt to decompose as $M_{\mathrm{HH}} =$ (local) +(cosmological)
to first order (general perts.)
Conformal factor $g_{ab} \to \Omega^2 \, g_{ab}$, $\Omega = a(\eta)$

Final result (with two methods) is

$$\tilde{M}_{\rm H} = \underbrace{\Omega M_{\rm H} - \frac{R\Omega_{,\eta}}{4\pi}\frac{\Omega_{,\eta}}{\Omega}\int_S \mu\,\phi_N}_{\text{local}} + \underbrace{\frac{R^3}{2}\frac{\Omega_{,\eta}^2}{\Omega}}_{\text{cosmological}}$$

Prain, Vitagliano, VF & Lapierre-Léonard, *Class. Quantum Grav.* 33, 145008

Now adapt to spherical symmetry $\rightarrow$

$$M_{\rm H} = ma + \frac{H^2 R^3}{2}(1-\phi) \simeq ma + \frac{H^2 R^3}{2}$$

with $m = \int d^3\vec{x}\,\nabla^2\phi$ Newtonian mass $\sim$ comoving length scale $ma \sim$ physical length scale. Criterion for a system on the verge of breaking down is now

local part $\quad ma = \frac{H^2 R^3}{2} \quad$ cosmological part $\rightarrow$

$$\boxed{R_c(t) = \left(\frac{2ma}{H^2}\right)^{1/3}}$$

Now $H^2 = 8\pi G\rho_{\rm DE}/3 \rightarrow R_c(t) = \left(\frac{3ma}{4\pi\rho_{\rm DE}}\right)^{1/3}$ and, if

$w =$ const., $\quad R_c = \left(\frac{3ma}{4\pi\rho_0}\right)^{1/3} a^{\frac{3w+4}{3}}$

Compare with Pavlidou, Tetradis & Tomaras

$$\frac{R_c}{R_c^{(PTT)}} = \left(\frac{|3w+1|}{2}\right)^{1/3} \approx 1 \qquad \text{if} \;\; w \approx -1$$

but now

- no ambiguities in “mass inside a sphere of radius R_c”;
- rigorous derivation of turnaround radius R_c
- important if you want to constrain w

Can express $R_c = R_c(z)$ and invert to obtain

$$\int dz \, \frac{w(z)+1}{z+1} = \ln\left[\left(\frac{3ma}{4\pi\rho} \right)^{1/3} \frac{1}{R(z)} \right]$$

If $w =$ const. reduces to

$$w(z) = -1 + \frac{\ln\left[\left(\frac{3ma}{4\pi\rho_0} \right)^{1/3} \frac{1}{R_c(z)} \right]}{\ln(z+1)}$$

constrain w if ma and R_c are known.

Turnaround radius in ST gravity

$$ds^2 = a^2(\eta)\left[-(1+2\psi)\,d\eta^2 + (1-2\phi)\left(dr^2 + r^2 d\Omega^2_{(2)}\right)\right],$$

$\phi = \phi(r), \psi = \psi(r)$. Massive test particles follow timelike geodesics

$$\frac{du^a}{d\tau} + \Gamma^a_{bc} u^b u^c = 0\,,$$

$u_c u^c = -1$ and the geodesic eq. give

$$\frac{du^0}{d\tau} + \frac{a_\eta}{a}(u^0)^2 + 2\psi' u^0 u^1 + \frac{a_\eta}{a}(1-2\phi-2\psi)(u^1)^2 = 0$$

$$\frac{du^1}{d\tau} + \psi'(u^0)^2 + \frac{2a_\eta}{a} u^0 u^1 - \phi'(u^1)^2 = 0$$

Areal radius is $R(t, r) = ar\sqrt{1 - 2\phi} \simeq ar\,(1 - \phi)$, further manipulation yields

$$\frac{d^2 R}{dt^2} = \left[\ddot{a}r + \frac{\dot{a}u^1}{au^0} + \frac{1}{au^0}\frac{d}{d\tau}\left(\frac{u^1}{u^0}\right)\right](1 - \phi)$$

Criterion locating the (unique) turnaround radius is $d^2 R/dt^2 = 0$, which becomes

$$\ddot{a}r - \frac{\psi'}{a} = 0$$

In terms of the *areal* turnaround radius,

$$R_c = a(t) r_c \left[1 - \phi(r_c)\right]$$

or, using the gravitational slip $\xi \equiv (\phi - \psi)/\phi$,

$$\ddot{a}\, R_c \left(1 + \phi_c\right) - \phi'_c \left(1 - \xi_c\right) + \phi_c \xi'_c = 0$$

Conclusions

- Turnaround radius is an opportunity to test gravity and the ΛCDM model.
- Split M_{H} for spherical perturbations of FLRW $\rightarrow$ rigorous derivation of R_c, small correction, much needed clarification of "mass".
- In modified gravity, no accepted M_{H}, use criterion $\ddot{R} = 0 \rightarrow$ eq. for R_c in ST gravity.
- Is it important? Astronomers claim that the upper bound on R_c in GR is exceeded by far in galaxy group NGC 5353/4 (Lee *et al.* 2015, *Astrophys. J.* 815, 43; Lee, arXiv:1603.06672). Wait and see!

THANK YOU