Extra Dimensions in Astrophysics

General Remarks
Gamma-Rays from Kaluza-Klein Gravitons
Kaluza Klein Dark Matter
Conclusions, Outlook

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General Remarks

In order to obtain effects of excitation of modes in extra dimensions, one needs to reach temperatures at least as high as the energy of the lowest excitation mode.

=> warped extra diension scenarios such as Randall-Sundrum type scenarios are often unconstrained by astrophysics

More generic constraints can thus be derived in ADD type scenarios with a torus topology:

$$\overline{M_{\mathrm{Pl}}}^{2} = (2\pi R)^{n}\, \overline{M_{4+n}}^{2+n}$$

$$\sum_{\mathrm{modes}} = \Omega_n R^n \int dm\, m^{n-1}$$

where m=graviton mass, R=radius of torus, M^{4+n}=fundamental gravity scale in 4+n dimensions, M_{pl}=observed reduced Planck scale

Gamma Rays from Kaluza Klein Gravitons

Following Hannestad and Raffelt, Phys.Rev. D67, 125008 (2003) and
Casse, Paul, Bertone, Sigl, Phys.Rev.Lett. 92, 111102 (2004)

Kaluza Klein gravitons can copioulsy be produced in nuclear gravi-bremsstrahlung as it occurs in hot neutron stars:

$$N + N \rightarrow N + N + g$$

The energy loss rate per volume for a single graviton mode is

$$Q \propto G_{\mathrm{N}} \sigma_{\mathrm{N}} n_{\mathrm{B}}^{2} T^{7/2} m_{\mathrm{N}}^{-1/2}$$

where $G_{\mathrm{N}} = \text{Newtons constant}$, $\sigma_{\mathrm{N}} = \text{nuclear cross section}$
$n_{\mathrm{B}} = \text{baryon density}$, $T = \text{temperature}$, $m_{\mathrm{N}} = \text{nucleon mass}$

This scaling is obvious because relative nucleon velocity ~ $(T/m_{\mathrm{N}})^{1/2}$.

Energy loss rate into all KK modes is a factor ~ $(RT)^n$ larger.

The massive Kaluza Klein modes leave the hot neutron star without further interaction once produced but remain trapped gravitationally around the star. They slowly decay with a rate

$$\tau_{2\gamma} = \frac{1}{2}\tau_{ee} = \tau_{\nu\bar{\nu}} \approx 6\times 10^{9}\,\mathrm{yr}\left(\frac{100\ \mathrm{MeV}}{m}\right)^{3}$$

At a distance d this implies a photon flux above energy E_0 of

$$F(>E_0) \propto (RT)^n \frac{G_{\mathrm{N}}\sigma_{\mathrm{N}}\, n_{\mathrm{B}}^2\, T^{11/2}}{m_{\mathrm{N}}^{1/2} d^2} V_{\mathrm{NS}}\,\Delta t_{\mathrm{NS}}\,\phi_n(E_0/T)$$

where V_{NS} = neutron star volume , Δt_{NS} = cooling time scale
$\phi(E_0/T)$ = function from detailed integrations

In numbers, this gives for a single neutron star

$$F(>E_0) \approx 8.1\times10^{-23}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\left(\frac{d}{\mathrm{kpc}}\right)^{-2} \times\left(\frac{T}{30\,\mathrm{MeV}}\right)^{11/2}\left(\frac{\rho_{\mathrm{B}}}{3\times10^{14}\,\mathrm{g\,cm}^{-3}}\right)\Omega_n(RT)^n I_n(E_0/T)$$

where ρ_{B} = average baryon density

Compare this with the galactic bulge flux measured by the Compton Gamma Ray Observatory:

$$F(>100\,\mathrm{MeV}) = 8\times10^{-7}\,\mathrm{photons\ cm}^{-2}\,\mathrm{s}^{-1}$$

Strongest constraints come from PSR J0953+0755 at 120 pc or from the Ensemble of ~ 7 x 10^8 galactic neutron stars at average distance ~ 8 kpc.

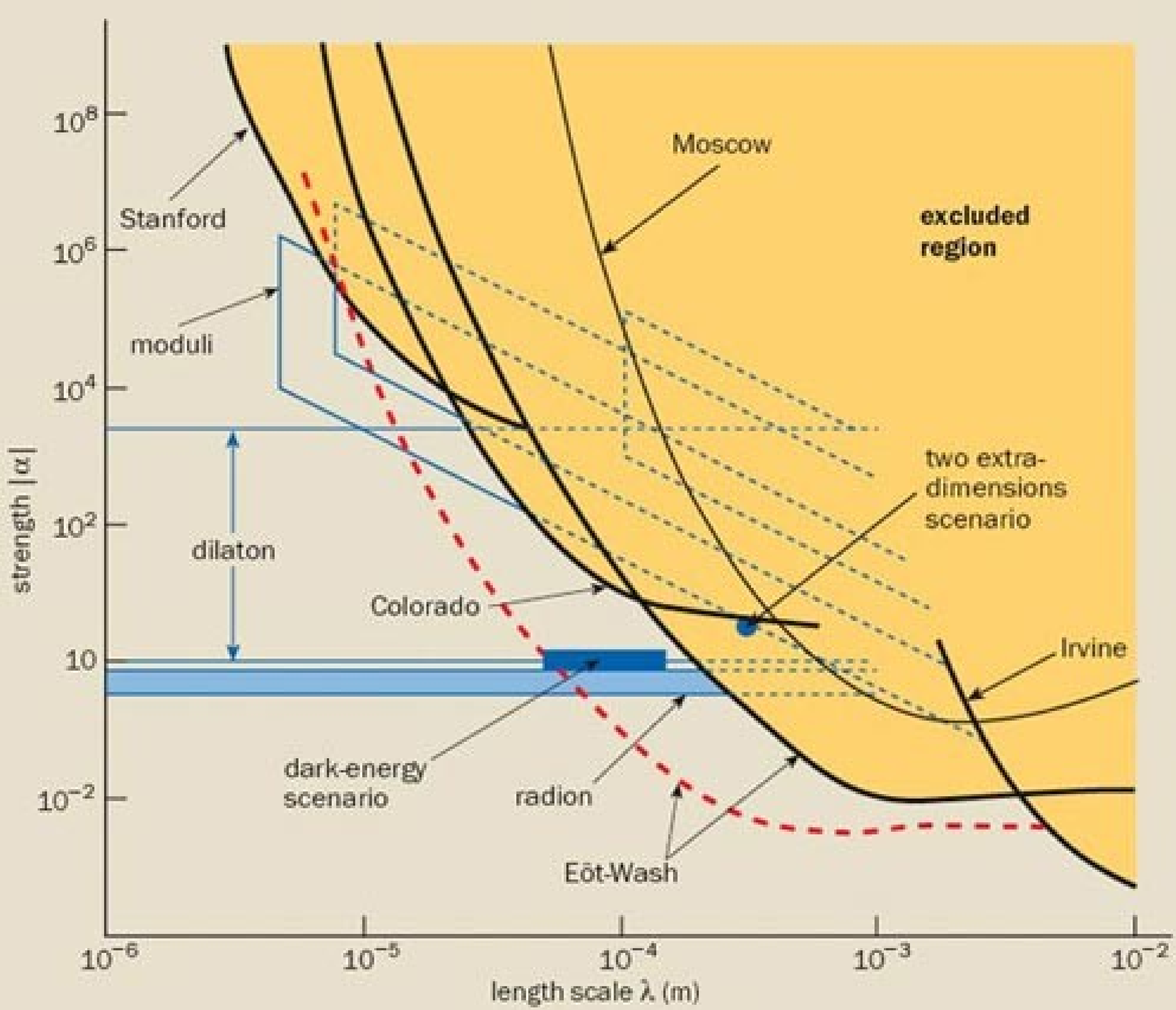
Resulting Constraints

TABLE I: Upper limits on the compactification radius R (in m) and corresponding lower limits on the fundamental energy scale $\bar{M}_{4+n}$ (in TeV), for $N_{NS} = 7 \times 10^8$ and T=30 MeV. We show for comparison, in parentheses, the limits obtained by Hannestad & Raffelt in Ref. [5], as they would be obtained for a distance to the NS of 0.12 kpc.

n	1	2	3	4	5	6	7
R	3.9×10^{-4} (7.7)	3.8×10^{-10} (5.3×10^{-8})	4.2×10^{-12} (1.1×10^{-10})	4.7×10^{-13} (5.5×10^{-12})	1.3×10^{-13} (9.2×10^{-13})	5.4×10^{-14} (2.8×10^{-13})	2.9×10^{-14} (1.2×10^{-13})
$\bar{M}_{4+n}$	7.8×10^4 (2.9×10^3)	4.5×10^2 (3.8×10^1)	1.9×10^1 (2.6)	2.2 (4.3×10^{-1})	4.7×10^{-1} (1.2×10^{-1})	1.47×10^{-1} (4.3×10^{-2})	5.9×10^{-2} (1.9×10^{-2})

Conclusion:

If fundamental gravity scale is ~ TeV, then either n > 4 or the compactification topology has to be more complex than a torus



For n=1, the astrophysical limit is < 4×10^{-4} m, overlapping with the distance at which Newtons law is directly measured.

Kaluza Klein Dark Matter

Following Servant and Tait, Nucl.Phys. B650, 391 (2003) and Bertone, Servant, Sigl, Phys.Rev. D68, 044008 (2003)

If Kaluza Klein (KK) parity is conserved, the lightest Kaluza Klein Particle (LKP) is a good dark matter candidate in Universal Extra Dimension (UED) models.

This is likely to be associated with the first KK excitation of the hypercharge gauge boson $B^{(1)}$. For mass m_X its annihilation cross section into fermion-antifermion pairs reads

$$\sigma v\left(B^{(1)}B^{(1)} \rightarrow f\,\bar{f}\right) = \frac{\alpha_1^2 N_c N_f \pi Y^4}{9 m_X^2}\left(8 - v^2\right)$$

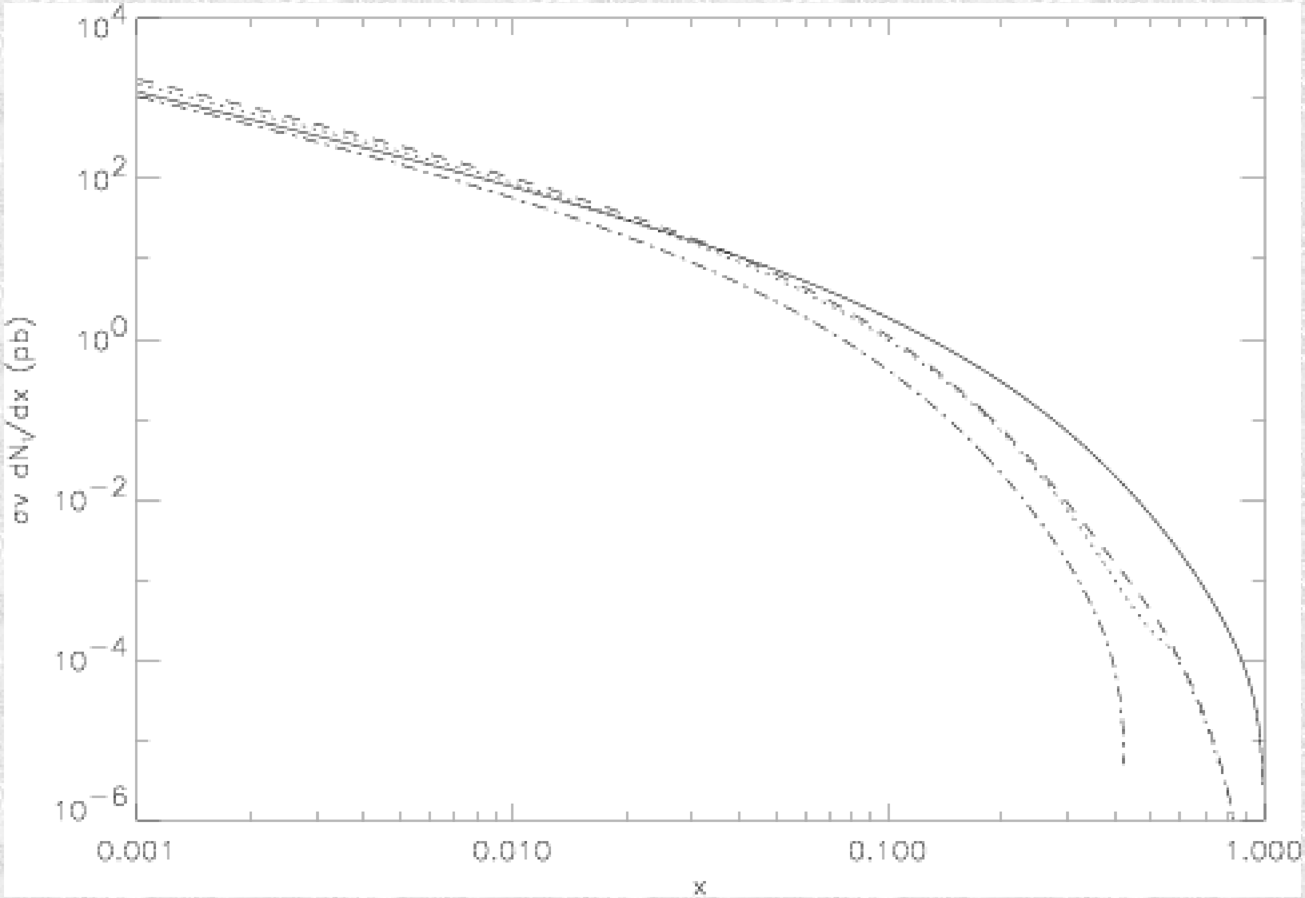
leading to the total annihilation cross section

$$\langle \sigma v \rangle_{\mathrm{ann}} \approx 1.8 \times 10^{-26} \left(\frac{\mathrm{TeV}}{m_X}\right)^2 \mathrm{cm}^3\,\mathrm{s}^{-1}$$

A correct relic abundance $\Omega_x h^2 \sim 0.105$ requires an annihilation cross section of

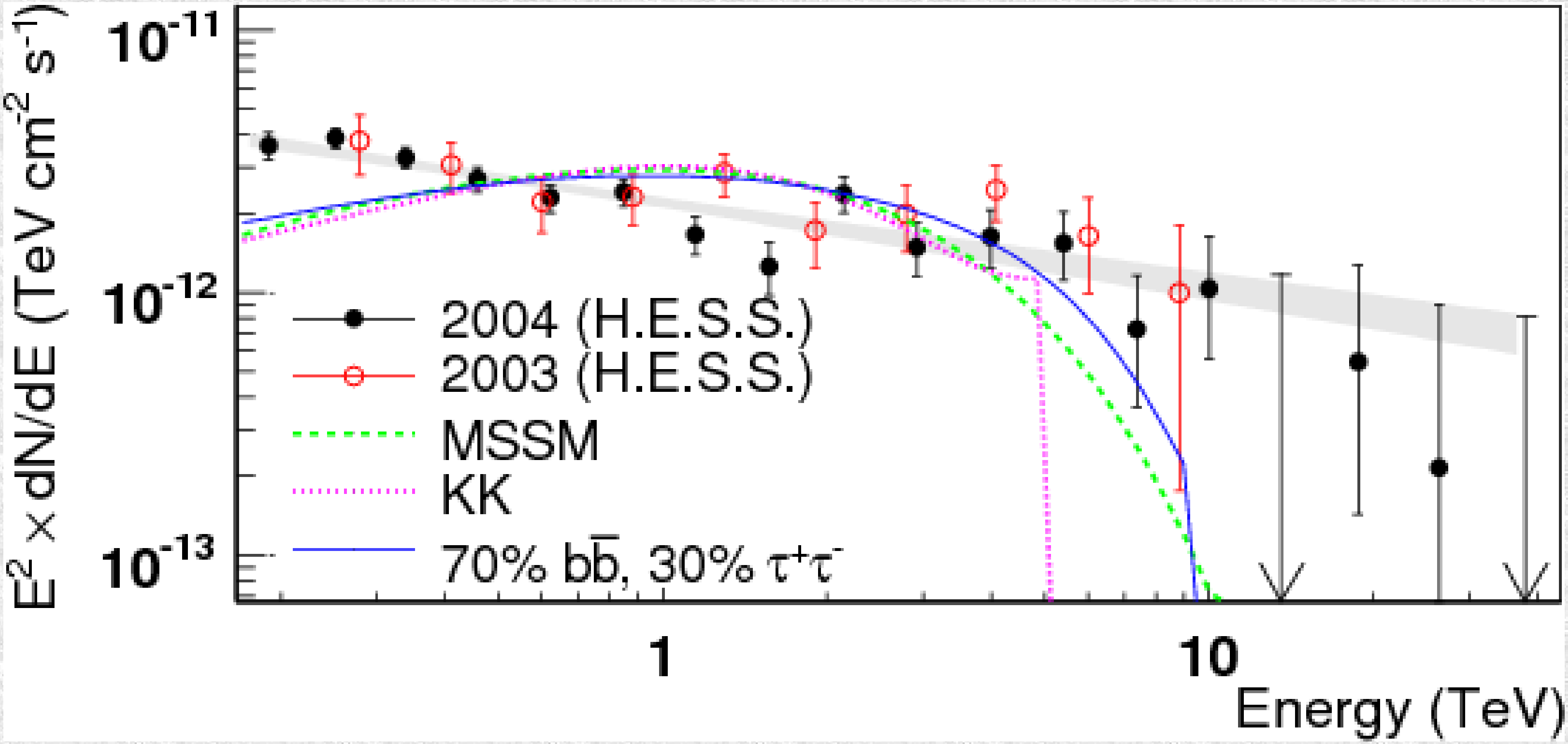
$$\langle \sigma v \rangle_{\rm ann} \approx 3 \times 10^{-26} \mathrm{cm}^3 \mathrm{s}^{-1}$$

Thus, masses 400 GeV < mX < 1.2 TeV lead to consistent relic densities, but are as yet unconstrained by accelerator data.



Annihilation spectra of γ-rays (solid), e+ (dashed), anti-ν_μ (dotted), ν_e and ν_μ(dash-dotted)

Galactic Centre gamma-ray Flux



The H.E.S.S. data extends to beyond 30 TeV which is would require unnaturally large dark matter masses; newest data consistent with acceleration with cut-off.

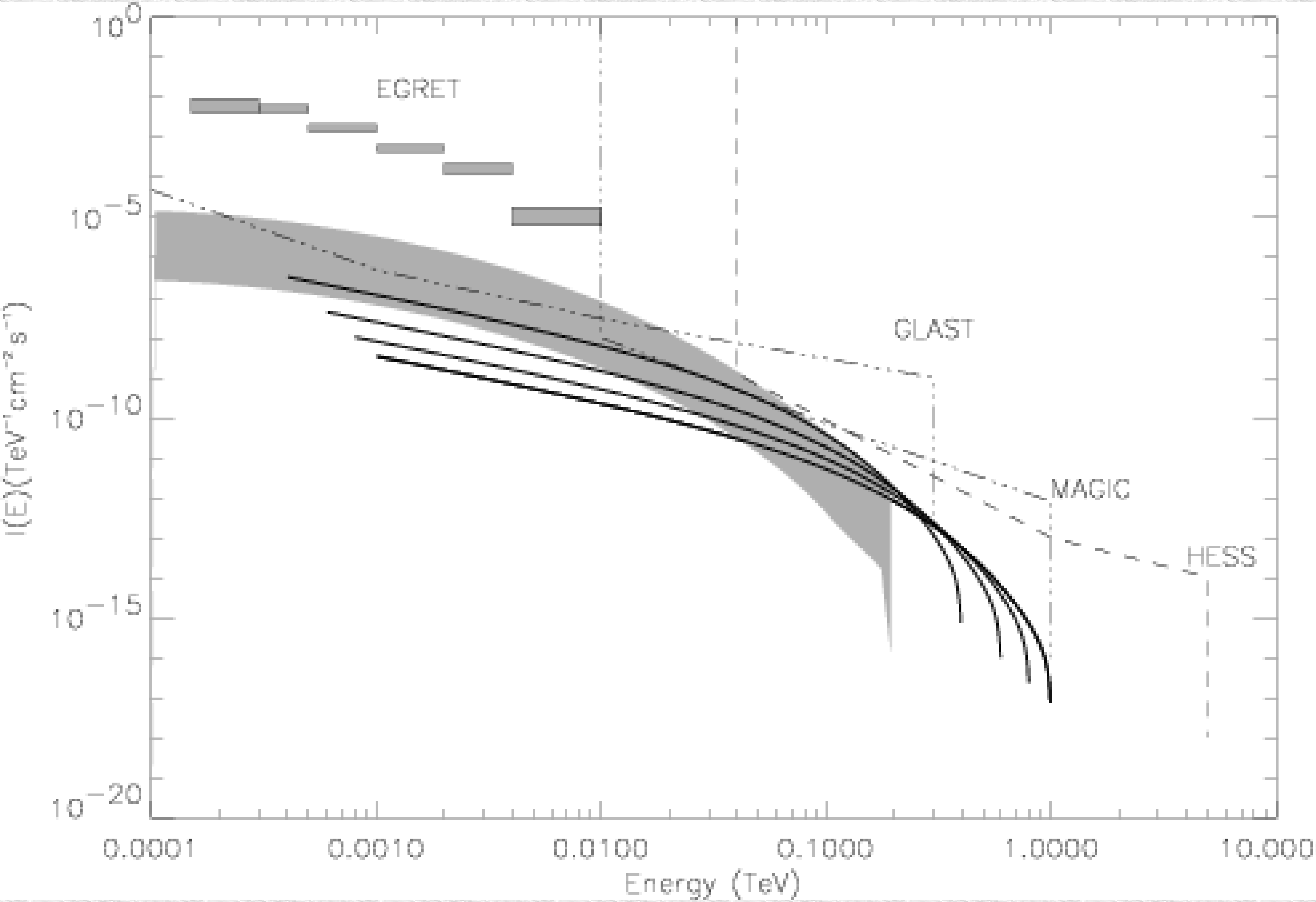
The galactic centre flux of species i produced with a spectrum dN_i/dE per annihilation can be written as

$$F_i(\Delta\Omega, E) \approx 5.6\times10^{-12}\,\frac{dN_i}{dE}\left(\frac{\langle\sigma v\rangle_{\mathrm{ann}}}{3\times10^{-26}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}\right)\left(\frac{\mathrm{TeV}}{m_X}\right)^2 \bar{J}(\Delta\Omega)\,\Delta\Omega\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$$

where $\bar{J}(\Delta\Omega)$ is the average of the quantity

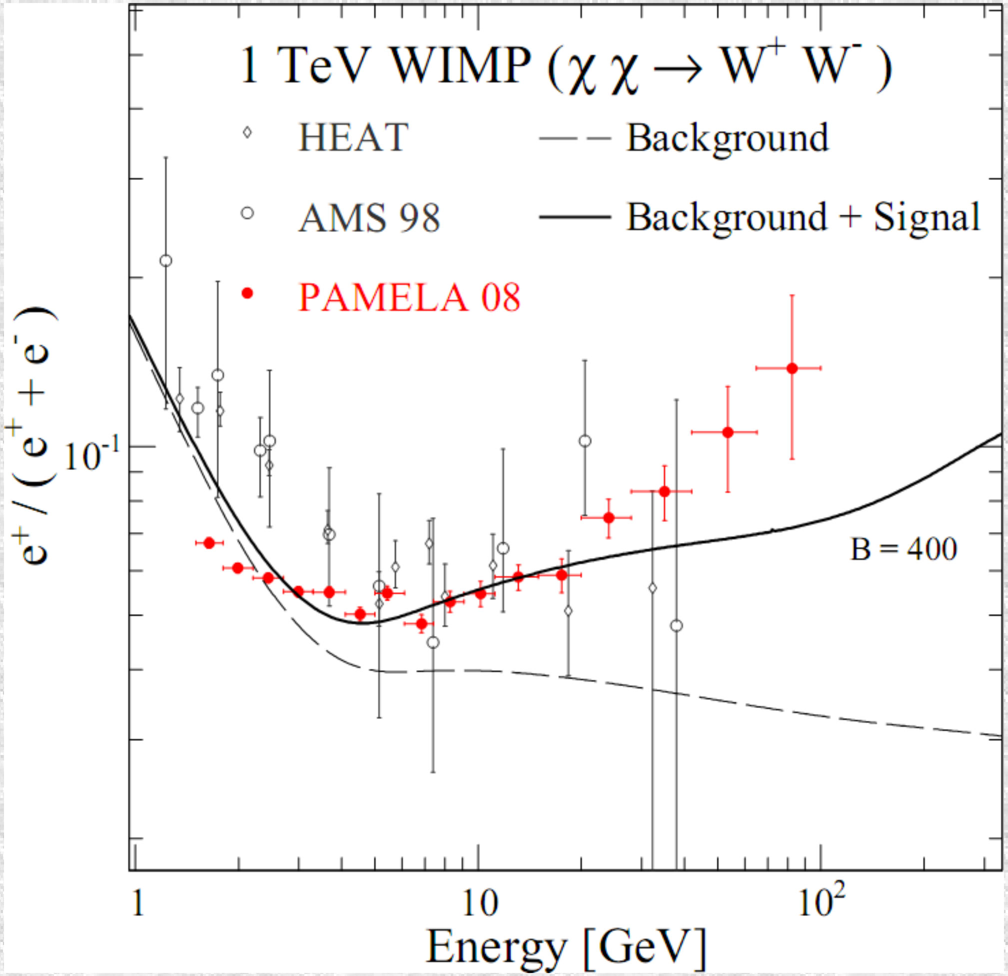
$$J(\psi) = \frac{1}{8.5\,\mathrm{kpc}}\left(\frac{1}{0.3\,\mathrm{GeV\,cm}^{-3}}\right)^2 \int_{\text{line of sight}} ds\,\rho^2[\boldsymbol{r}(s,\psi)]$$

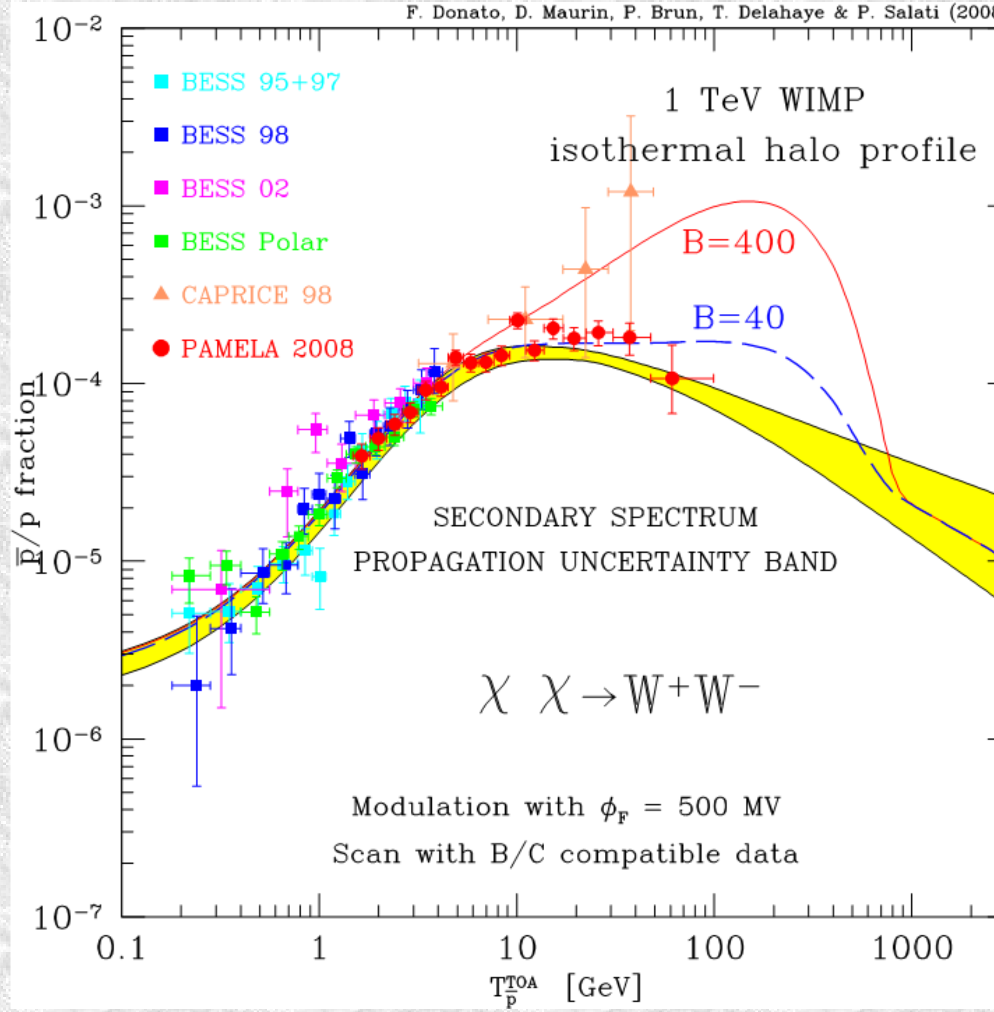
over a spherical region of solid angle $\Delta\Omega$, centered on $\psi = 0$.



Predicted γ-ray fluxes for (top to bottom) $m_X = 0.4, 0.6, 0.8,$ and 1 TeV and $\bar{J}\,(10^{-3}) = 500$

Galactic Positron Fraction Excess



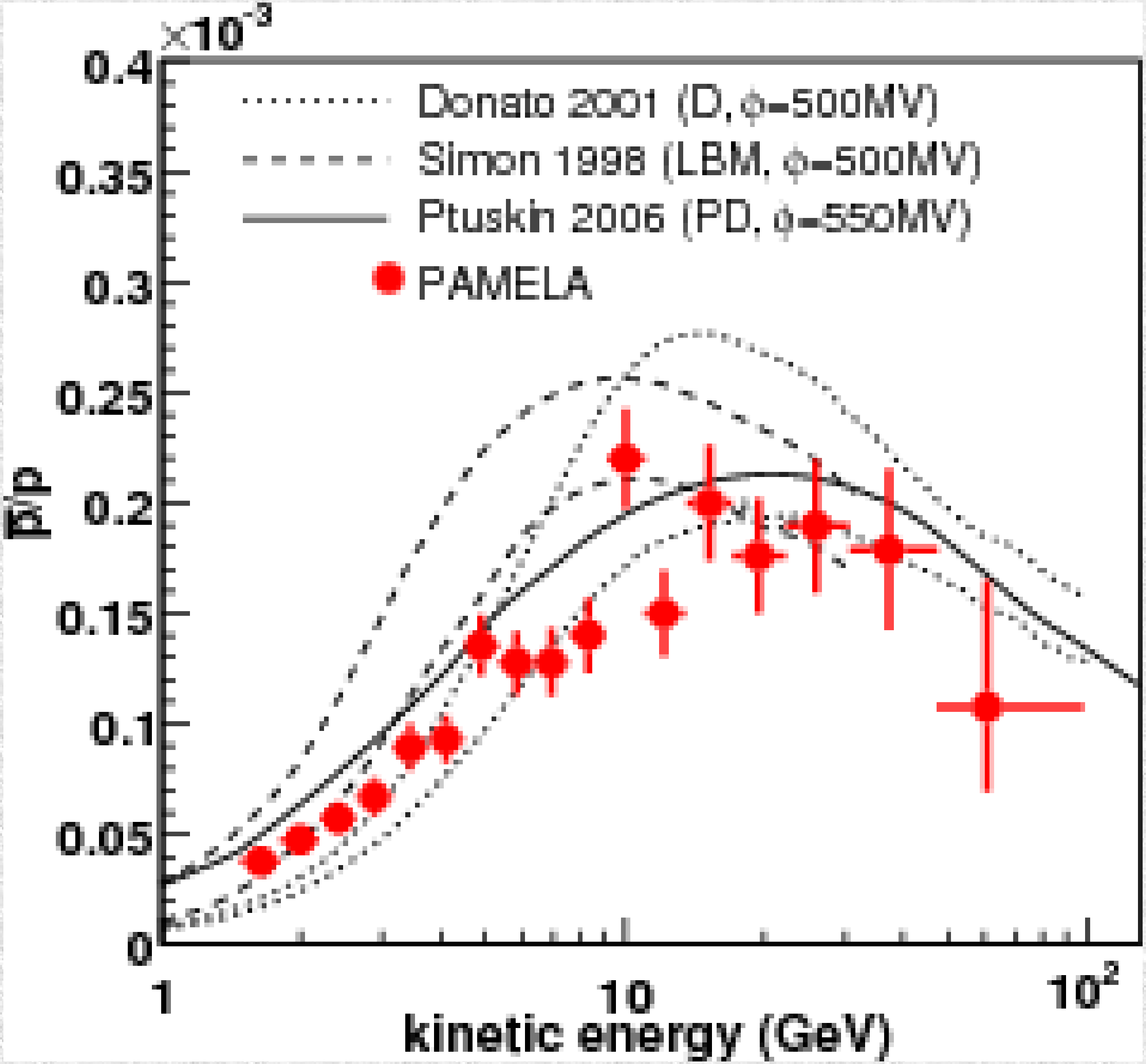


Positron fraction: Excess beyond expected secondary production from homogeneous cosmic ray source distribution

Antiproton fraction: No significant enhancement beyond expected secondary production by cosmic rays

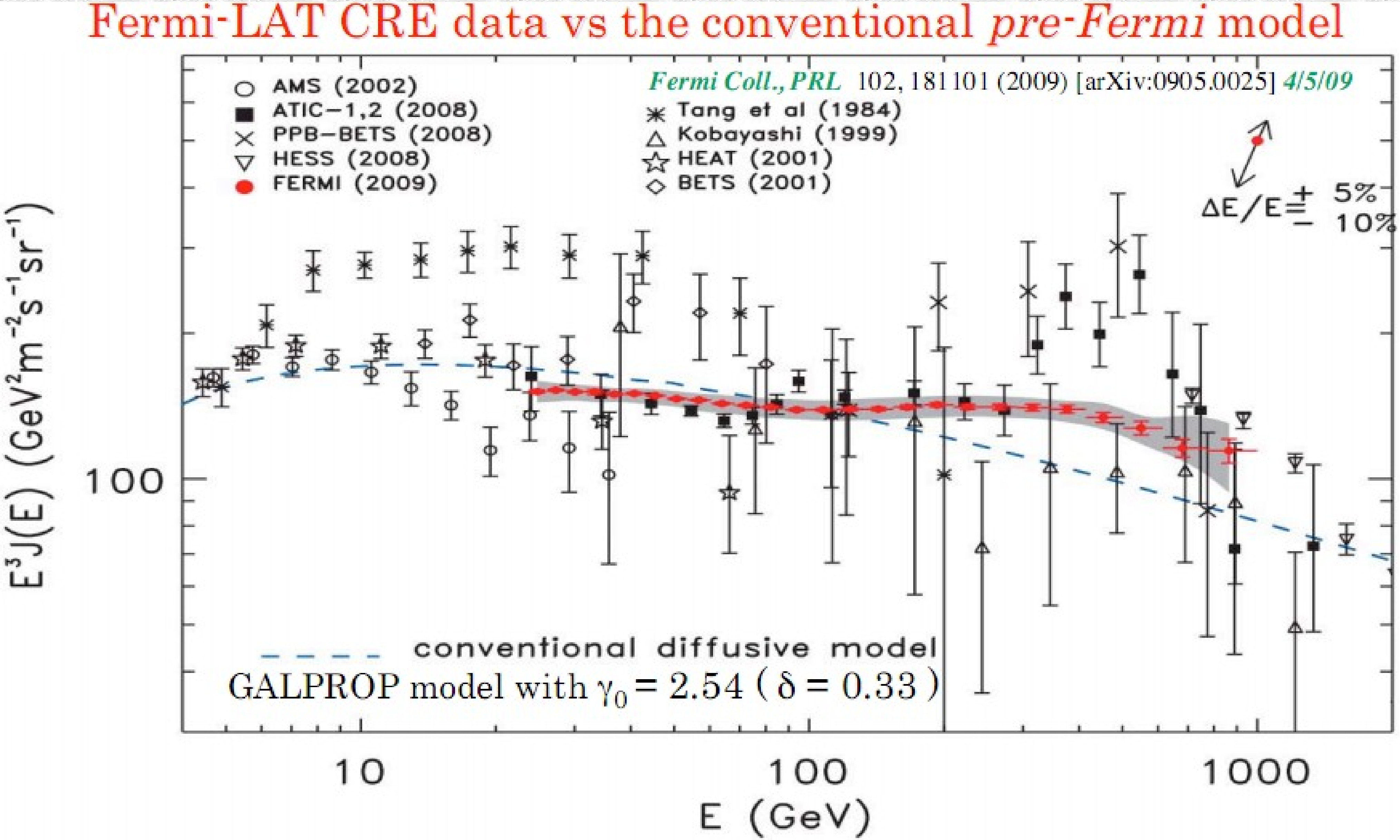
Donato et al., Phys.Rev.Lett.102, 071301 (2009)

But no significant enhancement of anti-proton fraction observed:



Pamela data, Adriani et al., Phys.Rev.Lett.102, 051101 (2009)

Galactic Electron+Positron Excess



Problems interpreting Positron Excess by annihilating Dark Matter

Requires annihilation cross section a „boost factor“ 100--1000 higher than the natural cross section required for dark matter produced by thermal freeze-out:

Option 1: small-scale dark matter clumps with large over-densities. Does not seem Consistent with astrophysics

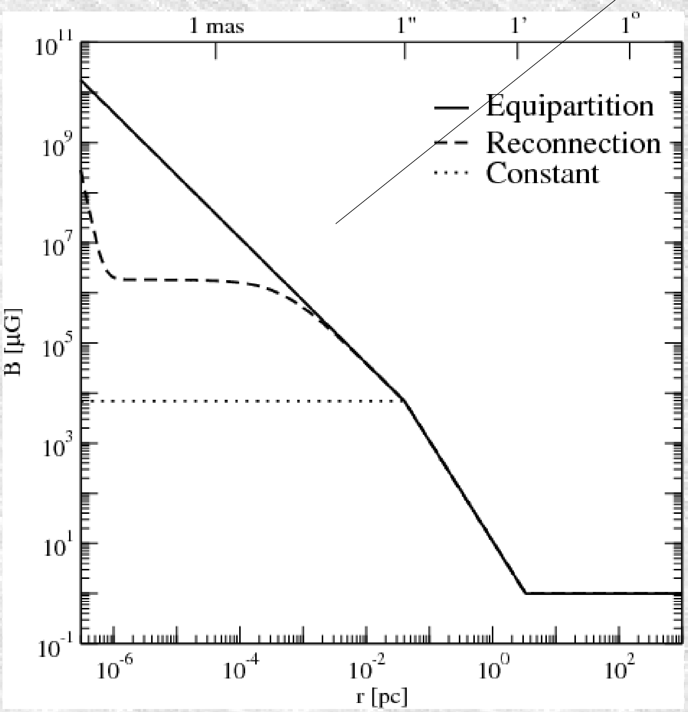
Option 2: “Sommerfeld enhancement“ at small relative velocities. Is not predicted In Universal Extra Dimension scenario.

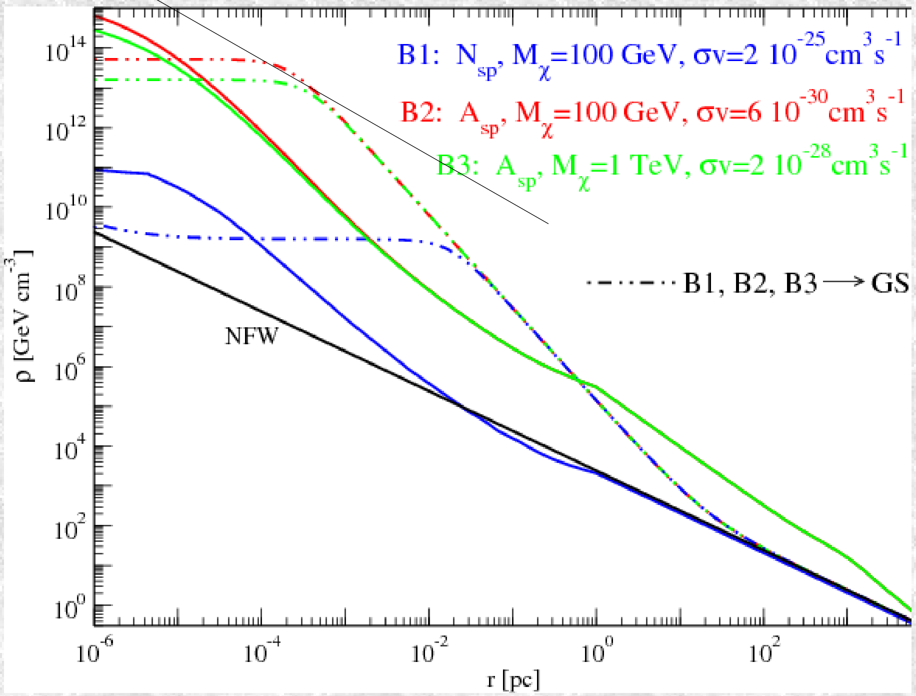
Both annihilation and decay scenarios require „leptophilic“ dark matter.

Some Constraints on Dark Matter Models from Galactic Centre Fluxes

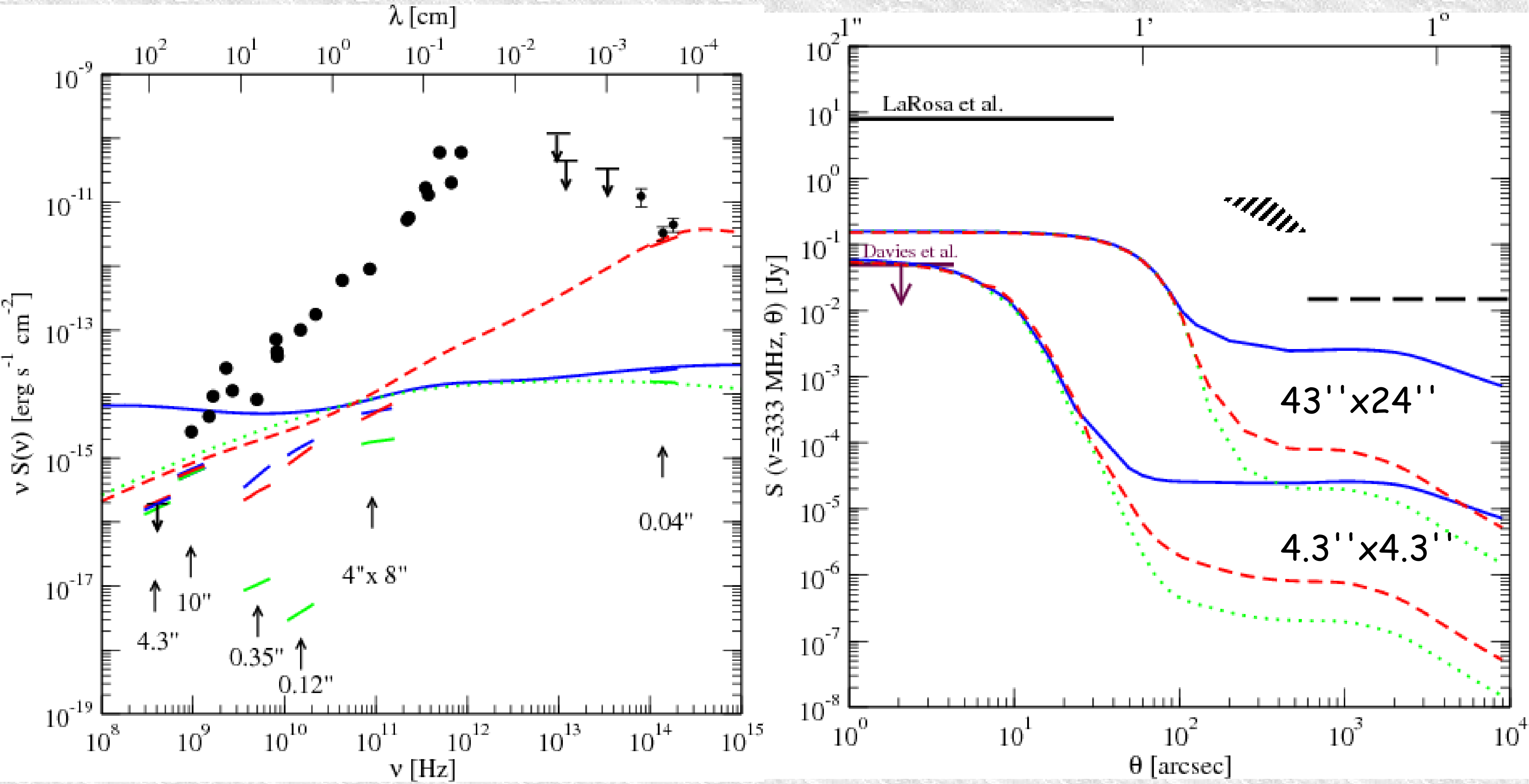
An example for annihilation scenarios from Regis and Ullio, Phys.Rev.D78, 043505 (2008):

	M_χ	σv	ann. mode	B	ρ
B1	100 GeV	$9 \cdot 10^{-26}$ cm^{3}s^{-1}	$b - \bar{b}$	Equipart.	N_{sp}
B2	100 GeV	$3 \cdot 10^{-30}$ cm^{3}s^{-1}	$b - \bar{b}$	Reconnect.	A_{sp}
B3	1 TeV	10^{-28} cm^{3}s^{-1}	$b - \bar{b}$	Constant	A_{sp}



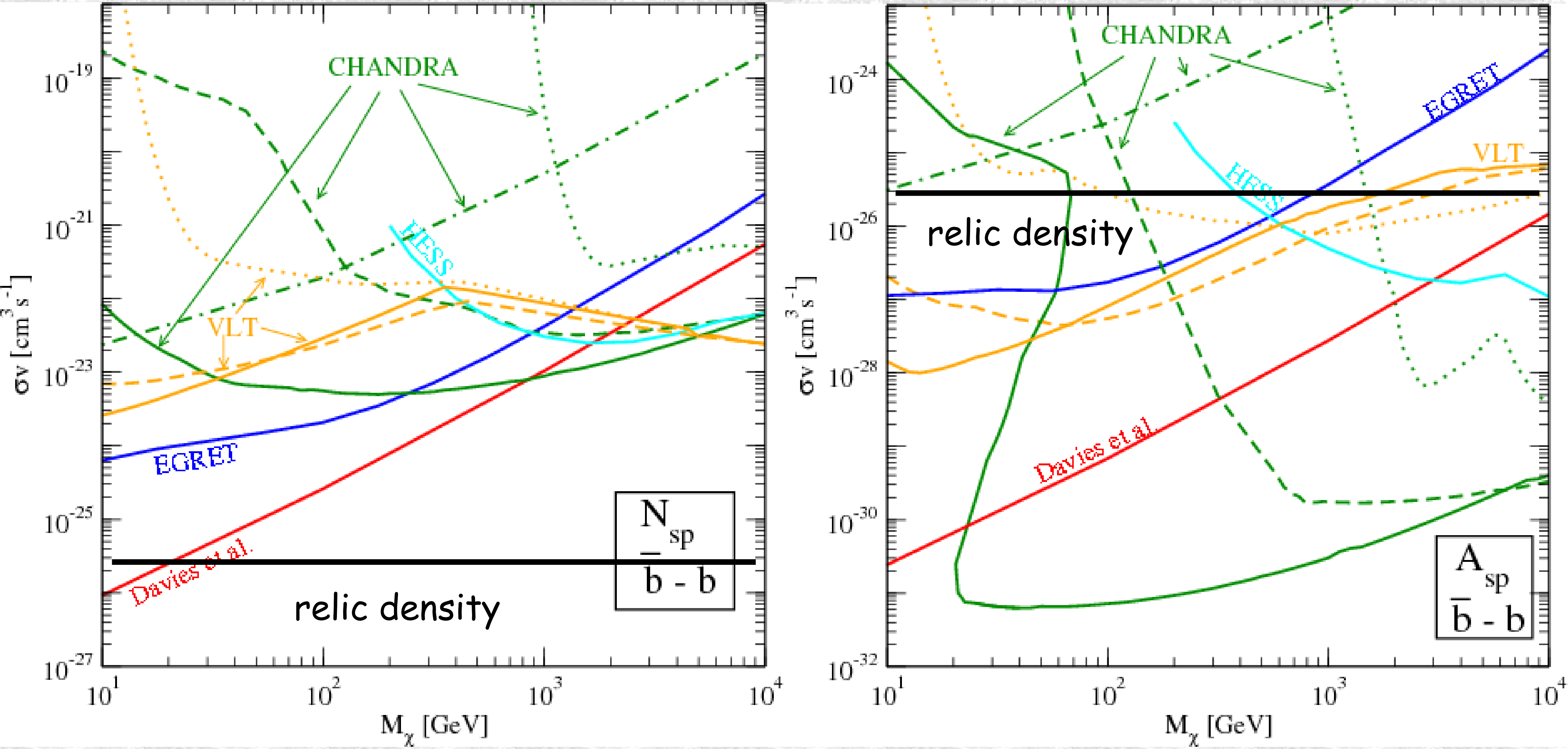


Comparison of observed and predicted fluxes close to the Galactic Centre:



Regis, Ullio, Phys.Rev.D78, 043505 (2008)

=> **Constraints**



Regis, Ullio, Phys.Rev.D78, 043505 (2008)

However, besides strong dependence on dark matter and magnetic field distribution close to Galactic Centre, both observed and predicted fluxes are largest in the centre, but best constraint may come from other directions, especially for decaying dark matter.

=> Generalizations to all-sky signals desired

Propagation Models

Galactic propagation is described by solving the diffusion-convection-energy loss equation:

$$\underbrace{\partial_t n = \nabla\left(D_{xx}\nabla n}_{\text{spatial diffusion}} - \underbrace{\boldsymbol{v}_c n}_{\text{convection}}\right) + \underbrace{\partial_p\left(p^2 D_{pp}\partial_p \frac{n}{p^2}\right)}_{\text{reacceleration}} - \partial_p\left[\underbrace{\dot{p}\, n}_{\text{energy loss}} - \underbrace{\frac{p}{3}(\nabla\cdot\boldsymbol{v}_c n)}_{\text{adiabatic compression/expansion}}\right] + \underbrace{Q(\boldsymbol{r}, p)}_{\text{source term}}$$

This equation is solved in a cylindrical slab geometry with suitable boundary Conditions.

Out of the resulting electron/positron distribution one can compute synchrotron emission (and also inverse Compton scattering) along any line of sight.

Parameters

Definition of diffusion coefficients:

$$D_{xx} = \frac{v}{c_0} D_0 \left(\frac{E/Z}{\mathrm{GV}} \right)^{\delta}$$

$$D_{pp} = \frac{4p^2 v_A^2}{3\delta(4-\delta^2)(4-\delta)D_{xx}}$$

Considering 5 different models:

Model	δ§	D_0 [kpc^2/Myr]	R [kpc]	L [kpc]	V_c [km/s]	dV_c/dz km/s/kpc	V_a [km/s]
MIN	0.85/0.85	0.0016	20	1	13.5	0	22.4
MED	0.70/0.70	0.0112	20	4	12	0	52.9
MAX	0.46/0.46	0.0765	20	15	5	0	117.6
DC	0/0.55	0.0829	30	4	0	6	0
DR	0.34/0.34	0.1823	30	4	0	0	32

Dark Matter Halo Profiles

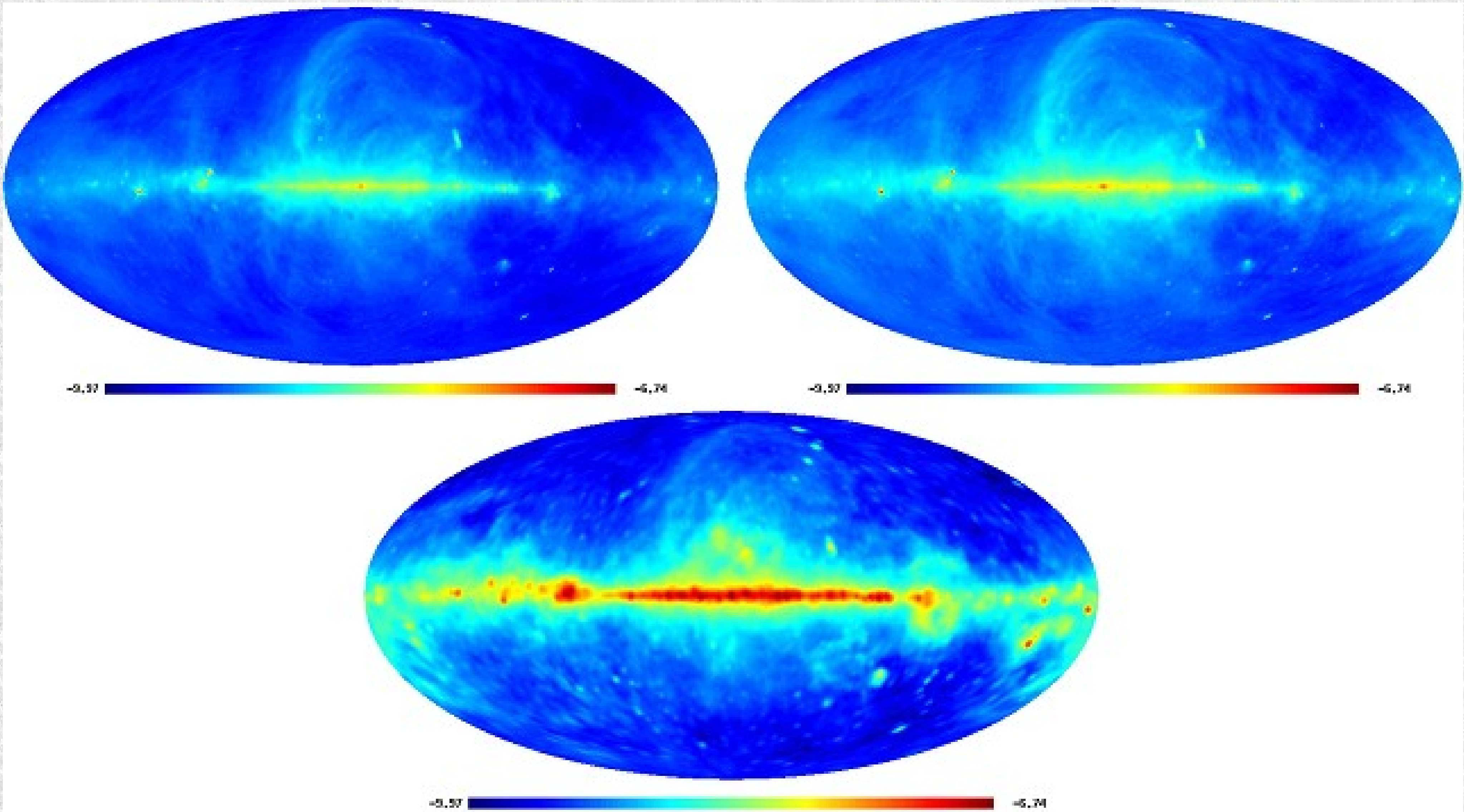
The radial profile is parametrized as:

$$\rho_X(r) = \frac{\rho_0}{(r/r_0)^\delta \left[1+(r/r_0)^\alpha\right]^{(\beta-\gamma)/\alpha}}$$

Considering 3 different halo models:

model	α	β	γ	R(kpc)
Kra	2	3	0.4	10
Iso	2	2	0	3.5
NFW	1	3	1	20

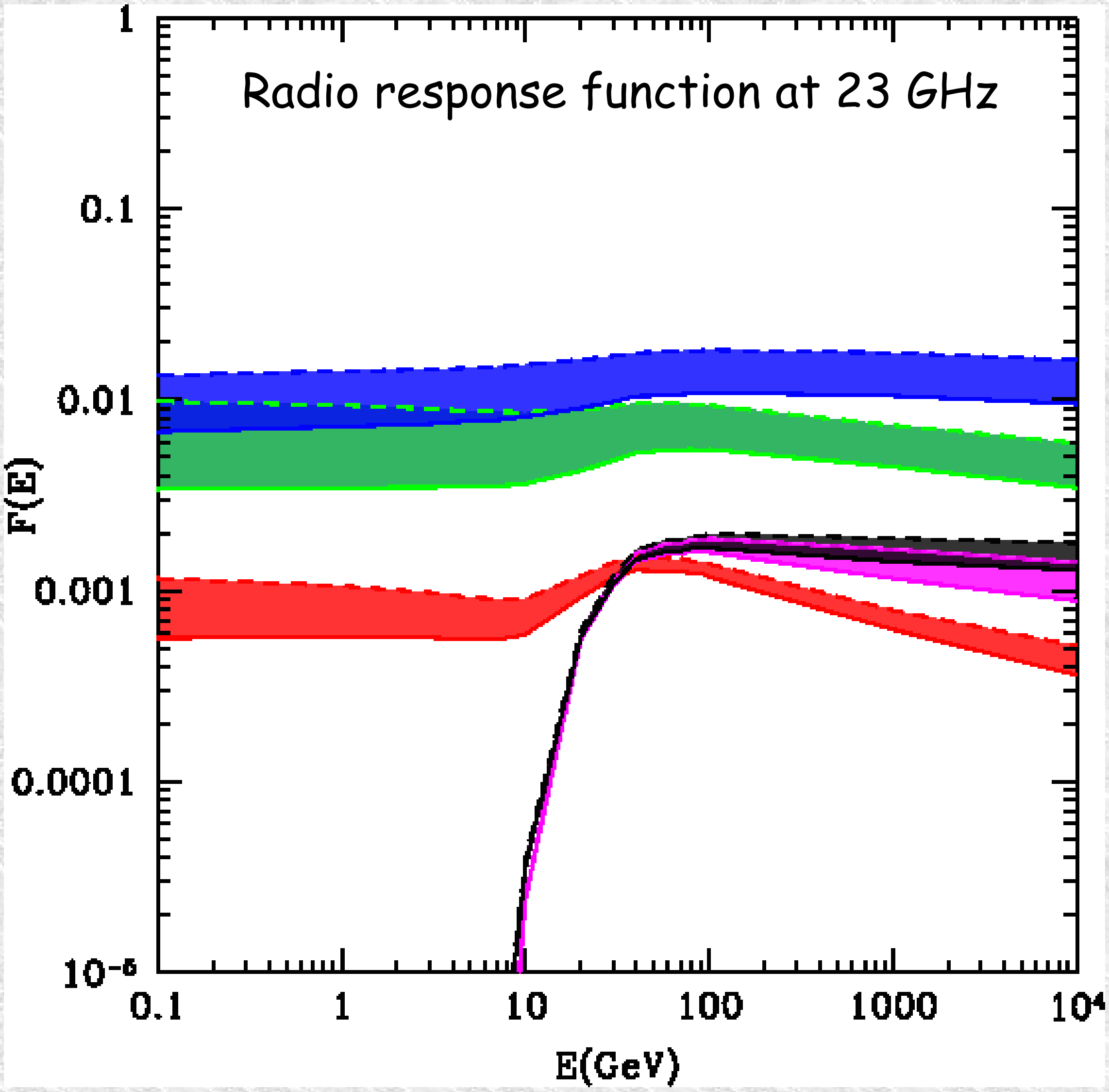
Radio maps at 408 Mhz, 1.42 Ghz, and 23 GHz



Comparison with prediction of signal for electrons/positrons of given energy allows to define a **response function**

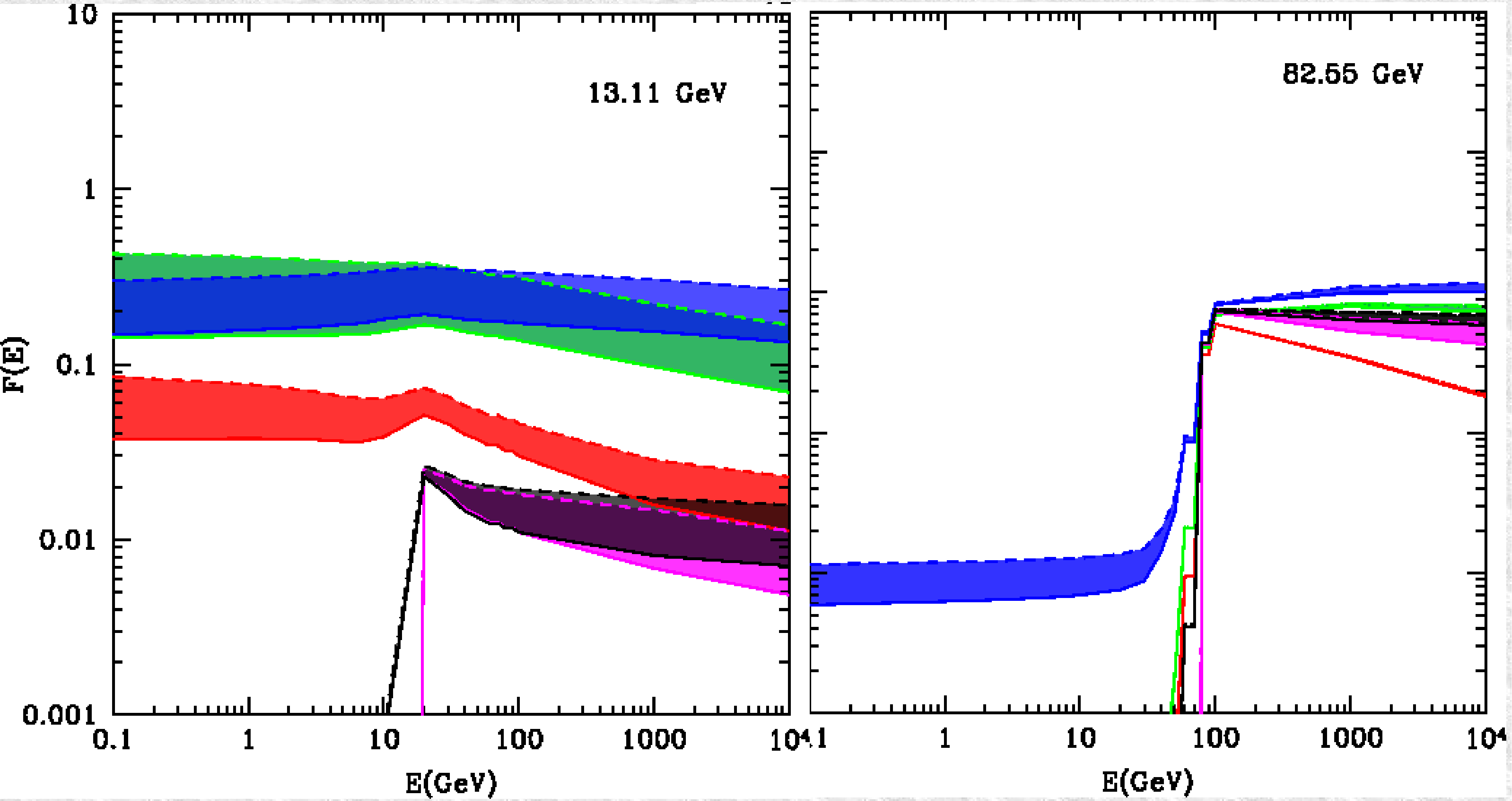
Synchrotron Response Functions
In Annihilating Dark Matter Scenarios

$$\int_{m_e}^{m_X} dE_0 F_r(\Omega, \nu; E_0) \frac{dN_+}{dE_0} \leq \left(\frac{3\times 10^{-26}\,\mathrm{cm^3\,s^{-1}}}{\langle \sigma v \rangle} \right) \left(\frac{m_X}{100\,\mathrm{GeV}} \right)^2$$



Positron Response Functions In Annihilating Dark Matter Scenarios

$$\int_{m_e}^{m_X} dE_0 F_p(E;E_0)\frac{dN_+}{dE_0} \le \left(\frac{3\times 10^{-26}\,\mathrm{cm}^3\mathrm{s}^{-1}}{\langle \sigma v\rangle}\right)\left(\frac{m_X}{100\,\mathrm{GeV}}\right)^2$$



Conclusions

1.) In toroidal compactification scenarios Kaluza Klein gravitons are produced copiously in hot neutron stars where temperatures of ~ 30 MeV are reached. They remain gravitationally bound outside the neutron star and their decays would produce observable gamma ray signals.

2.) Comparison with observations implies that if fundamental gravity scale is ~ TeV then either n > 4 extra dimensions must exist or compactification topology must be more complex than a torus.

3.) If Kaluza Klein parity is conserved, the lightest Kaluza Klein Particle (LKP) is a good dark matter candidate in Universal Extra Dimension models with unique predictions of cross sections as function of mass. Models producing the correct relic density are not yet constrained by accelerators, but are constrained by indirect detection signals