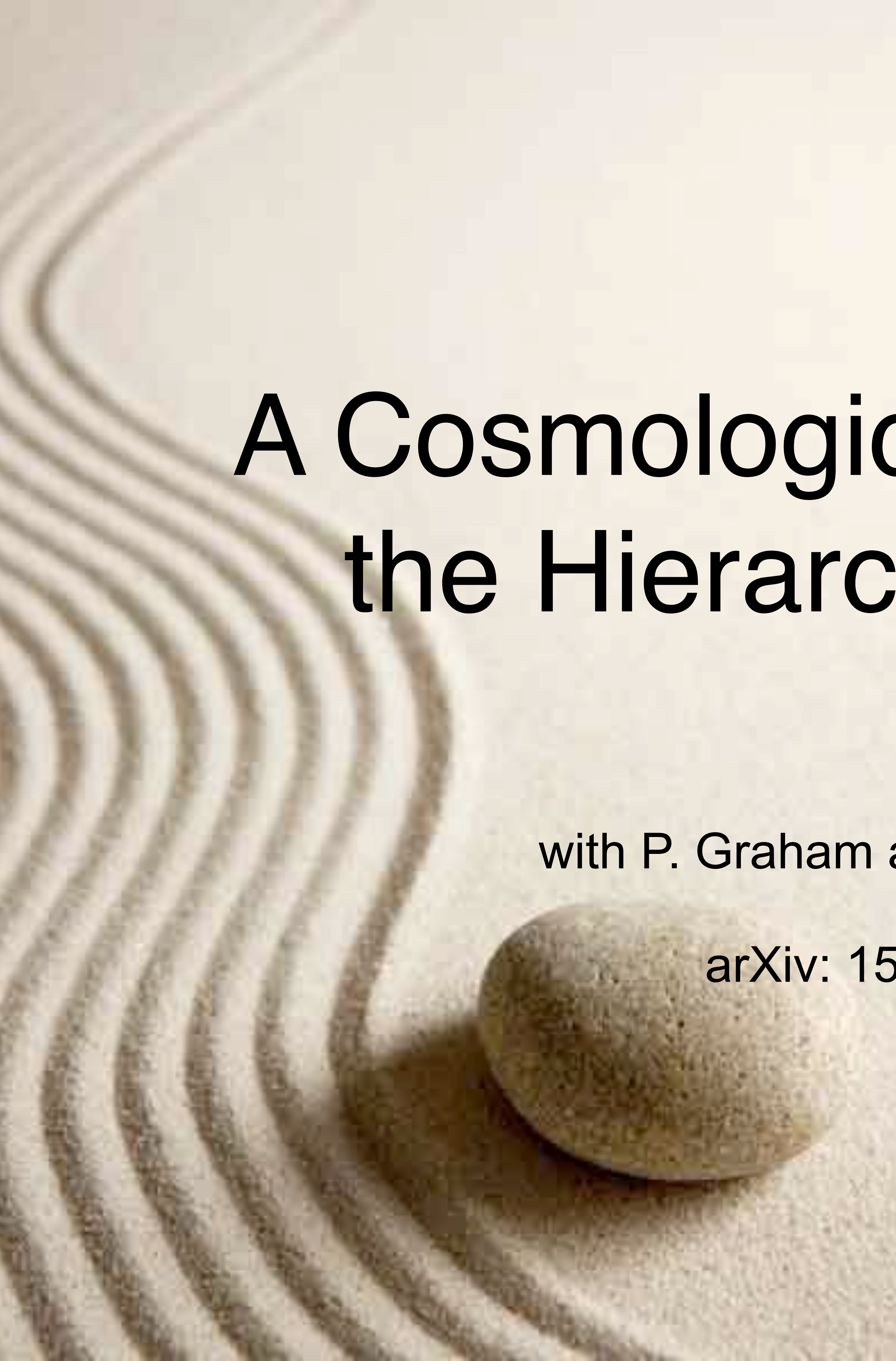


A Cosmological Solution to the Hierarchy Problem

with P. Graham and S. Rajendran

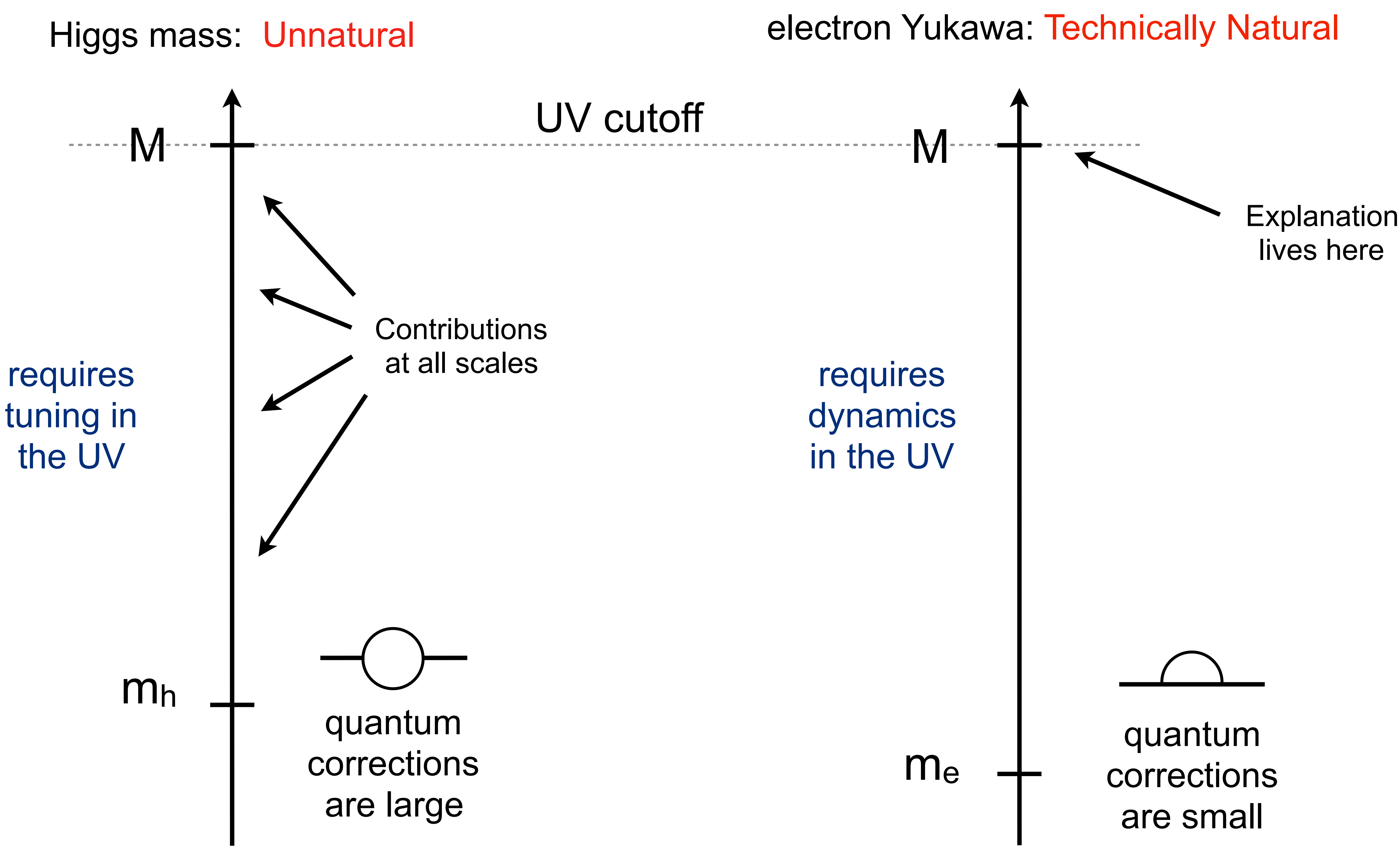
arXiv: 1504.07551

the Relaxion

The Hierarchy Problem

The Higgs mass in the standard model is sensitive to the ultraviolet.

Unnatural vs. Technically Natural in the SM



The Hierarchy Problem

The Higgs mass in the standard model is sensitive to the ultraviolet.

Two approaches to explain:

- New symmetry or new dynamics realized at the electroweak scale. (SUSY, composite Higgs, EOFT)
- An anthropic explanation for fine tuning of ultraviolet parameters. (Multiverse)

We Propose:
A Dynamical Solution

- Higgs mass-squared promoted to a field.
- The field evolves in time in the early universe.
- The mass-squared relaxes to a small negative value.
- The electroweak symmetry breaking stops the time-dependence.
- The small electroweak scale is fixed until today.

Caveats

The solution:

- is only technically natural. (there is a small parameter)

- requires large field excursions (larger than the scale that cuts off loops).

- requires a very long period of inflation.

- can only push the cutoff up to 10^8 GeV.

Simplest Model

Standard Model plus QCD axion

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 \qquad \cdots + \frac{\phi}{32\pi^2 f} G^{\mu\nu}\tilde{G}_{\mu\nu}$$

M is the cutoff.

The axion here is non-compact.

Simplest Model

Standard Model plus QCD axion

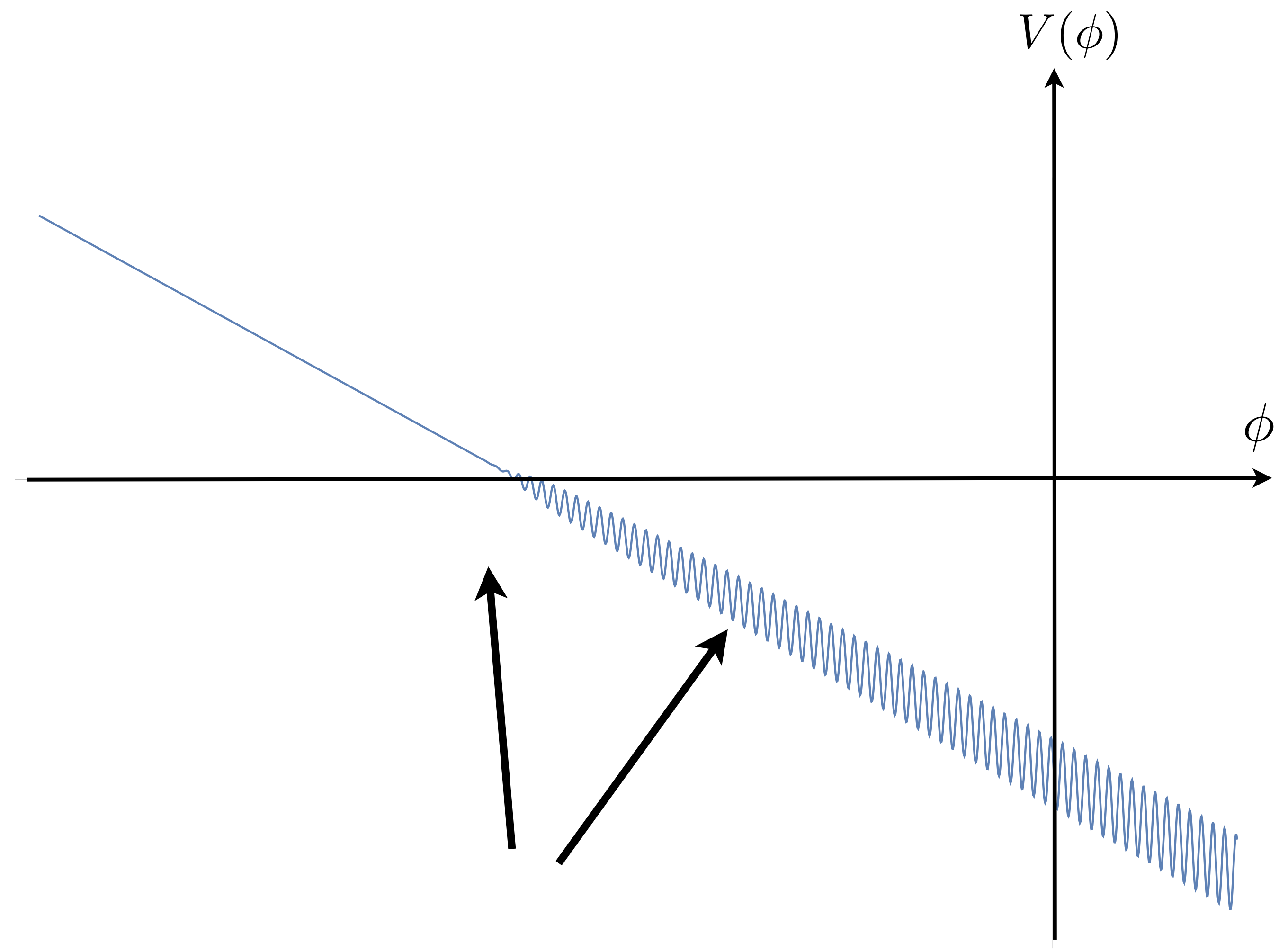
$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + gM^2\phi + g^2\phi^2 + \cdots + \Lambda^4 \cos\frac{\phi}{f}$$

Conservative effective field theory regime: $\phi \lesssim \dfrac{M^2}{g}$

(Assuming expansion of $V(g\phi)$ in powers of $\left(\frac{g\phi}{M^2}\right)$)

Chronology

- Take initial φ value such that $m_h^2 > 0$.
- *During inflation*, φ slow-rolls, scanning physical Higgs mass.
- φ hits value where m_h^2 crosses zero.
- Barriers grow until rolling has stopped.



Key: Barriers grow because they depend on the Higgs vev.

Higgs vev and the Periodic Potential

Barrier height (axion potential) can be approximated in the chiral Lagrangian (2 flavors):

$$V_{\mathrm{axion}}\left(\frac{\phi}{f}\right) \sim \Lambda^4 \cos\frac{\phi}{f}$$

where

$$\Lambda^4 \sim 4\pi f_\pi^3 m_u \propto \langle h \rangle$$

Barrier height grows with the Higgs vev!

Parameter Requirements

φ stops rolling and Higgs vev stops growing when slope turns around:

$$\partial_\phi(gM^2\phi + \Lambda^4\cos(\phi/f)) \sim 0$$

or

$$\boxed{gM^2 f \sim \Lambda^4}$$

$$\Lambda^4 \sim 100\ \mathrm{MeV}$$

fixed parameters

changes with Higgs vev

$$gM^2 f \sim 4\pi f_\pi^3 y_u \langle h \rangle$$

Parameter Requirements

1) Vacuum energy density during inflation $> M^4$

$$\boxed{H_{\rm infl} > \frac{M^2}{M_{\rm pl}}}$$

2) Barriers can form in Hubble volume:

$$\boxed{H_{\rm infl} < \Lambda}$$

Plugging in for g, and using 1) and 2):

$$\boxed{M^2 < \Lambda M_{\rm pl}}$$

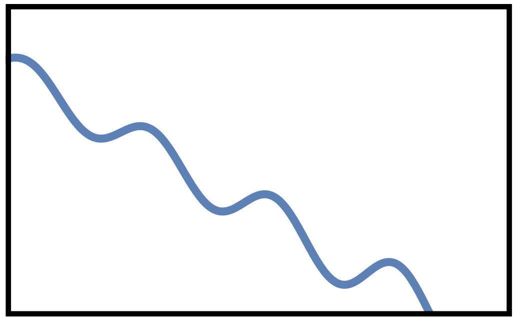
Bound on cutoff...

$$M < 3 \times 10^8 \text{ GeV}$$

However,...

$$\boxed{gM^2 f \sim \Lambda^4}$$

$$\theta_{\mathrm{QCD}} \simeq \pi/2$$



Prediction: $d_n \simeq few \times 10^{-16} e\,\mathrm{cm}$

Usual strong CP solutions don't work.

Solve Strong CP

Dynamical one -- Drop the slope:

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + \kappa\sigma^2\phi + gM^2\phi + \cdots + \Lambda^4 \cos\frac{\phi}{f}$$

inflaton - drops at end of inflation

$$gM^2 \simeq \theta \times \kappa\sigma^2 \longrightarrow$$

$$gM^2 f \sim \theta\,\Lambda^4$$

$$H_{\rm infl} > \theta^{-\frac{1}{2}}\,\frac{M^2}{M_{\rm pl}}$$

$$H_{\rm infl} < \Lambda$$

Bound on cutoff!

$$M^2 < \theta^{\frac{1}{2}} \Lambda M_{\text{pl}}$$

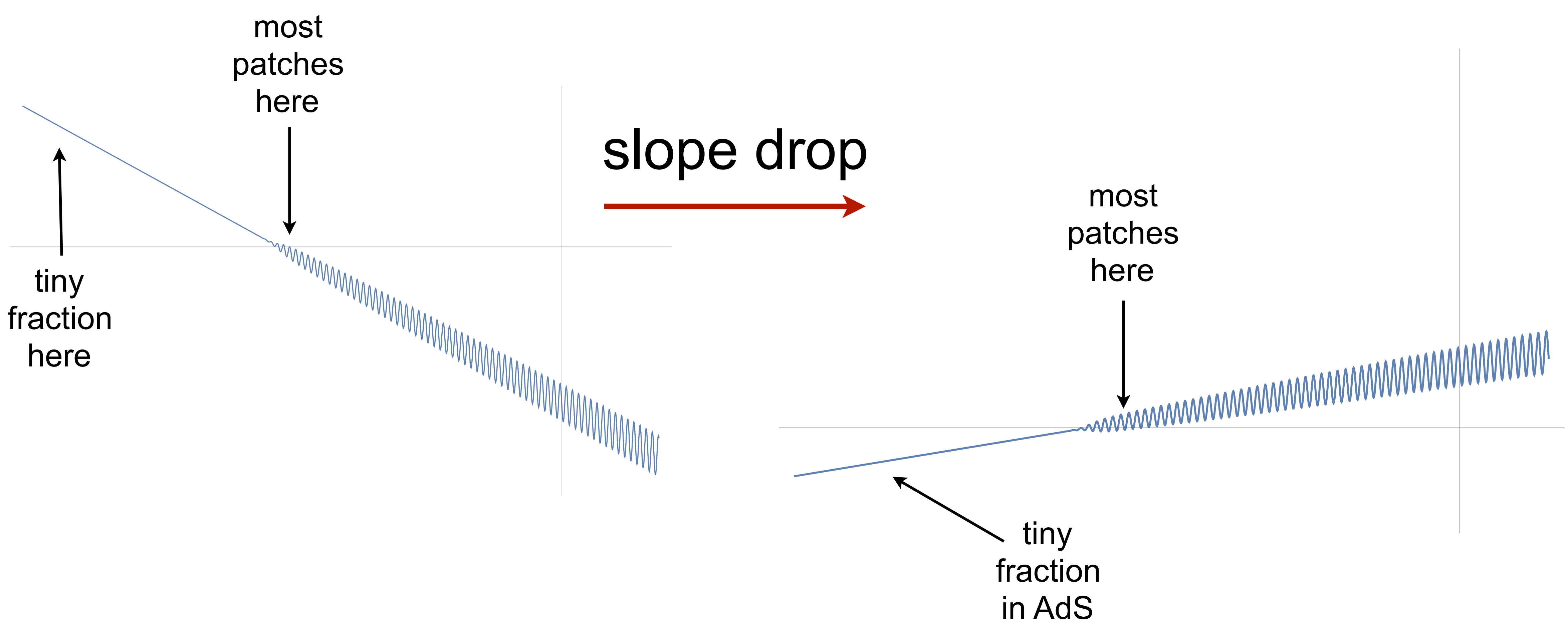
or

$$M < 1000 \text{ TeV} \left(\frac{\theta}{10^{-10}} \right)^{\frac{1}{4}}$$

Quantum vs. Classical evolution

Additional constraint can come from requiring classical evolution to dominate. $\longrightarrow$ $\frac{\dot{\phi}}{H_{\text{infl}}} > H_{\text{infl}}$

otherwise:



Solve Strong CP (2)
(Model 2)

Use a different strong group and couple ϕ to $G'^{\mu\nu}\tilde{G}'_{\mu\nu}$.

$$\mathcal{L} \supset m_L L L^c + m_N N N^c + y h L N^c + \tilde{y} h^\dagger L^c N$$

	$SU(3)$
L, N	$\square$
L^c, N^c	$\overline{\square}$

assume:

$$m_L \gg f_{\pi'} \gg m_N$$

NDA: $\Lambda^4 \simeq 4\pi f_{\pi'}^3 m_{N_1}$

Model 2

Use a different strong group and couple ϕ to $G'^{\mu\nu}\tilde{G}'_{\mu\nu}$.

Higgs induced: $\delta m_{N_1} \simeq \frac{y\tilde{y}\langle h\rangle^2}{m_L}$ "Bare": $m_N \gtrsim \frac{y\tilde{y}}{16\pi^2} m_L \log\frac{M}{m_L}$

Require: $$m_L < \frac{4\pi\langle h\rangle}{\sqrt{\log M/m_L}}$$

Bounds: $$m_L \gtrsim 250 \text{ GeV}$$

Parameter Requirements

$$\boxed{H_{\text{infl}} > \frac{M^2}{M_{\text{pl}}}}$$

$$\boxed{H_{\text{infl}}^3 < g M^2} \quad \frac{\dot{\phi}}{H_{\text{infl}}} > H_{\text{infl}}$$

Plugging in for g, ($gM^2 f \sim \Lambda^4$):

$$\boxed{M^6 < \frac{\Lambda^4 M_{\text{pl}}^3}{f}}$$

$$M < 3 \times 10^8 \text{ GeV} \left(\frac{f_{\pi'}}{30 \text{ GeV}}\right)^{\frac{3}{7}} \left(\frac{y\tilde{y}}{10^{-2}}\right)^{\frac{1}{7}} \left(\frac{250 \text{ GeV}}{m_L}\right)^{\frac{1}{7}} \left(\frac{M}{f}\right)^{\frac{1}{7}}$$

Inflation

To achieved the relaxed value,
inflation has to last long enough:

$$\Delta\phi \sim \frac{\dot{\phi}}{H_{\rm infl}} N \sim \frac{\partial_\phi V}{H_{\rm infl}^2} N \sim \frac{g M^2}{H_{\rm infl}^2} N$$

We require:

$$\Delta\phi \gtrsim \left(\frac{M^2}{g}\right)$$

$$N \gtrsim \frac{H_{\rm infl}^2}{g^2} \sim 10^{48}, 10^{37}$$ (Model 1,2 saturated)

Inflation

Minimum number of e-folds

$$N \gtrsim \frac{H_{\mathrm{infl}}^2}{g^2} \sim \frac{H_{\mathrm{infl}}^2 M^4 f^2}{\Lambda^8}$$

barriers stop roll: $\dot{\phi}^2 \sim \Lambda^4 < H\dot{\phi}f$ $\qquad f > \dfrac{\Lambda^2}{H}$

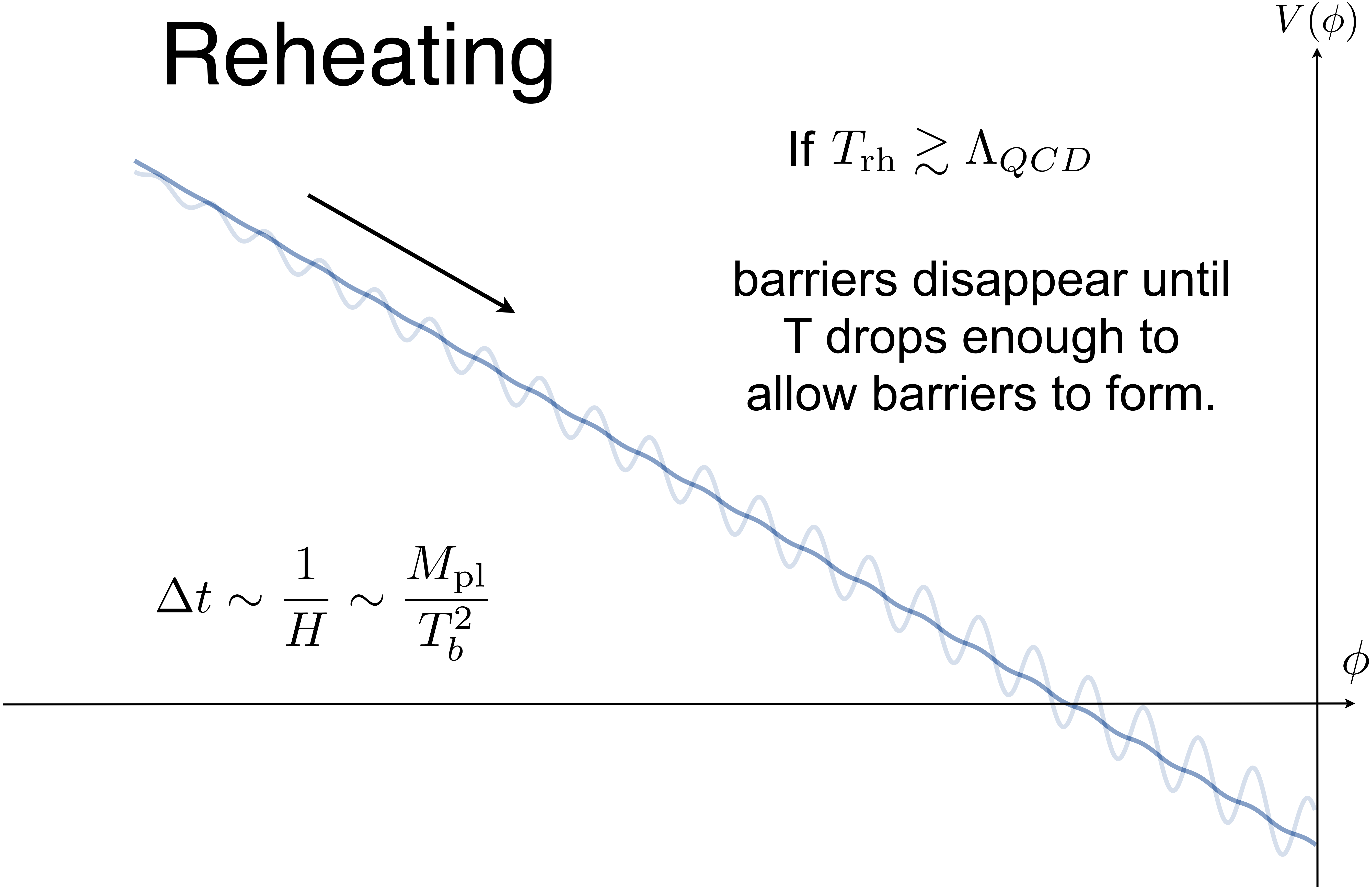
$$N > \left(\frac{M}{\Lambda}\right)^4$$

Reheating

If $T_{\rm rh} \gtrsim \Lambda_{QCD}$

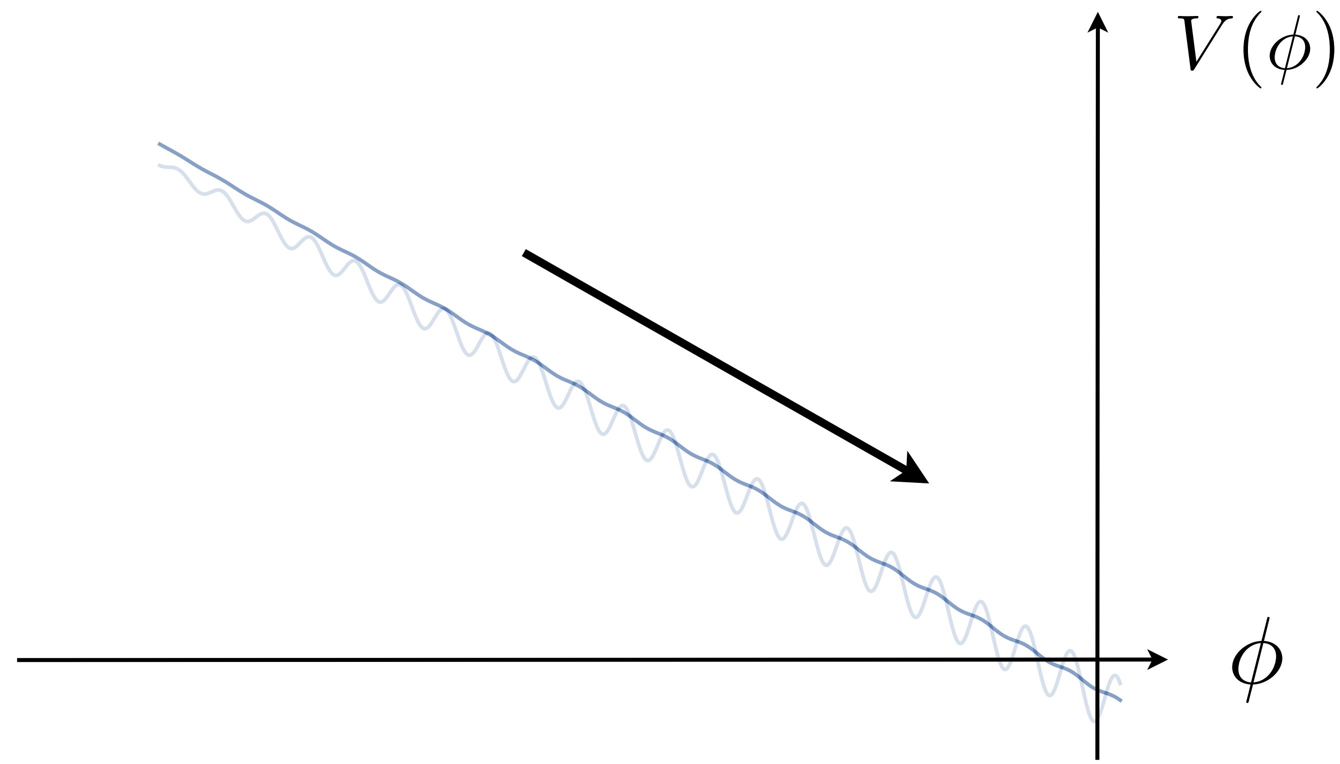
barriers disappear until
T drops enough to
allow barriers to form.

$$\Delta t \sim \frac{1}{H} \sim \frac{M_{\rm pl}}{T_b^2}$$



Reheating

QCD



KE gained per step with barriers turning on:

$$\dot{\phi}^2 \sim \theta \Lambda^4$$

Hubble friction loss (in one step):

$$H\dot{\phi}f \sim \frac{T_b^2}{M_{\rm pl}}\dot{\phi}f$$

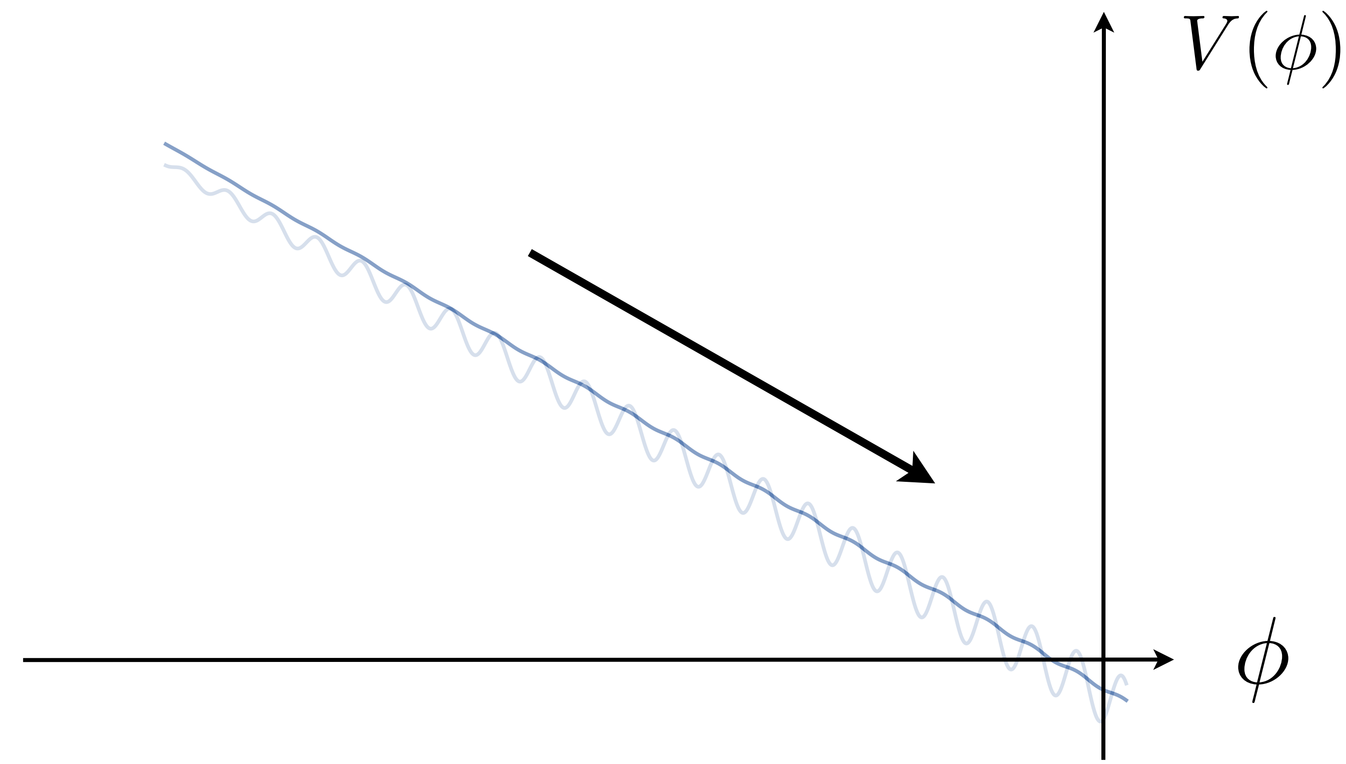
$$\dot{\phi} \sim \sqrt{\theta}\Lambda^2 \lesssim \frac{T_b^2}{M_{\rm pl}} f$$

$$\boxed{\begin{array}{c} T_b \sim 3 \text{ GeV} \\ \Lambda \sim 70 \text{ MeV} \end{array}}$$

$$f > 10^{10} \text{ GeV} \left(\frac{\theta}{10^{-10}}\right)^{1/2}$$

Reheating

non-QCD



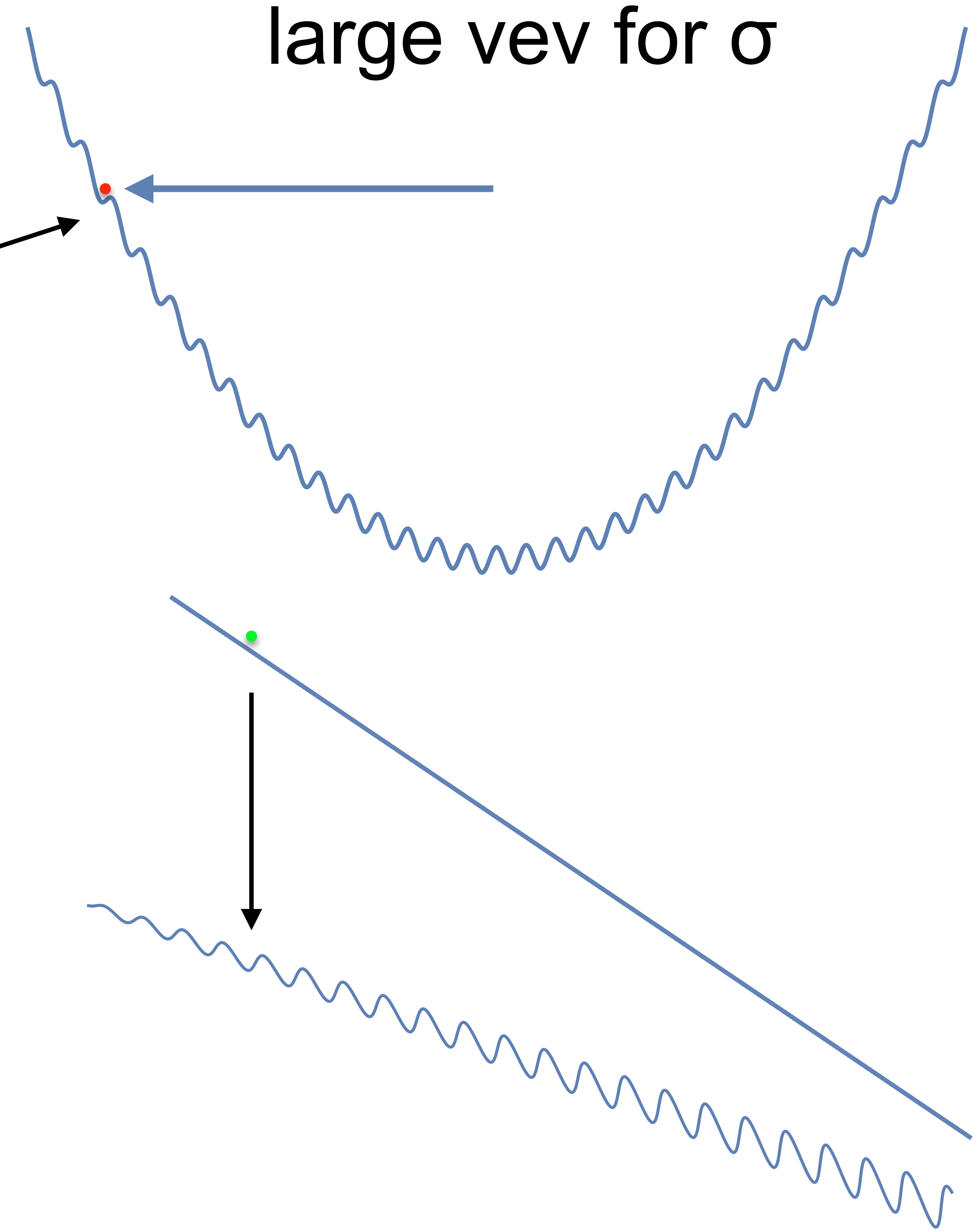
$$\dot{\phi} \sim \Lambda^2 \lesssim \frac{T_b^2}{M_{\rm pl}} f$$

$$f \gtrsim M_{\rm pl} \left(\frac{\Lambda}{T_b} \right)$$

$$\boxed{T_b \sim \sqrt{4\pi} f_{\pi'}}$$

$$f > 10^{17}\ \text{GeV} \left(\frac{y\tilde{y}}{10^{-2}} \right)^{\frac{1}{2}} \left(\frac{30\ \text{GeV}}{f_{\pi'}} \right)^{\frac{1}{2}} \left(\frac{300\ \text{GeV}}{m_L} \right)^{\frac{1}{2}}$$

Hybrid Relaxion/Inflaton



stuck until released by small higgs vev

add $\lambda\sigma^2\varphi$

Release of σ drops φ slope

and reheats the universe!

A Clockwork Axion

$$V(\phi) = \sum_{j=0}^{N} \left(-m^2 \phi_j^\dagger \phi_j + \frac{\lambda}{4} |\phi_j^\dagger \phi_j|^2 \right) + \sum_{j=0}^{N-1} \left(\epsilon \phi_j^\dagger \phi_{j+1}^3 + h.c. \right)$$

$$\phi_j \to U_j \equiv f e^{i\pi_j/(\sqrt{2}f)}$$

$$\frac{1}{2} \sum_{j=0}^{N} \partial_\mu \pi_j \partial^\mu \pi_j + \epsilon f^4 \sum_{j=0}^{N-1} e^{i(3\pi_{j+1} - \pi_j)/(\sqrt{2}f)} + h.c. + \cdots$$

A Clockwork Axion

$$M_{ij}^2 = \epsilon f^2 \begin{pmatrix} 1 & -3 & 0 & 0 & & & \\ -3 & 10 & -3 & 0 & \cdots & & \\ 0 & -3 & 10 & -3 & & & \\ 0 & 0 & -3 & 10 & & & \\ & & & & \ddots & & \\ & \vdots & & & & \ddots & \\ & & & & & 10 & -3 \\ & & & & & -3 & 9 \end{pmatrix}$$

eigenvalues:

$$m_\theta^2 = \epsilon f^2 (10 - 6\cos\theta), \quad 0 \le \theta < 2\pi$$

plus zero mode

$$\vec{a}_{(0)}^T = \mathcal{N} \begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{9} & \cdots & \frac{1}{3^N} \end{pmatrix}$$

An effective large ‘f’ from suppressed w.f. overlap: $\sim \dfrac{a}{32\pi^2 (3^N f)} H^{\mu\nu} \tilde{H}_{\mu\nu}$

To Do

- Better Inflation models - can one avoid inflation?

E. Hardy (2015)

- Better models?

J.R. Espinosa, C. Grojean, G. Panico, A. Pomarol, O. Pujolàs, G. Servant (2015)

- Phenomenology - New non-collider experiments?

See Surjeet's talk tomorrow!!

- Cosmological Constant - new solution?

Working on it…

Thank you!

Extra: Inflation

Single field: $V(\Phi) = m^2\Phi^2$

$$N = \int H dt \sim \int \frac{H^2}{\partial_\Phi V} d\Phi \sim \frac{\Phi_i^2}{M_{\rm pl}^2}$$

Classical rolling:

$$\frac{\dot\Phi}{H_{\rm infl}} < H_{\rm infl} \longrightarrow \frac{m\Phi_i^2}{M_{\rm pl}^3} < 1 \longrightarrow V(\Phi_i) < \frac{M_{\rm pl}^4}{N}$$

$$\longrightarrow N < \left(\frac{M_{\rm pl}}{M}\right)^4 (\times\theta)$$

$$N \gtrsim \frac{H_{\rm infl}^2}{g^2} \longrightarrow M < 10^5, 10^{8.75}\ {\rm GeV}$$

Reheating requires additional dynamics (e.g., hybrid)

T-dependence of barriers