Theoretical introduction to mini black holes

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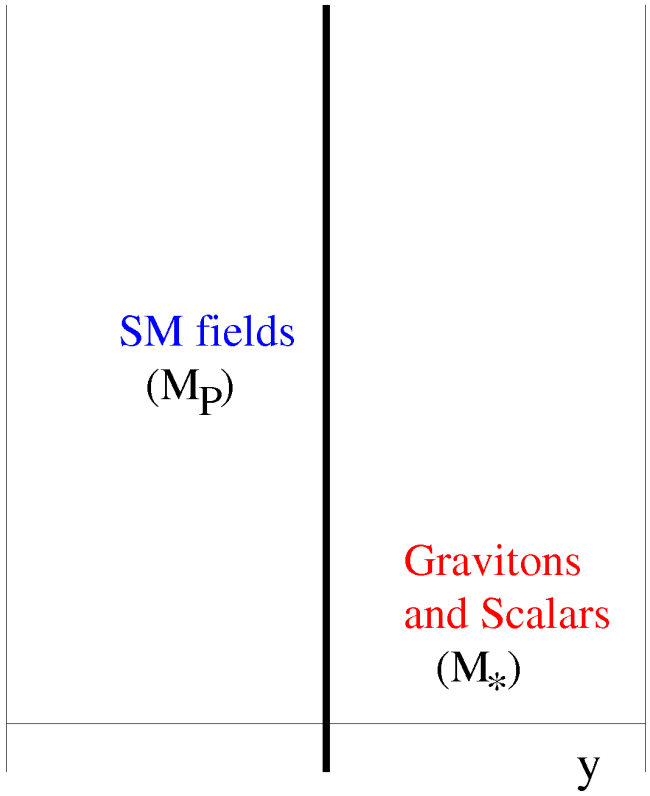
Outline

1. Modelling Mini Black Holes
2. Formation of Mini Black Holes
3. Evolution of Mini Black Holes
4. Conclusions

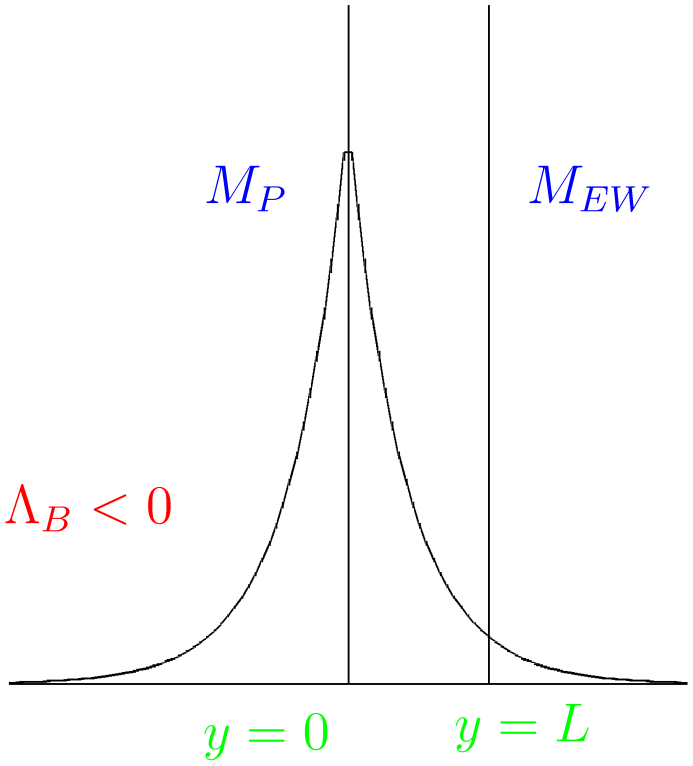
Brane worlds

ADD model



$$M_P^2 \sim M_*^{n+2} R^n$$

RS I model

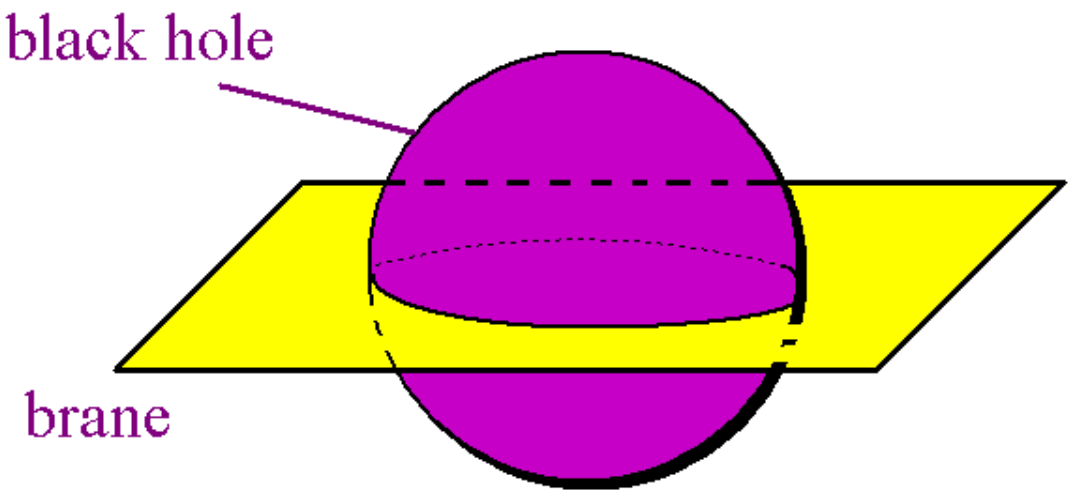


$$M_P^2 = \frac{M_*^3}{k}\left(1 - e^{-2kL}\right)$$

[Figures taken from Kanti, `arXiv:0802.2218 [hep-th]`]

Modelling mini black holes in ADD

- Metric of higher-dimensional black holes in general relativity is known [Myers and Perry, *Annals Phys.* **172**, 304 (1986)]
- Take a 'slice' through a higher-dimensional black hole to give a brane black hole



Modelling mini black holes in ADD

Myers-Perry higher-dimensional black hole

$$
\begin{aligned}
ds^2 &= \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 + \frac{2a\mu \sin^2\theta}{\Sigma r^{n-1}} dt\, d\varphi - \frac{\Sigma}{\Delta_n} dr^2 - \Sigma\, d\theta^2 \\
&\quad - \left(r^2 + a^2 + \frac{a^2 \mu \sin^2\theta}{\Sigma r^{n-1}}\right) \sin^2\theta\, d\varphi^2 - r^2 \cos^2\theta\, d\Omega_n^2
\end{aligned}
$$

where

$$
\Delta_n = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \qquad \Sigma = r^2 + a^2 \cos^2\theta
$$

Black hole mass M and angular momentum J:

$$
M = \frac{(n+2)\, A_{n+2} \mu}{16\pi G_{4+n}}, \qquad J = \frac{2aM}{n+2}
$$

Modelling mini black holes in ADD

Slice of Myers-Perry black hole

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\end{aligned}
$$

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$$
\Delta_n = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \qquad \Sigma = r^2 + a^2 \cos^2\theta
$$

and n is the number of extra dimensions.

Usual Kerr black hole

Set $n = 0$ in the above metric

Modelling mini black holes in RS

- Exact solution known for black holes on a 2+1-dimensional brane
  [Emparan et al, hep-th/9911043]
- Perturbative solutions in RS I for small black hole on the brane
  [Karasik et al, gr-qc/0309076]
- Small black hole solutions found numerically
  [Kudoh et al, gr-qc/0301089]
- Large static black holes on the brane do not exist
  - must be time-dependent
  [Emparan et al, hep-th/0206155]
- Large static black holes should exist but may be dynamically unstable
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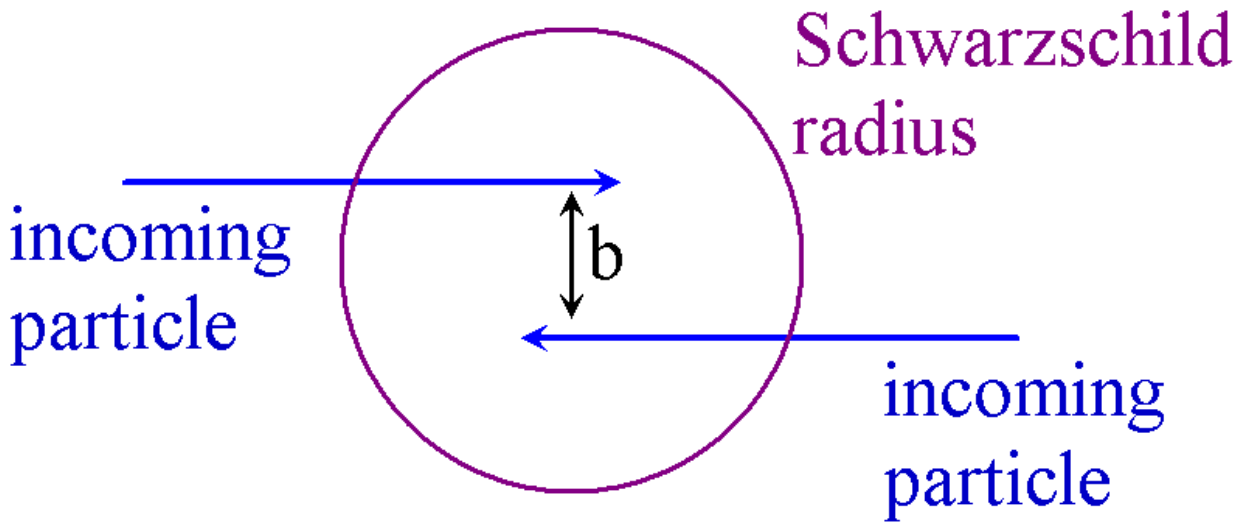
Particle collisions and the hoop conjecture

Two particles with centre-of-mass energies greater than M_*, and impact parameter b:



4D hoop conjecture [Thorne, *Magic Without Magic* 231 (1972)]

A black hole is formed when a mass M gets compacted into a region whose circumference in every direction is less than $2\pi r_H(M)$

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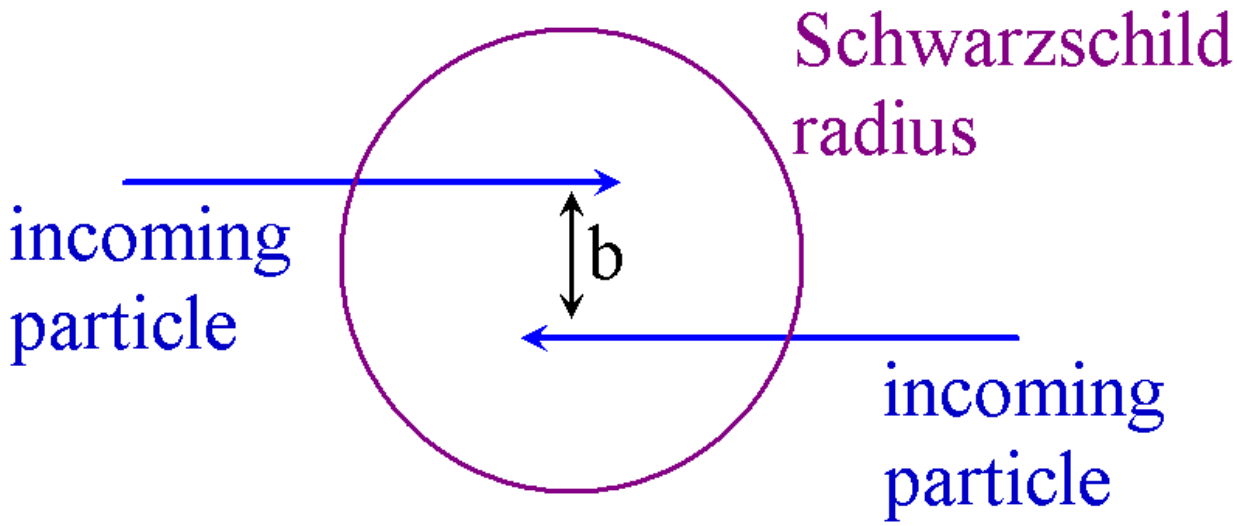

Hyperhoop conjecture [Ida and Nakao, gr-qc/0204082]

A black hole is formed when a mass M gets compacted into a region whose $D-3$-dimensional volume in every direction is less than $\alpha_D M$

Modified hyperhoop conjecture required when extra dimensions are compactified [Yoo et al, arXiv:0906.0689 [gr-qc]]

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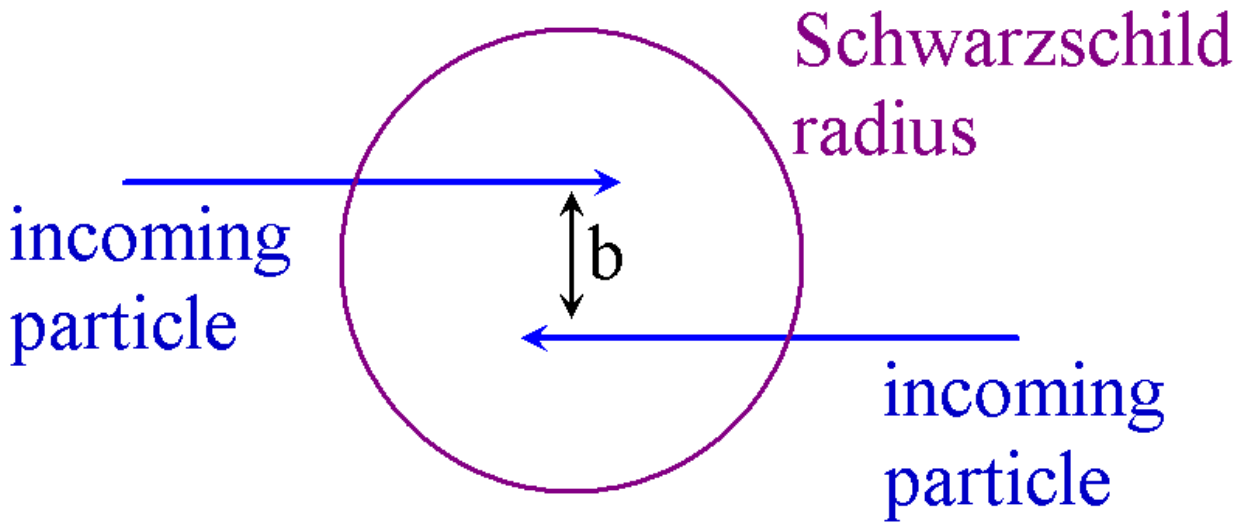


Compton wavelength [Meade and Randall, arXiv:0708.3017 [hep-ph]]

Compton wavelength of colliding particle of energy $E/2$ must lie within the Schwarzschild radius:

$$4\pi/E < r_h(E)$$

Therefore $E/M_* \gtrsim 10$ in order for black holes to form

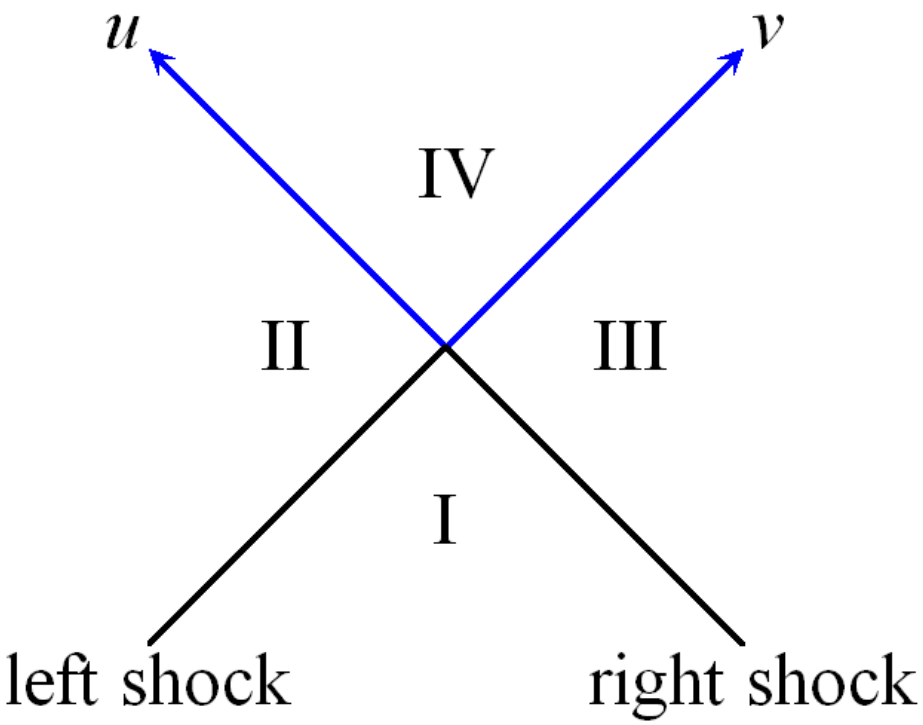
Modelling the formation of mini black holes

- Classical process
- Model particles as infinitely-boosted black holes
- Search for apparent horizon
- Maximum impact parameter b_{max}
- Parton-level production cross-section

$$\sigma \sim \pi b_{max}^2 \sim 3\pi r_h^2$$

[Yoshino and Rychkov, `hep-th/0503171`]

Aichelberg-Sexl shock waves
[Aichelberg and Sexl, GRG **2** 303 (1971)]



Open issues in mini black hole formation

- Recent work on full numerical relativity simulations in four dimensions, with $\gamma \sim 1 - 3$
  [Shibata et al, `arXiv:0810.4735 [gr-qc]`]
  [Sperhake et al, `arXiv:0907.1252 [gr-qc]`]
- Higher-dimensional numerical relativity simulations yet to be developed
  [Shibata and Yoshino, `arXiv:0907.2760 [gr-qc]`]
- Effects of brane tension, thickness and extra dimension geometry yet to be completely modelled
- Quantum gravity effects are not fully understood
  [Giddings and Rychkov, `hep-th/0409131`]
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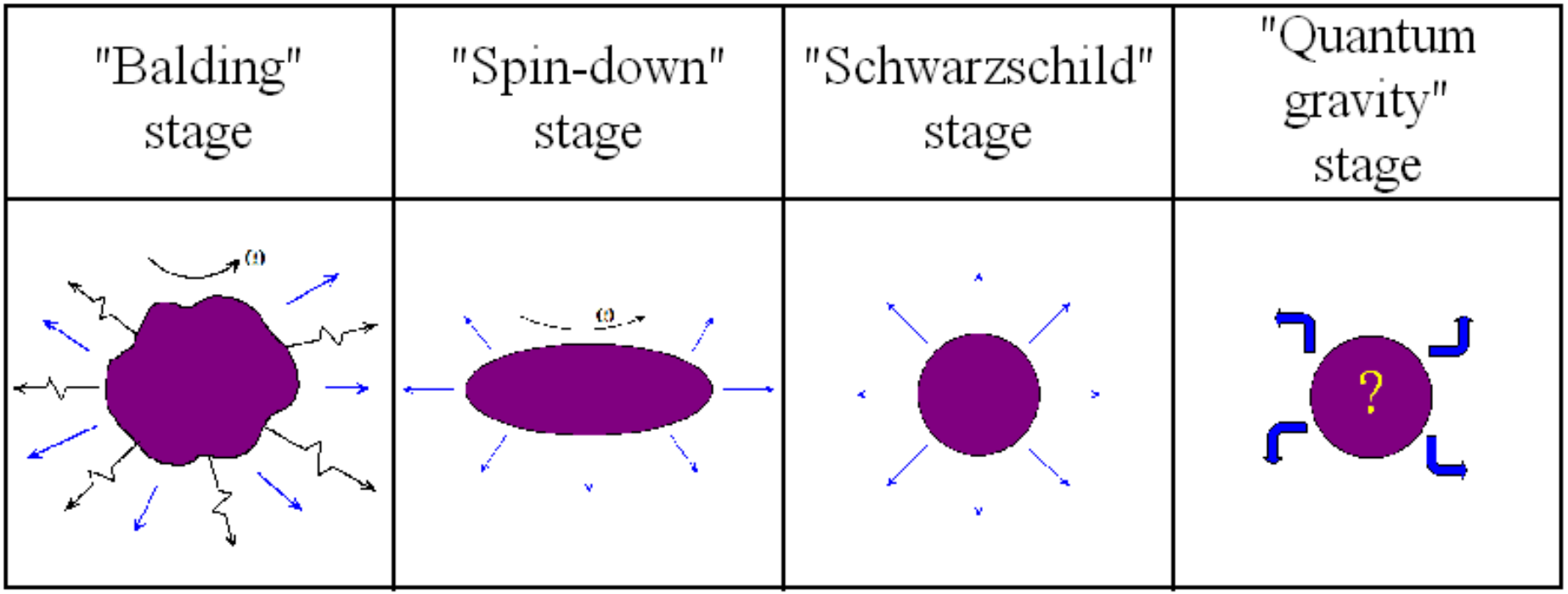
Stages in the evolution of brane black holes

Black holes formed will be rapidly rotating, highly asymmetric, and have gauge field hair

Four stages of subsequent evolution:



Balding phase

Limits on amount of energy lost in gravitational radiation

- Colliding shock waves: $\leq 30\% \quad (n = 0), \qquad \leq 40\% \quad (n = 7)$
  [Yoshino and Rychkov, `hep-th/0503171`]
- Four-dimensional numerical relativity: $\leq 35\% \quad (n = 0)$
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Angular momentum of formed black hole

- Angular momentum of black holes with $n > 1$ potentially unbounded
- Limited by maximum impact parameter
- Colliding shock waves: $j \sim 0.93 \quad (n = 1)$
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Quantum fields on brane black holes

Quantum field theory in curved space-time

- Black hole geometry is fixed and classical
- Quantum fields (scalars, fermions, gauge bosons, gravitons) propagate on this background

Quantum field modes

- "Master" equation for fields of spin 0, $\frac{1}{2}$, 1 and 2 on Kerr [Teukolsky, *Phys. Rev. Lett.* **29** 1114 (1972); *Astrophys. J.* **185** 635 (1973)]
- Expand field Ψ in terms of modes of frequency ω:

$$\Psi = \sum_{\omega \ell m} R_{s\omega\ell m}(r) S_{s\omega\ell m}(\theta) e^{-i\omega t} e^{im\varphi}$$

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Computing Hawking radiation

Differential emission rates, integrated over all angles:

$$\frac{d^2}{dt\,d\omega}\begin{pmatrix} N \\ E \\ J \end{pmatrix} = \frac{1}{4\pi}\sum_{\text{modes}} \frac{|\mathcal{A}_{s\omega\ell m}|^2}{e^{\tilde{\omega}/T_H} \mp 1}\begin{pmatrix} 1 \\ \omega \\ m \end{pmatrix}$$

where $\tilde{\omega} = \omega - m\Omega_H$

Grey-body factor $|\mathcal{A}_{s\omega\ell m}|^2$

- Emitted radiation is not precisely thermal
- Interaction of emitted quanta with gravitational potential around the black hole

What we know about the Hawking radiation phases

"Spin-down" phase

- Brane emission - scalars, fermions, gauge bosons done
- Bulk emission - scalars done
- Graviton emission - partial results only

"Schwarzschild" phase

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"Black holes radiate mainly on the brane"

[Emparan, Horowitz and Myers, hep-th/0003118]

Ratio of bulk/brane emission for scalars, $n = 2$

$a_* = 0.0$	$a_* = 0.2$	$a_* = 0.4$	$a_* = 0.6$	$a_* = 0.8$	$a_* = 1.0$
19.9%	18.6%	15.3%	11.7%	9.0%	7.1%

[Casals et al, arXiv:0801.4910 [hep-th]]

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Open issues in mini black hole evolution

- Complete computation of graviton radiation
  - Requires full gravitational perturbation equations for rotating higher-dimensional black holes
  - Recent work only for tensor-type gravitational perturbations with $n \geq 3$
    [Doukas et al, `arXiv:0906.1515 [hep-th]`]
    [Kanti et al, `arXiv:0906.3845 [hep-th]`]
- Effects of brane tension, thickness and warped extra dimensions not fully modelled
- Realistic evolution will be a stochastic process
  - Individual quanta emitted rather than a continuum
  - Black hole will recoil, possibly even come off the brane
  - Black hole may not have time to approach thermal equilibrium between emissions
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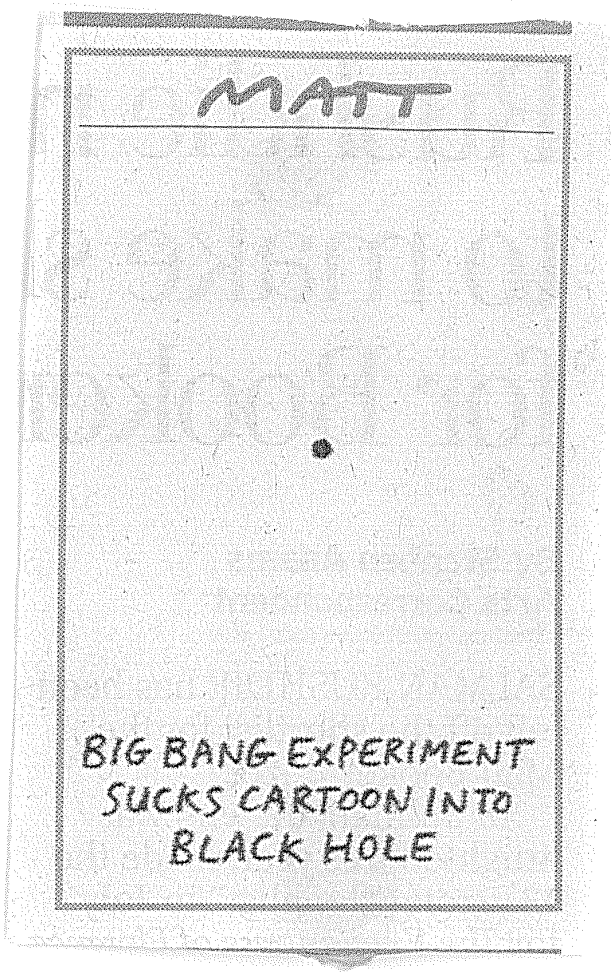
Conclusions

- General metric for black hole on the brane remains elusive
- Formation of mini black holes merits further study, particularly including spin and charge
- Semi-classical Hawking radiation analysis complete apart from graviton modes for rotating black hole
- End-point of black hole evolution not fully understood



Recent reviews

[Kanti, arXiv:0903.2147 [hep-th]]
[Gregory, arXiv:0804.2595 [hep-th]]
[Kanti, arXiv:0802.2218 [hep-th]]