

Lecture 12 – Instabilities I

ACCELERATOR PHYSICS

HT 2016

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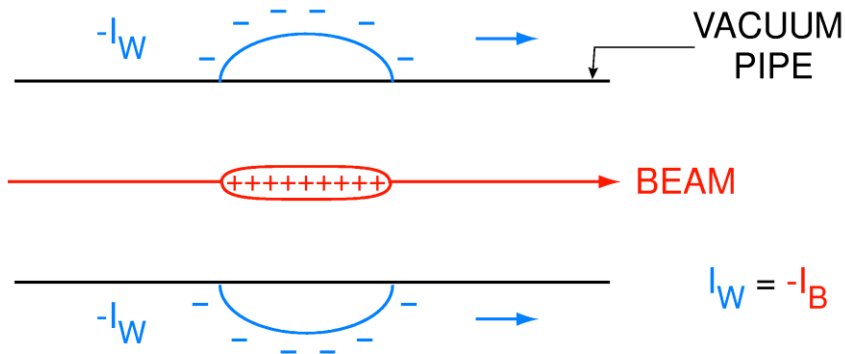
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Instabilities I

- ◆ 1. **General Comment on Instabilities**
- ◆ 2. **Negative Mass Instability**
- ◆ 3. **Driving terms (second cornerstone)**
- ◆ 4. **A cavity-like object is excited**
- ◆ 5. **Equivalent circuit**
- ◆ 6. **Above and below resonance**
- ◆ 7. **Laying the bricks in the wall (row 1)**
- ◆ 8. **Laying the bricks in the wall (row 2)**
- ◆ 9. **By analogy with the negative mass**

Impedance of the wall



Wall current I_w due to circulating bunch
 Vacuum pipe not smooth, I_w sees an
IMPEDANCE :

Resistive = in phase.
 capacitive lags,
 inductive leads

$$\text{Impedance } Z = Z_r + iZ_i$$

$$\text{Induced voltage } V \sim I_w Z = -I_B Z$$

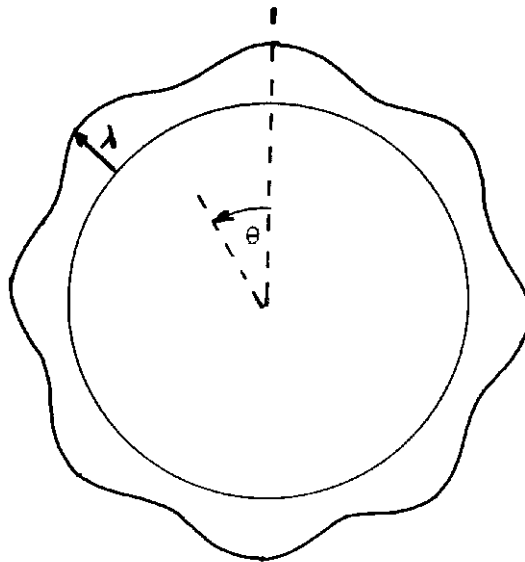
V acts back on the beam
 \Rightarrow INSTABILITIES
 INTENSITY DEPENDENT

Test: If Initial Small Perturbation is :

INCREASED? **INSTABILITY**
 DECREASED? **STABILITY**

Test on instability

- ◆ **Postulate a simple form for its effect**
- ◆ **Test whether such a pattern is self sustaining.**
 - » examine forces set up by such a pattern
 - » if they reinforce its shape – it is sure to grow exponentially
 - » it starts from some random component of noise.

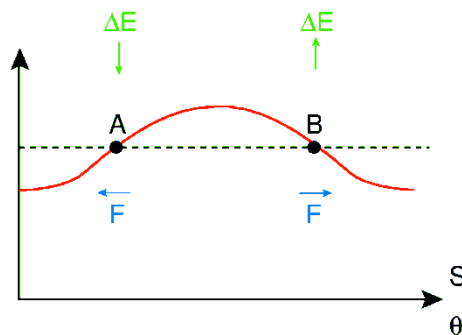
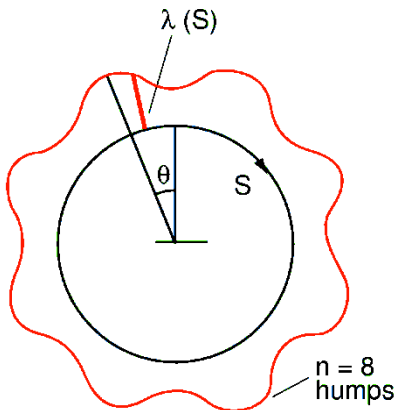


Typical perturbation
Local line density of charge around a synchrotron

Negative Mass Instability

Un-bunched (=coasting) beam in a proton/ion ring, travels around ring with angular frequency ω_0

Line density $\lambda(s)$ [particles/m] is modulated around the synchrotron



Line density modulation

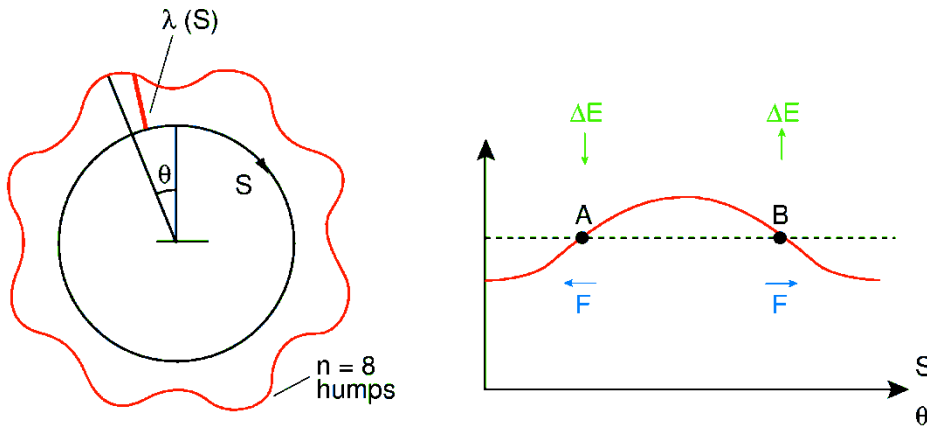
Zooming in one modulation

“mode” with $n=8$ humps

- ◆ **Azimuthal modulation replotted in Cartesian form.**
- ◆ **A finds itself with a larger charge density behind it than in front of it, pushing it forward.**
- ◆ **Conversely particle B will be decelerated by the mountain of charge in front of it.**

$$E = - \left[\frac{e}{4\pi\epsilon_0\gamma^2} \right] \frac{\partial\lambda}{\partial s}$$

"Negative Mass" Instability Qualitative



WILL THE HUMPS INCREASE OR ERODE?

The **self-force** F (proportional to $-\partial\lambda/\partial s$)

$$E = - \left[\frac{e}{4\pi\epsilon_0\gamma^2} \right] \frac{\partial\lambda}{\partial s}$$

Increases energy of particles in B
Decreases energy of particles in A

STABLE

$\gamma < \gamma_t$: if $\Delta E \uparrow$ then $\Delta\omega_0 \uparrow$ A and B move away from the hump **eroding** the mountain

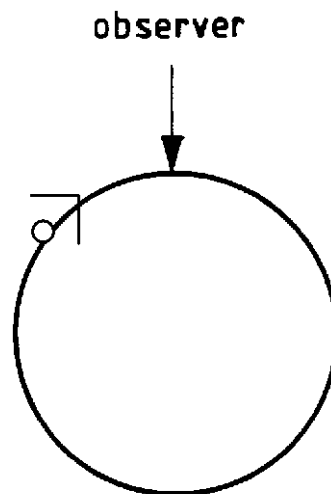
UNSTABLE

$\gamma > \gamma_t$: if $\Delta E \uparrow$ then $\Delta\omega_0 \downarrow$ A and B move towards the hump **enhancing** the mountain

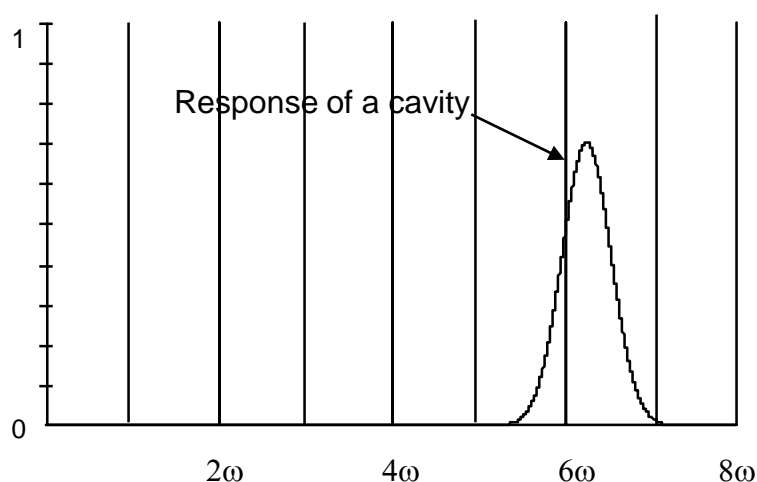
Driving terms (second cornerstone)

- ◆ **Fourier analysis of a circulating delta function bunch of charge passing an observer.**

$$I = \sum I_n e^{in\omega_0 t}$$



- ◆ **Produces a fundamental at the revolution frequency plus all higher harmonics are in equal strength**

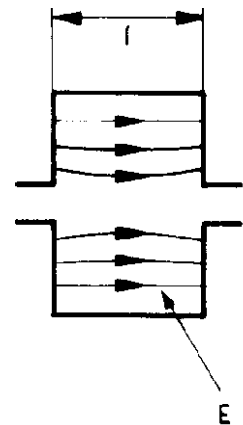


- ◆ **Spectrum from a bunch showing response of an r.f. cavity**

A cavity-like object is excited

- ◆ Local enlargement in the beam tube which can resonate like a cavity
- ◆ Voltage experienced has same form as the current which excites it

$$I = \hat{I} e^{-i\omega t}$$
$$U = \hat{U} e^{-i\omega t}$$



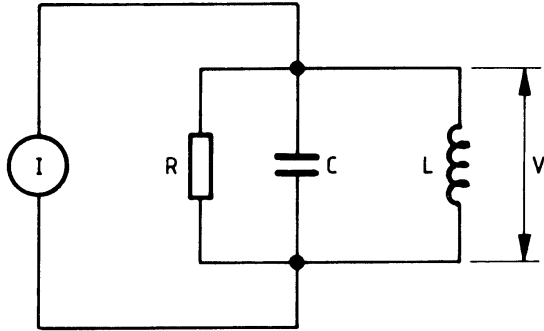
- ◆ Impedance

$$Z = X + iY = \frac{U}{I}$$

- ◆ Relates force on particles to the Fourier component of the beam current which excites the force.
- ◆ A complex quantity
 - REAL if the voltage and current are in phase
 - IMAGINARY if 90 degrees or "i" between voltage and current (L = +, C = -)
 - different from r.f. wave by 90 degrees!

Equivalent circuit

◆ AC resonant circuit



◆ Elementary relations

$$\omega_r = \frac{1}{\sqrt{LC}}$$
$$Q = R\sqrt{\frac{C}{L}} = \frac{R}{L\omega_r} = RC\omega_r$$

◆ Differential equation for voltage and current

$$\ddot{V} + \frac{\omega_r}{Q}\dot{V} + \omega_r^2 V = \omega_r \frac{R}{Q} \dot{I}$$

◆ Solution:

$$V = V_0 e^{-\alpha t} \sin \left[\omega_r \sqrt{1 - \frac{1}{4Q^2}} t + \mu \right]$$

Above and below resonance

$$V = V_0 e^{-\alpha t} \omega s \left[\omega_r \sqrt{1 - \frac{1}{4Q^2}} t + \mu \right]$$

- ◆ Damping rate

$$\alpha = \frac{\omega_r}{2Q}$$

- ◆ Books on AC circuit theory will show that if

$$I = \hat{I} e^{i\omega t}$$

- ◆ Impedance seen by the generator is:

$$Z(\omega) = R \left[\frac{1 - iQ \left(\frac{\omega^2 - \omega_r^2}{\omega\omega_r} \right)}{1 + Q^2 \left(\frac{\omega^2 - \omega_r^2}{\omega\omega_r} \right)^2} \right]$$

- ◆ When ω is below resonance, the reactive component is inductive or positive
- ◆ After crossing zero it becomes negative and capacitive when the driving frequency is above the natural resonance of the oscillator.

Laying the bricks in the wall (row 1)

- ◆ Instead of the line density λ , the beam current I which we write as

$$I = I_0 + I_1 e^{i(n\theta - \Omega t)}$$

n is the number of humps

ω , angular frequency felt by an antenna in the wall

dI/ds drives instability

- ◆ we can ignore constant current and write:

$$\frac{dI}{ds} = \frac{1}{R} \cdot \frac{dI}{d\theta} = \frac{in}{R} I_1 e^{i(n\theta - \Omega t)}$$

$$I_1 e^{i(n\theta - \Omega t)} = \frac{R}{in} \frac{dI}{ds}$$

- ◆ A cavity-like object presents an impedance, $X + iY$, at this frequency- hence the V/turn:

$$\begin{aligned} (X + iY) I_1 e^{-\Omega t} &= R \left(\frac{X + iY}{in} \right) \frac{dI}{ds} \\ &= -\frac{iR}{n} (X + iY) \frac{dI}{ds} = \frac{R}{n} (Y - iX) \frac{dI}{ds} \end{aligned}$$

Laying the bricks in the wall (row 2)

◆ **Accelerating voltage:**

$$V = \frac{R}{n} (Y - iX) \frac{dI}{ds}.$$

Compare with the acceleration for negative mass instability which was:

$$-\left(\frac{e}{4\pi\epsilon_0\gamma^2} \right) \frac{d\lambda}{ds}$$

◆ **Thus we have the same effect as the negative mass**

Above transition.- Instability when Y is negative, i.e. capacitive.

Inductive impedance causes instability below transition

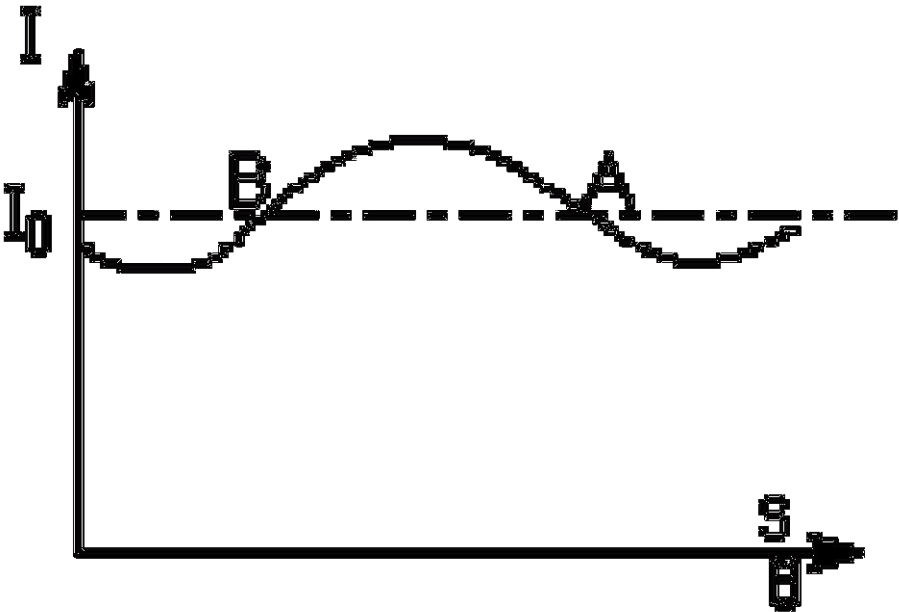
SUMMARY

We have calculated the slope of a harmonic of I

Multiplying by Z gives a voltage

Effect is like negative mass

By analogy with the negative mass



Summary

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