Exact analytic solution for non-linear density fluctuation in a ΛCDM universe

Jinn-Ouk Gong

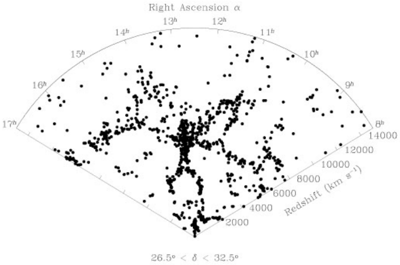
APCTP, Pohang 37673, Korea

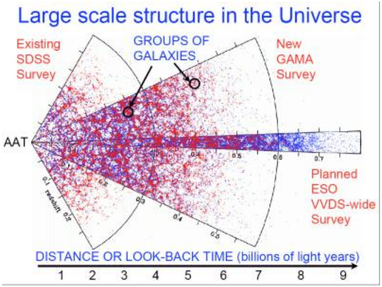
12th International Workshop on Dark Side of the Universe
Bergen, Norway
26th July, 2016

Based on J. Yoo and JG, JCAP **1607**, 017 (2016) [1602.06300 [astro-ph.CO]]

Outline

Why GR in LSS?





- Very early surveys (80s, CfA): about 1,000 galaxies, $z \sim 0.02$
- Modern surveys (00s, SDSS): 3 million spectra, z up to 0.7
- Future surveys (next decade): 50 million objects up to $z \sim 2$

DESI, HETDEX, LSST, Euclid, WFIRST... will probe huge volumes

Why ΛCDM in non-linear regime?

- Λ (or any kind of DE) was negligible at very early times
- Non-linearities are developed when Λ becomes significant, so its effects emerge more prominently at non-linear level
- Λ is the simplest form of DE, so first to study
- No explicit analytic NL study is available yet!

What are the effects of Λ in non-linear regime of LSS?

Basic GR non-linear equations

The full non-linear equations are more complex (Bardeen 1980) e.g.

$$\text{E-constraint}: \underbrace{R - \overline{K}^i{}_j \overline{K}^j{}_i + \frac{2}{3}K^2}_{\text{geometry}} = \underbrace{16\pi G E}_{\text{matter}, \delta}$$

$$\text{E-conserv:} \underbrace{\frac{E_{,0}}{N} - K\left(E + \frac{S}{3}\right)}_{\text{matter evolution}, \dot{\delta}} = \frac{E_{,i}N^i}{N} + \overline{K}^i{}_j \overline{S}^j{}_i - \frac{\left(N^2 J^i\right)_{;i}}{N^2}$$

In EdS universe, very simple: $T_{\mu\nu} = \rho_m u_\mu u_\nu \rightarrow E = \rho_m, J_i = S_{ij} = 0$

Newtonian correspondence in GR

- Pressure ∼ kinetic E ≫ rest mass: relativistic (e.g. photons)
- EdS contains pressureless matter, so Newtonian picture works

(Hwang & Noh 2007; Jeong, JG, Noh & Hwang 2011; Hwang, Noh, Jeong, JG and Biern 2015)

- In the comoving gauge ($T^0{}_i = 0$), coordinate time = proper time

(Yoo 2014; Yoo & JG 2016a; 2016b)

Why is ΛCDM also special?

- Λ is a constant thus no perturbation
- Time slicing is entirely determined by matter, identical to EdS

(Yoo & JG 2016b)

Battle plan

Combining continuity & energy constraint eqs

$$\mathcal{H}\delta' + \frac{3}{2}\mathcal{H}^2\Omega_m\delta = \frac{a^2}{4}\left(R - \overline{K}^{ij}\overline{K}_{ij} + \frac{2}{3}\kappa^2 + 4HN^i\delta_{,i} + 4H\delta\kappa\right)$$

1. Growing solution $\delta = H\int dt\mathcal{H}^{-2} \times (\text{RHS})$
2. Split RHS as $\text{RHS} = \text{RHS}^{(1)} + \text{RHS}^{(2)} + \text{RHS}^{(3)} + \cdots$ with

$$\text{RHS}^{(n)}(t,\boldsymbol{x}) \equiv \sum_I \text{RHS}_I^{(n)}(t,\boldsymbol{x}) = \sum_I X_{mI}^{(n)}(\boldsymbol{x})T_{mI}(t)$$

[n: n-th order, I: t-dep, $m(\leq n)$: growth factor $\propto D_1^m$ in EdS]

3. With $\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots$ each analytic solution is given by

$$\delta^{(n)} = \sum_I \delta_{mI}^{(n)}(t,\boldsymbol{x}) = \sum_I D_{mI}(t)X_{mI}^{(n)}(\boldsymbol{x}) \quad \text{with} \quad D_{mI}(t) = H\int dt\frac{T_{mI}(t)}{\mathcal{H}^2}$$

Linear and second order solutions

1. At linear order $\text{RHS}^{(1)} = -\Delta\varphi^{(1)}(\boldsymbol{x}) \equiv X_1^{(1)}(\boldsymbol{x})$, so with $\varphi^{(1)} \equiv \mathscr{R}$

$$\delta^{(1)}(t,\boldsymbol{x}) = D_1(t)X_1^{(1)}(\boldsymbol{x}) = -D_1\Delta\mathscr{R} \equiv \delta_1^{(1)} \quad \text{with} \quad D_1(t) \equiv H\int\frac{dt}{\mathscr{H}^2}$$

2. At 2nd order $\left[f_1 \equiv d\log D_1/d\log a \text{ and } \Sigma_1 \equiv 1+3\Omega_m/(2f_1)\right]$

$$\text{RHS}^{(2)} \sim \left(\text{constant}, \frac{1}{\mathscr{H}^2 f_1\Sigma_1}, \frac{1}{\mathscr{H}^2\Sigma_1^2}, \frac{1}{\mathscr{H}^2 f_1\Sigma_1^2}\right)$$

Thus other than D_1 (from const RHS) 3 new growth factors

$$D_{2A} = \frac{7}{5}\int dt D_1^2 f_1\Sigma_1\,, \quad D_{2B} = \frac{7}{2}H\int dt D_1^2 f_1^2\,, \quad D_{3C} = \frac{7}{2}H\int dt D_1^2 f_1$$

Thus the 2nd order solution consists of 4 different t-dep

$$\delta^{(2)}(t,\boldsymbol{x}) = D_1X_1^{(2)} + \sum_{I=A}^{C} D_{2I}X_{2I}^{(2)} \equiv \delta_1^{(2)} + \sum_{I=A}^{C}\delta_{2I}^{(2)}$$

3rd order solutions (1/2)

At 3rd order, RHS has various time dependences: e.g. $\varphi^{(3)}$ reads

$$\varphi^{(3)} \sim \left(\text{constant}, D_1, D_1^2, D_{2I}\right)$$

Accordingly we have components proportional to D_1 and D_{2I}:

$$\delta^{(3)} \supset \underbrace{\delta_1^{(3)}}_{\propto D_1} + \underbrace{\delta_{2A}^{(3)}}_{\propto D_{2A}} + \underbrace{\delta_{2B}^{(3)}}_{\propto D_{2B}} + \underbrace{\delta_{2C}^{(3)}}_{\propto D_{2C}} \equiv \delta_1^{(3)} + \delta_2^{(3)}$$

And the contributions $\propto D_1^3$ in EdS:

$$\delta^{(3)} \supset \sum_{I=D}^{F} D_{3I} X_{3I}^{(3)} + \sum_{I=A}^{C} \sum_{i=a}^{d} D_{3Ii} X_{3Ii}^{(3)} \equiv \delta_3^{(3)}$$

These new solutions can be obtained analytically

3rd order solutions (2/2)

New growth factors that all scale as D_1^3 in EdS:

$$D_{3D} = \frac{9}{5} H \int dt D_1^3 f_1 \Sigma_1 \quad \text{with} \quad X_{3D}^{(3)} = \text{too long!} \quad (1)$$

$$D_{3E} = \frac{9}{2} H \int dt D_1^3 f_1 \quad \text{with} \quad X_{3E}^{(3)} = \text{too long!} \quad (2)$$

$$D_{3F} = \frac{9}{2} H \int dt D_1^3 f_1^2 \quad \text{with} \quad X_{3F}^{(3)} = \text{too long!} \quad (3)$$

and those coming from $\delta_{2I}^{(2)}$ that also scale as D_1^3 in EdS:

$$D_{3Ia} = \frac{9}{5} H \int dt D_1 f_1 \Sigma_1 D_{2I} \quad \text{with} \quad X_{3Ia}^{(3)} = -\frac{5}{18} \left[\left(\mathscr{R}^{,ij} \Delta^{-1} \partial_j + \Delta \mathscr{R} \Delta^{-1} \partial^i \right) X_{2I}^{(2)} \right]_{,i}$$

$$D_{3Ib} = \frac{9}{4} H \int dt D_1 D_{2I} f_{2I} \quad \text{with} \quad X_{3Ib}^{(3)} = -\frac{4}{9} \left(\Delta \mathscr{R} \Delta^{-1} X_{2I}^{(2),i} \right)_{,i}$$

$$D_{3Ic} = \frac{9}{2} H \int dt D_1 f_1 D_{2I} \quad \text{with} \quad X_{3Ic}^{(3)} = -\frac{2}{9} \left(X_{2I}^{(2)} \mathscr{R}^{,i} \right)_{,i}$$

$$D_{3Id} = \frac{9}{4} H \int dt D_1 f_1 D_{2I} f_{2I} \quad \text{with} \quad X_{3Id}^{(3)} = \frac{2}{9} \left(\mathscr{R}^{,ij} \Delta^{-1} \partial_i \partial_j - \Delta \mathscr{R} \right) X_{2I}^{(2)}$$

Previous GR solutions

1-loop power/bi-spectrum of δ (Jeong et al. 2011, Biern et al. 2014)

1. Initial condition at $t = t_i$ is set by δ rather than geometry
2. Linear initial condition: $\delta(t_i) = \delta_1^{(1)}(t_i)$

t-dep \ pert order	1st	2nd	3rd
$\sim D_1$ in EdS	$\delta_1^{(1)}$		
$\sim D_1^2$ in EdS		$\delta_2^{(2)}$	
$\sim D_1^3$ in EdS			$\delta_3^{(3)}$

$$\delta(t,\boldsymbol{x}) = \frac{2}{5\mathcal{H}^2} c(\boldsymbol{x}) + \cdots \text{ with} \qquad \text{assuming linear energy constraint}$$

$$c(\boldsymbol{x}) = -\Delta\mathscr{R}$$

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$\sim D_1^3$ in EdS			$\delta_3^{(3)}$

$\delta(t,\boldsymbol{x}) = \dfrac{2}{5\mathscr{H}^2} c(\boldsymbol{x}) + \cdots$ with**out** assuming linear energy constraint

$$c(\boldsymbol{x}) = -\Delta\mathscr{R} + \frac{3}{2}\mathscr{R}^{,i}\mathscr{R}_{,i} + 4\mathscr{R}\Delta\mathscr{R} - 3\mathscr{R}\left(3\mathscr{R}^{,i}\mathscr{R}_{,i} + 4\mathscr{R}\Delta\mathscr{R}\right) + \cdots$$

δ is intrinsically non-linear in $\mathscr{R}$ so is a sensitive probe to the primordial non-linearity

Conclusions

- As galaxy surveys become deeper and deeper, fully GR description is relevant
- With a non-zero cosmological constant Λ:
  - Proper-time hypersurface provides Newtonian intuition
  - Perturbative analytic solutions can be obtained
  - Initial non-linearity in δ in terms of $\mathscr{R}$
- Directly connected to inflation: $\mathscr{R}$ is the primordial curvature perturbation and is of fundamental impotance in inflation