# Lattice Higgs physics: the b quark mass

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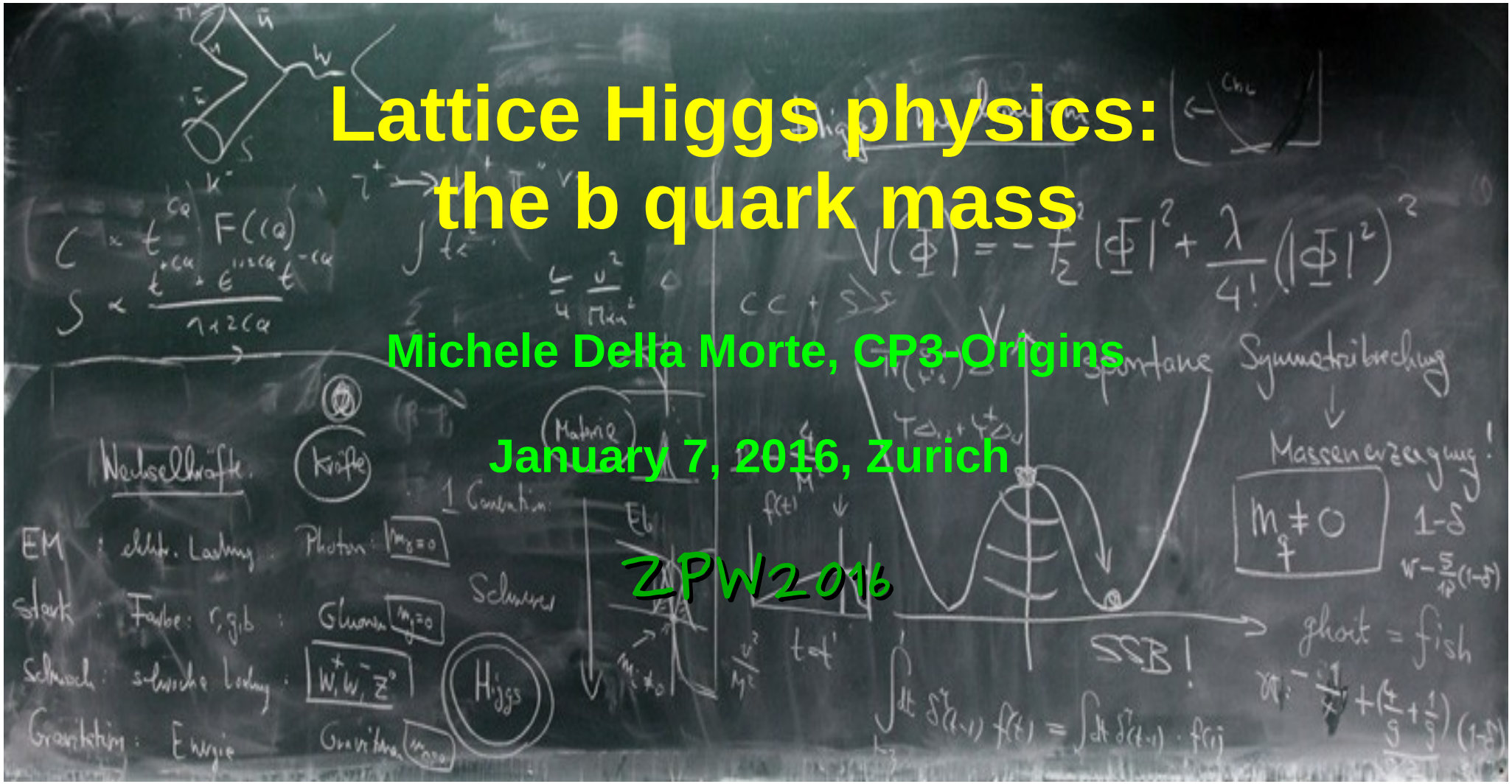
January 7, 2016, Zurich

ZPW2016




CP3 Origins
Cosmology & Particle Physics

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# Outline

- Higgs physics
- b quark mass: lattice QCD input to Higgs physics
- (Non-perturbative) HQET on the lattice: a retrospective
  - Signal to noise problem
  - Non-perturbative matching between QCD and HQET, why and how
- The b-quark mass, results and review

# SM Very Predictive

- Very precise predictions
  - Couplings to fermions proportional to mass $\frac{m_f}{v} H \bar{f} f$
  - Couplings to massive gauge bosons proportional to (mass)$^2$

$$2m_W^2 \frac{H}{v} W_\mu^+ W^{-\mu} + m_Z^2 \frac{H}{v} Z_\mu Z^\mu$$

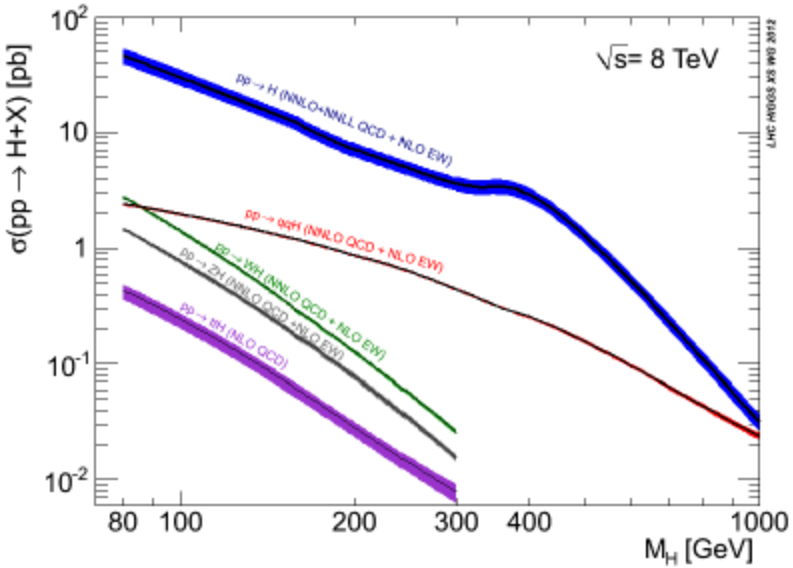
  - Couplings to massless gauge bosons at 1-loop

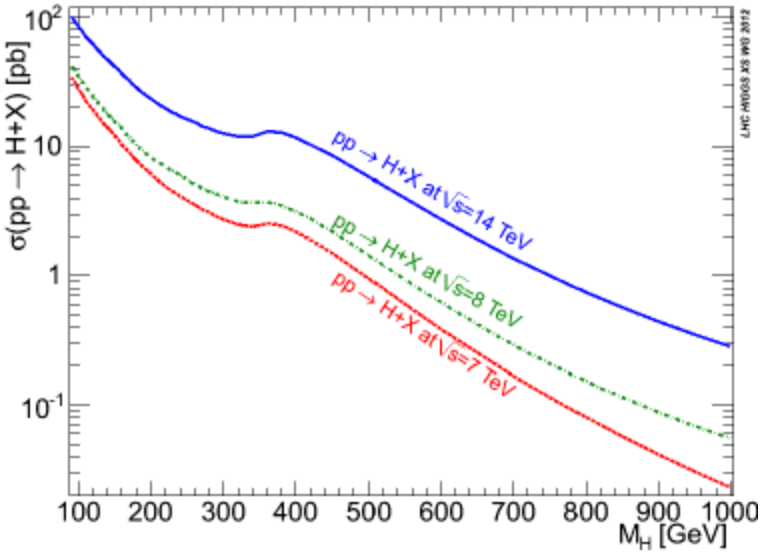
$$\kappa_g \frac{\alpha_s}{12\pi} \frac{H}{v} G^A_{\mu\nu} G^{A,\mu\nu} + \kappa_\gamma \frac{\alpha}{8\pi} \frac{H}{v} F_{\mu\nu} F^{\mu\nu} + \kappa_{Z\gamma} \frac{\alpha}{8\pi s_W} \frac{H}{v} F_{\mu\nu} Z^{\mu\nu}$$

  - Higgs self-couplings proportional to $m_H^2$

$$\frac{m_H^2}{2} H^2 + \frac{m_H^2}{2v} H^3 + \frac{m_H^2}{8v^2} H^4$$

# Precise Calculations




Reduction of scale uncertainty at $N^3LO$:

13 TeV $\sigma(m_H=125\text{ GeV})=43.14\text{ pb}^{+2.71\%}_{-4.45\%}$ 3 Loops!

Increase of +2.2% from NNLO rate [Anastasiou, Duhr, Dulat, Herzog, Mistlberger, arXiv:1503.06056]

## At 13 TeV: factors 2-4 increases in rates

# Largest Higgs BR is to b's

S. Dawson (BNL)
CP3, May, 2015

- Sensitive to $m_b$: $\Gamma(H \to b\bar{b}) = \frac{G_F N_c}{4\sqrt{2}\pi} m_H \beta^3 M_b^2$

- QCD included to $N^3LO$ for H→bb predictions

Input values for Higgs BR fits

| Parameter | Central Value | Uncertainty |
|---|---|---|
| $\alpha_s(M_Z)$ | 0.119 | ±0.002 (90%CL) |
| $m_b$ | 4.49 GeV | ±0.06 GeV |
| $M_t$ | 172.5 GeV | ±2.5 GeV |

$M_b$ is pole mass calculated with 1 loop running of $m_b(m_b)$=4.16 GeV

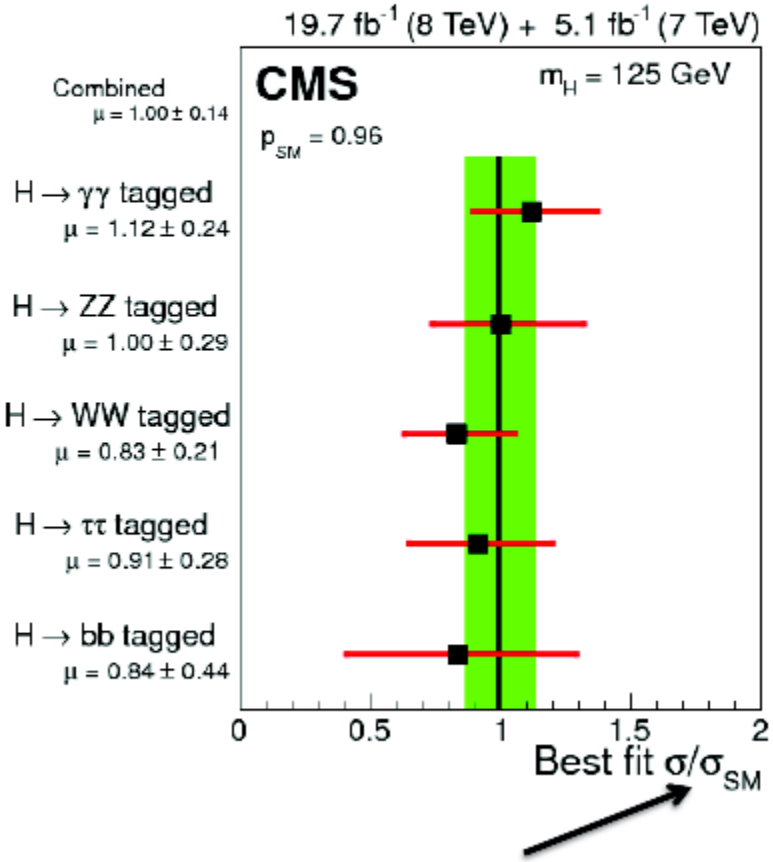
$$\frac{\delta\Gamma_b}{\Gamma_b} \sim \pm 3\%$$

Also, dominating uncertainty in:

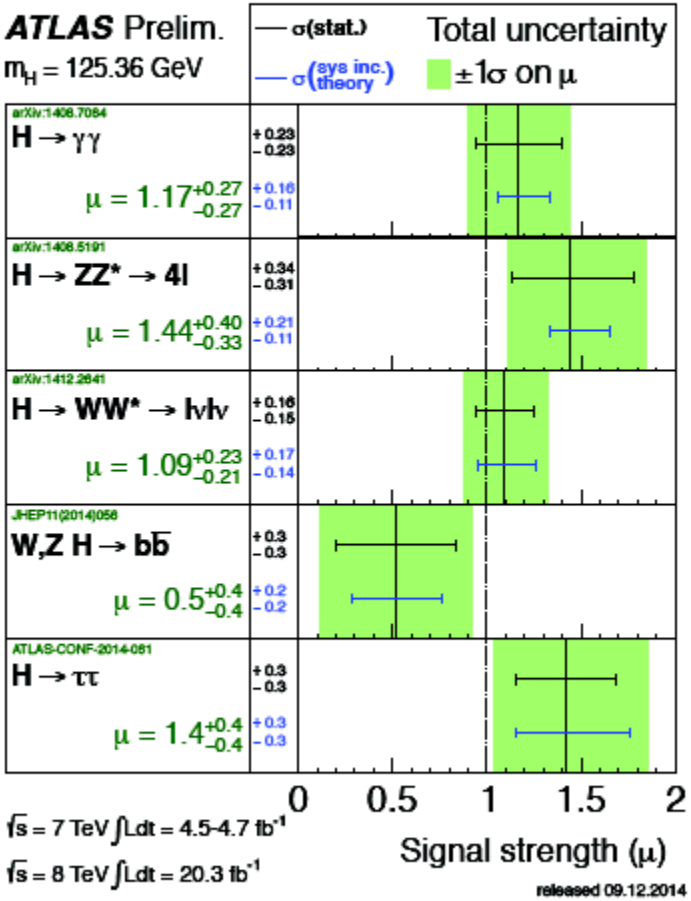
$$\Gamma_H(m_H = 125\ GeV) = 4\ MeV \pm 4\%$$

S. Dawson (BNL)
CP3, May, 2015

# Consistent with SM Hypothesis




Requires theory input!

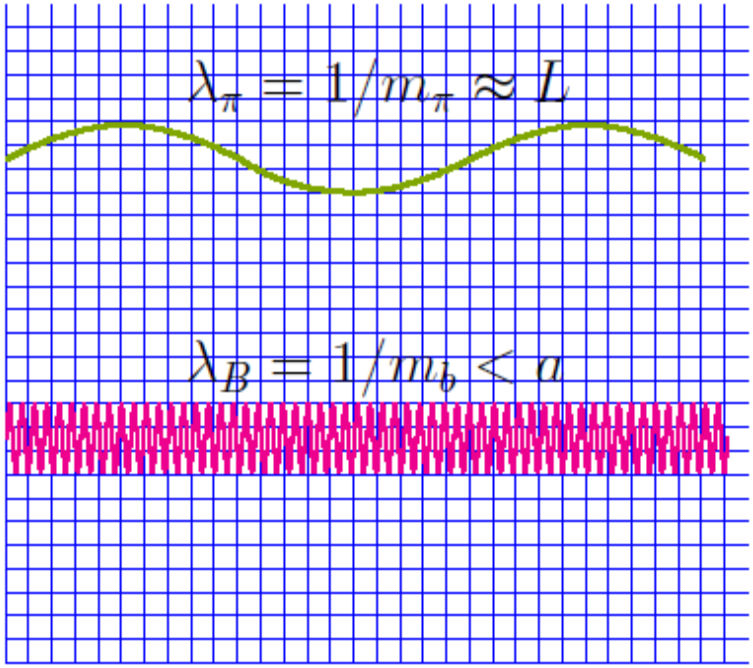



Errors soon to be dominated by theory

# Heavy quarks on the lattice

- finite volume effects are mainly triggered by the light degrees of freedom. The usual requirement is $m_{PS}L > 4$ and $m_{PS}$ is now getting to the physical point $\Rightarrow L \simeq 7$ fm.
- cutoff effects are related to the heavy quark mass. $a << 1/m_b \simeq 0.03$ fm .

$\Rightarrow L/a \simeq 200$ is needed to have those systematics under control !!
Integrating out the heavy quark mass in this case is useful !!




In addition the autocorrelation of observables grows as $1/a^n$ with $n \geq 2$ [Schäfer, Sommer and Virotta '10, Lüscher and Schäfer, '11]

Different methods have different systematics. It is crucial to compare results from a variety of them. Briefly:

Most approaches directly apply some EFT (typically valid in a particular kinematic regime)

- NRQCD [Thacker, Lepage 1991]: Expansion in $v_h$ and in $1/m_h$.
  Dim. 5 ops at leading order $\Rightarrow$ non-renormalizable.
  One has to look for a *window* where cutoff effects $(O(a^n))$and power divergences $(O(1/a^m))$ are both small. Typically $am_h \geq 1$.
- Combinations of HQET and Symanzik effective theory:
  - O(a) improved HQET [ALPHA ...]
  - First Symanzik EFT, then HQET (expand in $1/m_h$ the improvement coefficients) $\rightarrow$ Fermilab action [El-Khadra, Kronfeld, Mackenzie 1996], RHQ [Christ, Li, Lin 2007] and Tsukuba action [Aoki, Kuramashi, Tominaga 2003].

Having introduced operators of higher dimensions all these theories produce power divergences (also in the Fermilab approach when $m_h \rightarrow \infty$ at fixed $a$).

The continuum limit exists only if these divergences are subtracted non-perturbatively. At any order in $g_0^2$:

$$\frac{g_0^{2n}}{a} \approx \frac{1}{\ln(a)^n a} \to \infty \quad \text{as } a \to 0$$

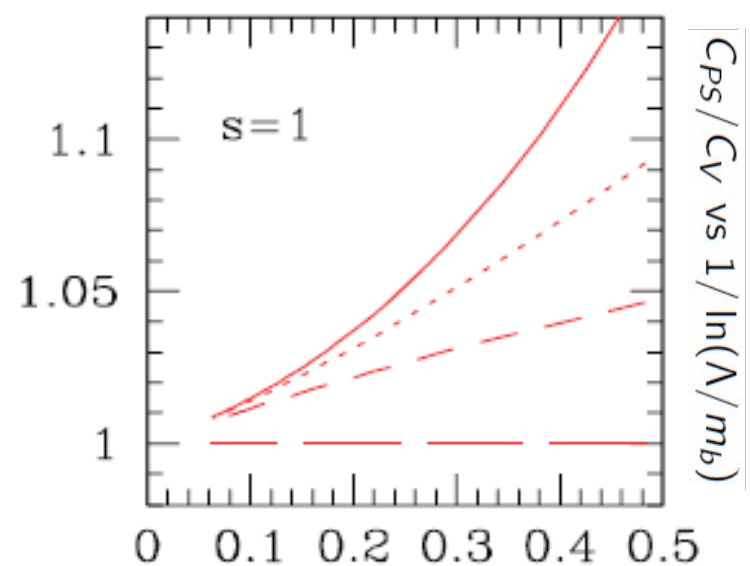
We devised a completely non-perturbative setup for lattice HQET.

Non (directly) EFT based approaches

- HISQ [HPQCD 2011]: at lattice spacings of $a \approx 0.05$ fm and $L/a \approx 100$ as currently produced by MILC, $am_b \approx 1$ so one can simulate directly at $m_b/2$ and then extrapolate to the $b$ (using HQET).
  Getting there but autocorrelations seem a severe problem ...
- Interpolation method [Guazzini, Sommer, Tantalo 2006, ...]: using data around the charm and results in the static limit + fits in powers of $1/m_h$.

  In both methods matching factors from PT are used to define obervables with the proper scaling (removing $\ln(m_h)$).

[R. Sommer, 2010]

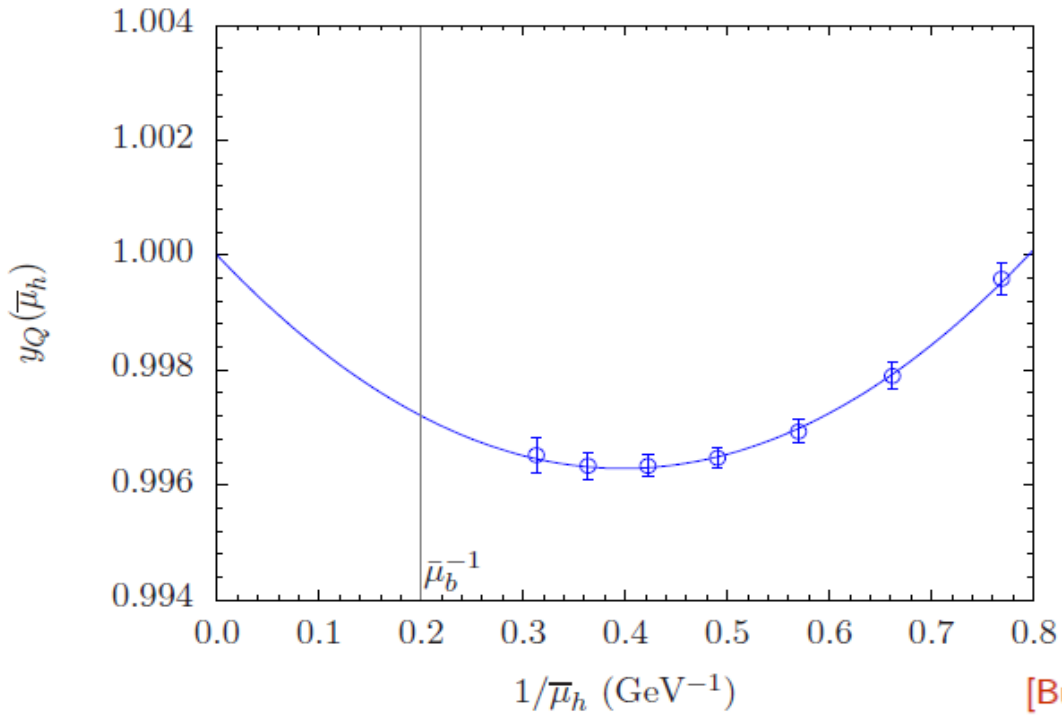



[Chetyrkin and Grozin 2003, Broadhurst and Grozin '91, '95, Bekavac et al. 2010]

Those factors *mostly* drop out in the Ratio Method [ETM 2010 ...]:

$$P(m_b) = P(m_c)\,\frac{P(\lambda m_c)}{P(m_c)}\,\frac{P(\lambda^2 m_c)}{P(\lambda m_c)}\ldots$$

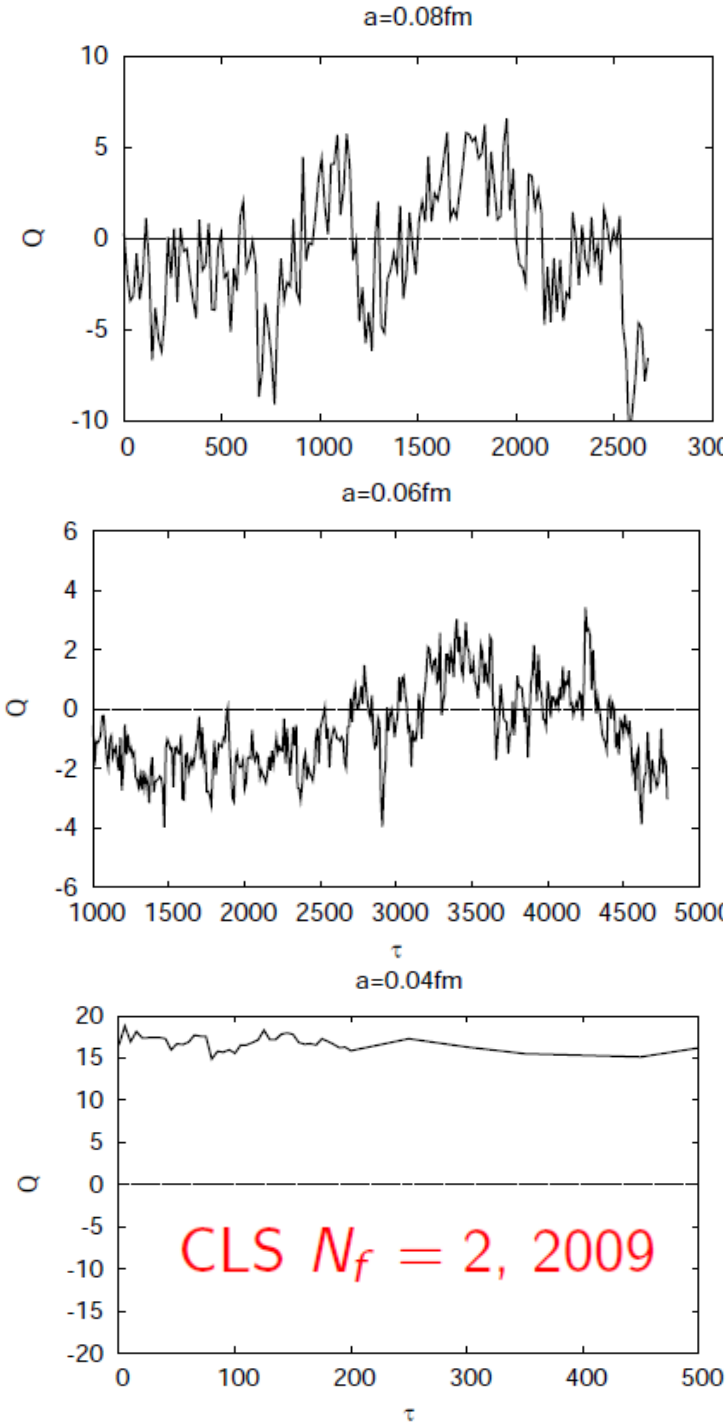
The ratios have static limit=1.



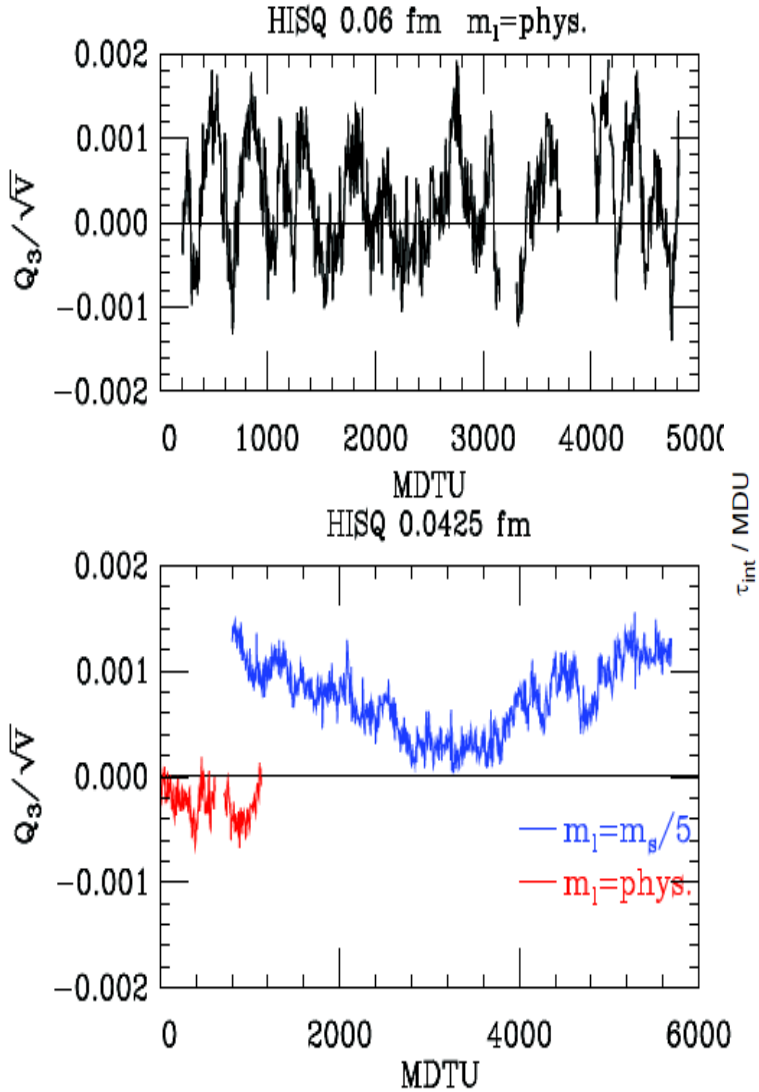

[Bussone et al., 2014]

- At the masses explored the $1/m_h^2$ corrections seem to be as big as $1/m_h$.

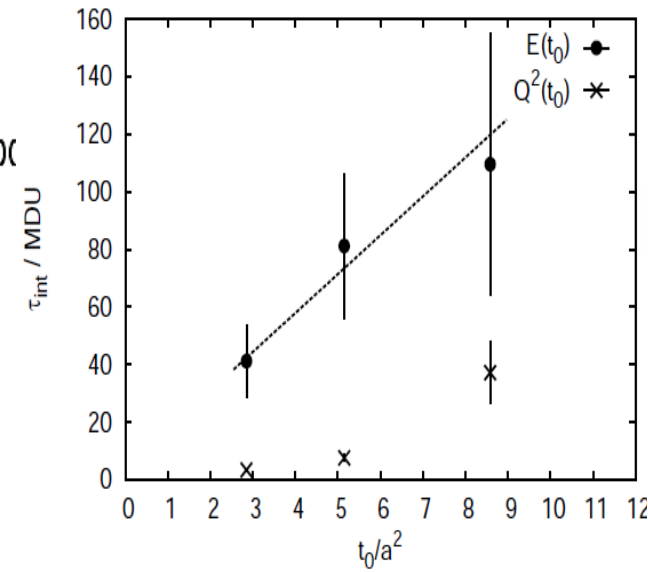
# Autocorrelations




CLS $N_f = 2$, 2009




De Tar, Lattice2015




CLS $N_f = 2 + 1$, 2015

open boundary conditions

# Noise to signal ratio in HQET

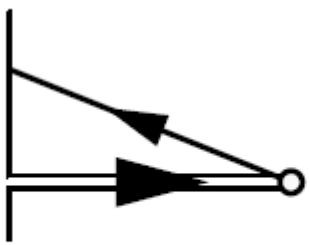
Field content [Eichten and Hill 1990]: $P_+\psi_h = \psi_h\,, \quad \bar{\psi}_h P_+ = \bar{\psi}_h\,, \quad P_+ = \frac{1+\gamma_0}{2}$

Action: $S_h^W = a^4 \sum_x \bar{\psi}_h(x)(D_0^W + \delta m_W)\psi_h(x)$ [MDM, Shindler, Sommer 2005]

with the covariant derivative

$$D_0^W \psi_h(x) = \frac{1}{a}\left[\psi_h(x) - W^\dagger(x - a\hat{0}, 0)\psi_h(x - a\hat{0})\right]$$

$\delta m_W$ cancels the divergence of the self energy. Let us consider a (SF) two-point function with $A^{stat}$ in the bulk, and set $\delta m_W = 0$



$x_0=0$



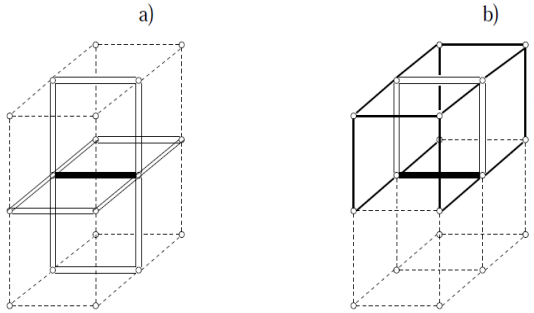
T

$$f_A^{\rm stat}(x_0) \propto e^{-E_{\rm stat} x_0}$$

with $E_{\rm stat} \sim E_{self} + O(a^0) \sim \frac{1}{a} e^{(1)}\, g_0^2 + \ldots$

$e^{(1)}$ depends on the regularization and $\delta m_W = -E_{self} + O(a^0)$.

We just changed the gauge parallel transporter (HYP smearing), following [Hasenfratz, Knechtli 2001]



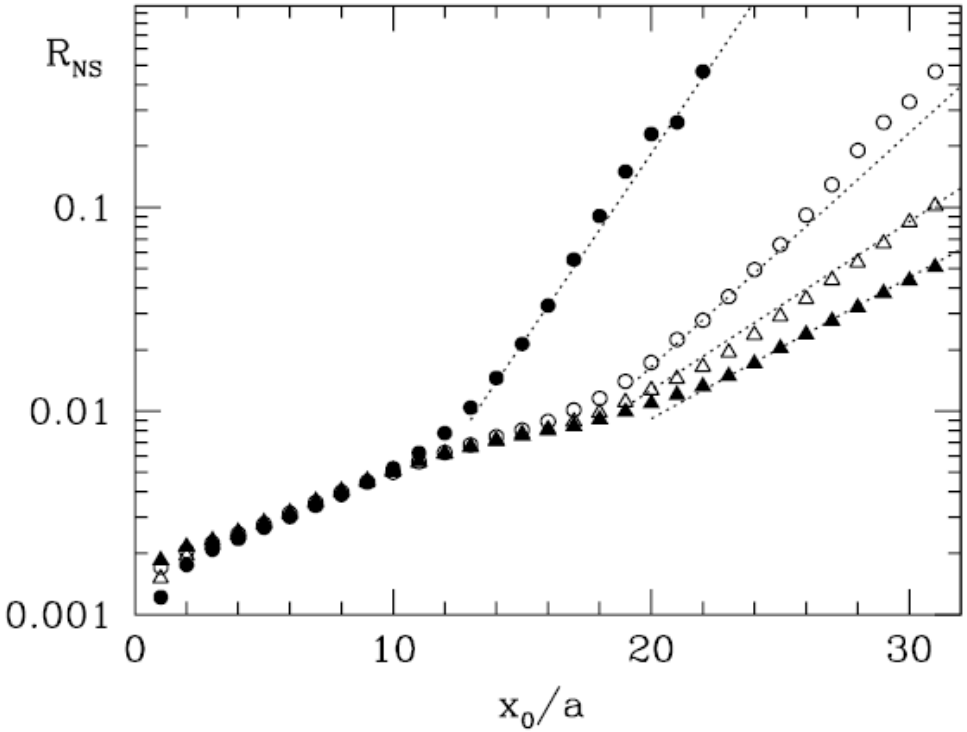




**Figure 2:** *The ratio $R_{NS}$ for the correlation function $f_A^{stat}(x_0)$ for a statistics of 5000 measurements on a $16^3 \times 32$ lattice at $\beta = 6$. Filled circles refer to $S_h^{EH}$, empty circles to $S_h^A$ ($S_h^s$ gives similar results), empty (filled) triangles to $S_h^{HYP1}$ ($S_h^{HYP2}$).*

This smearing is now used in all applications of HQET, and also in dynamical simulations [BMW].

# NLO in $1/m_h$

Symmetries of the static action

- In the static action no $\gamma$-matrices appear, the interactions do not change the spin of the heavy quark ($SU(2)$ spin symmetry).
- also, $\psi_h(x) \to e^{i\eta(\vec{x})}\psi_h(x)$ is a symmetry (local flavor number conservation).

both broken at $O(1/m_h)$:

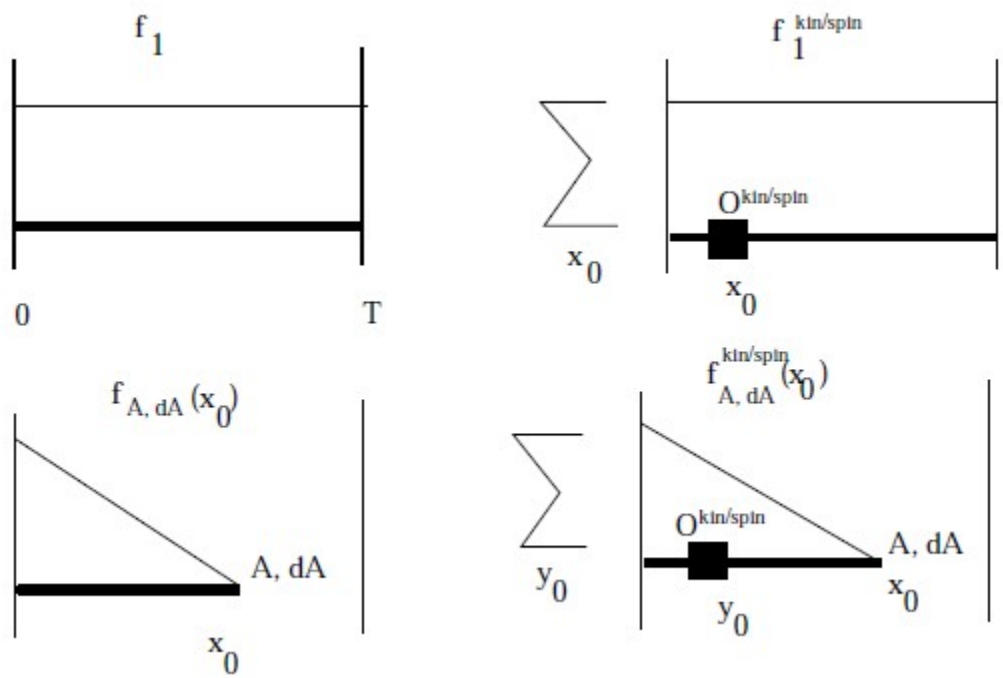
$$S_{HQET} = a^4 \sum_x \left\{ \bar{\psi}_h(D_0 + m_{\rm bare})\psi_h + \omega_{spin}\bar{\psi}_h(-\sigma \mathbf{B})\psi_h + \omega_{kin}\bar{\psi}_h\left(-\frac{1}{2}\mathbf{D}^2\right)\psi_h + \ldots \right\}$$

Analogously for operators

$$\begin{aligned}
A_0^{\rm HQET}(x) &= Z_{\rm A}^{\rm HQET}[A_0^{\rm stat}(x) + \sum_{i=1}^{2} c_{\rm A}^{(i)} A_0^{(i)}(x)] \,, \\
A_0^{(1)}(x) &= \overline{\psi}_{\rm l}\tfrac{1}{2}\gamma_5\gamma_i(\nabla_i^{\rm S} - \overleftarrow{\nabla}_i^{\rm S})\psi_{\rm h}(x) \,, \\
A_0^{(2)}(x) &= -\tilde{\partial}_i A_i^{\rm stat}(x)/2 \,, \quad A_i^{\rm stat}(x) = \overline{\psi}_{\rm l}(x)\gamma_i\gamma_5\psi_{\rm h}(x)
\end{aligned}$$

- NB. The static theory is not renormalizable by power-counting (static prop. $\propto \delta(\vec{x}-\vec{y})$). Still, only dim. 4 ops. in the action. In [Grinstein, '90] it has been shown that QCD correlators are reproduced to all orders in $\alpha_s$ at LO in $1/m_h$.

- Next to leading order terms in the $1/m_h$ expansion are not included in the action, that would produce couplings of negative dimension. They are treated as insertions into correlation functions evaluated in the static theory and renormlaized order by order.



Are there enough coeffs to make correlators finite ? YES [MDM, Garron, Papinutto, Sommer 2006], take the expansion of

$$C_{\mathrm{AA}}(x_0) = Z_{\mathrm{A}}^2 a^3 \sum_{\mathbf{x}} \left\langle A_0(x)(A_0)^\dagger(0) \right\rangle$$

- $m_{bare}$ has to absorb a $1/a^2$ divergence from the mixing between the kinetic operator and $\bar{\psi}_h \psi_h$
- $Z_A^{HQET}$ absorbs a $1/a$ divergence from the <u>contact term</u> between the kinetic operator and $A_0^{stat}$.

**Why matching should be performed non-perturbatively ?**

Let us consider the example

$$m_{B^*}^2 - m_B^2 = C_{mag}(m_b/\Lambda_{\mathrm{QCD}}) \, \langle B|\bar{\psi}_h \sigma \mathbf{B} \psi_h|B^*\rangle_{RGI} \times (1 + \mathrm{O}(1/m_b))$$

$C_{mag}(m_b/\Lambda_{\mathrm{QCD}})$ has a perturbative expansion. The truncation at O(n-1)

$$\simeq \alpha(m_b)^n \simeq \left\{ \frac{1}{2b_0 \ln(m_b/\Lambda_{\mathrm{QCD}})} \right\}^n >> \frac{\Lambda_{\mathrm{QCD}}}{m_b} \quad \text{as } m_b \to \infty$$

The PT corrections to the leading term are larger than the $1/m_b$ ones !

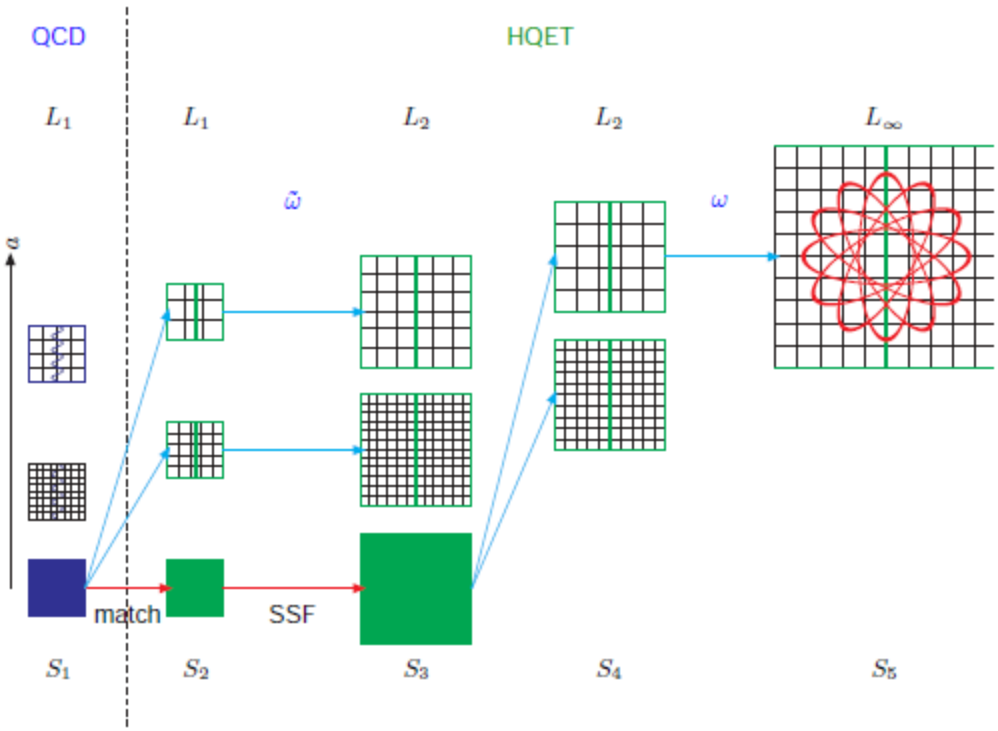
# More on the actual matching procedure

[ALPHA, arXiv:1001.4783, 1004.2661, 1006.5816 and 1203.6516, 1311.5498]

- $a = f(\beta)$, $\beta = 6/g_0^2$. large $\beta$ = small $a$.
- The parameters are renormalization factors. They depend on $a$ but not on $L$.
- $L/a$ can't be arbitrarily large.
- Eventually we want them for $a \simeq 0.1 - 0.05$ fm (large volumes for phenomenology).
- Idea: at small $L$ and very fine $a$ we simulate HQET and QCD with a relativistic b-quark. We get the parameters by matching 19 suitable quantities [MDM et al., arXiv:1312.1566]

$$\Phi_i^{\rm QCD}(m_{\rm b}, 0) = \Phi_i^{\rm HQET}(\omega_{\ldots}, c^{(j)}, Z_{\ldots}^{\rm HQET}, a)$$

- By a sequence of evolution (in $L$, fixed $a$) and matching (continuum vs finite $a$, fixed $L$) steps in HQET, one can obtain the parameters at larger $a$.



# The b quark mass in HQET at $O(1/m_h)$

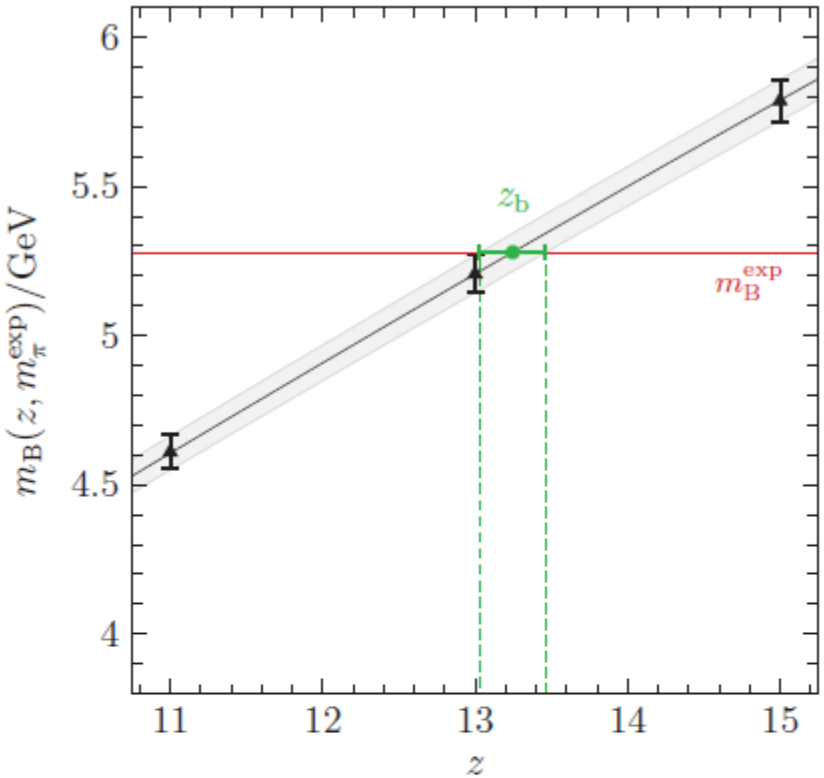
We generate $N_f = 2$ dynamical configurations, with NP $O(a)$ improved Wilson fermions and plaquette gauge action.

| $\beta$ | $a$[fm] | $L/a$ | $m_\pi$[MeV] | $m_\pi L$ | #cfgs | $\frac{\#\text{cfgs}}{\tau_{\text{exp}}}$ | id |
|---|---|---|---|---|---|---|---|
| 5.2 | 0.075 | 32 | 380 | 4.7 | 1012 | 122 | A4 |
| | | 32 | 330 | 4.0 | 1001 | 164 | A5 |
| | | 48 | 280 | 5.2 | 636 | 52 | B6 |
| 5.3 | 0.065 | 32 | 440 | 4.7 | 1000 | 120 | E5 |
| | | 48 | 310 | 5.0 | 500 | 30 | F6 |
| | | 48 | 270 | 4.3 | 602 | 36 | F7 |
| | | 64 | 190 | 4.1 | 410 | 17 | G8 |
| 5.5 | 0.048 | 48 | 440 | 5.2 | 477 | 4.2 | N5 |
| | | 48 | 340 | 4.0 | 950 | 38 | N6 |
| | | 64 | 270 | 4.2 | 980 | 20 | O7 |

The b-quark mass is determined by computing, as a function of the heavy quark mass $m_h$ used in the matching, the large-volume quantity

$$m_B(m_h) = m_{bare}(m_h) + E^{stat} + \omega_{spin}(m_h)E^{spin} + \omega_{kin}(m_h)E^{kin}$$

and then solving $m_B(m_h) = m_B^{exp}$, with $m_h$ as unknown.




$$z = L_1 m_h \ ,$$

[ MDM and ALPHA, arXiv:1311.5498]

| $N_f$ | Ref. | $M$ | $\overline{m}_{\overline{MS}}(\overline{m}_{\overline{MS}})$ | $\overline{m}_{\overline{MS}}(4\,\mathrm{GeV})$ | $\overline{m}_{\overline{MS}}(2\,\mathrm{GeV})$ |
|---|---|---|---|---|---|
| 0 | [36] | 6.76(9) | 4.35(5) | 4.39(6) | 4.87(8) |
| 2 | this work | 6.58(17) | 4.21(11) | 4.25(12) | 4.88(15) |
| 5 | PDG13 [1] | 7.50(8) | 4.18(3) | 4.22(4) | 4.91(5) |

Convergence at lower scales may be due to the common low-energy input ($m_B$).

# Other recent computations [see Sanfilippo, LAT14 for less recent ones]

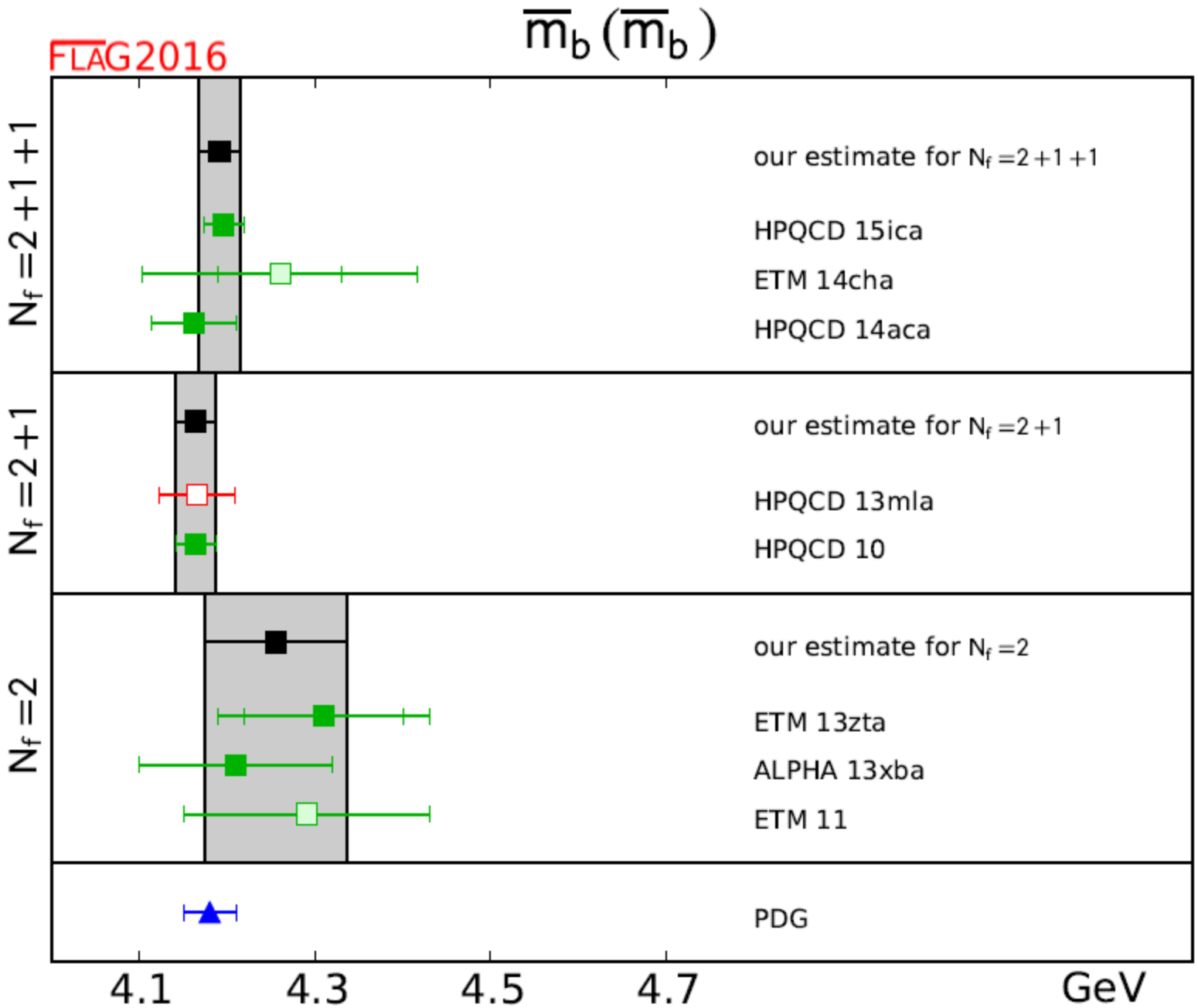
- ETMC [1411.0484], $N_f = 2+1+1$, ratio (or Tor Vergata) method with $P = \frac{m_B(h\ell)}{m_h^{pole}}$. $\mathrm{N^3LO}$ PT is used to convert pole mass to $\overline{\mathrm{MS}}$.

- HPQCD [1408.4169], $N_f = 2+1+1$, HISQ MILC confs. Based on fitting moments of heavy-heavy PS two-point functions to obtain $m_c$ and $\alpha_s$
  - Low moments (n=4,... 10) should be short distance dominated.
  - (Gluon) condensates enter the OPE.
  - Lattice data for moments up to $n = 10$ are simultaneously fitted including $O(\alpha_s^{15})$ [$\mathrm{N^3LO}$ known] and $O(am_h)^{42}$.
  - Over 40 params, with priors, and 92 data points.
  - Stability of the results including about half the $O(\alpha_s)$ and $O(a)$ terms.
  - The approach gives $m_{\eta_h}$ as a function of $m_h$. By extrapolating to $m_{\eta_b}$ from $\approx 3m_c$ HPQCD gets $m_b/m_c$ and eventually $m_b$ with about 1% error.

# PRELIMINARY FLAG 2016 b-mass

| Collaboration | Ref. | $N_f$ | publication status | chiral extrapolation | continuum extrapolation | finite volume | renormalization | heavy quark treatment | $\overline{m}_b(\overline{m}_b)$ | $m_b/m_c$ |
|---|---|---|---|---|---|---|---|---|---|---|
| HPQCD 15ica | [132] | 2+1+1 | A | ★ | ★ | ★ | ★ | ✓ | 4.196(23)† | |
| ETM 14cha | [133] | 2+1+1 | C | ○ | ★ | ○ | ★ | ✓ | 4.26(7)(14) | 4.40(6)(5) |
| HPQCD 14aca | [39] | 2+1+1 | A | ★ | ★ | ★ | – | ✓ | 4.162(48) | 4.528(14)(52) |
| HPQCD 13mla | [134] | 2+1 | A | ■ | ○ | – | – | ✓ | 4.166(43) | |
| HPQCD 10 | [94] | 2+1 | A | ★ | ★ | ★ | – | ✓ | 4.164(23)* | 4.51(4) |
| ETM 13zta | [135] | 2 | A | ○ | ★ | ○ | ★ | ✓ | 4.31(9)(8) | |
| ALPHA 13xba | [136] | 2 | A | ★ | ★ | ★ | ★ | ✓ | 4.21(11) | |
| ETM 11 | [137] | 2 | A | ○ | ★ | ○ | ★ | ✓ | 4.29(14) | |
| PDG | [103] | | | | | | | | 4.18(3) | |

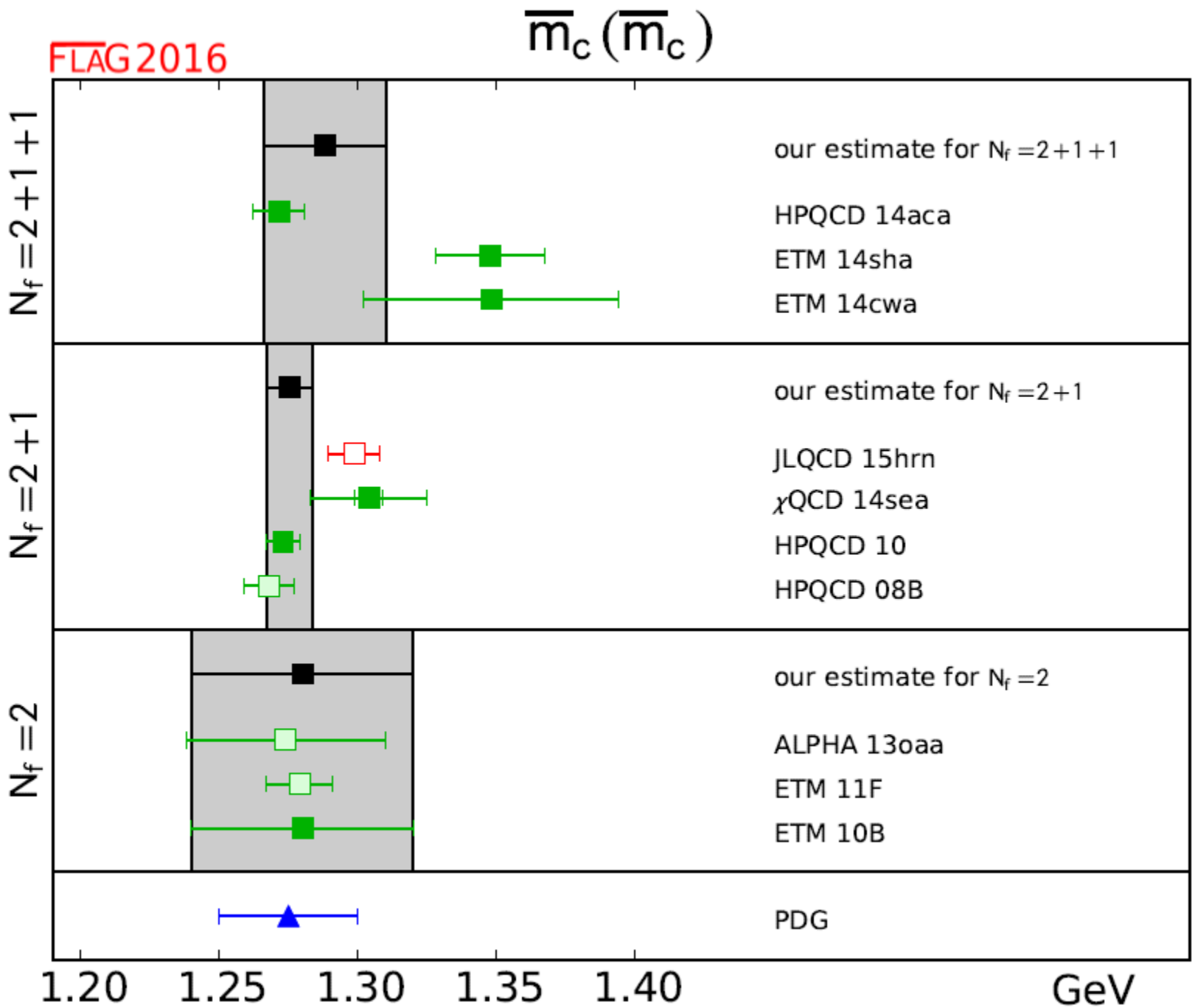
† Warning: only two pion points are used for chiral extrapolation.
* The number that is given is $m_b(10\ \text{GeV}, N_f = 5) = 3.617(25)$ GeV.


$\overline{m}_b(\overline{m}_b)$
FLAG2016
$N_f = 2+1+1$
our estimate for $N_f = 2+1+1$
HPQCD 15ica
ETM 14cha
HPQCD 14aca
$N_f = 2+1$
our estimate for $N_f = 2+1$
HPQCD 13mla
HPQCD 10
$N_f = 2$
our estimate for $N_f = 2$
ETM 13zta
ALPHA 13xba
ETM 11
PDG
4.1
4.3
4.5
4.7
GeV

# PRELIMINARY FLAG 2016 c-mass

| Collaboration | Ref. | $N_f$ | publication status | chiral extrapolation | continuum extrapolation | finite volume | renormalization | $\overline{m}_c(\overline{m}_c)$ | $\overline{m}_c(3\text{ GeV})$ |
|---|---|---|---|---|---|---|---|---|---|
| HPQCD 14aca | [39] | 2+1+1 | A | ★ | ★ | ★ | – | 1.2715(95) | 0.9851(63) |
| ETM 14sha | [127] | 2+1+1 | A | ○ | ★ | ○ | ★ | 1.3478(27)(195) | 1.0557(22)(153) |
| ETM 14cwa | [23] | 2+1+1 | A | ○ | ★ | ○ | ★ | 1.348(46) | 1.058(35) |
| JLQCD 15hrn | [125] | 2+1 | C | ○ | ★ | ★ | – | 1.2987(21)(90) | 0.9948(16)(69) |
| $\chi$QCD 14sea | [128] | 2+1 | A | ○ | ○ | ○ | ★ | 1.304(5)(20) | 1.006(5)(22) |
| HPQCD 10 | [94] | 2+1 | A | ★ | ★ | ★ | – | 1.273(6) | 0.986(6) |
| HPQCD 08B | [104] | 2+1 | A | ○ | ★ | ○ | – | 1.268(9) | 0.986(10) |
| ALPHA 13oaa | [129] | 2 | C | ★ | ○ | ★ | ★ | 1.274(36) | 0.976(28) |
| ETM 11F | [126] | 2 | C | ○ | ★ | ○ | – | 1.279(12)/1.296(18)* | 0.979(09)/0.998(14)* |
| ETM 10B | [79] | 2 | A | ○ | ★ | ○ | ★ | 1.28(4) | 1.03(4) |
| PDG | [103] | | | | | | | 1.275(25) | |

* Two results are quoted.


FLAG 2016
$\overline{m}_c(\overline{m}_c)$
$N_f = 2+1+1$
our estimate for $N_f = 2+1+1$
HPQCD 14aca
ETM 14sha
ETM 14cwa
$N_f = 2+1$
our estimate for $N_f = 2+1$
JLQCD 15hrn
$\chi$QCD 14sea
HPQCD 10
HPQCD 08B
$N_f = 2$
our estimate for $N_f = 2$
ALPHA 13oaa
ETM 11F
ETM 10B
PDG
1.20
1.25
1.30
1.35
1.40
GeV

# Conclusions

- Higgs less of a portal to New Physics than we hoped.

- Even to establish that precise lattice results in the b-sector are needed.

- Tensions in B-physics ( R(D*), $B^0 \rightarrow K^0 \mu^+\mu^-$ ) need lattice inputs to assess significances.

- HQET on the lattice:
  NP subtraction of power divergences
  (conceptually a 'must' as $a \rightarrow 0$).

  NP matching at $O(1/m_h)$
  (conceptually a 'must' as $m_h \rightarrow \infty$)

- b-quark mass

- Now tackling form factors