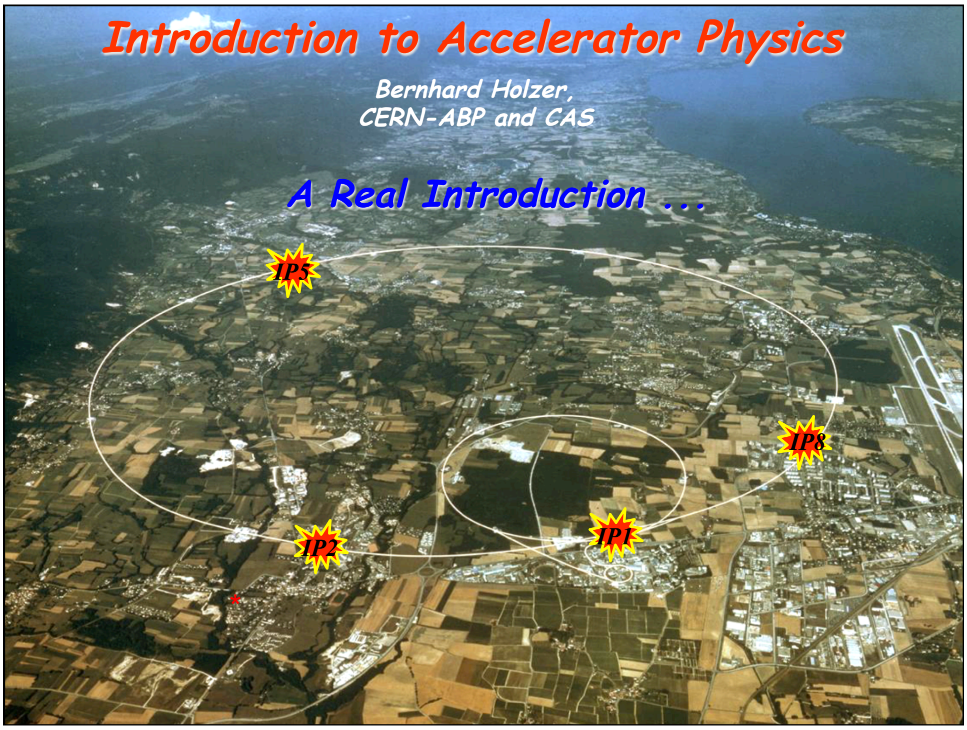
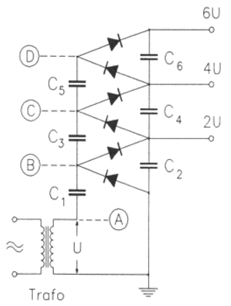


Introduction to Accelerator Physics

Bernhard Holzer,
CERN-ABP and CAS

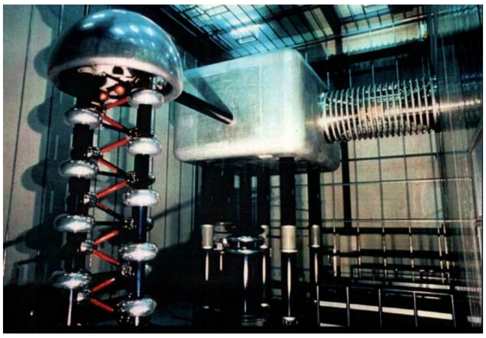
A Real Introduction ...

1.) Electrostatic Machines:
The Cockcroft-Walton Generator



***1928:** Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam*

***1932:** First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV*



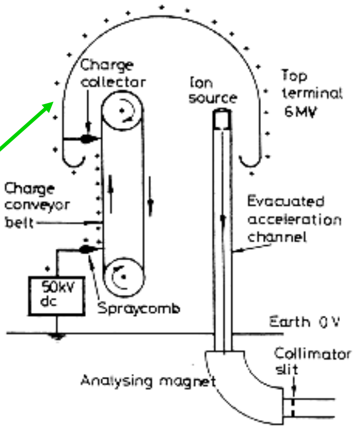
Particle source: Hydrogen discharge tube on 400 kV level
Accelerator: evacuated glas tube
Target: Li-Foil on earth potential

***Technically:** rectifier circuit, built of capacitors and diodes (Greinacher)*

Problem:
DC Voltage can only be used once

2.) Electrostatic Machines:
(Tandem -) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



** Terminal Potential: $U \approx 12 \ldots 28$ MV*
using high pressure gas to suppress discharge (SF_6)

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

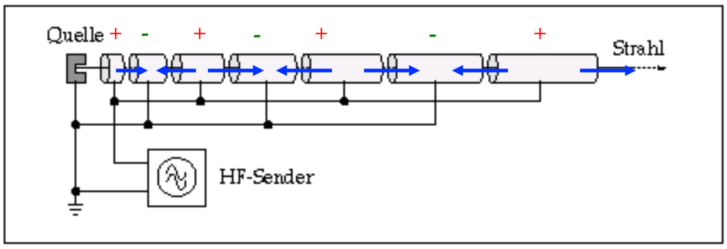
The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

3.) The first RF-Accelerator: „Linac“

1928, Wideroe: how can the acceleration voltage be applied several times to the particle beam

schematic Layout:



Energy gained after n acceleration gaps

$$E_n = n * q * U_0 * \sin\psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
$\boldsymbol{U_0}$ *Peak voltage of the RF System*
$\boldsymbol{\Psi_S}$ *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped“ to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

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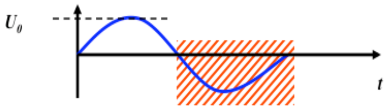
Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$$\rightarrow v_i = \sqrt{2E_i/m}$$

$$l_i = \frac{1}{v_{rf}} * \sqrt{\frac{i * q * U_0 * \sin\psi_s}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: $\approx$ ***20 MeV per Nucleon*** $\beta \approx 0.04 \ldots 0.6$***, Particles: Protons/Ions***

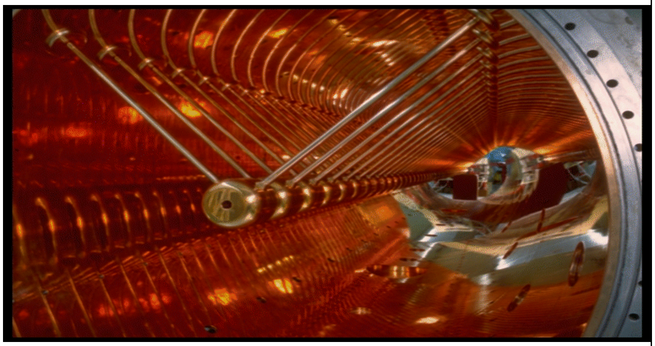
Accelerating structure of a Proton Linac (DESY Linac III)

$E_{total} = 988\,MeV$

$m_0c^2 = 938\,MeV$

$p = 310\,MeV\ /\ c$

$E_{kin} = 50\,MeV$



Beam energies

Energy Gain per „Gap“: $W = q\,U_0 \sin \omega_{RF} t$

1.) reminder of some relativistic formula

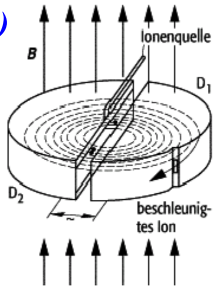
rest energy $E_0 = m_0c^2$

total energy $E = \gamma * E_0 = \gamma * m_0c^2$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

momentum $E^2 = c^2p^2 + m_0^2c^4$

3.) The Cyclotron: (Livingston / Lawrence ~1930)



Idea: B = const, RF = const
Synchronisation particle / RF via orbit

Lorentzforce

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \longrightarrow B * R = p / q$$

increasing radius for increasing momentum
➔ Spiral Trajectory

revolution frequency

$$\omega_z = \frac{v}{R} = \frac{q}{m} * B_z$$

the cyclotron (rf-) frequency is independent of the momentum

rf-frequency = h revolution frequency,   h = “harmonic number”*

Cyclotron:

exact equation for revolution frequency:

$$\omega_z = \frac{v}{R} = \frac{q}{\boxed{\gamma} * m} * B_z$$

1.) if $v << c \Rightarrow \gamma \cong 1$

2.) γ increases with the energy
$\Rightarrow$ no exact synchronism


Cyclotron SPIRAL at GANIL

"synchronisation" with the acceleration potential is established
via the spiraling orbit length

Syn on

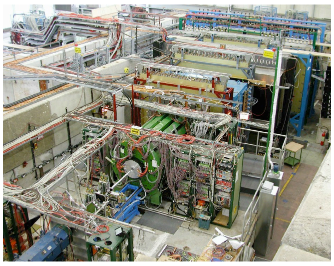
B = constant
$\gamma\, \omega_{RF}$ = constant
ω_{RF} decreases with time

$$\omega_s(t) = \omega_{rf}(t) = \frac{q}{\gamma(t) * m_0} * B$$

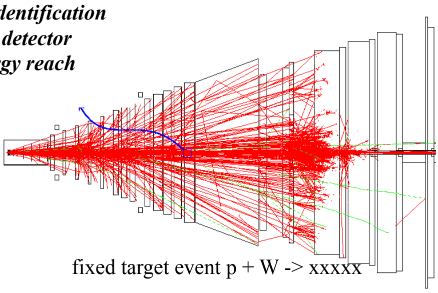
keep the synchronisation condition by varying the rf frequency

---

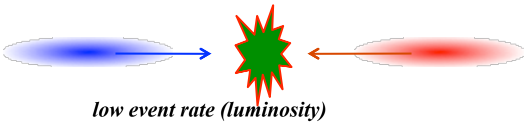
Fixed target experiments:


HARP Detector, CERN

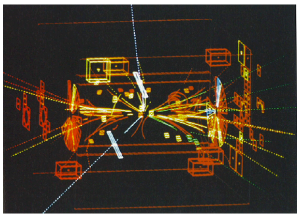
high event rate
easy track identification
asymmetric detector
limited energy reach


fixed target event p + W -> xxxxx

Collider experiments:



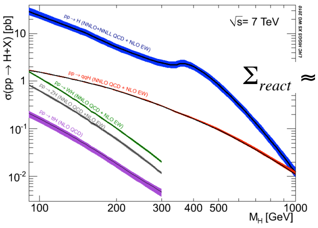
low event rate (luminosity)
challenging track identification
symmetric detector
$E_{lab} = E_{cm}$



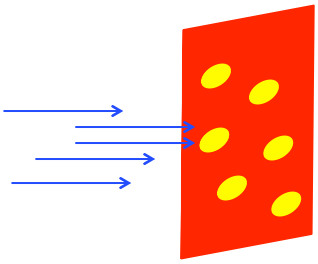
Z_0 boson discovery at the UA2 experiment (CERN).
The Z_0 boson decays
into a e+e− pair, shown as white dashed lines.

Problem: Our particles are VERY small !!

Overall cross section of the Higgs:



$$\Sigma_{react} \approx 1\, pb$$

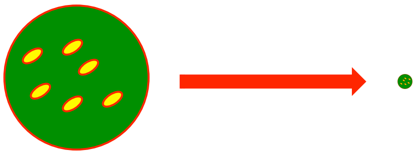


$$1b \; = 10^{-24}\, cm^2$$

$$1pb = \quad 10^{-12} * 10^{-24}\, cm^2 = 1/mio * 1/mio * 1/mio * 1/mio * 1/mio * 1/10000\ mm^2$$

The particles are "very small"

The only chance we have:
compress the transverse beam size … at the IP



LHC typical:

σ = 0.1 mm → 16 µm

1.) Introduction and Basic Ideas

„ ... in the end and after all it should be a kind of circular machine"
→ need transverse deflecting force

Lorentz force

$$\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines:

$$v \approx c \approx 3 * 10^8\, {}^{m}\!/\!_{s}$$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q * \underbrace{300 \frac{MV}{m}}$$

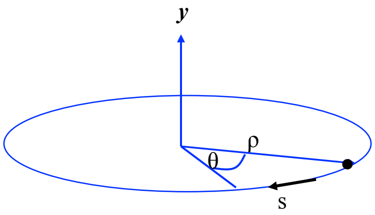
equivalent E
electrical field:

Technical limit for electrical fields:

$$E \le 1 \frac{MV}{m}$$

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $F_L = e\,v\,B$

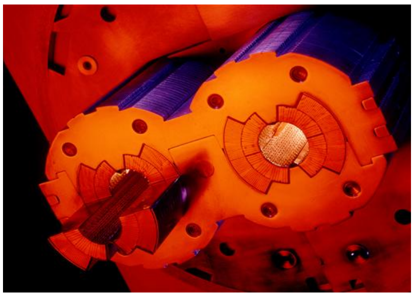
centrifugal force $F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$

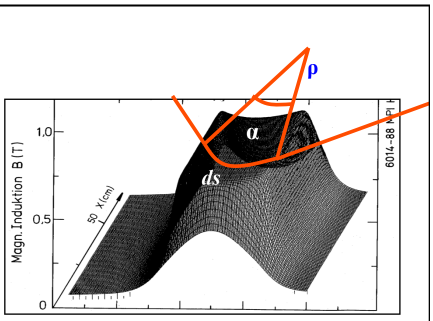
$$\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\,\cancel{v}\,B$$

$$\boxed{\frac{p}{e} = B\,\rho}$$

$B\rho$ = "beam rigidity"

The Magnetic Guide Field





field map of a storage ring dipole magnet

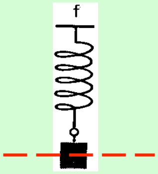
$\rho = 2.8\ km$ $\longrightarrow$ $2\pi\rho = 17.6\ km$
$\approx 66\%$

$B \approx 1 \ldots 8\ T$

rule of thumb: $\boxed{\frac{1}{\rho} \approx 0.3\,\frac{B[T]}{p[GeV/c]}}$ ***„normalised bending strength“***

Focusing Properties and Quadrupole Magnets

classical mechanics:
pendulum



there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2 x}{dt^2} = -c * x$$

general solution: free harmonic oszillation $x(t) = A * \cos(\omega t + \varphi)$

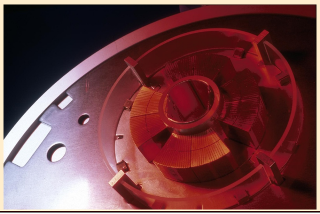
this is how grandma's Kuckuck's clock is working!!!

Storage Rings: ***linear increasing Lorentz force to keep trajectories in vicinity of the ideal orbit***

linear increasing magnetic field $B_y = g\, x \qquad B_x = g\, y$

$$F(x) = q * v * B(x)$$

LHC main quadrupole magnet $g \approx 25 \ldots 220 \; T/m$



Focusing forces and particle trajectories:

normalise magnet fields to momentum
*(remember: **B*ρ = p / q**)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

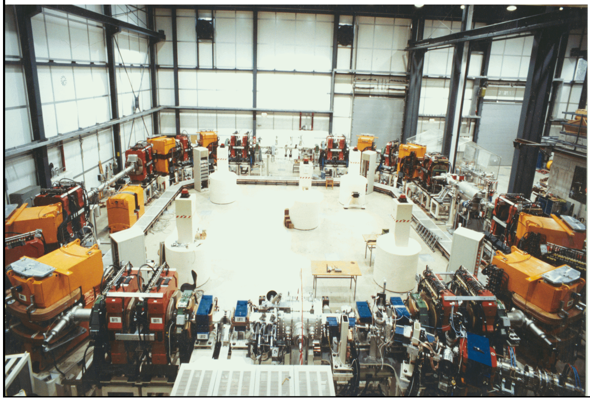
Quadrupole Magnet

$$k := \frac{g}{p/q}$$

3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}\,m\,x^2} + \cancel{\frac{1}{3!}\,n\,x^3} + \ldots$$

only terms linear in x, y taken into account *dipole fields*
quadrupole fields



Example:
heavy ion storage ring TSR

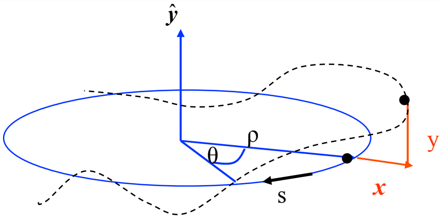
Separate Function Machines:

Split the magnets and optimise them according to their job:

bending, focusing etc

* *man sieht nur*
dipole und quads → linear

The Equation of Motion:



* ***Equation for the horizontal motion:***

$$x'' + x\,(\frac{1}{\rho^2} + k) = 0$$

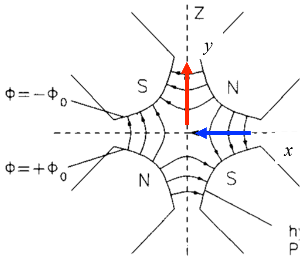
x = particle amplitude
x' = angle of particle trajectory (wrt ideal path line)

* ***Equation for the vertical motion:***

$\frac{1}{\rho^2} = 0$ ***no dipoles … in general …***

$k \;\leftrightarrow\; -k$ ***quadrupole field changes sign***

$$y'' - k\,y = 0$$



4.) Solution of Trajectory Equations

Define … hor. plane: $K = 1/\rho^2 + k$

… vert. Plane: $K = -k$

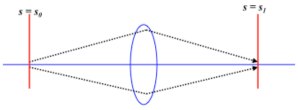
$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad \boxed{x'' + K\,x = 0}$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: ***Hor. Focusing Quadrupole $K > 0$:***

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$



For convenience expressed in matrix formalism:

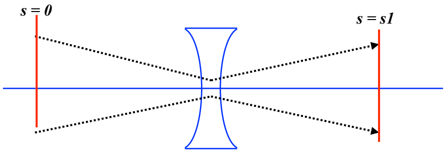
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$\boxed{M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}\,l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}\,l\right) \\ -\sqrt{|K|} \sin\left(\sqrt{|K|}\,l\right) & \cos\left(\sqrt{|K|}\,l\right) \end{pmatrix}}$$

---

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



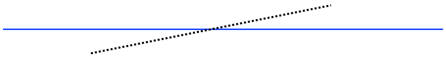
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$\boxed{M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}} \sinh\sqrt{|K|}\,l \\ \sqrt{|K|} \sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}}$$

drift space:

$K = 0$



$$x(s) = x'_0 * s$$

$$\boxed{M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}}$$

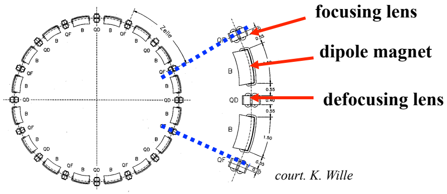
! with the assumptions made, the motion in the horizontal and vertical planes are independent „ … the particle motion in x & y is uncoupled"

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

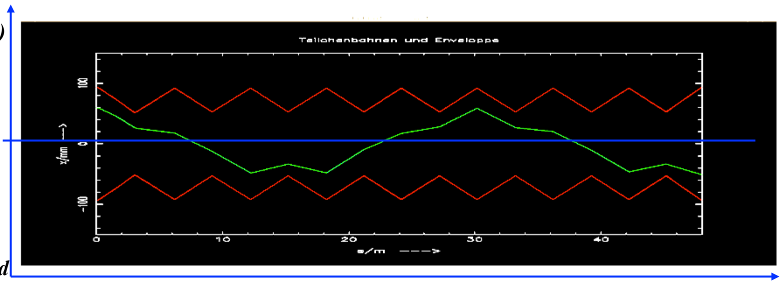
$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*}.....$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



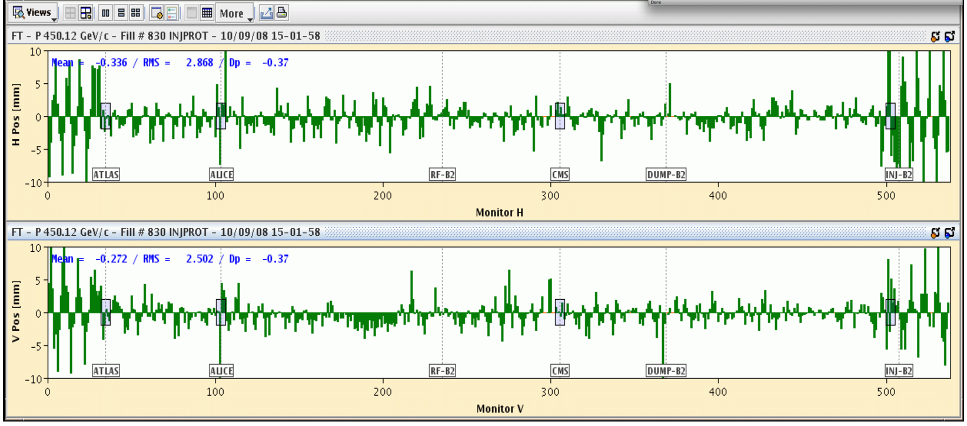
typical values
in a strong
foc. machine:
x ≈ mm, x´ ≤ mrad

LHC Operation: Beam Commissioning

The transverse focusing fields create a harmonic oscillation of the particles with a well defined "Eigenfrequency" which is called tune

First turn steering "by sector:"

YASP DV LHCRING / INJ-TEST-NB / beam 2



FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

Mean = -0.336 / RMS = 2.868 / Dp = -0.37

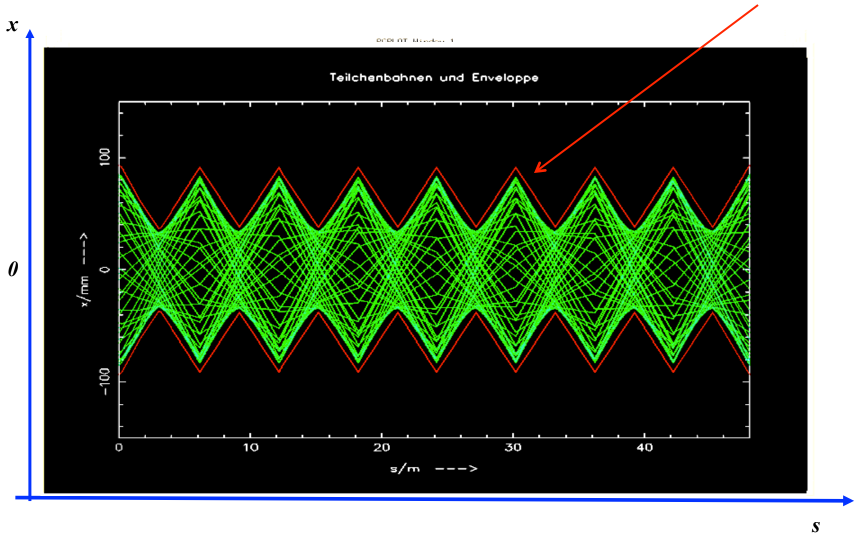
FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

Mean = -0.272 / RMS = 2.502 / Dp = -0.37

Question: what will happen, if the particle performs a second turn ?

... or a third one or ... 10^{10} turns

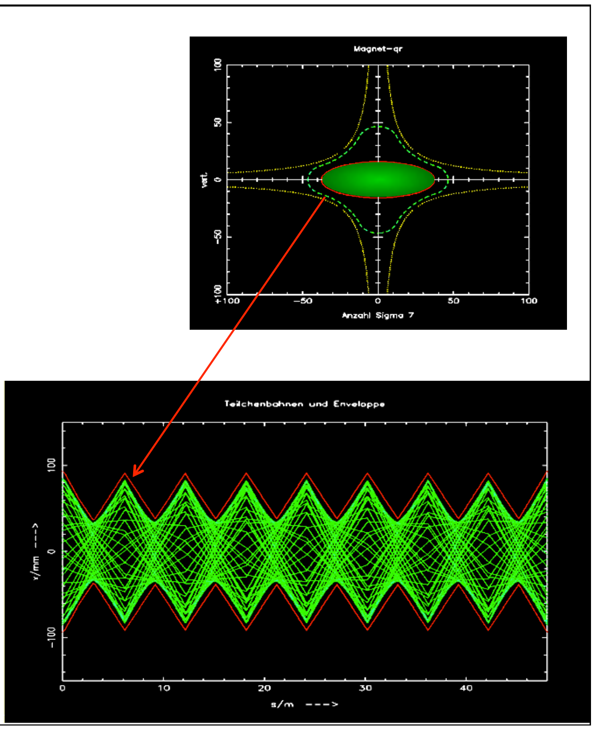
$\sigma = \sqrt{\varepsilon\beta}$



The Beta Function

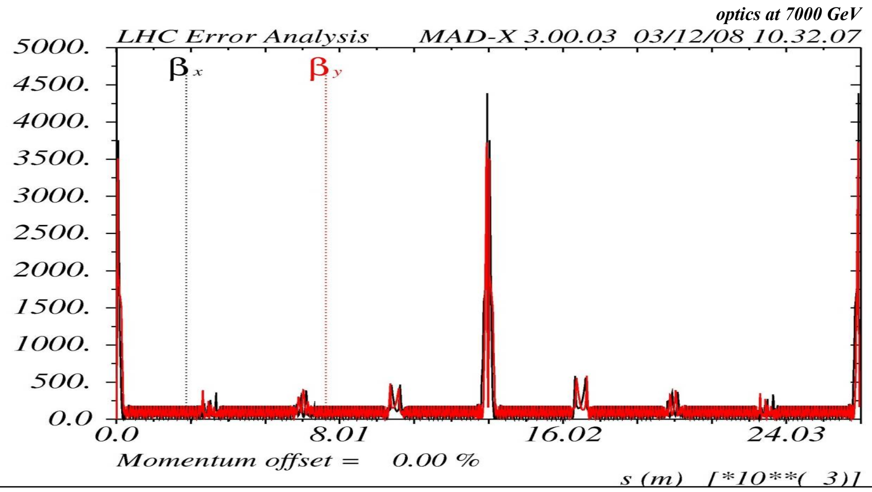
β determines the beam size
... the envelope of all particle trajectories at a given position "s" in the storage ring under the influence of all (!) focusing fields.

It reflects the periodicity of the magnet structure.



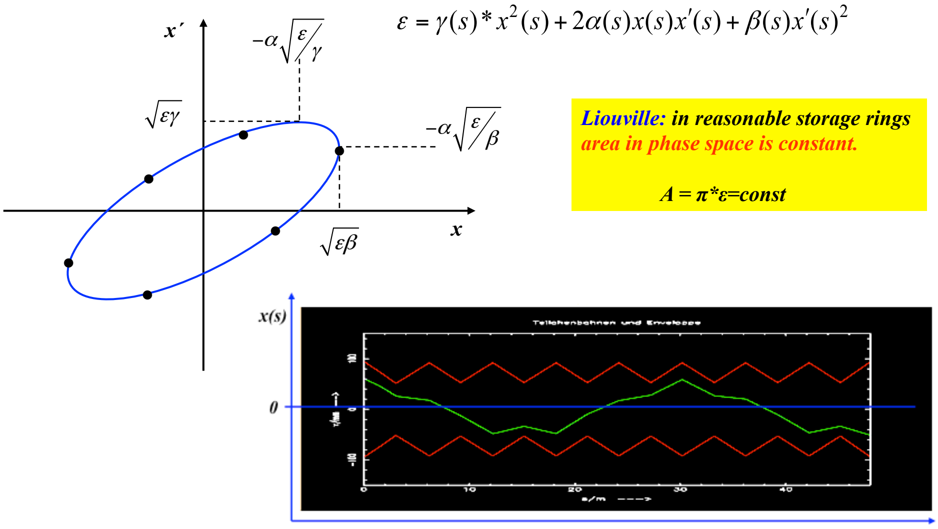
The Beta Function: Lattice Design & Beam Optics

The beta function determines the maximum amplitude a single particle trajectory can reach at a given position in the ring.
It is determined by the focusing properties of the lattice and follows the periodicity of the machine.



Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$



***Liouville:* in reasonable storage rings**
area in phase space is constant.

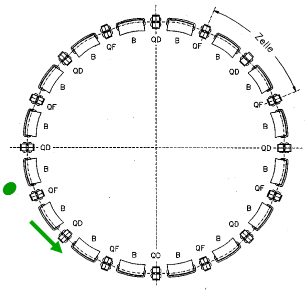
A = π*ε=const

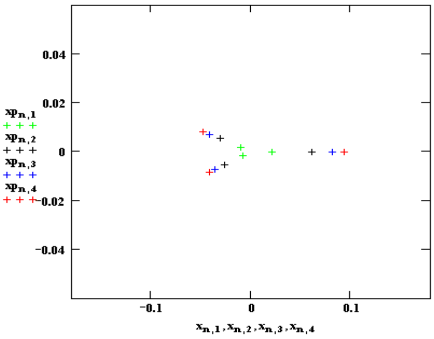
***ε beam emittance** = **woozilycity** of the particle ensemble, **intrinsic beam parameter**,*
cannot be changed by the foc. properties.
***Scientifiquely spoken:** area covered **in transverse x, x´ phase space** … and it is constant !!!*

Particle Tracking in a Storage Ring

Calculate x, x´ for each accelerator element according to matrix formalism and plot x, x´at a given position „s“ in the phase space diagram

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



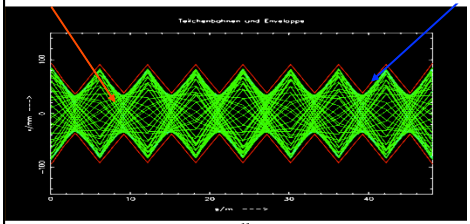


A beam of 4 particles

– each having a slightly different emittance:

Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \qquad \hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



single particle trajectories, N ≈ 10 ¹¹ per bunch

Gauß Particle Distribution:

$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

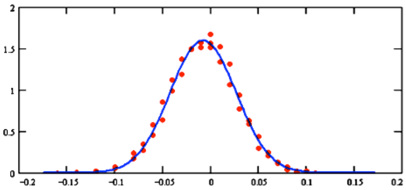
particle at distance 1 σ from centre ↔ 68.3 % of all beam particles

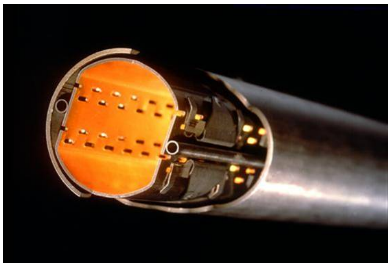
LHC:

$$\beta = 180\,m$$

$$\varepsilon = 5*10^{-10}\,m\,rad$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5*10^{-10}m*180m} = 0.3\,mm$$



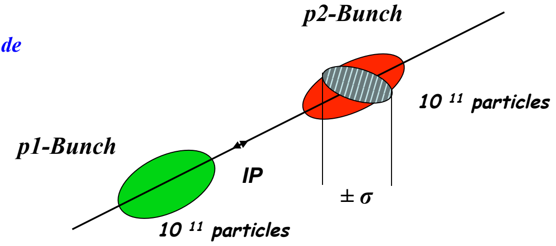


*aperture requirements: r ₀ = 17 * σ*

5.) Luminosity

Ereignis Rate:“Physik“ pro Sekunde

$$R = L * \Sigma_{react}$$



Example: Luminosity run at LHC

$\beta_{x,y} = 0.55\ m$

$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m$

$\sigma_{x,y} = 17\ \mu m$

$f_0 = 11.245\ kHz$

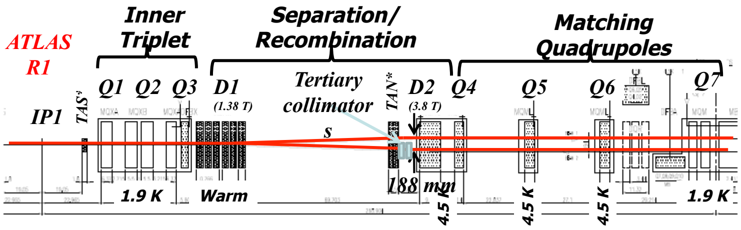
$n_b = 2808$

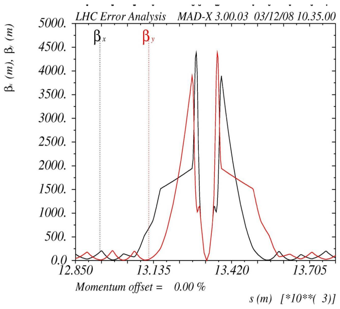
$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$I_p = 584\ mA$

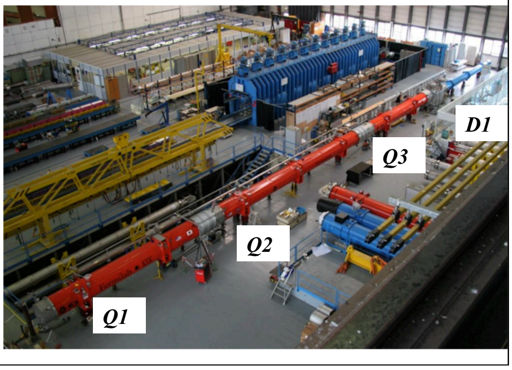
$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

The LHC Mini-Beta-Insertions





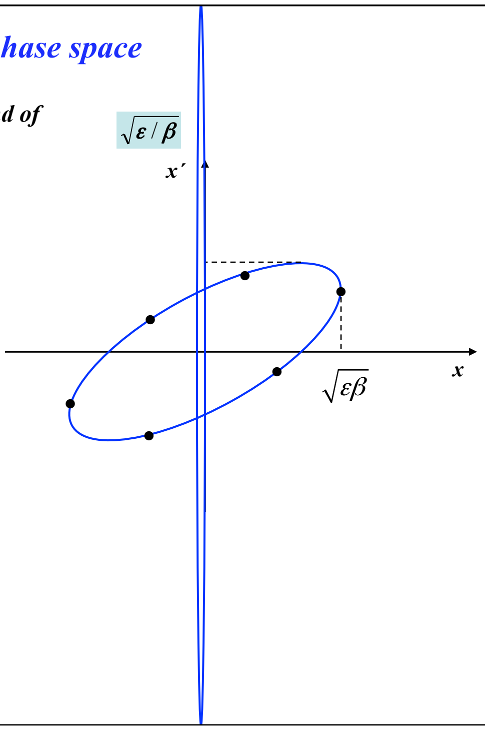
mini β optics



Mini-Beta-Insertions in phase space

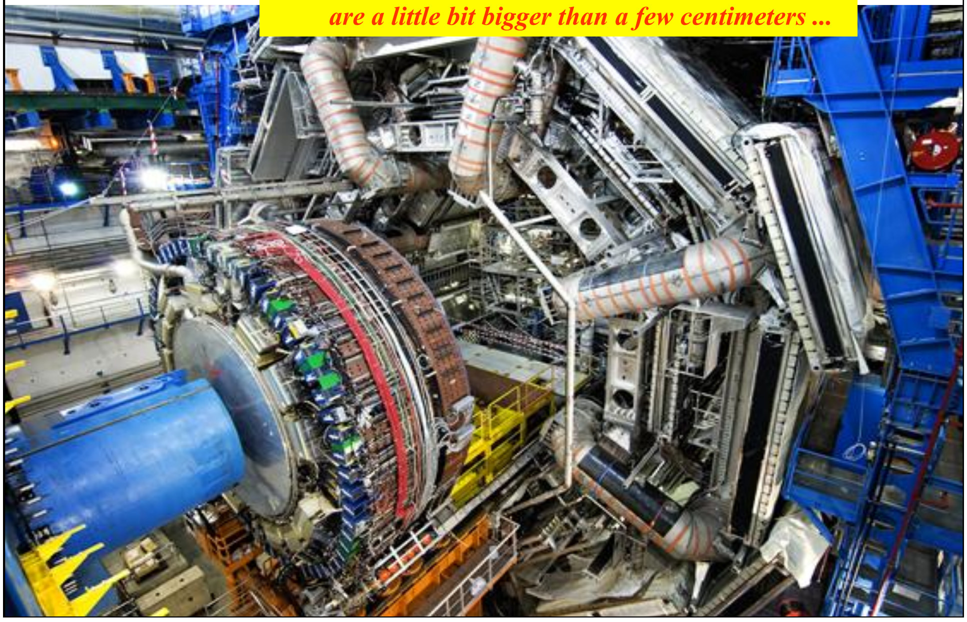
A mini-β insertion is always a kind of
special symmetric drift space.
→ greetings from Liouville

the smaller the beam size
the larger the bam divergence

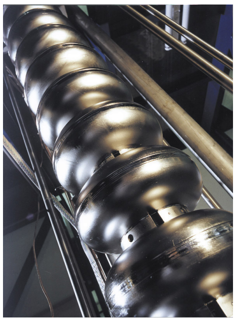


... clearly there is an

... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...

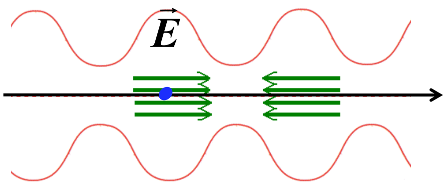


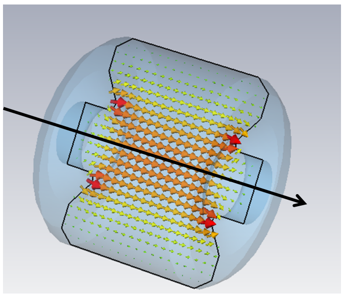
III. The Acceleration



Where is the acceleration?

Install an RF accelerating structure in the ring:





B. Salvant
N. Biancacci

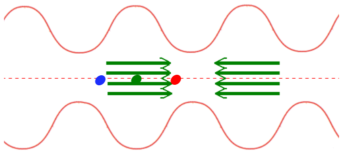
The Acceleration & "Phase Focusing"

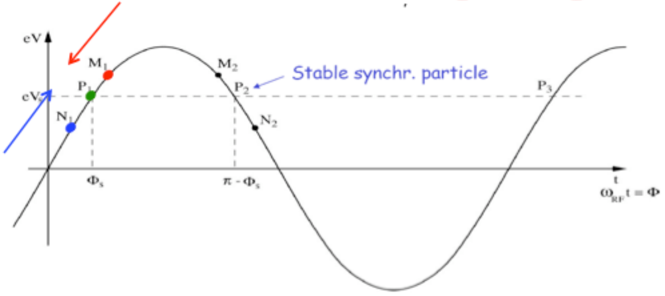
$\Delta p/p \neq 0$ below transition

ideal particle ●

particle with $\Delta p/p > 0$ ● *faster*

particle with $\Delta p/p < 0$ ● *slower*





Focussing effect in the longitudinal direction keeping the particles close together
... forming a "bunch"

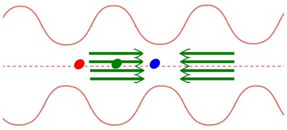
oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0 \cos\phi_s}{E_s}}$ *≈ some Hz*

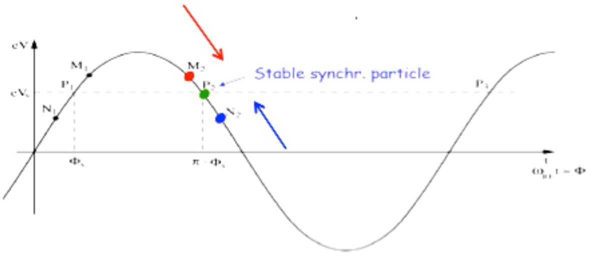
The Acceleration above transition

ideal particle ●

particle with $\Delta p/p > 0$ ● *heavier*

particle with $\Delta p/p < 0$ ● *lighter*



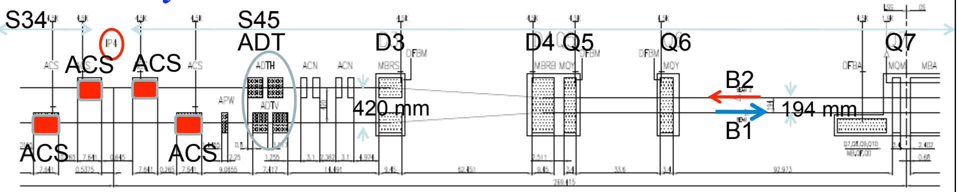


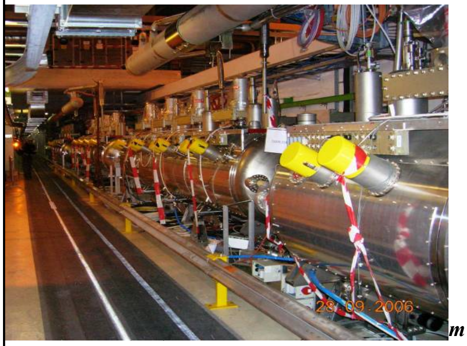
Focussing effect in the longitudinal direction
keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4



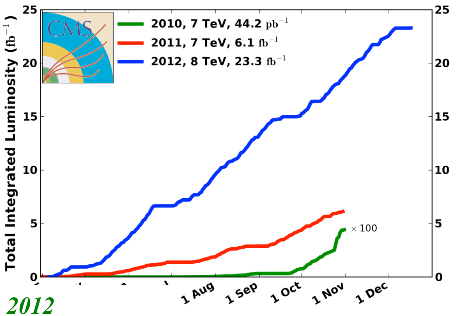


m

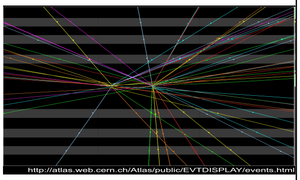
Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	*10^{-3}*	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*M Hz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

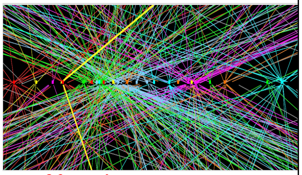
And still...
The LHC Performance in Run 1



	Design	*2012*
Momentum at collision	*7 TeV /c*	*4 TeV/c*
Luminosity	10^{34} *cm* $^{-2}$ *s* $^{-1}$	$7.7 \cdot 10^{33}$ *cm* $^{-2}$ *s* $^{-1}$
Protons per bunch	1.15×10^{11}	1.50×10^{11}
Number of bunches/beam	*2808*	*1380*
Nominal bunch spacing	*25 ns*	*50ns*
Normalized emittance	*3.75 µm*	*2.5µm*
*beta **	*55 cm*	*60 cm*
rms beam size (arc)	*300 µm*	*350 µm*
rms beam size IP	*17 µm*	*20 µm*


http://atlas.web.cern.ch/Atlas/public/EVTDISPLAY/events.html

2 vertices

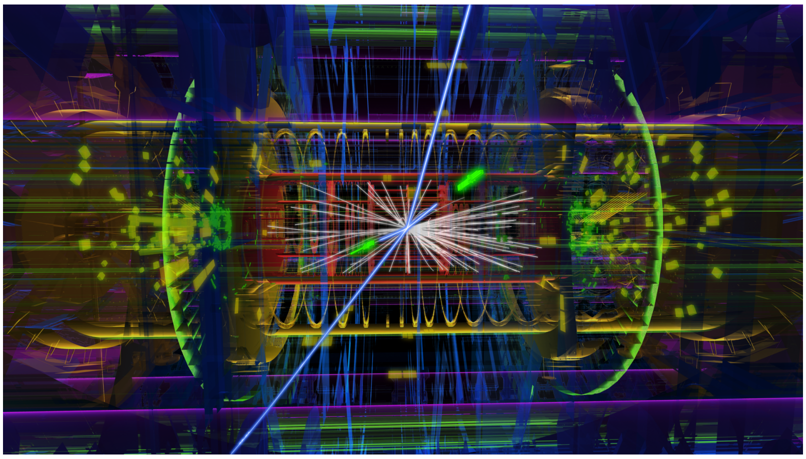


20 vertices

1.) Where are we ?

**** Standard Model of HEP***
**** Higgs discovery***

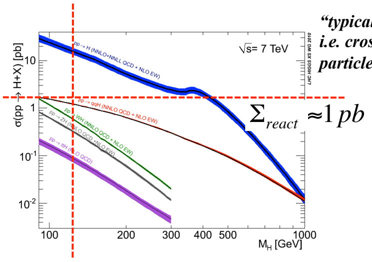
High Light of the HEP-Year 2012 / 13 naturally the HIGGS



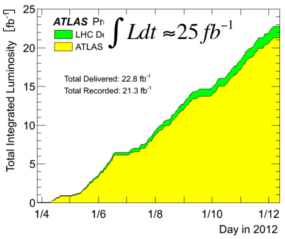
ATLAS event display: Higgs => two electrons & two muons

The High light of the year

***production rate of events* is *determined by* the *cross section* Σ_{react}**
and a parameter L that is given by the design of the accelerator:
… the luminosity



"typical particle size"
i.e. cross section for
particle production



accumulated
collision rate
in LHC run 1

$$1b = 10^{-24}\,cm^2 \quad = \quad 1/mio * 1/mio * 1/mio * \frac{1}{100}\,mm^2$$

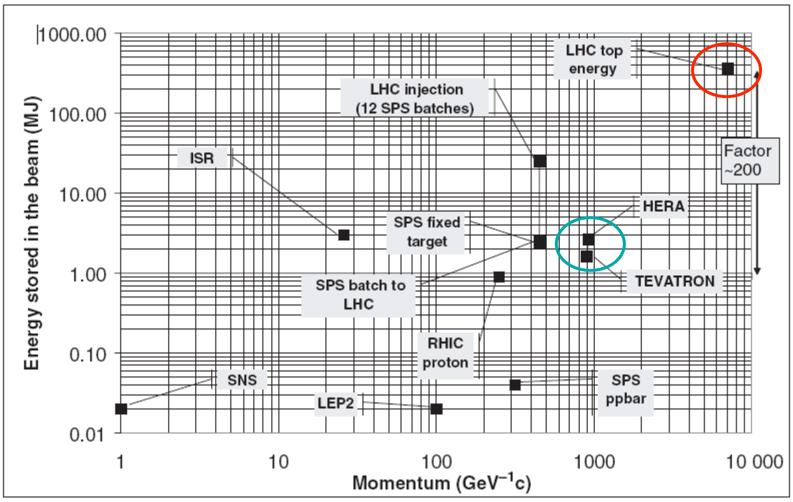
The particles are "very small"

$$R = L * \Sigma_{react} \quad \approx 10^{-12}b \cdot 25\,\frac{1}{10^{-15}b} = some\,1000\,H$$

During collider run we had in Run 1 …
1400 bunches circulating,
with 800 Mio proton collisions per second in the experiments
and collected only 450 Higgs particles in three years.

LHC Operation:
Machine Protection & Safety

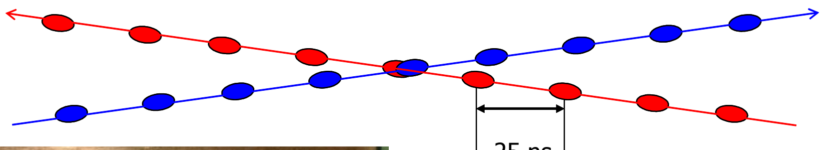
Energy Stored in the Beam of different Storage Rings

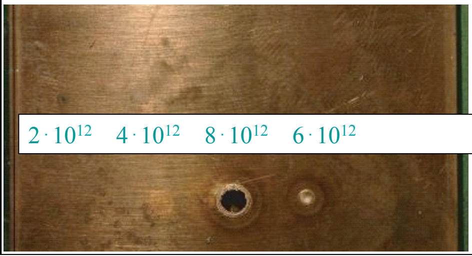


LHC Operation:
Machine Protection & Safety

Energy stored in magnet system	10	GJ
Energy stored in one main dipole circuit	1.1	GJ
Energy stored in one beam	362	MJ

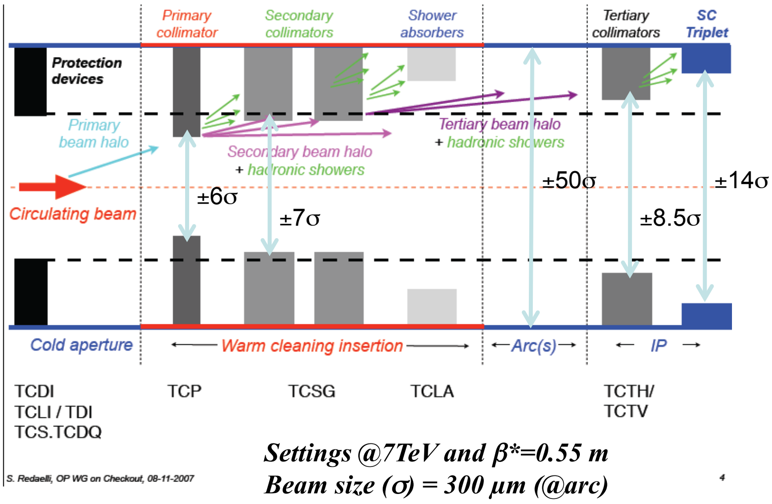
Enough to melt 500 kg of copper





$2 \cdot 10^{12}$ $4 \cdot 10^{12}$ $8 \cdot 10^{12}$ $6 \cdot 10^{12}$ 450 GeV p Strahl

LHC Aperture and Collimation


Primary collimator
Secondary collimators
Shower absorbers
Tertiary collimators
SC Triplet
Protection devices
Primary beam halo
Secondary beam halo
+ hadronic showers
Tertiary beam halo
+ hadronic showers
Circulating beam
±6σ
±7σ
±50σ
±8.5σ
±14σ
Cold aperture
←Warm cleaning insertion→
←Arc(s)→
← IP →

TCDI
TCLI / TDI
TCS.TCDQ

TCP

TCSG

TCLA

TCTH/
TCTV

S. Redaelli, OP WG on Checkout, 08-11-2007

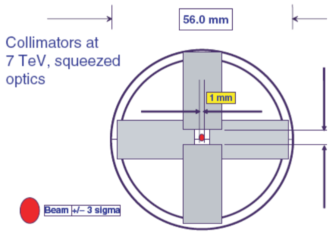
4

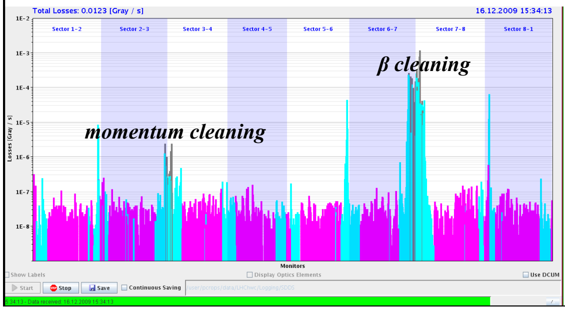
Settings @7TeV and β*=0.55 m
Beam size (σ) = 300 µm (@arc)
Beam size (σ) = 17 µm (@IR1, IR5)

LHC Operation:
Machine Protection & Safety

... Komponenten des Machine Protection Systems :

beam loss monitors
QPS
permit server
orbit control
power supply control
collimators
online on beam check of all (?)
hardware components
a fast dump
the gaussian beam profile

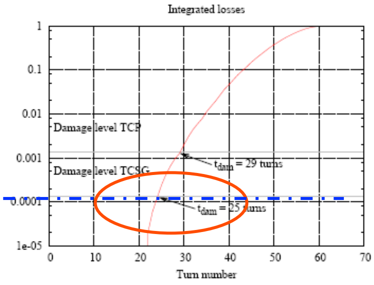



Total Losses: 0.0123 [Gray / s]
16.12.2009 15:34:13

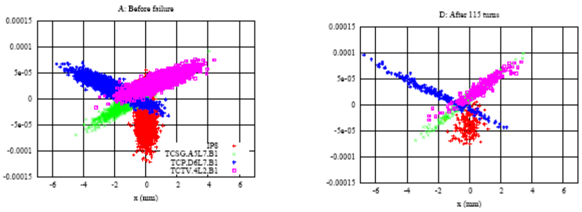
/user/pcrops/data/LHChwc/Logging/SDDS
5:34:13 - Data received: 16.12.2009 15:34:13

β cleaning

momentum cleaning



LHC Operation: Machine Protection & Safety



What will happen in case of Hardware Failure

Phase space deformation in case of failure of RQ4.LR7
(A. Gómez)

Short Summary of the studies:

quench in sc. arc dipoles: τ_{loss} *=20 - 30 ms*
BLM system reacts in time, QPS is not fast enough

quench in sc. arc quadrupoles: τ_{loss} *=200 ms*
BLM & QPS react in time

failure of nc. quadrupoles: $\tau_{det} = 6$ *ms*
$\tau_{damage} = 6.4$ *ms*
failure of nc. dipole: $\tau_{damage} = 2$ *ms*
→ FMCM installed

Energy stored in the magnets

~ 10 Gjoule* (only in the main dipoles) corresponds to …

… an aircraft carrier at battle-speed of 55 km/h

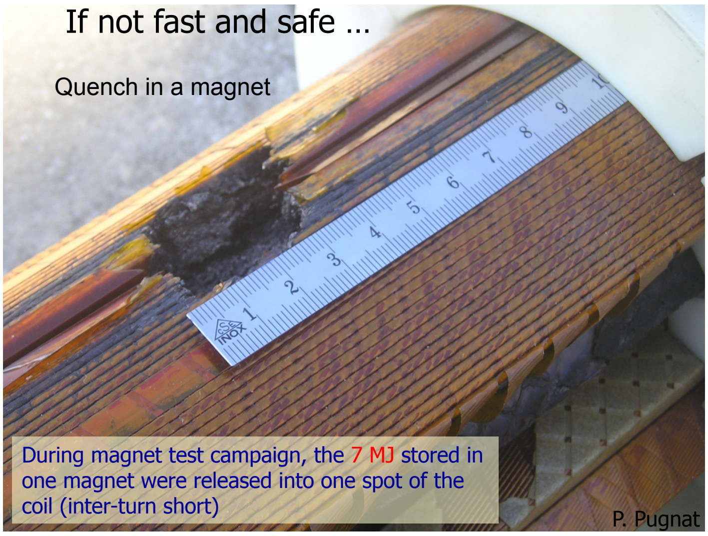
The energy of ~3 Tons TNT
The energy of 370 kg dark chocolate

More important than the amount of energy is …

How fast (an safe) can this energy be released?

$^*E=1/2LI^2$
L: inductance ~0.1 Henry for LHC dipoles

Energy stored in the magnets: quench



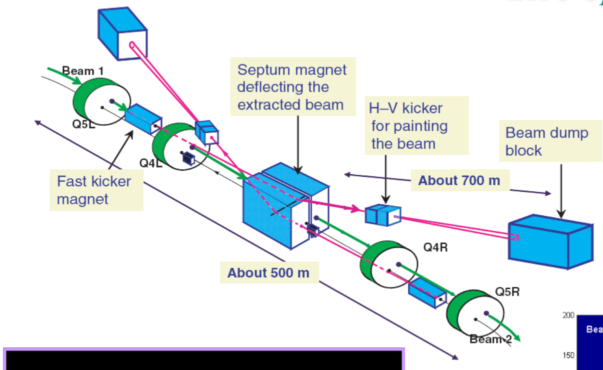
If not fast and safe …

Quench in a magnet

During magnet test campaign, the 7 MJ stored in one magnet were released into one spot of the coil (inter-turn short)

P. Pugnat

LHC Operation: Dump System



Septum magnet deflecting the extracted beam

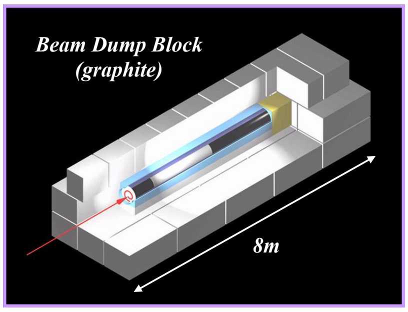
H–V kicker for painting the beam

Beam dump block

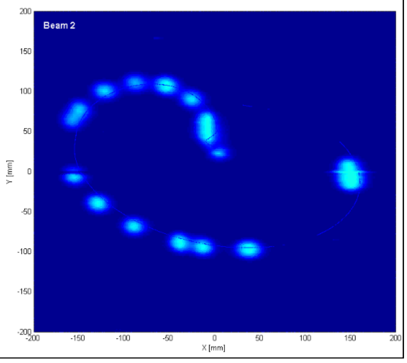
Fast kicker magnet

About 700 m

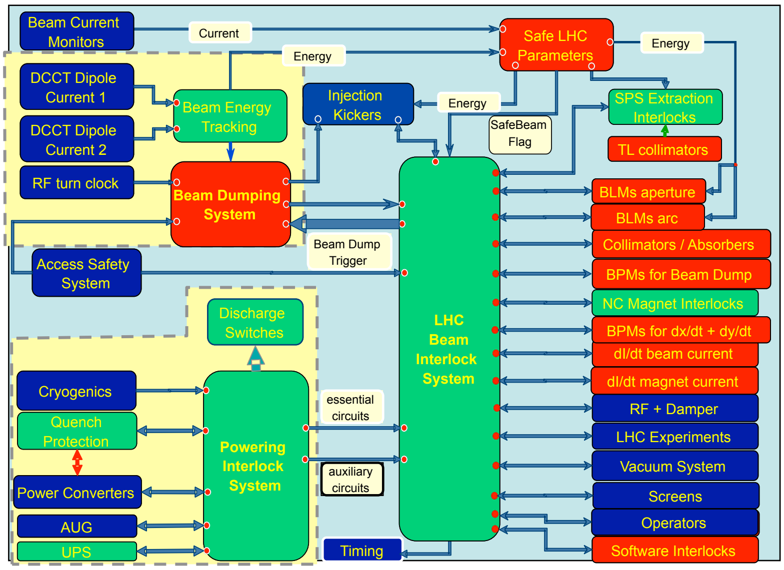
About 500 m



Beam Dump Block (graphite)



LHC Operation: Machine Protection & Safety



Beam Current Monitors

Current

Energy

Safe LHC Parameters

Energy

DCCT Dipole Current 1

DCCT Dipole Current 2

RF turn clock

Beam Energy Tracking

Injection Kickers

Energy

SafeBeam Flag

SPS Extraction Interlocks

TL collimators

Beam Dumping System

Beam Dump Trigger

Access Safety System

Discharge Switches

LHC Beam Interlock System

BLMs aperture

BLMs arc

Collimators / Absorbers

BPMs for Beam Dump

NC Magnet Interlocks

BPMs for dx/dt + dy/dt

dI/dt beam current

dI/dt magnet current

RF + Damper

LHC Experiments

Vacuum System

Screens

Operators

Software Interlocks

Cryogenics

Quench Protection

Power Converters

AUG

UPS

Powering Interlock System

essential circuits

auxiliary circuits

Timing

... no comment

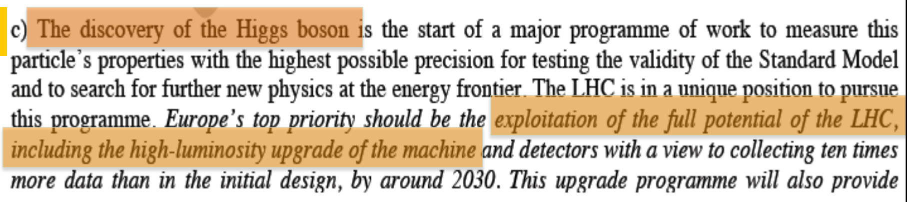
2.) Where do we go ?

- ***Physics beyond the Standard Model***
- ***Dark Matter / Dark Energy***

Future Projects

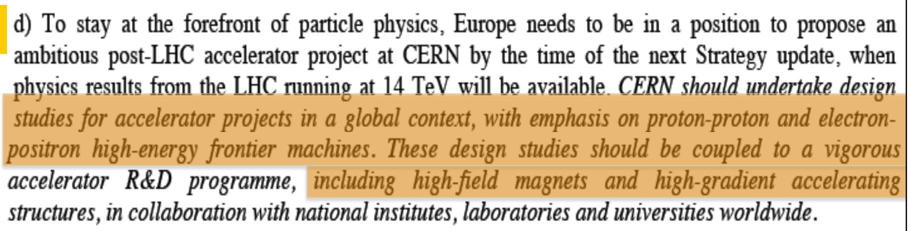
Recommendations from European Strategy Group

#1



c) The discovery of the Higgs boson is the start of a major programme of work to measure this particle's properties with the highest possible precision for testing the validity of the Standard Model and to search for further new physics at the energy frontier. The LHC is in a unique position to pursue this programme. *Europe's top priority should be the exploitation of the full potential of the LHC, including the high-luminosity upgrade of the machine and detectors with a view to collecting ten times more data than in the initial design, by around 2030. This upgrade programme will also provide*

#2



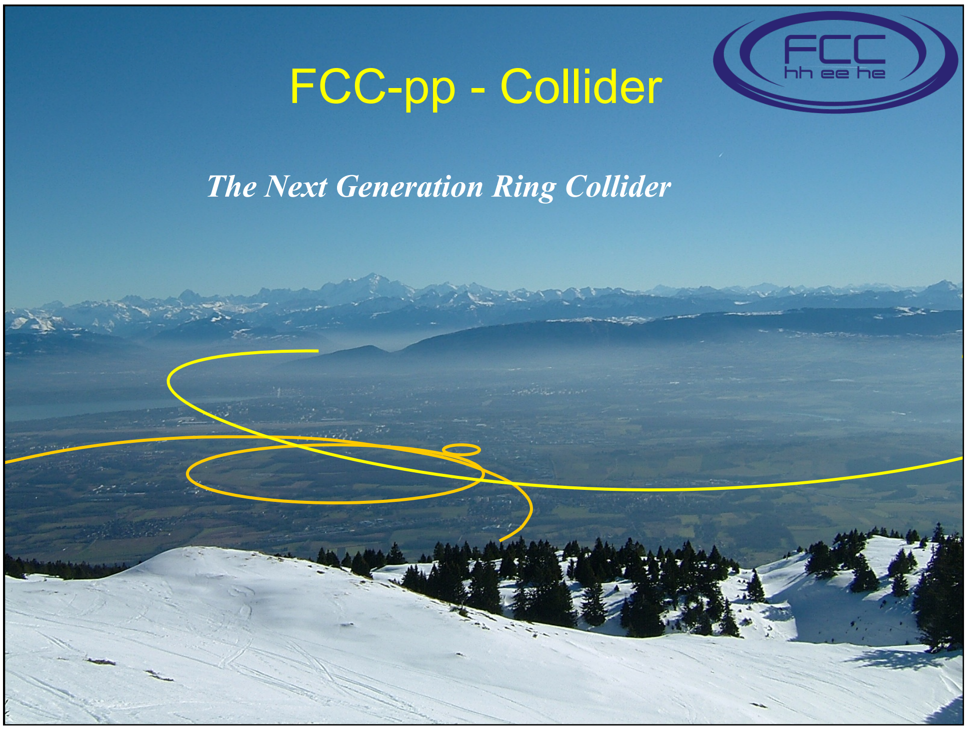
d) To stay at the forefront of particle physics, Europe needs to be in a position to propose an ambitious post-LHC accelerator project at CERN by the time of the next Strategy update, when physics results from the LHC running at 14 TeV will be available. *CERN should undertake design studies for accelerator projects in a global context, with emphasis on proton-proton and electron-positron high-energy frontier machines. These design studies should be coupled to a vigorous accelerator R&D programme, including high-field magnets and high-gradient accelerating structures, in collaboration with national institutes, laboratories and universities worldwide.*

→Proton –Proton Colliders *=> e+/e- colliders*

LHC / HL-LHC, HE-LHC ***TLEP, CLIC***

4.) Push for higher energy: FCC

**** increasing the ring size***
**** stronger magnets***



FCC-pp - Collider

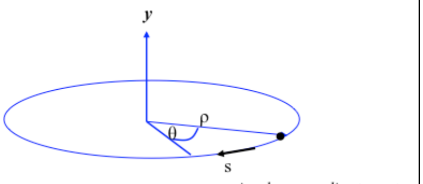
The Next Generation Ring Collider

Maximum Beam Energy in a Storage Ring:

For a given magnet technology it is the size of the machine that defines the maximum particle momentum

... and so the energy

$\cancel{E = mc^2}$ $E^2 = (pc)^2 + m^2c^4$



circular coordinate system

Condition for an ideal circular orbit:

Lorentz force $F_L = e\,v\,B$

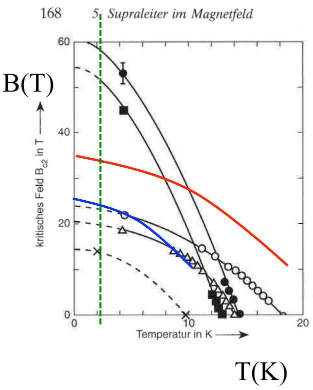
centrifugal force $F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$

$$\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\,\cancel{v}\,B$$

$$\frac{p}{e} = B\,\rho$$

$B\,\rho =$ *"beam rigidity"*

Two key players in sc magnet technology: NbTi and Nb_3Sn



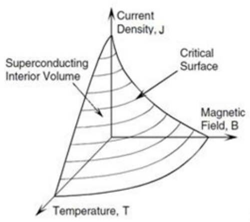
critical field in NbTi and Nb_3Sn

Abb. 77 Oberes kritisches Feld einiger Hochfeldsupraleiter.

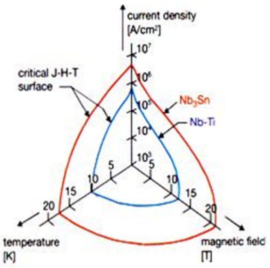
–○–○–○–	Nb_3Sn, Drahtdurchmesser 0,5 mm [127]
–△–△–△–	V_3Ga, Sinterprobe [127]
–×–×–×–	Nb_{50} Ti_{50} [128]
–■–■–■–	$PbMo_{6,35}S_8$ [130]
–●–●–●–	$PbGd_{0,3}Mo_6S_8$ [130]

(siehe auch Ø. Fische
Otaniemi 1975, Band
Publ. Comp. 1975).

General



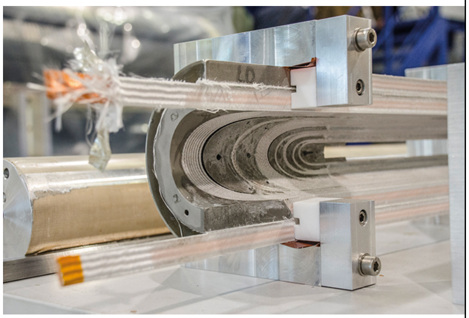
NbTi and Nb_3Sn



The Push for Higher Beam Energy

NbTi LHC standard dipoles,
8.3 T

Nb_3Sn FCC type dipole coils,
11 T – 16 T



it is a simple scaling wrt LHC:

circumference 100km /27km
→ Factor 3.7

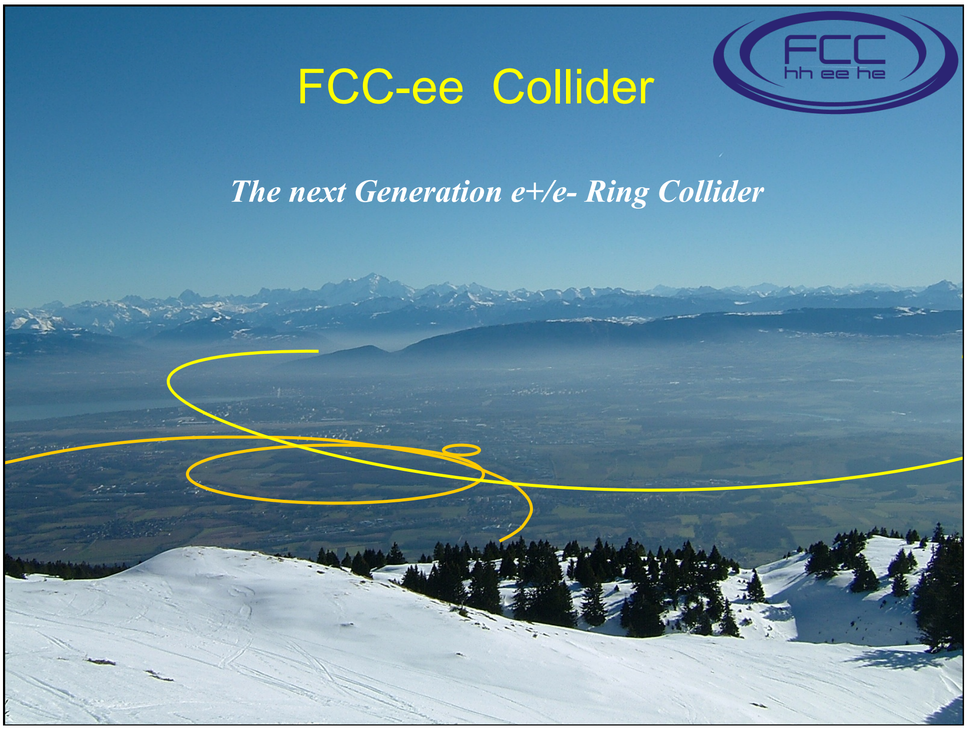
dipole field: 16 T / 8.3 T
→ Factor 1.93

LHC energy E_{cm} = 2 7 TeV * 7.1*

FCC energy E_{cm} = 100 TeV centre of mass

5.) High Energy Lepton Colliders

** Limited by Synchrotron Radiation*
** and RF Power*

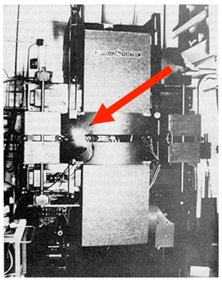


FCC-ee Collider

The next Generation e+/e- Ring Collider

Synchrotron Radiation

In a circular accelerator charged particles lose energy via emission of intense light.



$$P_s = \frac{2}{3}\alpha\hbar c^2 \boxed{\frac{\gamma^4}{\rho^2}}$$ ***radiation power***

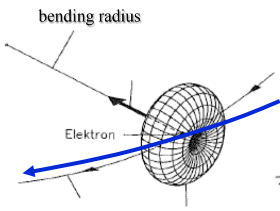
$$\Delta E = \frac{4}{3}\pi\alpha\hbar c \boxed{\frac{\gamma^4}{\rho}}$$ ***energy loss***

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$ ***critical frequency***

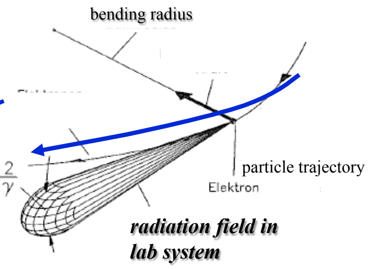
$$\alpha \approx \frac{1}{137}$$

$$\hbar c \approx 197\, MeV\ fm$$

1946 observed for the first time in the General Electric Synchrotron



radiation field in electron rest frame

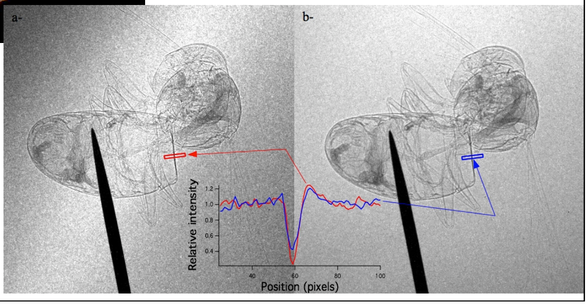


radiation field in lab system

court. K. Wille

Synchrotron Radiation as useful tool

structure analysis with highest resolution Ribosome molecule

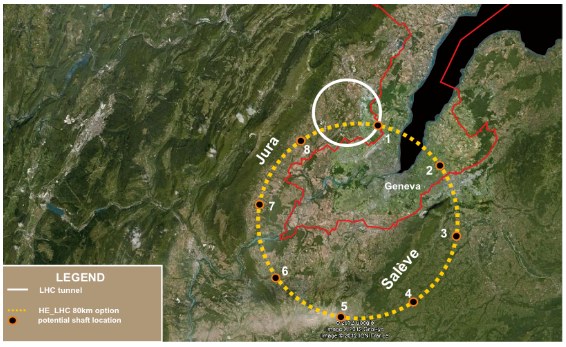


Absorption Line Radiographie

Planning the next generation e+ / e- Ring Colliders

Design Parameters FCC-ee

E = 175 GeV / beam
L = 100 km



$$\Delta U_0(keV) \approx \frac{89 * E^4(GeV)}{\rho}$$

$$\Delta U_0 \approx 8.62\ GeV$$

$$\Delta P_{sy} \approx \frac{\Delta U_0}{T_0} * N_p = \frac{10.4 * 10^6 eV * 1.6 * 10^{-19} Cb}{263 * 10^{-6} s} * 9 * 10^{12}$$

$\Delta P_{sy} \approx 47\ MW$

Circular e+ / e- colliders are severely limited by synchrotron radiation losses and have to be replaced for higher energies by linear accelerators

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6.) Push for higher energy

- ***go linear***
- ***higher acceleration gradients***

Lepton Colliders: Linear / Storage Rings

Avoid bending forces → go linear

Storage Ring: ***dipole magnets***

synchrotron radiation

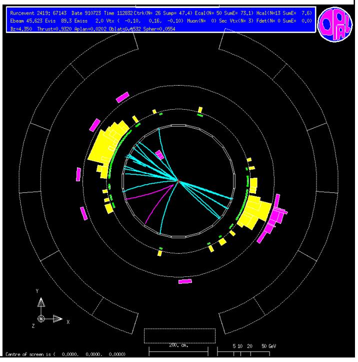
energy loss per turn

$$P_\gamma = \frac{c\,C_\gamma}{2\pi}\frac{E^4}{\rho^2}\ ,\quad C_\gamma = 8.9*10^{-5}\,m/GeV^3$$

high RF power to compensate losses

very efficient,

turn by turn acceleration



Linear Collider: ***no synchr. Radiation***

limited efficiency:

N^{10}-1 particles are lost after the collision

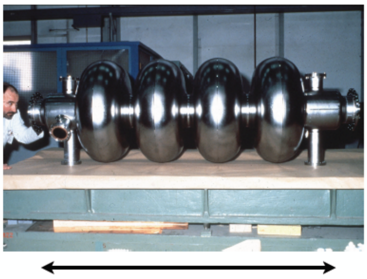
need highest acceleration gradient

"one turn" machines"

lepton colisions are "*clean*"

Plasma Wake Acceleration

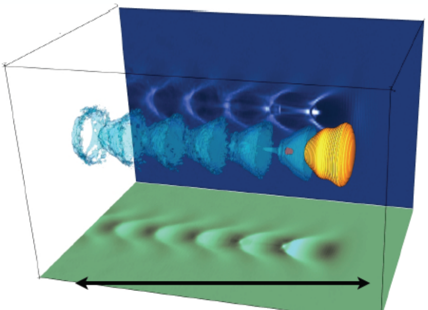
RF Cavity



1 m => 50 MeV Gain

Electric field < 100 MV/m

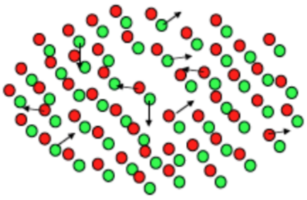
Plasma Cavity



1mm => 100 MeV

Electric field > 100 GV/m

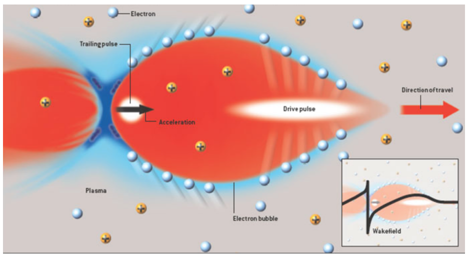
Study of High Gradient Acceleration Techniques



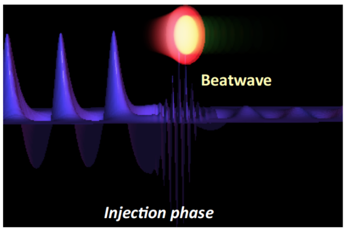
Plasma oscillation frequency:

$$\omega_{\rm pe} = \sqrt{\frac{n_e e^2}{m^* \varepsilon_0}},$$

Intense Laser light creates a plasma beat wave, that separates the electrons from the heavy (and so much slower) ions. A quasi electron free region (bubble) is created and as consequence a large electric field that can be used to accelerate particles.



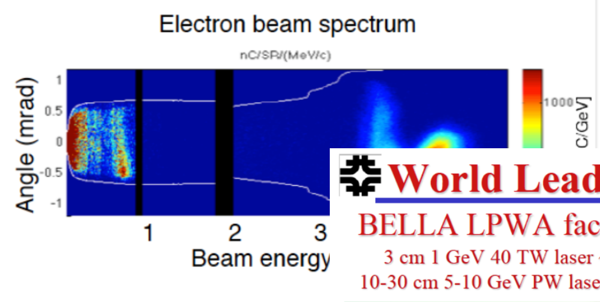
The first laser creates the accelerating structure, a second laser beam is used to heat electrons

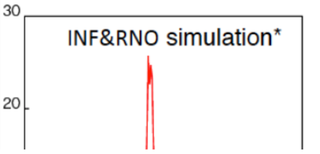


Theory : E. Esarey *et al.*, PRL **79**, 2682 (1997), H. Kotaki *et al.*, PoP **11** (2004)
Experiments : J. Faure *et al.*, Nature **444**, 737 (2006)

4.25 GeV beams have been obtained from 9 cm plasma channel powered by 310 TW laser pulses (15 J)

*C. Benedetti et al., proceedings of AAC2010, proceedings of ICAP2012

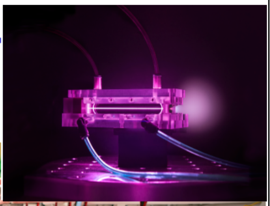


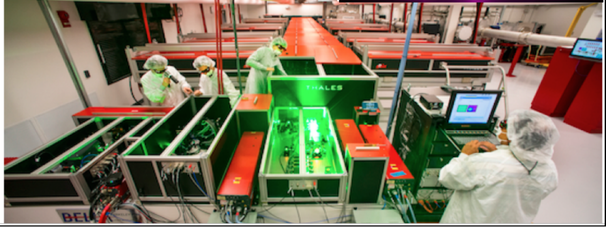


World Leader

BELLA LPWA facility:

3 cm 1 GeV 40 TW laser ~1Hz
10-30 cm 5-10 GeV PW laser, ~1 Hz





There is no such thing as a free lunch !!
High power Lasers are not small and not efficient.

→ ***work in progress***

court. M. Ferrario, CAS 2015, Warsaw

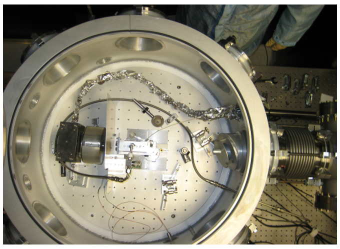
Study of High Gradient Acceleration Techniques

Plasma Wake Acceleration
particle beam driven / LASER driven

Incoming laser pulse (or pulse of particles) creates a travelling plasma wave in a low-pressure gas

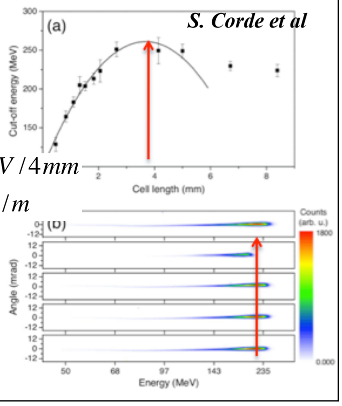
Plasma wake field gradient accelerates electrons that 'surf' on the plasma wave

Field Gradients up to 100 GeV /m observed



Plasma cell Univ. Texas, Austin
E_e =2 GeV

$$\Delta E/\Delta s = 200\,MeV/4mm$$
$$= 50\,GeV/m$$

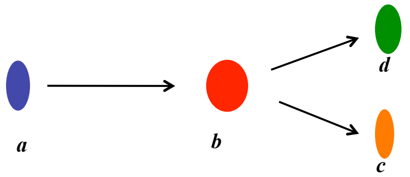


S. Corde et al

Fixed Target Machines
The (Problem of the) Centre of Mass Energy

Fixed Target experiments

accelerated particle beam hits a target at rest $a + b \rightarrow c + d$



Lab system: $p_b^{lab} = 0,\ E_b^{lab} = m_b c^2$

Centre of mass system: $p_b^{cm} + p_b^{cm} = 0$

relativistic total energy $E^2 = p^2c^2 + (mc^2)^2$

and for a single particle as well as for system of particles the overall rest energy is constant

... invariance of the 4momentum scalar product

$$\sum_i E_i^2 - \sum_i p_i^2 c^2 = \left(Mc^2\right)^2 = const$$

$$\left(E_a^{cm} + E_b^{cm}\right)^2 - \left(p_a^{cm} + p_b^{cm}\right)^2 c^2 = \left(E_a^{lab} + E_b^{lab}\right)^2 - \left(p_a^{lab} + p_b^{lab}\right)^2 c^2$$

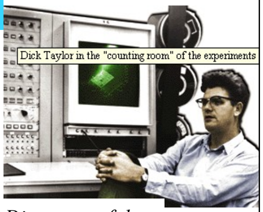
The (Problem of the) Centre of Mass Energy

Fixed Target experiments:

$$\left(E_a^{cm}+E_b^{cm}\right)^2-\left(\underbrace{p_a^{cm}+p_b^{cm}}_{=0}\right)^2c^2=\left(E_a^{lab}+E_b^{lab}\right)^2-\left(\underbrace{p_a^{lab}+p_b^{lab}}_{=p_a^{lab}}\right)^2c^2$$

$$W^2=\left(E_a^{cm}+E_b^{cm}\right)^2=\left(E_a^{lab}+m_bc^2\right)^2-\left(p_a^{lab}c\right)^2$$

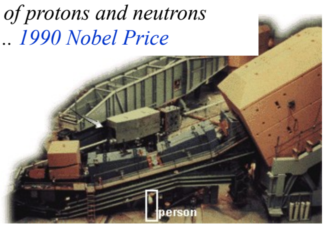
$$=2E_a^{lab}m_bc^2+\left(m_a^2+m_b^2\right)c^4$$



for $E_a^{lab}>>m_ac^2,\ m_bc^2$

$$\text{=>}\quad W\approx\sqrt{2E_a^{lab}m_bc^2}$$

Taylor/Kendall/Friedman: Discovery of the quark structure of protons and neutrons 1966-1978 ..... 1990 Nobel Price



For high energies in the centre of mass system,
fixed target machines are not effective.
... → need for colliding beams

→ go for particle colliders

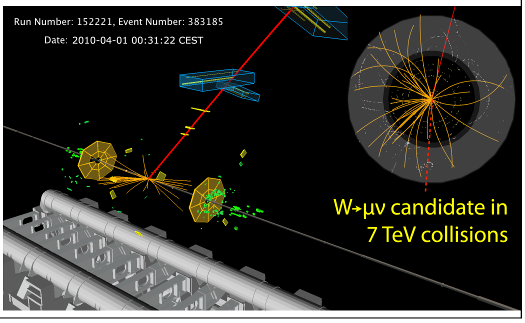
The (Problem of the) Centre of Mass Energy

Colliding Beams experiments:

$$\left(E_a^{cm}+E_b^{cm}\right)^2-\left(\underbrace{p_a^{cm}+p_b^{cm}}_{=0}\right)^2c^2=\left(E_a^{lab}+E_b^{lab}\right)^2-\left(\underbrace{p_a^{lab}+p_b^{lab}}_{p_a^{lab}=-p_b^{lab}=0}\right)^2c^2$$

$$W^2=\left(E_a^{cm}+E_b^{cm}\right)^2$$

$$\text{=>}\quad W=2E_a^{lab}$$



The full lab energy is available
in the center of mass system.
Prize to pay: we have to build colliders
... beam sizes = μm