Precision predictions for Higgs production at the LHC

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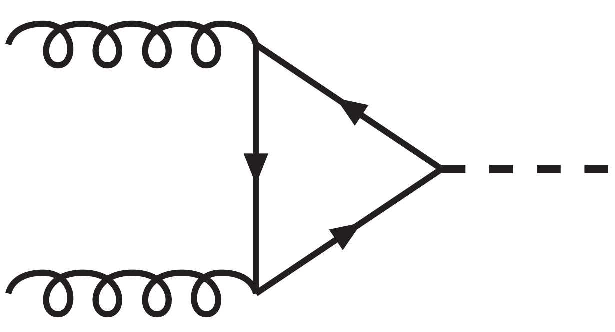
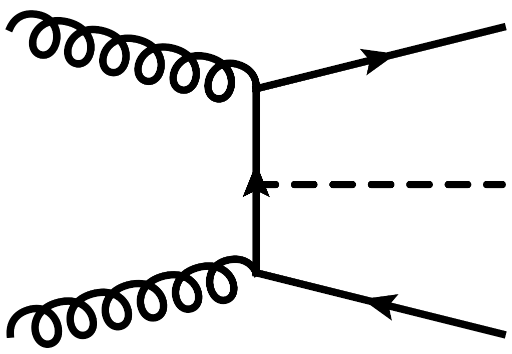
Higgs production at the LHC

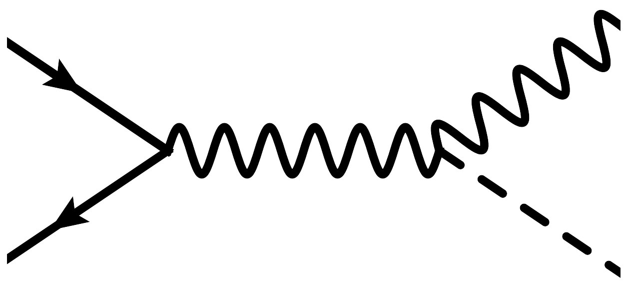
- Establishing whether the BEH mechanism and its boson is SM-like will be of outmost importance for the run of the LHC.
- Higgs-boson production modes at the LHC:



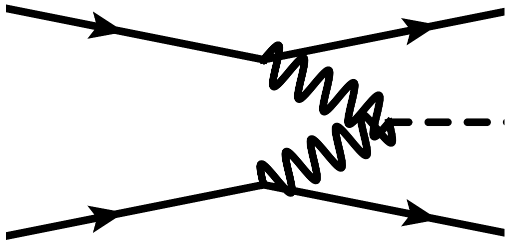
Gluon fusion



TTH



Higgs strahlung



VBF

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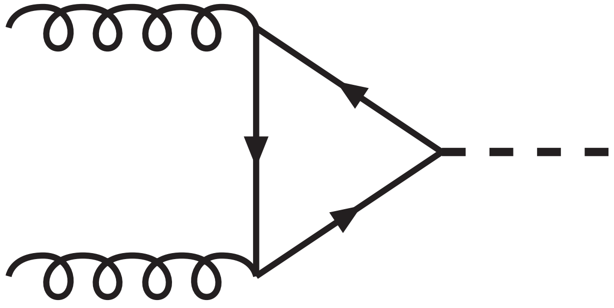


Gluon fusion

TTH

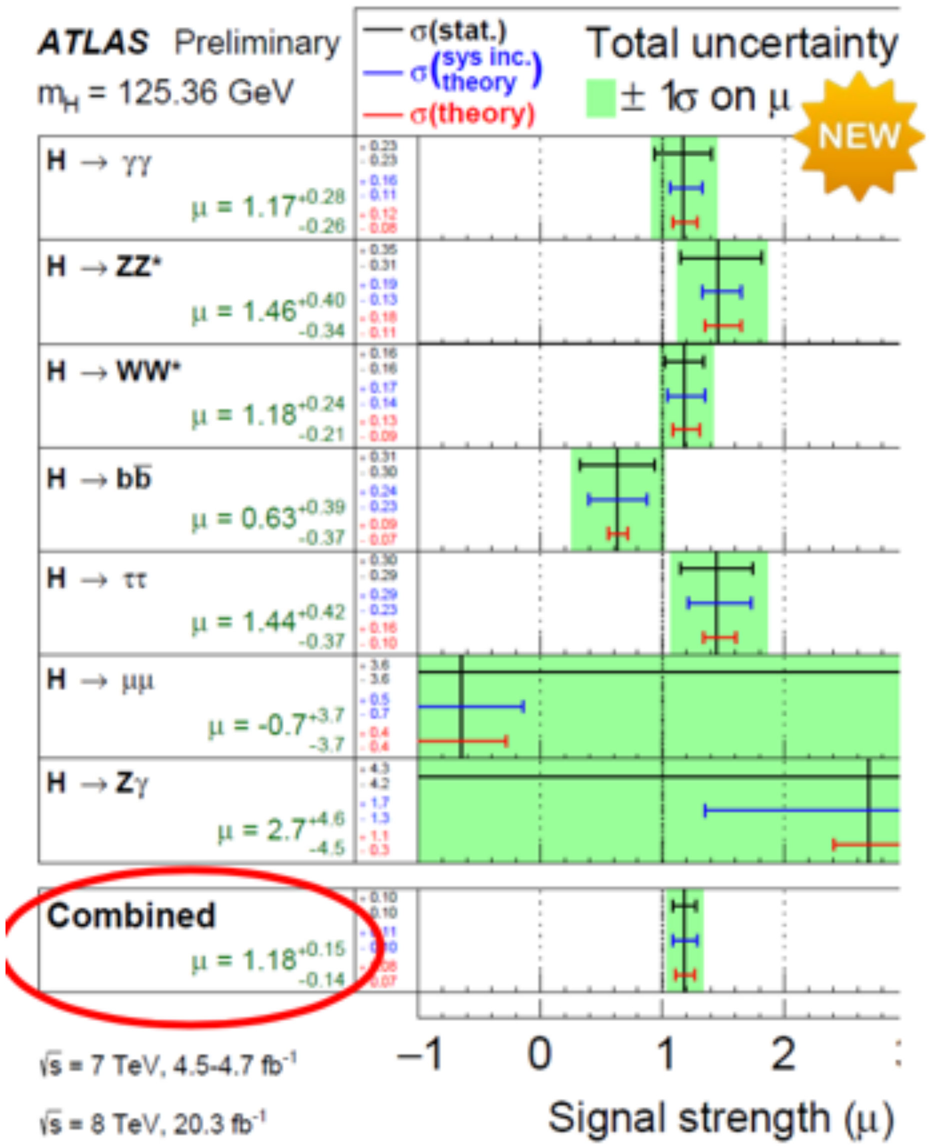
Higgs strahlung

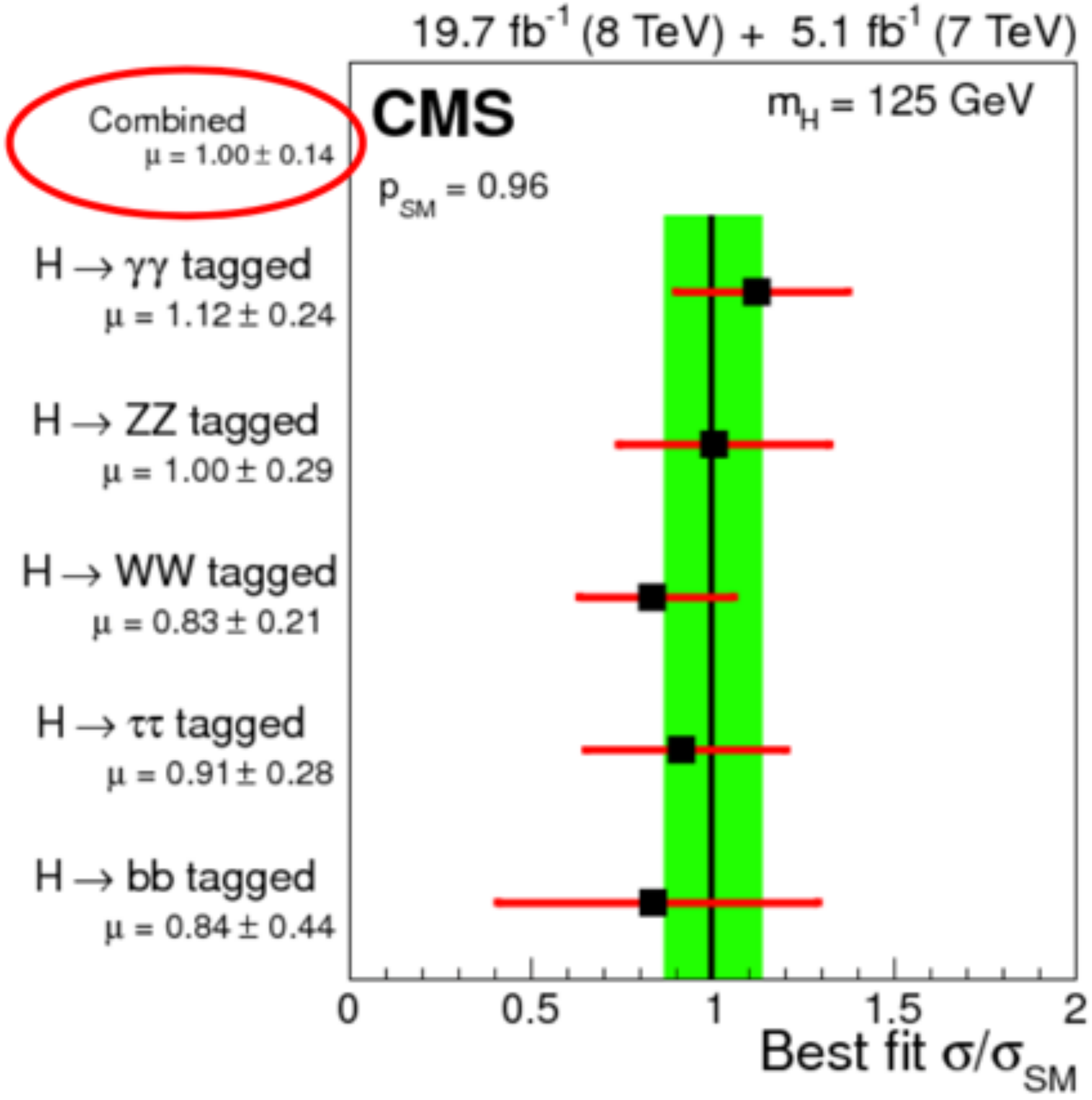
VBF

Dominant production mode at the LHC

- We want to know the gluon-fusion cross section precisely!

Higgs production at the LHC





$$\mu_{ATLAS} = 1.18^{+0.15}_{-0.14}$$

$$\mu_{CMS} = 1.00 \pm 0.14$$

$$\text{stat.} = {}^{+0.10}_{-0.10}$$

$$\text{sys. (inc. theo.)} = {}^{+0.11}_{-0.10}$$

$$\text{theory} = {}^{+0.08}_{-0.07}$$

[M. Dührssen @ Moriond EW 2015]

Higgs production at the LHC

- Aim: Combine recent computation of N3LO cross section in the large mt limit with other known effects.
    - ➡ finite quark-mass effects.
    - ➡ state-of-the-art PDFs.
    - ➡ electroweak corrections.
    - ➡ resummation of threshold logarithms.
- Assess residual uncertainty on the cross section at the LHC:
    - ➡ Scale, PDF, aS.
    - ➡ Truncation of threshold series.
    - ➡ Missing higher orders (N4LO and beyond).
    - ➡ Missing N3LO PDFs.
    - ➡ Scheme for quark masses + parametric uncertainties.

Outline

- The N3LO cross section in the large mt limit:
  - ➡ Convergence of the threshold expansion.
  - ➡ Scale variation.
  - ➡ Effects beyond N4LO.
- Other corrections:
  - ➡ Quark mass effects.
  - ➡ Electroweak corrections.
- PDF uncertaintites.
- Final prediction for the cross section

The N3LO cross section in the large mt limit

Scale variation
&
higher orders in QCD

The large mt limit

- In the limit $m_t \to \infty$, the Higgs boson couples directly to gluons:

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1\, H\, G^a_{\mu\nu}\, G_a^{\mu\nu}$$

- In this limit, the cross section is known
  - ➡ at NLO. [Dawson; Djouadi, Spira, Zerwas]
  - ➡ at NNLO. [Anastasiou, Melnikov; Harlander, Kilgore; Ravindran, Smith, van Neerven]
  - ➡ at N3LO. [Anastasiou, Dulat, CD, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger]

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  - ➡ at N3LO. [Anastasiou, Dulat, CD, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger]

- The N3LO cross section is only known as an expansion around threshold:

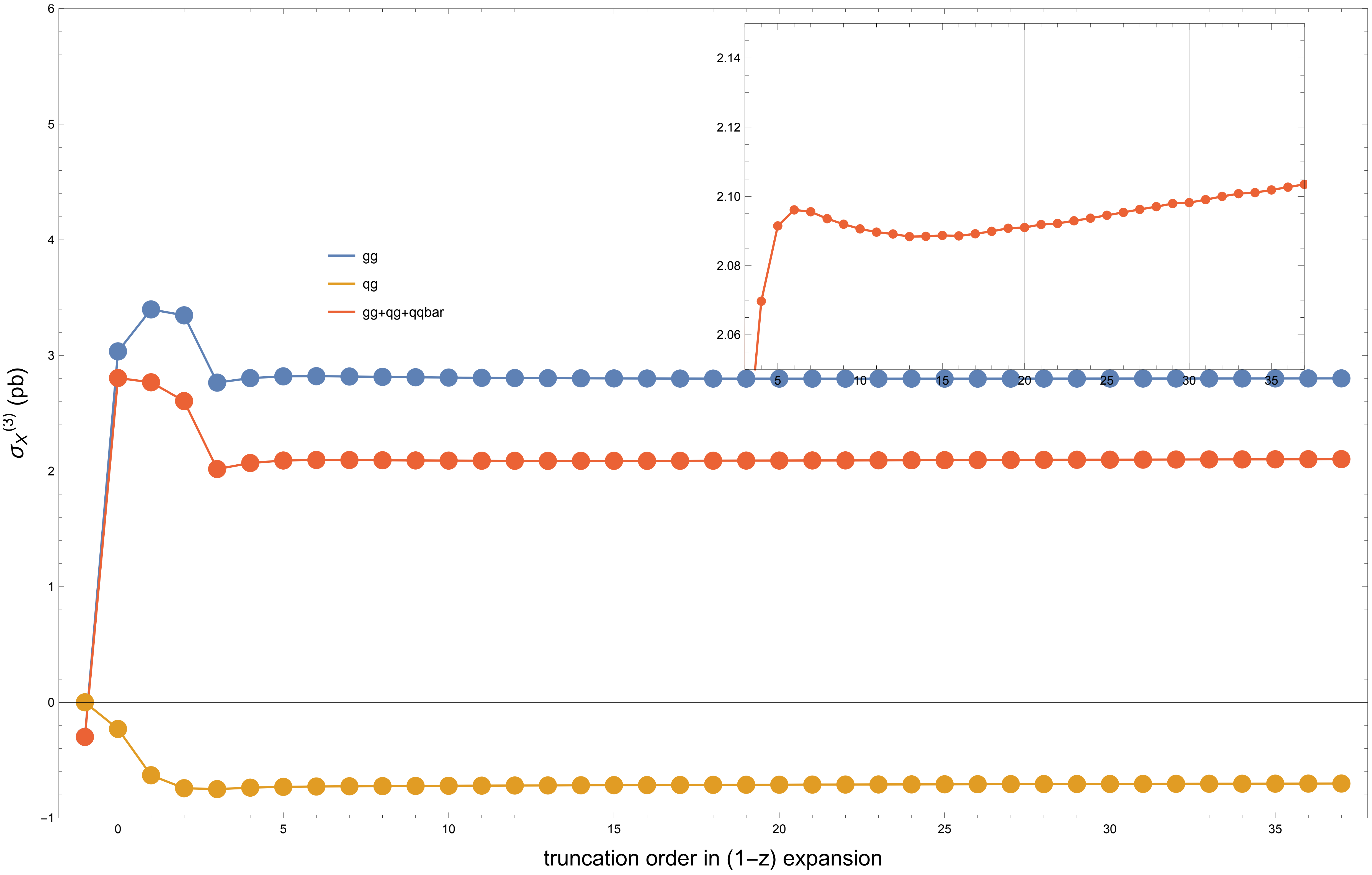
$$\sigma = \tau \sum_{ij} \int_\tau^1 \frac{dz}{z}\, \mathcal{L}_{ij}(\tau/z)\, \frac{\hat{\sigma}_{ij}(z)}{z}$$

$$z = \frac{m_H^2}{\hat{s}}$$

$$\hat{\sigma}(z) = \sigma_{-1} + \sigma_0 + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2$$

$$\tau = \frac{m_H^2}{S} \simeq 10^{-4}$$

Truncation uncertainty



Truncation uncertainty

- We estimate the uncertainty from the truncation of the series as:

Conservative factor

$$\boxed{5} \times \frac{\sigma^{(3)}_{EFT}(30) - \sigma^{(3)}_{EFT}(20)}{\sigma^{\mathrm{N^3LO}}_{EFT}} = 0.25\%$$

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- This is consistent with known exact results for logarithms:

$$\sigma^{(3)}_{EFT} = f_0(z) + f_1(z)\,\log(1-z) + f_2(z)\,\log^2(1-z) + f_3(z)\,\log^3(1-z)$$

$$+ f_4(z)\,\log^4(1-z) + f_5(z)\,\log^5(1-z)$$

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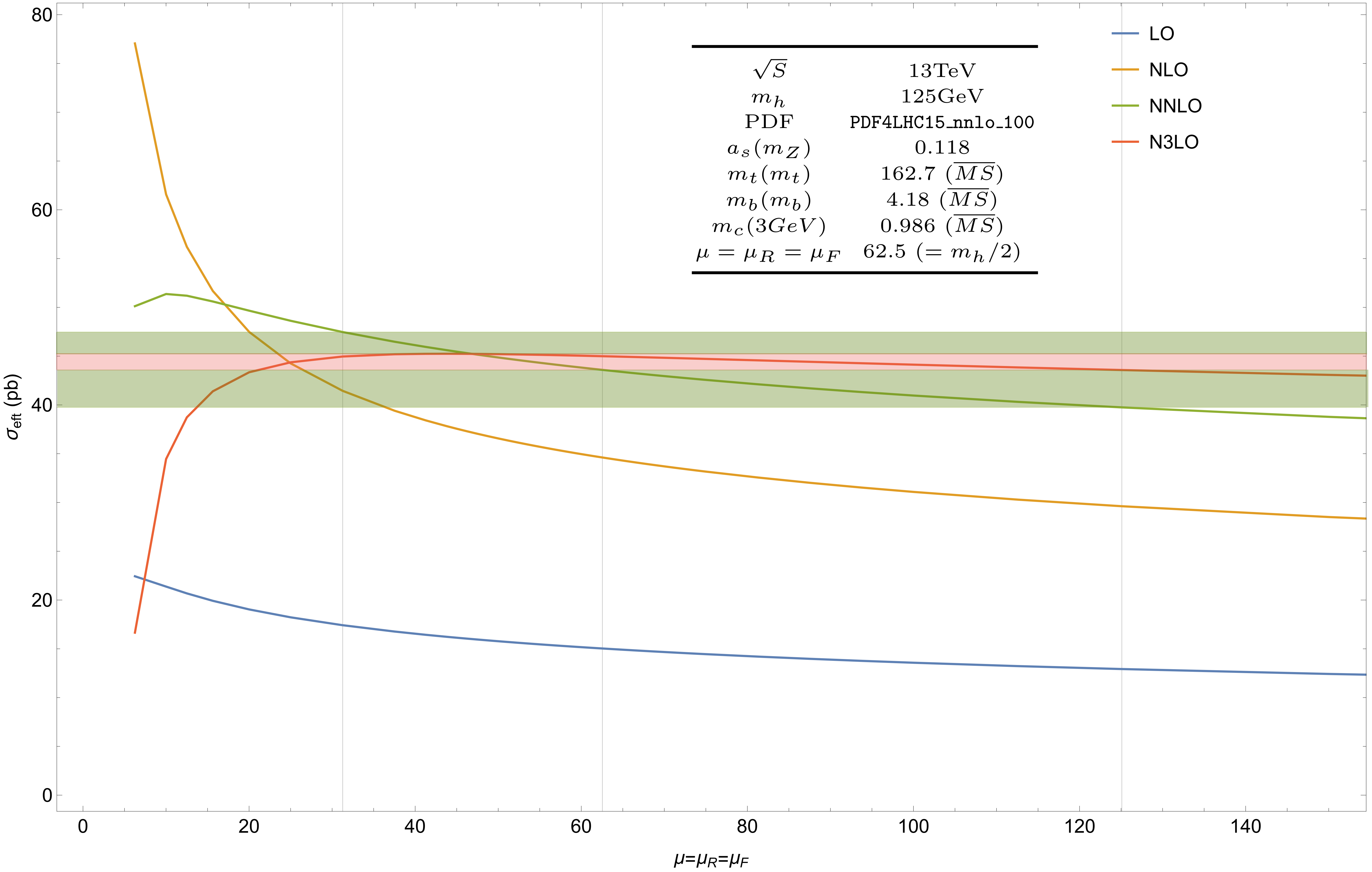
Known exactly

$$\sigma^{(3)}_{EFT}\Big|_{\text{expansion}} - \sigma^{(3)}_{EFT}\Big|_{\text{full logs}} = 0.006\,\text{pb}$$

- The threshold expansion gives a reliable result for the N3LO cross section!

Scale variation



$\sqrt{S}$	13TeV
m_h	125GeV
PDF	PDF4LHC15_nnlo_100
$a_s(m_Z)$	0.118
$m_t(m_t)$	162.7 $(\overline{MS})$
$m_b(m_b)$	4.18 $(\overline{MS})$
$m_c(3GeV)$	0.986 $(\overline{MS})$
$\mu = \mu_R = \mu_F$	62.5 $(= m_h/2)$

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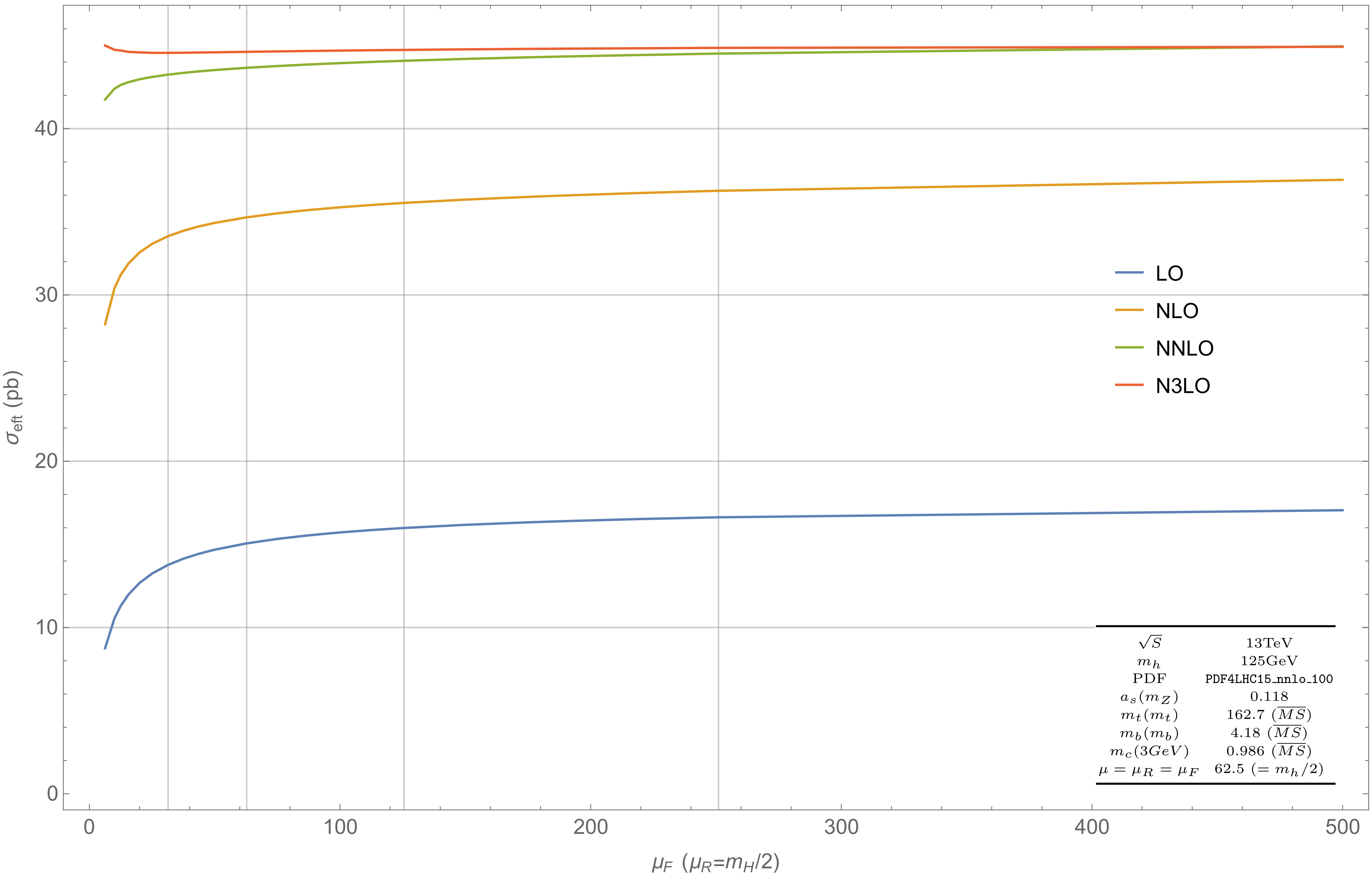

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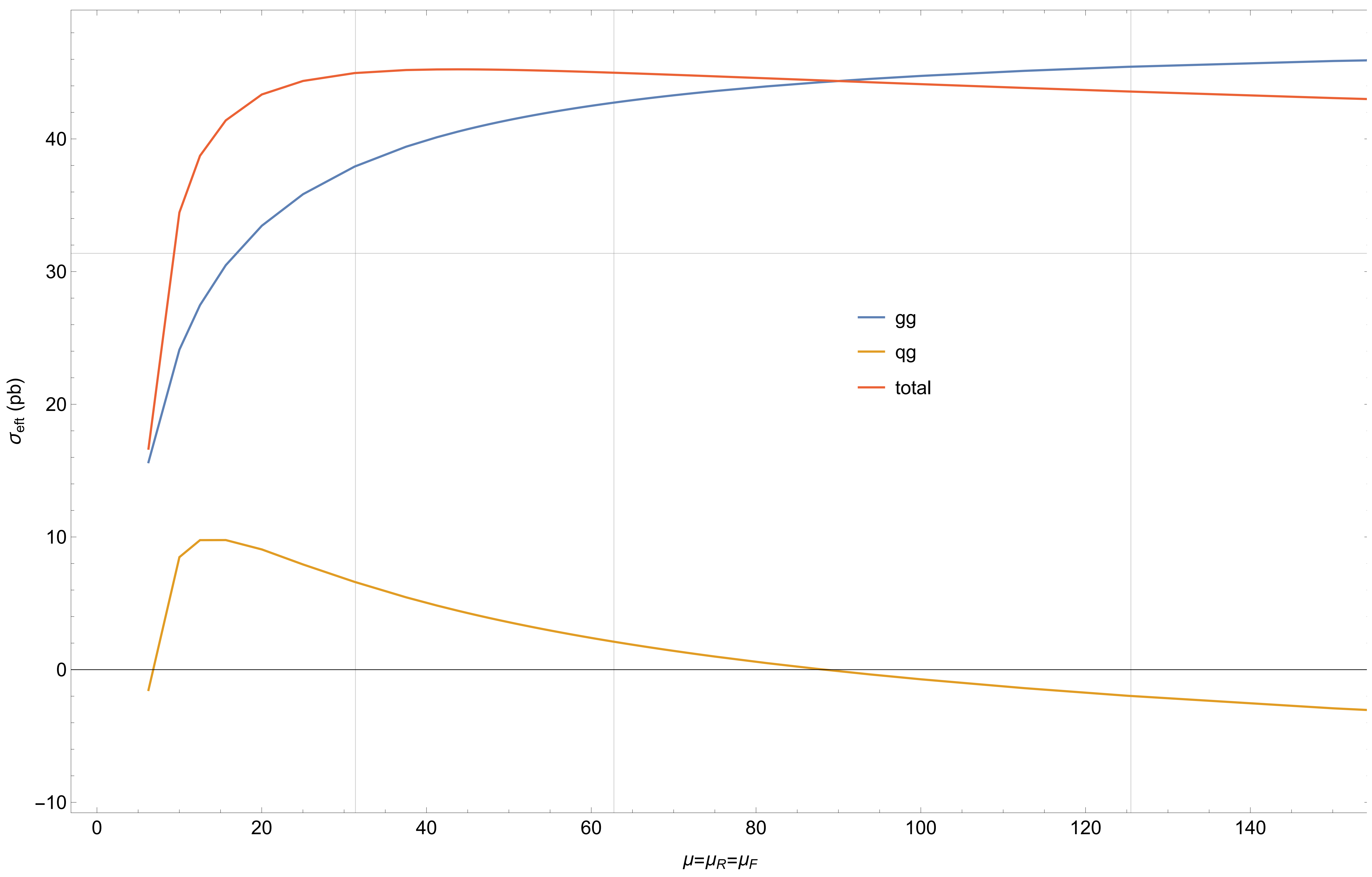

Scale variation

- Scale variation at N3LO almost entirely due to renormalisation scale.
- Scale uncertainty $\mu \in [m_H/4, m_H]$ per order:

$$\Delta^{\text{scale}}_{EFT,k} = \pm \frac{\sigma^{\text{max}}_{EFT,k} - \sigma^{\text{min}}_{EFT,k}}{\sigma^{\text{max}}_{EFT,k} + \sigma^{\text{min}}_{EFT,k}} \, 100\%$$

		$\Delta^{\text{scale}}_{EFT,k}$
LO	$(k=0)$	$\pm 22.0\%$
NLO	$(k=1)$	$\pm 19.2\%$
NNLO	$(k=2)$	$\pm 9.5\%$
N^3LO	$(k=3)$	$\pm 1.9\%$

- Important question: Is scale variation a reliable estimator of missing higher-order corrections?

➡ We know that it is not at low orders!

Missing higher orders

- We estimate the effect of missing higher orders in different ways.
  - ➡ Factorisation of the Wilson coefficient.
  - ➡ Threshold resummation in Mellin space (using different prescriptions).
  - ➡ Threshold resummation in SCET (including π^2 resummation).

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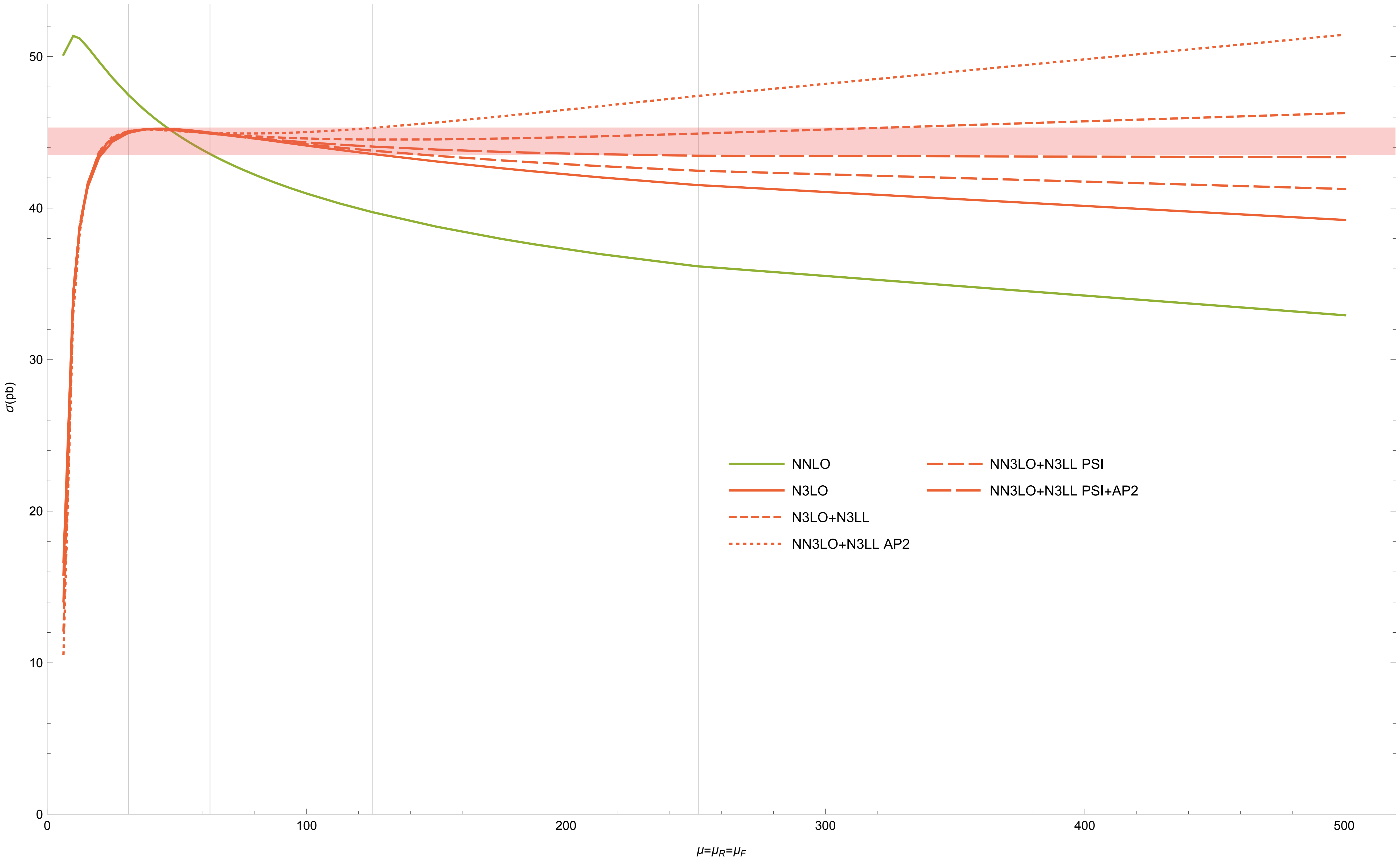
- In all cases, we find that the higher-order effects are captured by the scale variation with $\mu \in [m_H/4, m_H]$.
  - ➡ Scale variation is a reliable estimator of missing QCD corrections for scales $\mu \in [m_H/4, m_H]$.
  - ➡ Outside this interval different prescriptions may differ wildly.

Threshold resummation



Other corrections

Quark masses
&
Electroweak corrections

Quark-mass effects

- At LO and NLO, we know the exact result including all quark mass effects.
  ➡ EFT works well if rescaled by the LO ratio.

$$R_{LO} \equiv \frac{\sigma^{LO}_{ex;t}}{\sigma^{LO}_{EFT}}$$

σ^{LO}_{EFT}	15.05	σ^{NLO}_{EFT}	34.66
$R_{LO}\,\sigma^{LO}_{EFT}$	16.00	$R_{LO}\,\sigma^{NLO}_{EFT}$	36.84
$\sigma^{LO}_{ex.;t}$	16.00	$\sigma^{NLO}_{ex;t}$	36.60
$\sigma^{LO}_{ex.;t+b}$	14.94	$\sigma^{NLO}_{ex;t+b}$	34.96
$\sigma^{LO}_{ex.;t+b+c}$	14.83	$\sigma^{NLO}_{ex;t+b+c}$	34.77

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- At NNLO, we do not know any quark mass effects exactly.
  ➡ Can include top-mass corrections as 1/mt expansion.
  [Harlander, Mantler, Marzani, Ozeren]
  ➡ We do not know t-b interference at NNLO.

$$\Delta^{\rm NNLO}_{rEFT}\,\frac{\Delta^{\rm NLO}_{t+b} - \Delta^{\rm NLO}_{t}}{\Delta^{\rm NLO}_{t}} \simeq \pm 0.38 \rm pb$$

Electroweak corrections

- Exact NLO EW corrections are known. [Actis, Passarino, Sturm, Uccirati]
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➡ Modified Wilson coefficient.

$$C_{QCD} \to C_{QCD} + \underbrace{\lambda_{EW}}_{\text{NLO EW}}(1 + \underbrace{C_{1w} a_s}_{\text{EW-QCD in EFT approach}} + C_{2w} a_s^2 + \ldots)$$

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  - ➡ Numerical impact is similar to 'complete factorisation' for EW corrections, ~5.1%.
- We estimate the uncertainty of this approach by varying the mixed QCD-EW Wilson coefficient,

$$\lambda_{EW}(1 + C_{1w}a_s + \ldots) \to \lambda_{EW}(1 + y_\lambda \cdot C_{1w}a_s + \ldots)$$

  - ➡ Cross section varies by +0.4% / -0.2% if y_λ is varied by a factor 3.

PDF + aS uncertainties

PDF + aS uncertainty

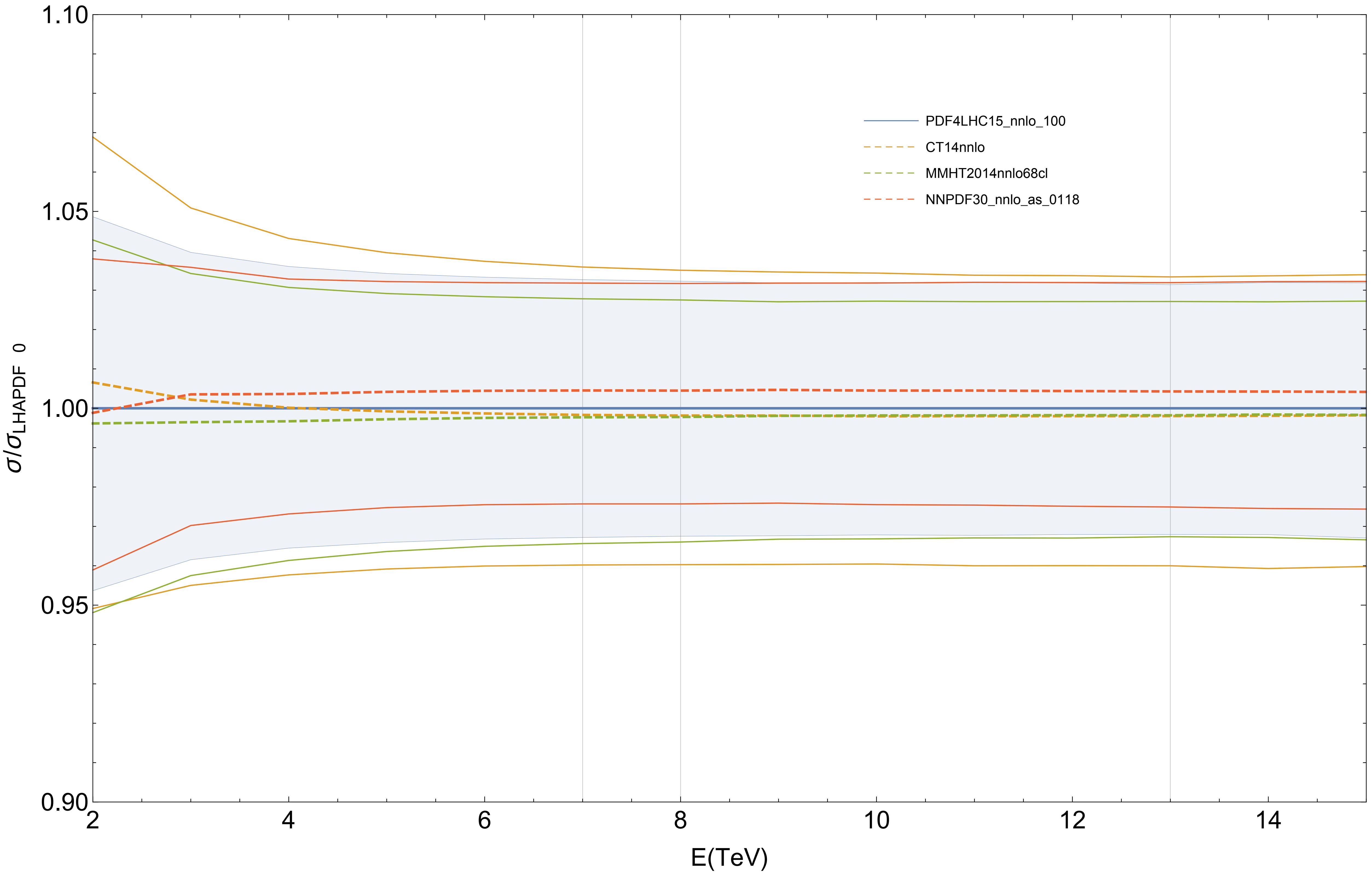
- We follow the PDF4LHC recommendation:
  - ➡ PDF and α_s error are added in quadrature

$$\delta_{\mathrm{PDF}+\alpha_s} = \sqrt{\delta^2_{\mathrm{PDF}} + \delta^2_{\alpha_s}}$$

$$\delta_{\alpha_s} = \frac{\sigma(\alpha_s = 0.1195) - \sigma(\alpha_s = 0.1165)}{2}$$

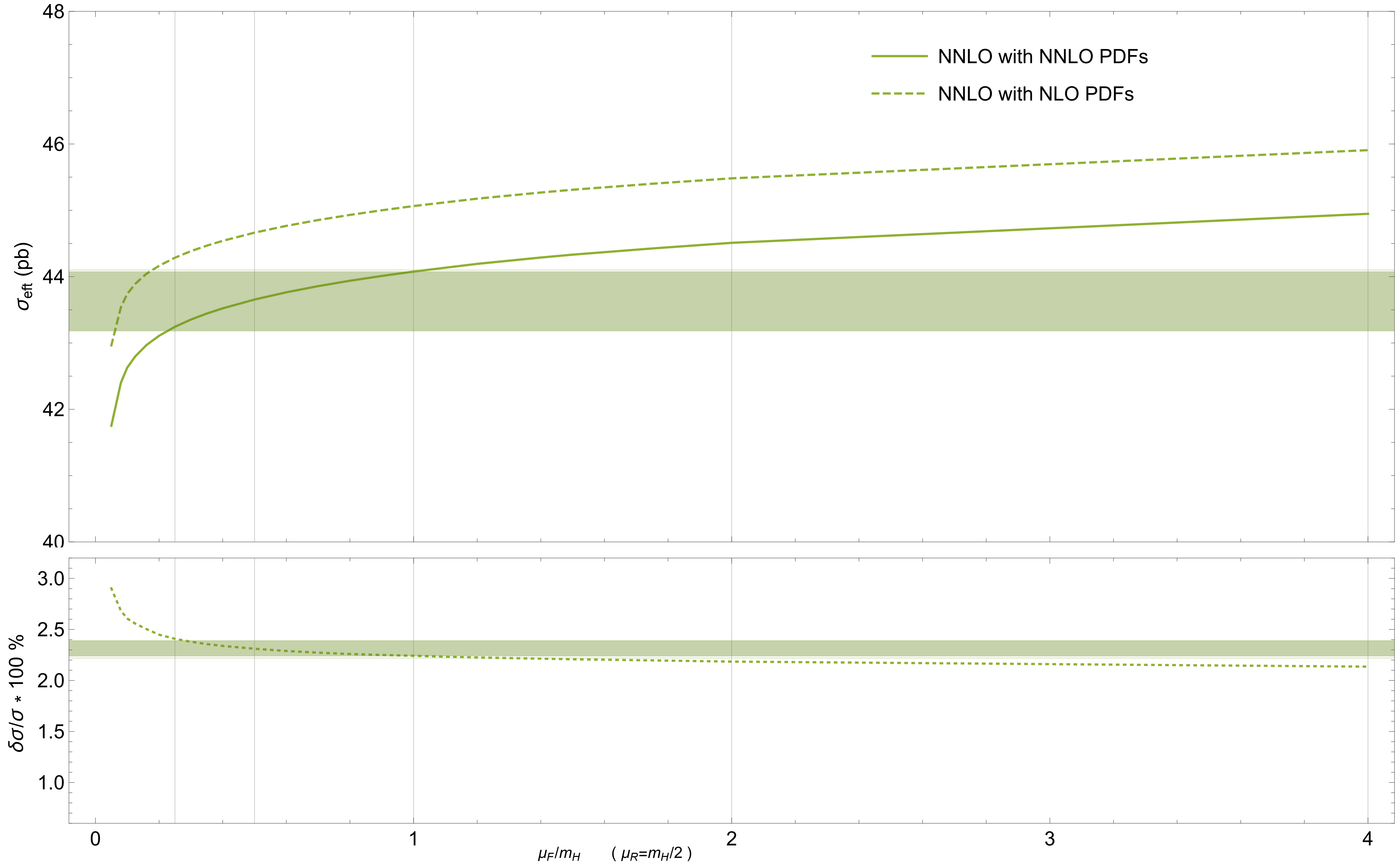
  - ➡ δ_{PDF} obtained by using Hessian method.
- We also studied other PDF sets (ABM, HERAPDF).
  - ➡ At LHC energies, HERAPDF in good agreement with PDF4LHC, while ABM 7-9% lower.

PDF + aS uncertainty



Missing N3LO PDFs

- All our predictions were made using NNLO PDFs.



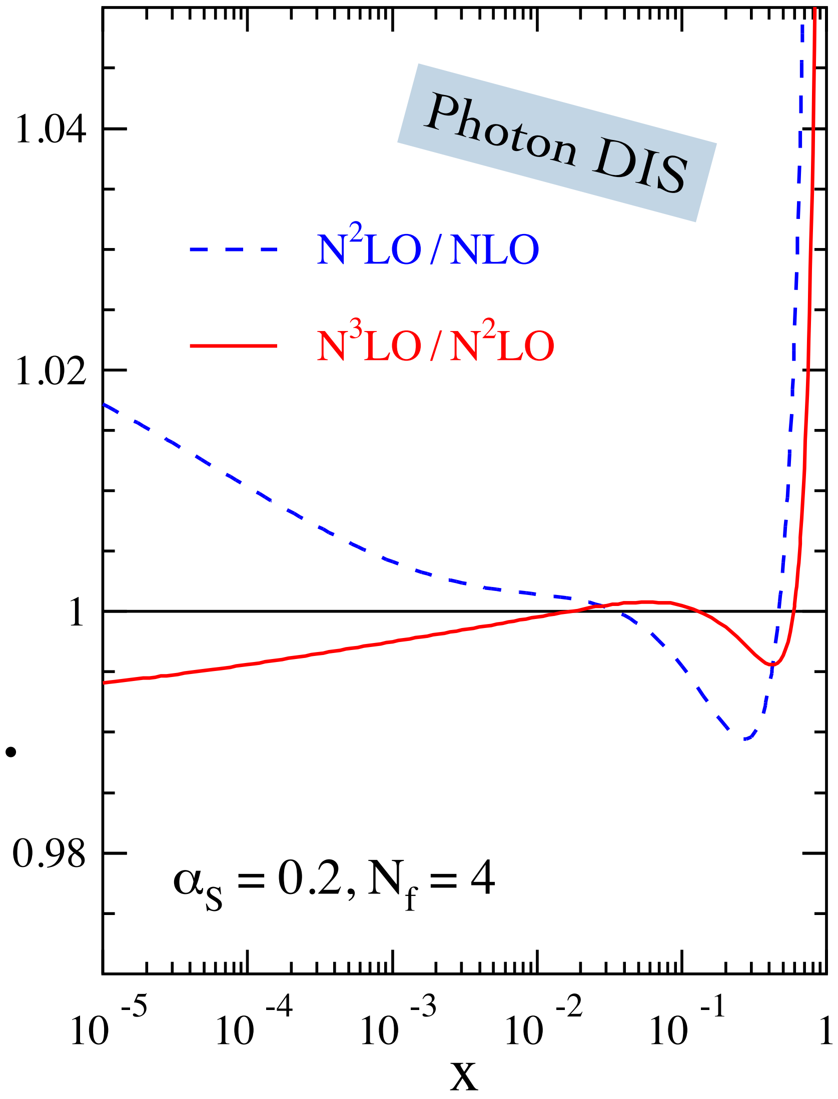
Missing N3LO PDFs

- Using NLO PDFs at NNLO results in a 2-2.5% error at NNLO.
- From this, we estimate the uncertainty of using NNLO PDFs at N3LO

$$\pm\frac{1}{2}\,\frac{\sigma^{\rm NNLO}_{\rm NNLO\ PDF}-\sigma^{\rm NNLO}_{\rm NLO\ PDF}}{\sigma^{\rm NNLO}_{\rm NLO\ PDF}}\,\sigma^{\rm N^3LO}_{\rm NNLO\ PDF}$$

$$\simeq \pm 0.55\,{\rm pb} \simeq \pm 1.15\%$$

- The factor 1/2 takes into account that this estimate is most likely overly conservative.
  - ➡ cf. convergence pattern of DIS.



[Moch, Vermaseren, Vogt]

Summary

$\sigma[\text{pb}]$	δ_{PDF}	δ_{α_s}	δ_{scale}	δ_{trunc}	$\delta_{\text{PDF-TH}}$	δ_{EW}	δ_{tb}	δ_{1/m_t}
48.48	±0.90pb	±1.26pb	$^{+0.09}_{-1.11}$ pb	±0.12	±0.56	±0.48	±0.34	±0.48
	±1.86%	±2.60%	$^{+0.2}_{-2.3}$ %	±0.25%	±1.15%	±1.00%	±0.70%	±1.00%

- Scale choice $\mu_F = \mu_R \in [m_H/4, m_H]$
- δ_{PDF} and δ_{α_s} are computed using the PDF4LHC recommendation.
- We have also considered parametric uncertainties on quark masses, and change of renormalisation scheme.
  - ➡ Negligible.
- We do not include threshold resummation effects.
  - ➡ Captured in N3LO scale variation.
- Combination of errors:
  - ➡ PDF and aS in quadrature.
  - ➡ the rest is added linearly.

Summary

$$\sigma = 48.48 \pm 1.55\,^{+2.07}_{-3.09}\,\text{pb} = 48.48\,\text{pb} \pm 3.19\%\,^{+4.27\%}_{-6.37\%}$$

- Most precise prediction of the Higgs cross section to date!
- Perturbative stability of the cross section under control.
  - ➡ Scale variation gives a reliable estimate of higher-order QCD corrections.
- Places where we can improve:
  - ➡ top-bottom interference at NNLO in QCD.
  - ➡ N3LO PDFs.
  - ➡ Exact mixed QCD-EW corrections.
  - ➡ NNLO corrections including exact top-mass dependence.