The Vainshtein mechanism in general disformal gravity

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One of the possible explanations for accelerated expansion of the late-time universe is based on the modification of general relativity on large scales.

Introduction

One of the possible explanations for accelerated expansion of the late-time universe is based on the modification of general relativity on large scales.

Deviations from general relativity have to be suppressed inside the solar system due to screening mechanism.

Conformal and Disformal transformations

Starting from the action for scalar-tensor theory

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}C(\phi)R - \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - V(\phi)\right], \quad (1)$$

where $M_p \equiv 1/\sqrt{8\pi G}$ and $C(\phi)$ is an arbitrary function of the scalar field ϕ.

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we can use the conformal transformation,

$$\bar{g}_{\mu\nu} = C(\phi)g_{\mu\nu} , \qquad (2)$$

to transform the above action to

$$S=\int d^4x\sqrt{-\bar{g}}\left[\frac{M_p^2}{2}\bar{R}-\frac{1}{2}\partial_\mu\bar{\phi}\partial^\mu\bar{\phi}-U(\bar{\phi})\right], \qquad (3)$$

where $\bar{g}$ and $\bar{R}$ are computed from $\bar{g}_{\mu\nu}$ and

$$\frac{\partial\phi}{\partial\bar{\phi}}=\frac{C}{\sqrt{C+3C_\phi^2/2}}, \quad \text{and} \quad U(\bar{\phi})=\frac{V(\phi)}{C^2(\phi)}\bigg|_{\phi=\phi(\bar{\phi})}. \qquad (4)$$

Actions for general scalar-tensor theories, such as the Horndeski and GLPV theories, can contain

- second-order derivatives of the field, e.g., $\partial_\alpha \partial^\alpha \phi$
- non-minimal derivative coupling $G_{\alpha\beta}\partial^\alpha\phi\partial^\beta\phi$

[a]

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Subclass of the Horndeski and GLPV theories can be transformed to the Einstein frame via

$$\bar{g}_{\mu\nu} = \underbrace{C(\phi) g_{\mu\nu}}_{1st part is: Conformal Tr} + \underbrace{D(\phi, X)\partial_\mu\phi\partial_\nu\phi}_{2nd part: Disformal Tr}, \quad (5)$$

General disformal gravity

We apply the purely disformal transformation,

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi\,, \tag{6}$$

to the Einstein-Hilbert action.

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The resulting action can be written in the GLPV form as

$$S = \underbrace{\int d^4x\sqrt{-g}\sum_{i=2}^{4} L_i}_{actionforgravity} + \underbrace{\int d^4x\sqrt{-g}\,\mathcal{L}_m(g_{\alpha\beta}, \psi)}_{actionformatter}\,, \tag{7}$$

where

$$L_2 = P(\phi, X) + XC_{3,\phi}\,, \quad L_3 = (C_3 + 2XC_{3,X})\,\Box\phi\,, \tag{8}$$

$$\begin{aligned} L_4 &= B_4 R - \frac{B_4 + A_4}{X}\left[(\Box\phi)^2 - \nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi\right] \\ &\quad + \frac{2\,(B_4 + A_4 - 2XB_{4,X})}{X^2}\,(\nabla^\mu\phi\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi\,\Box\phi \\ &\quad - \nabla^\mu\phi\nabla_\mu\nabla_\nu\phi\nabla_\sigma\phi\nabla^\nu\nabla^\sigma\phi)\,. \end{aligned} \tag{9}$$

Here, $X \equiv \partial_\alpha\phi\partial^\alpha\phi$,

$$C_3 = -\frac{M_p^2}{4}\int \frac{D_{,\phi}dX}{\sqrt{1+DX}}\,,\ B_4 = \frac{M_p^2}{2}\sqrt{1+DX}\,,\ A_4 = -\frac{M_p^2/2}{\sqrt{1+DX}}\,.$$

Equations for the field profile

The line element for the static and spherically symmetric background :

$$ds^2 = -e^{2\Psi(r)}dt^2 + e^{2\Phi(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) . \qquad (10)$$

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The weak gravitional field limit:

$$|\Phi| \ll 1\,, \quad |\Psi| \ll 1\,. \quad (11)$$

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[2] A. De Felice, R. Kase, S. Tsujikawa [arXiv:1508.06364]

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The fifth force $\propto \partial\phi/\partial r$.

The Vainshtein radius

For concreteness, we choose the disformal coupling as

$$D = M^{-4\lambda_2 - 4} e^{-\lambda_1 \phi} (-X)^{\lambda_2} \,, \tag{12}$$

where λ_1, λ_2 and M are the constant parameters,

The Vainshtein radius

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and choose the Lagrangian for the field as

$$P = \frac{1}{2} M_k^{4-4\lambda_3} (-X)^{\lambda_3} - M_v^4 e^{-\lambda_4 \phi}\,, \tag{13}$$

where lm_3, λ_4, M_k and M_v are the constant parameters.

The differential equation for the field:

$$\phi'' + \frac{2}{\tilde{r}}\phi' \quad = \quad A(\tilde{r})\tilde{\rho}_m + \frac{B(\tilde{r})}{1 + F(\tilde{r})}\,, \qquad (14)$$

where a prime denotes $d/d\tilde{r}$, $\tilde{r} \equiv rH_0$ and $\tilde{\rho}_m \equiv \rho_m/H_0^2$, The differential equation for the field:

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We define the Vainshtein radius $\tilde{r}_v$ as

$$F(\tilde{r}_v) = 1\,. \quad (15)$$

Solution for the equation of the scalar field when $\tilde{r} < \tilde{r}_s$,

$$\phi' \simeq c, , \quad \text{where} c \ll 1 \, . \tag{16}$$

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r_s can be of the order of the radius of the Milky Way.

Conclusions

We find that the Vainshtein mechanism in general disformal gravity can work if the kinetic terms of the scalar field in the theory take non-canonical forms. Using the constraint from local gravity experiments, we show that General Relativity is recovered inside the Vainshtein radius which can be of the order of the radius of the Milky Way.