Small nonassociative corrections to the SUSY generators and cosmological constant

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DE models

Dark energy models:

- Quintessence: dark energy is in the form of a time varying scalar field.
- K-essence: the modification of the scalar field canonical kinetic energy term.
- Chaplygin gas: eos $p = -A/\rho^{\alpha}$.
- $f(R)$ gravities.
- Brane cosmology models.
- ...
- Cosmological constant.

Cosmological constant - fundamental constant

All these models are dynamic in the sense that the present value of the cosmological constant Λ is explained in a dynamic way by using either some kind of matter or modified gravity. Another way is to postulate that Λ is indeed a fundamental constant. Following this way, it would be very interesting to understand what kind of physics is behind such approach.

From non-associativity to new fundamental constant

One can propose the idea that the cosmological constant Λ can be connected with the appearance of nonassociativity in physics. In this model, the constant Λ controls the smallness of NA effects in quantum physics: the dimensionless quantity $l_{Pl}^2\Lambda \approx 10^{-120}$ shows where the NA effects may occur. It may happen on the huge scale $\ell_0 = 1/\sqrt{\Lambda} \approx 10^{26}\mathrm{m}$ (that means that there exists a maximal length ℓ_0)

Main idea

How it can happen ? Main idea: the generalization of associative SUSY to non - associative SUSY leads to the appearance of a new fundamental constant that has the dimension cm^{-1}.

In standard SUSY the operators $Q_{a,\dot{a}}$ and the momentum operator $P_\mu = -i\hbar\partial_\mu$ are connected by the expression

$$\{Q_a, Q_{\dot{a}}\} = 2\sigma^\mu_{a\dot{a}} P_\mu .$$

Let us recall the definition of an associator

$$[A, B, C] = (AB)\,C - A\,(BC)\,,$$

Example of associator

$$[Q_a, Q_b, Q_c] = \frac{\hbar}{\ell_0}\zeta_1\left(Q_a\epsilon_{bc} - Q_c\epsilon_{ab}\right)$$

$\ell_0 = \Lambda^{-1/2}$

How it works ?

There exists a maximal length ℓ_0 or that in Nature there exists a minimal 4D scalar curvature (a unique Lorentz invariant quantity having the dimensions cm^{-2}): $R_{min} \approx \Lambda$. It immediately leads to a very simple explanation for the acceleration of the present Universe: the Universe reaches the minimally possible curvature and has to stay in this state.

From classical physics to NA physics

Physically, in this model the appearance of the Λ constant is connected with the breaking of non-associativity.

classical physics

$\downarrow$ breaking of commutativity $\rightarrow \hbar$

quantum physics

$\downarrow$ breaking of associativity $\rightarrow \ell_0 = \Lambda^{-1/2}$

NA physics

A few mathematical consequences

For small NA corrections:

$$\tilde{Q}_a = Q_a + \xi Q_{1,a} + \dots$$

where $\xi = \left(L_{Pl}/\ell_0\right)^{1/3}$ is a small NA parameter.

A few mathematical consequences

A generalized uncertainty principle:

$$\left[x^\mu, \tilde{P}_\nu\right] = i\hbar\left[\delta^\mu_\nu + \frac{\xi}{P_0^2}\left(\delta^\mu_\nu P^2 + 2P^\mu P_\nu\right)\right]$$

q - deformed commutator:

$$\left[x^\mu, \tilde{P}_\nu\right]_q = q x^\mu \tilde{P}_\nu - \frac{1}{q}\tilde{P}_\nu x^\mu = i\hbar\delta^\mu_\nu .$$

Conclusions

- One can introduce non-associativity into SUSY.
- Breaking of non-associativity leads to a new fundamental constant with the dimension cm^{-1} that can be connected with the Λ constant.
- Small NA corrections appear in Heisenberg uncertainty principle: either as generalized uncertainty principle or as q - deformed commutator.

Thanks for your attention !