An introduction to Magnets for Accelerators

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John Adams Institute
Accelerator Course

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This is an introduction to magnets as building blocks of a synchrotron / transfer line

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//
// MADX Example 2: FODO cell with dipoles
// Author: V. Ziemann, Uppsala University
// Date: 060911

TITLE,'Example 2: FODO2.MADX';

BEAM, PARTICLE=ELECTRON,PC=3.0;

DEGREE:=PI/180.0;                    // for readability

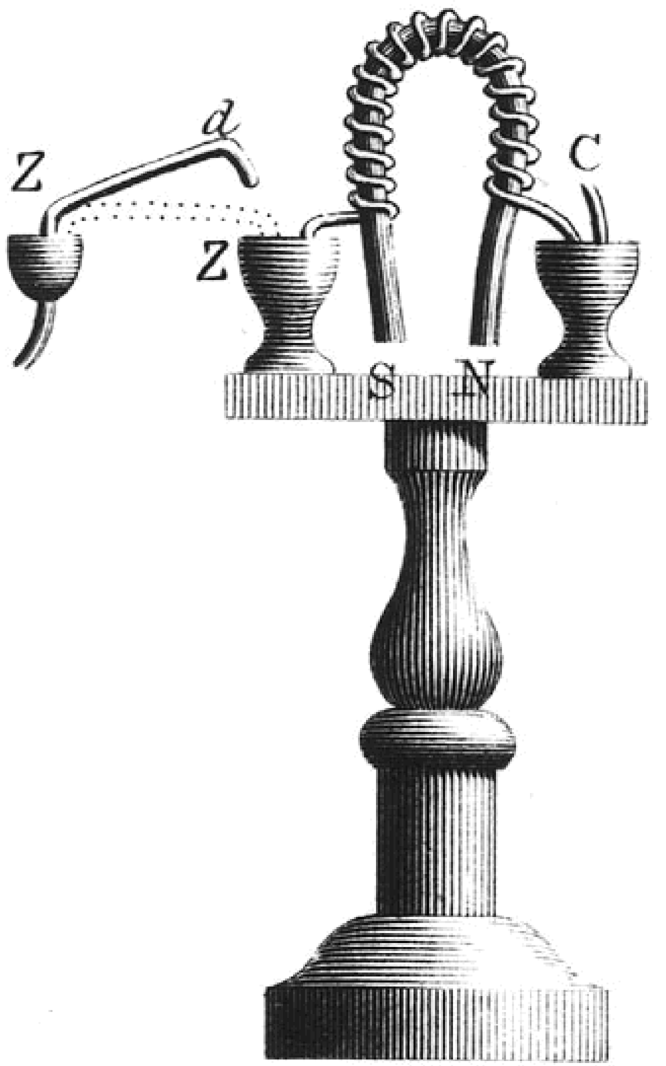
QF: QUADRUPOLE,L=0.5,K1=0.2;         // still half-length
QD: QUADRUPOLE,L=1.0,K1=-0.2;        // changed to full length
B: SBEND,L=1.0,ANGLE=15.0*DEGREE;    // added dipole

FODO: SEQUENCE,REFER=ENTRY,L=12.0;
  QF1:   QF,      AT=0.0;
  B1:    B,       AT=2.5;
  QD1:   QD,      AT=5.5;
  B2:    B,       AT=8.5;
  QF2:   QF,      AT=11.5;
ENDSEQUENCE;
```

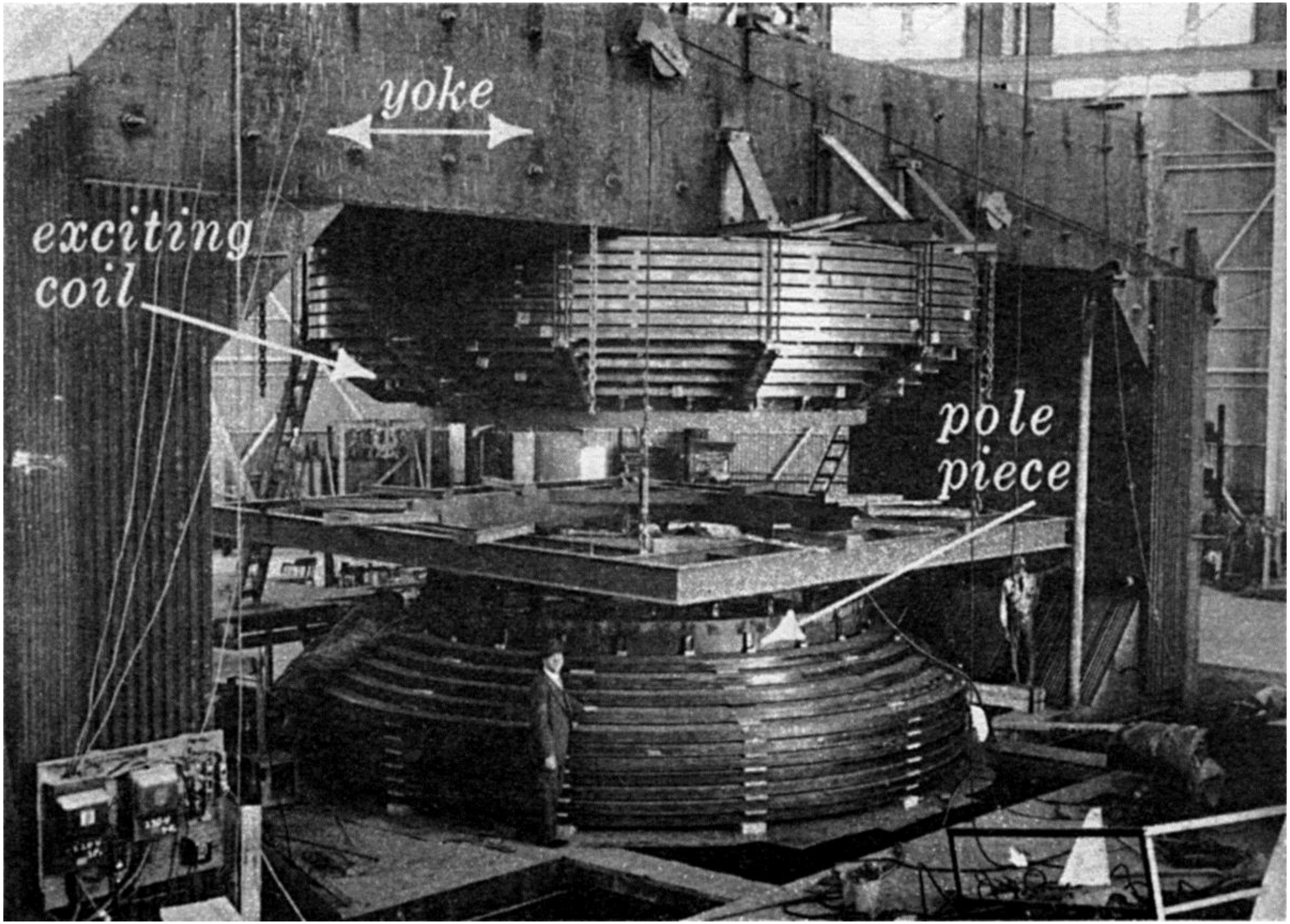
These are a few choices for further reading

1. N. Marks, Magnets for Accelerators, J.A.I. Jan. 2015
2. D. Tommasini, Practical Definitions & Formulae for Normal Conducting Magnets, Sept. 2011
3. Lectures about magnets in CERN Accelerator Schools
4. Special CAS edition on magnets, Bruges, Jun. 2009
5. Superconducting magnets for particle accelerators in U.S. Particle Accelerator Schools
6. J. Tanabe, Iron Dominated Electromagnets
7. P. Campbell, Permanent Magnet Materials and their Application
8. K.-H. Mess, P. Schmüser, S. Wolff, Superconducting Accelerator Magnets
9. M. N. Wilson, Superconducting Magnets

According to history, the first electromagnet (not for accelerators!) was built in England in 1824 by William Sturgeon



The working principle is the same as this large magnet, of the 184'' (4.7 m) cyclotron at Berkeley (picture taken in 1942)



This short course is organized in several blocks

1. Introduction
2. Jargon and mathematical concepts
3. Thought experiment

4. Basics for the design of resistive magnets
5. A glimpse on the design of superconducting magnets

6. Guided magnetic design (with 2D FEM simulations)

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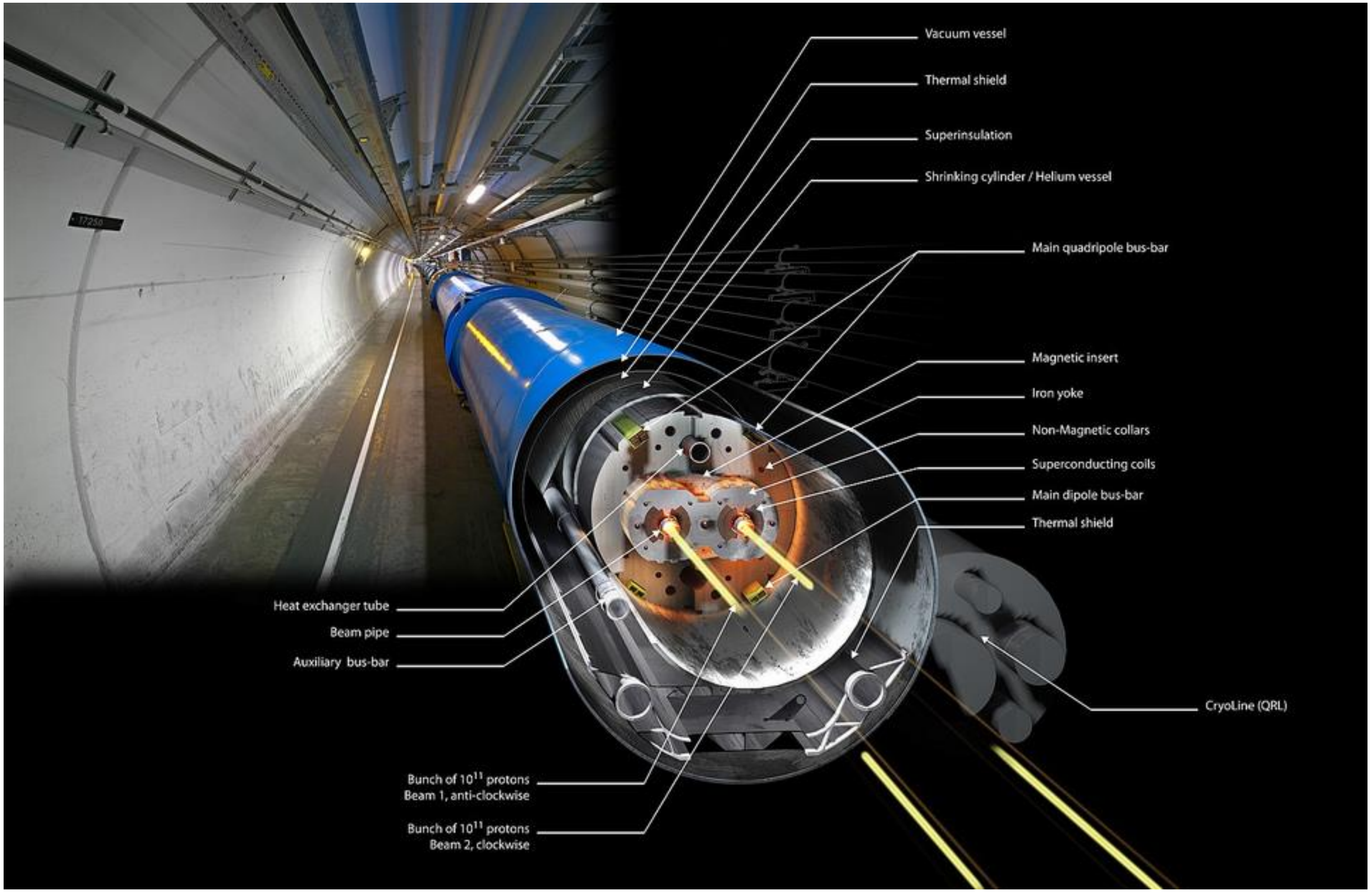
Introduction

There are several types of magnets found in synchrotrons (and transfer lines) – based on what they do to the beam

dipole	solenoid
quadrupole	combined function bending
sextupole	corrector
octupole	skew magnet
kicker	undulator / wiggler

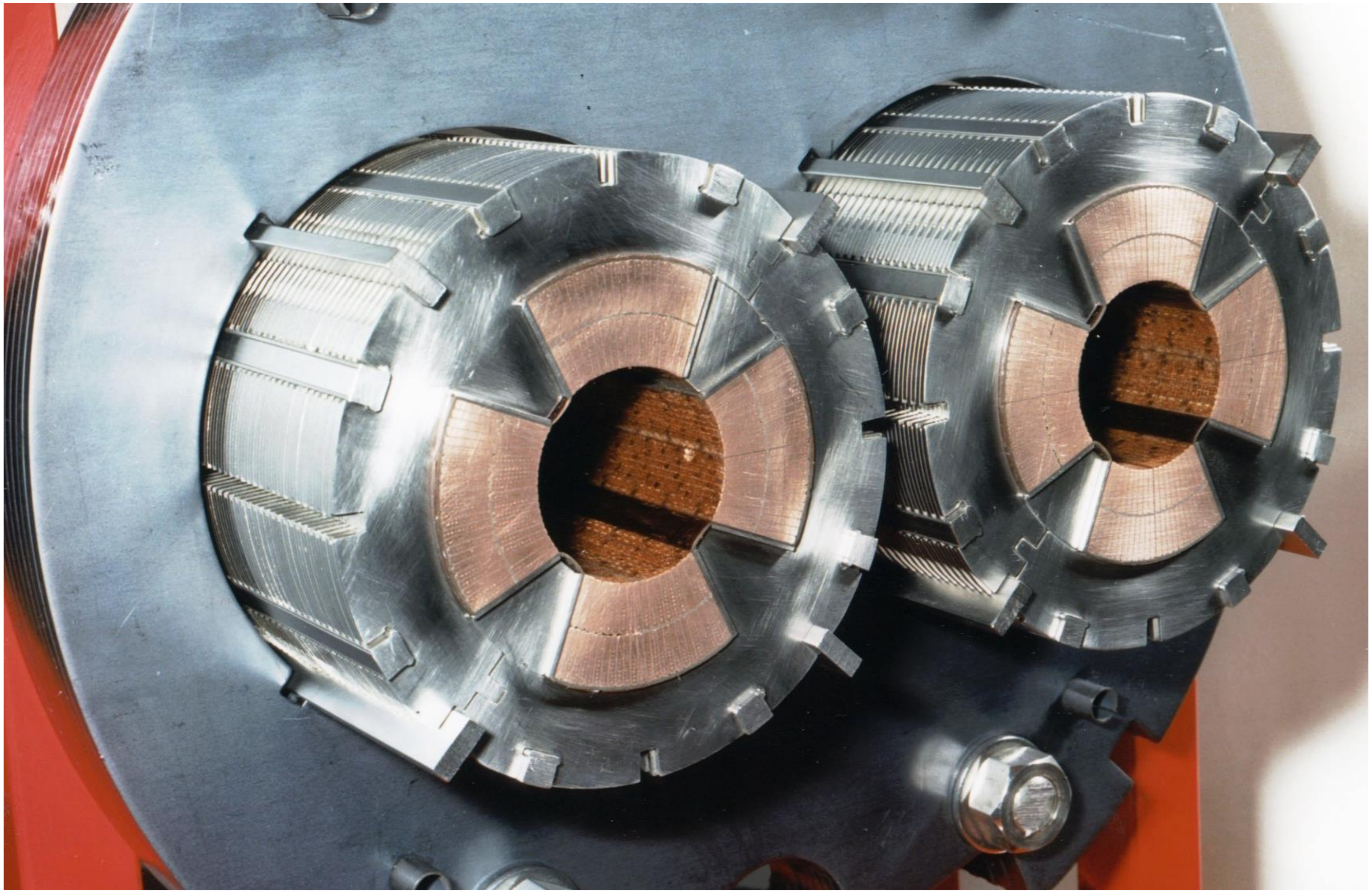
This is a main dipole of the LHC at CERN: 8.3 T × 14.3 m



These are main dipoles of the SPS at CERN: 2.0 T × 6.3 m

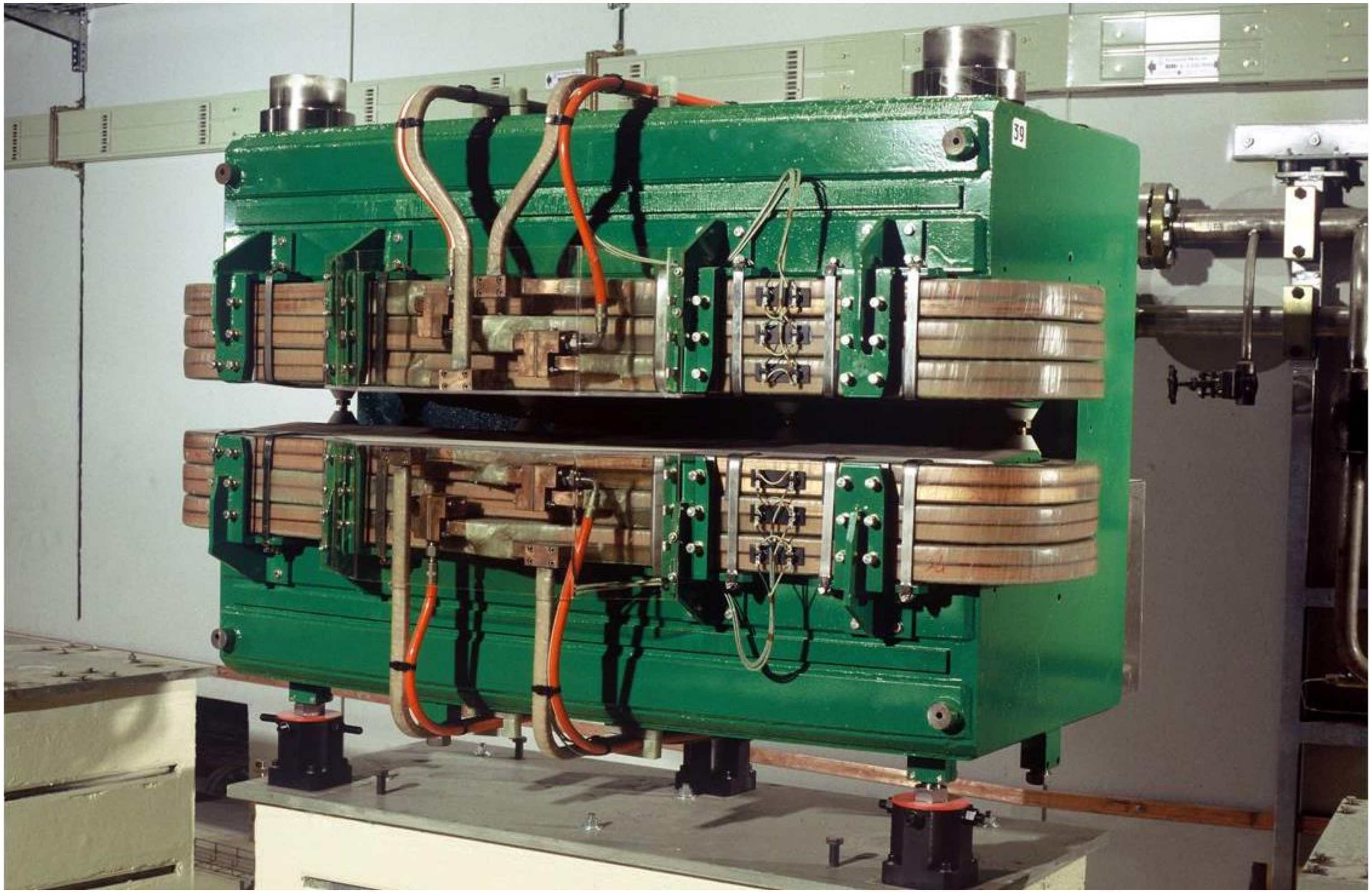
This is a cross section of a main quadrupole of the LHC at CERN:

223 T/m × 3.2 m

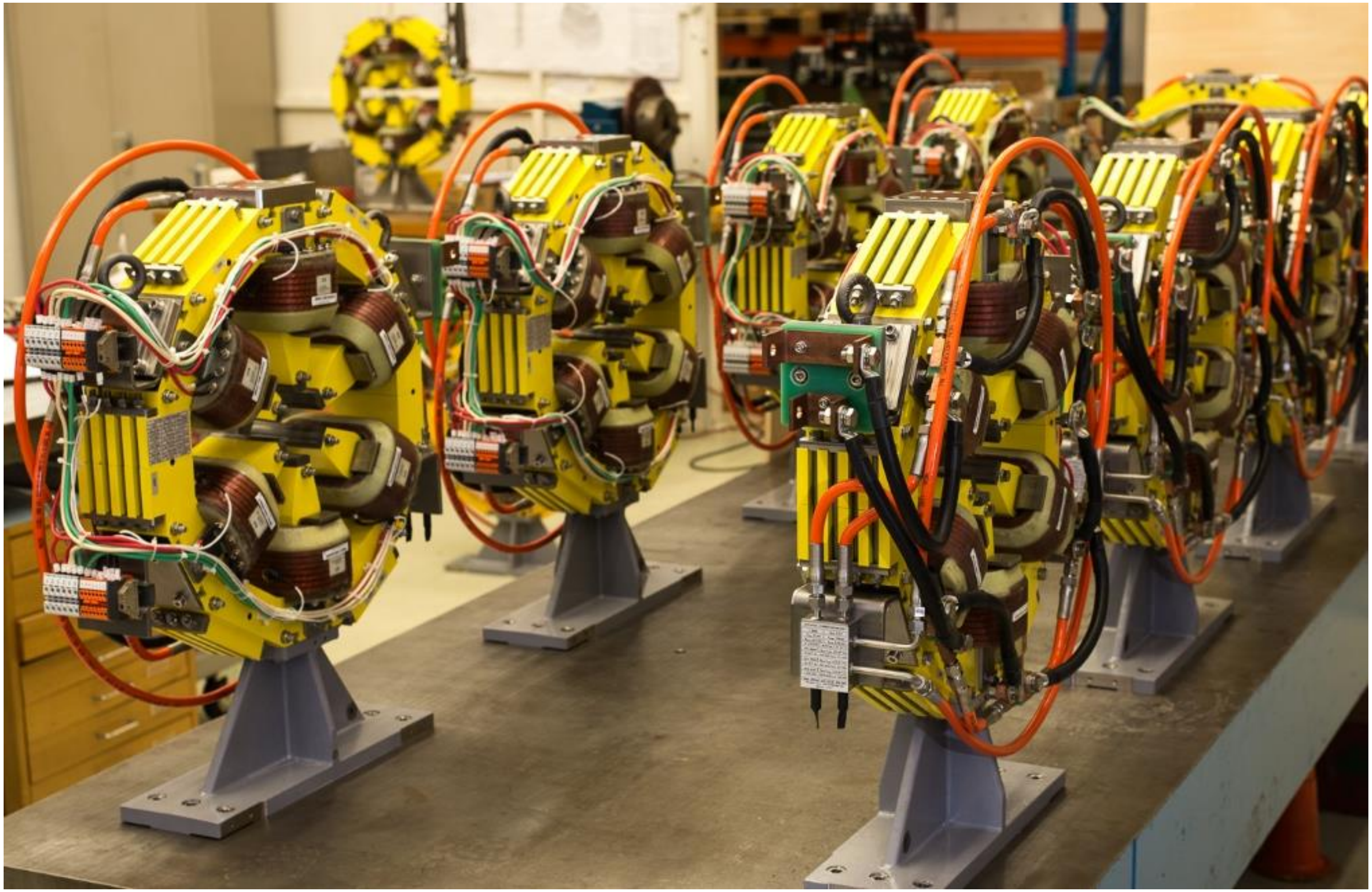


These are main quadrupoles of the SPS at CERN: 22 T/m × 3.2 m

This is a combined function bending magnet of the ELETTRA light source



These are sextupoles (with embedded correctors) of the main ring of the SESAME light source



There are several types of magnets found in synchrotrons and transfer lines – based on technology

electromagnet	permanent magnet
iron dominated	coil dominated
normal conducting (resistive)	superconducting

static	cycled / ramped slow pulsed	fast pulsed

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Jargon and mathematical concepts

Nomenclature

B	magnetic field B field magnetic flux density magnetic induction	T (Tesla)
H	H field magnetic field strength magnetic field	A/m (Ampere/m)
μ_0	permeability of vacuum	$4\pi \cdot 10^{-7}$ H/m (Henry/m)
μ_r	relative permeability	dimensionless
μ	permeability, $\mu = \mu_0 \mu_r$	H/m

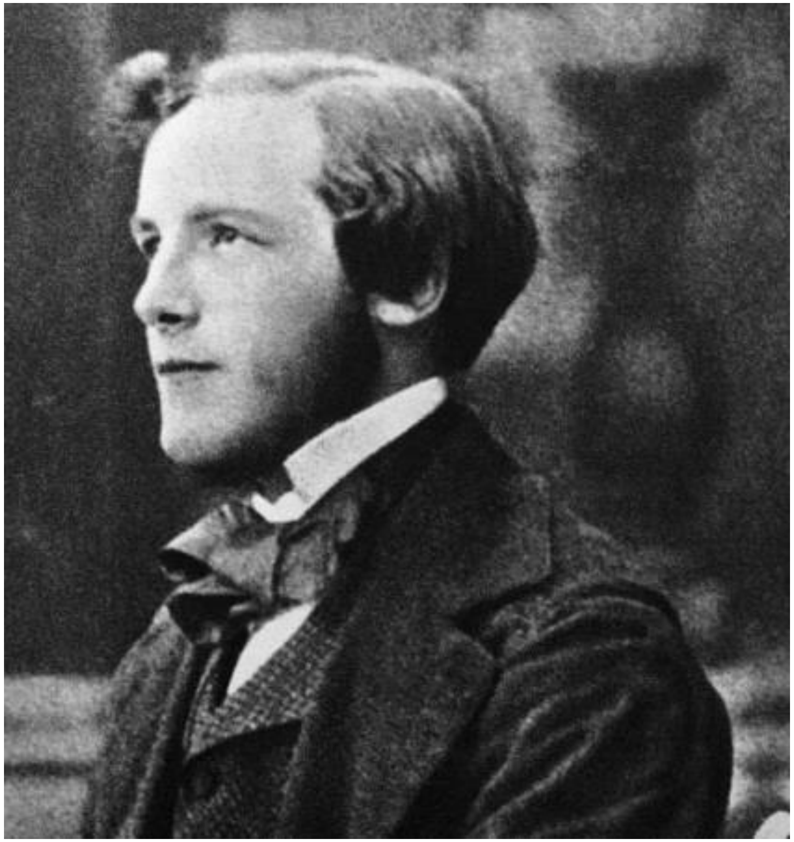
Magnetostatic fields are described by (these versions of) Maxwell's equations, coupled with a law describing the material

$$\operatorname{div} \vec{B} = 0$$

$$\oint_S \vec{B} \cdot \overrightarrow{dS} = 0$$

$$\operatorname{rot} \vec{H} = \vec{J}$$

$$\oint_C \vec{H} \cdot \overrightarrow{dl} = \int_S \vec{j} \cdot \overrightarrow{dS} = NI$$



$$\vec{B} = \mu_0 \mu_r \vec{H}$$

The Lorentz force is the main link between electromagnetism and mechanics

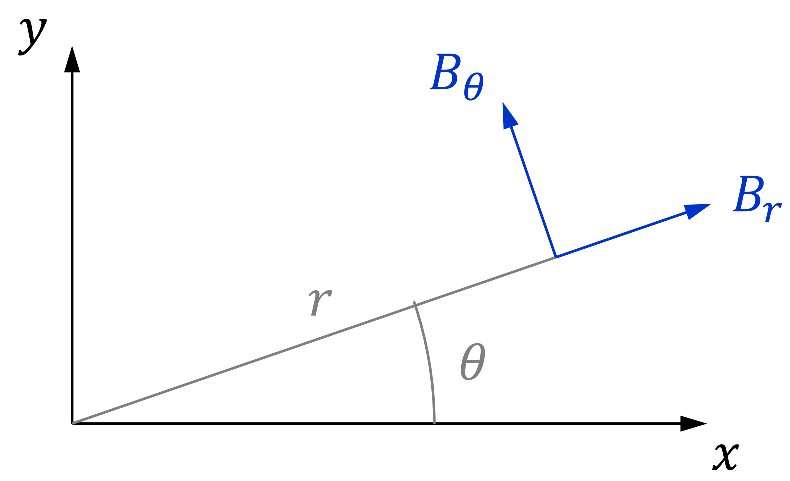
$$\vec{F} = q\left[\vec{E} + \left(\vec{v} \times \vec{B}\right)\right]$$ for the beam

$$\vec{F} = I\vec{\ell} \times \vec{B}$$ for the forces on conductors

In synchrotrons / transfer lines the B field as seen from the beam is usually expressed as a series of multipoles

$$B_r = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} [B_n \sin(n\theta) + A_n \cos(n\theta)]$$

$$B_\theta = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} [B_n \cos(n\theta) - A_n \sin(n\theta)]$$

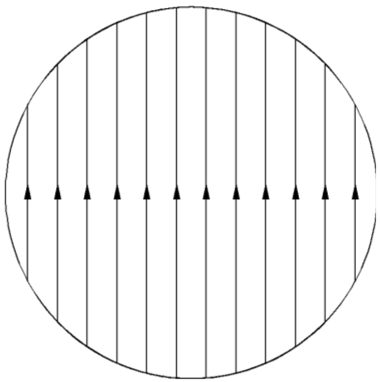


$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R}\right)^{n-1}$$

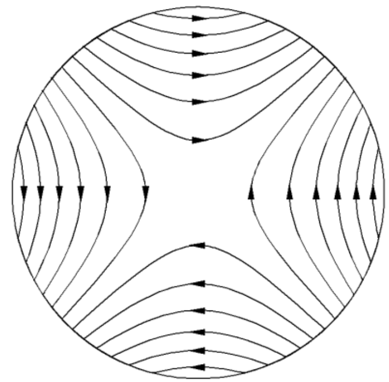
$$z = x + iy = re^{i\theta}$$

Each multipole term has a corresponding magnet type

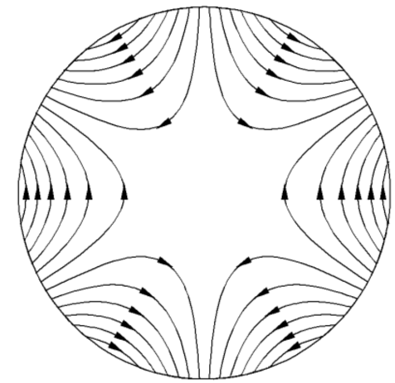
B_1: normal dipole



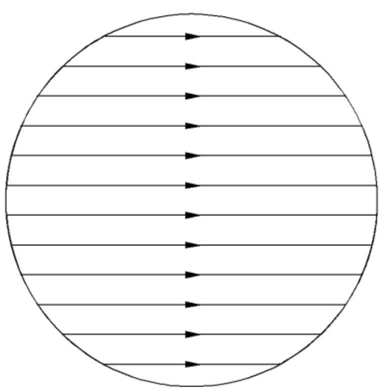
B_2: normal quadrupole



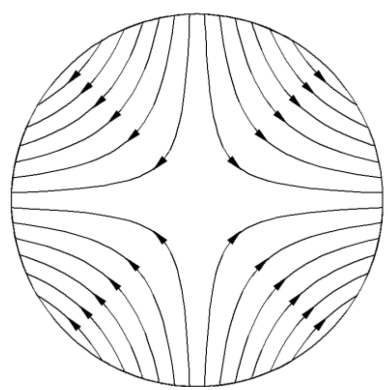
B_3: normal sextupole



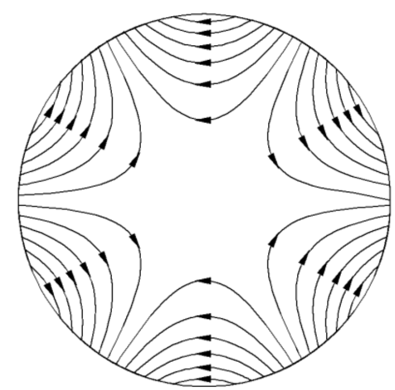
A_1: skew dipole



A_2: skew quadrupole

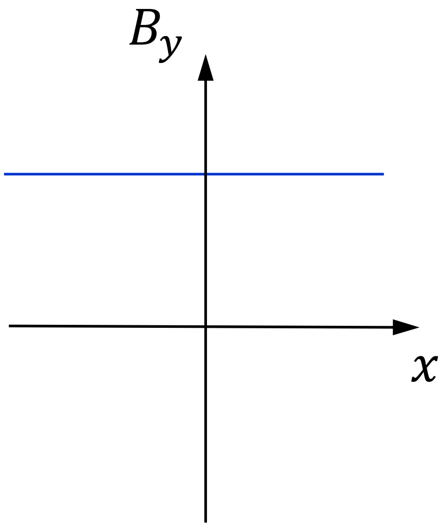


A_3: skew sextupole

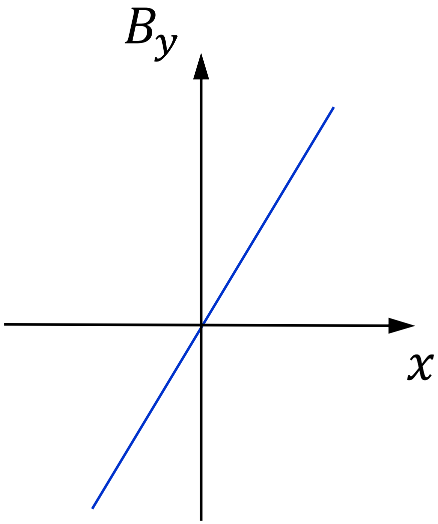


The field profile in the horizontal plane follows a polynomial expansion

$$B_y(x) = \sum_{n=1}^{\infty} B_n \left(\frac{x}{R}\right)^{n-1} = B_1 + B_2 \frac{x}{R} + B_3 \frac{x^2}{R^2} + \cdots$$

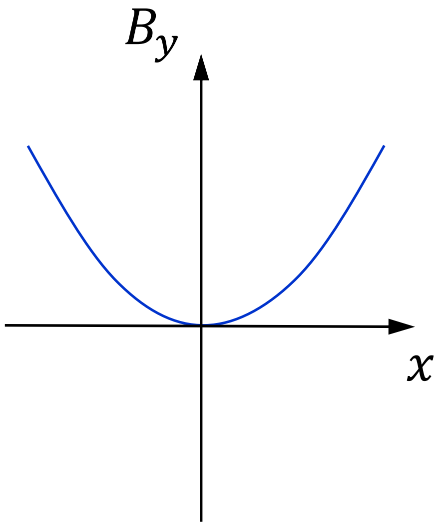


B_1: dipole



B_2: quadrupole

$$G = \frac{B_2}{R} = \frac{\partial B_y}{\partial x}$$



B_3: sextupole

Usually, for optics calculation, the field or multipole component is given, together with the (magnetic) length; these are a few definitions from MAD-X

Dipole

bend angle α [rad] & length L [m]

k_0 [1/m] & length L [m] obsolete

$k_0 = B / (B\rho)$ $B = B_1$

Quadrupole

quadrupole coefficient k_1 [1/m^2] × length L [m]

$k_1 = (dB_y/dx) / (B\rho)$

$G = dB_y/dx = B_2/R$

Sextupole

sextupole coefficient k_2 [1/m^3] × length L [m]

$k_2 = (d^2B_y/dx^2) / (B\rho)$

$(d^2B_y/dx^2)/2! = B_3/R^2$

We can now translate the MAD-X entries into (purposeful) magnetic quantities

```
BEAM, PARTICLE=ELECTRON,PC=3.0;
DEGREE:=PI/180.0;
QF: QUADRUPOLE,L=0.5,K1=0.2;
QD: QUADRUPOLE,L=1.0,K1=-0.2;
B: SBEND,L=1.0,ANGLE=15.0*DEGREE;
```

$(B\rho) = 10^9/c*PC = 10\text{^}9/299792485*3.0 = 10.01$ Tm

dipole (SBEND)

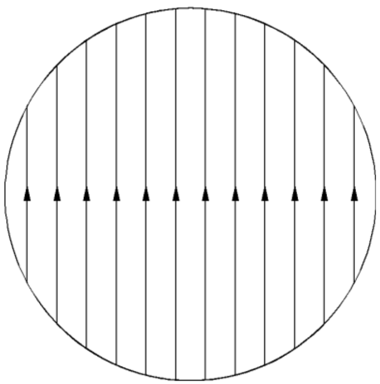
$B = |ANGLE|/L*(B\rho) = (15*pi/180)/1.0*10.01 = 2.62$ T

quadrupole

$G = |K1|*(B\rho) = 0.2*10.01 = 2.00$ T/m

The harmonic decomposition is very handy to describe the field quality, that is, deviations of the actual B vs. the ideal one

(normal) dipole



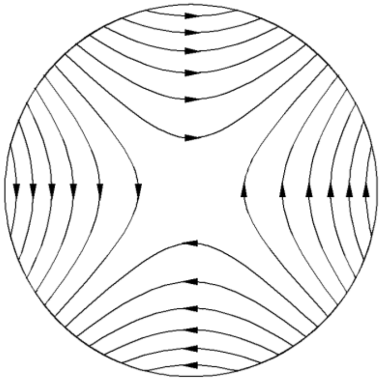
$$\vec{B}_{id}(x,y) = B_1\vec{J}$$

$$B_y(z) + iB_x(z) =$$
$$= B_1 + \frac{B_1}{10000}\left[ia_1 + (b_2 + ia_2)\left(\frac{z}{R}\right) + (b_3 + ia_3)\left(\frac{z}{R}\right)^2 + (b_4 + ia_4)\left(\frac{z}{R}\right)^3 + \cdots\right]$$

$$b_2 = 10000\frac{B_2}{B_1} \quad b_3 = 10000\frac{B_3}{B_1} \quad a_1 = 10000\frac{A_1}{B_1} \quad a_2 = 10000\frac{A_2}{B_1} \quad \ldots$$

The same expression can be written for a quadrupole

(normal) quadrupole



$$\vec{B}_{id}(x,y) = B_2[x\vec{j} + y\vec{i}]\frac{1}{R}$$

$$B_y(z) + iB_x(z) =$$

$$= B_2\frac{z}{R} + \frac{B_2}{10000}\left[ia_2\left(\frac{z}{R}\right) + (b_3 + ia_3)\left(\frac{z}{R}\right)^2 + (b_4 + ia_4)\left(\frac{z}{R}\right)^3 + \cdots\right]$$

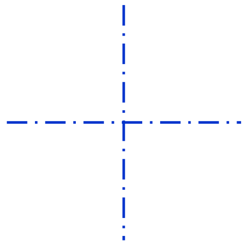
$$b_3 = 10000\frac{B_3}{B_2} \quad b_4 = 10000\frac{B_4}{B_2} \quad a_2 = 10000\frac{A_2}{B_2} \quad \ldots$$

The so-called *allowed* / *not-allowed* harmonics refer to some terms that shall / shall not cancel out for design symmetries

<u>fully symmetric dipoles</u>

allowed: b_3, b_5, b_7, b_9, etc.

not-allowed: all the others



<u>half symmetric dipoles</u>

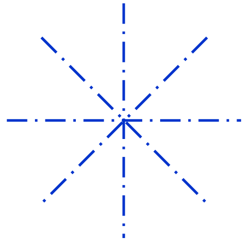
allowed: b_2, b_3, b_4, b_5, etc.

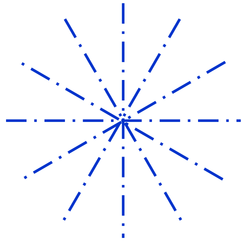
not-allowed: all the others

<u>fully symmetric quadrupoles</u>

allowed: b_6, b_{10}, b_{14}, b_{18}, etc.

not-allowed: all the others





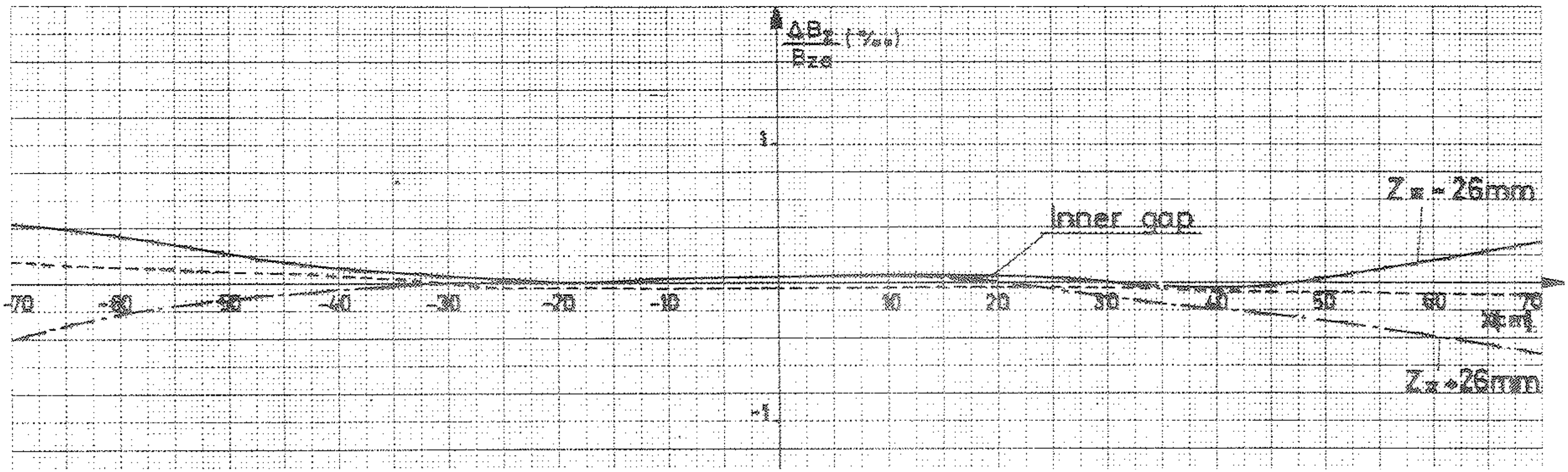
<u>fully symmetric sextupoles</u>

allowed: b_9, b_{15}, b_{21}, etc.

not-allowed: all the others

The field quality is often also expressed by a ΔB/B plot

$$\frac{\Delta B}{B} = \frac{B(x,y) - B_{id}(x,y)}{B_{id}(x,y)}$$



ΔB/B can (usually) be expressed from the harmonics, this is the expansion for a dipole

$$B_{y,id}(x) = B_1$$

$$B_y(x) = B_1 + \frac{B_1}{10000}\left[b_2\left(\frac{x}{R}\right) + b_3\left(\frac{x}{R}\right)^2 + b_4\left(\frac{x}{R}\right)^3 + \cdots\right]$$

$$\frac{\Delta B}{B}(x) = \frac{1}{10000}\left[b_2\left(\frac{x}{R}\right) + b_3\left(\frac{x}{R}\right)^2 + b_4\left(\frac{x}{R}\right)^3 + \cdots\right]$$

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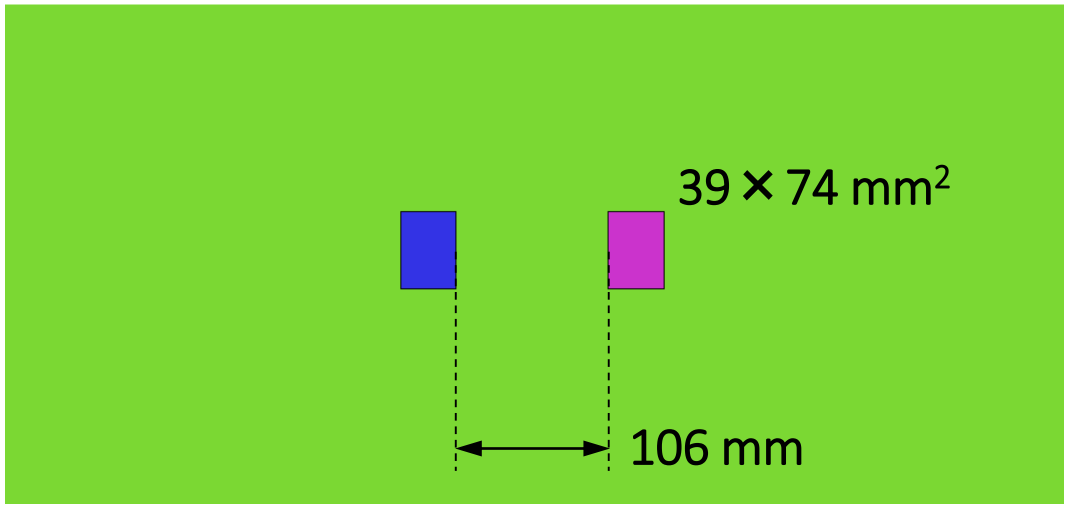
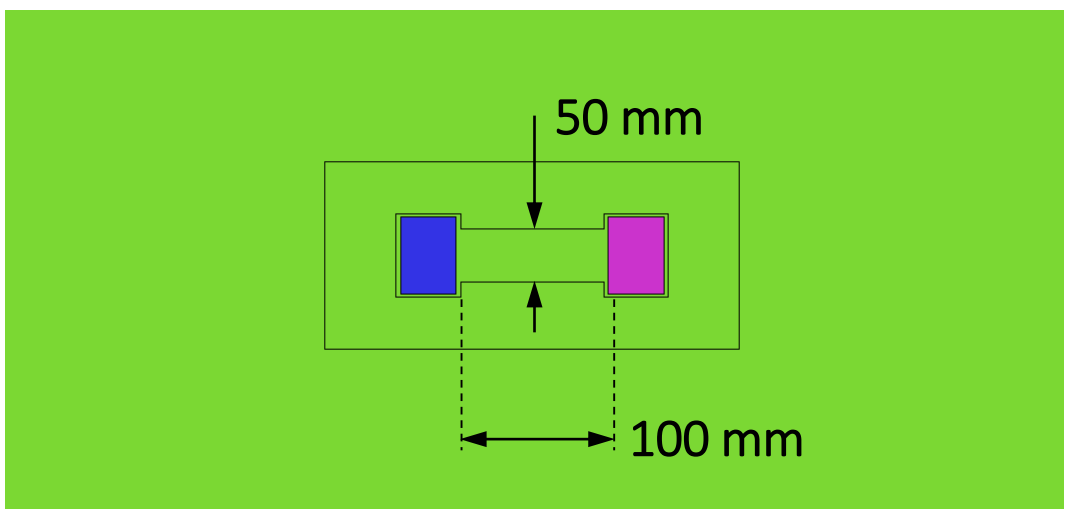
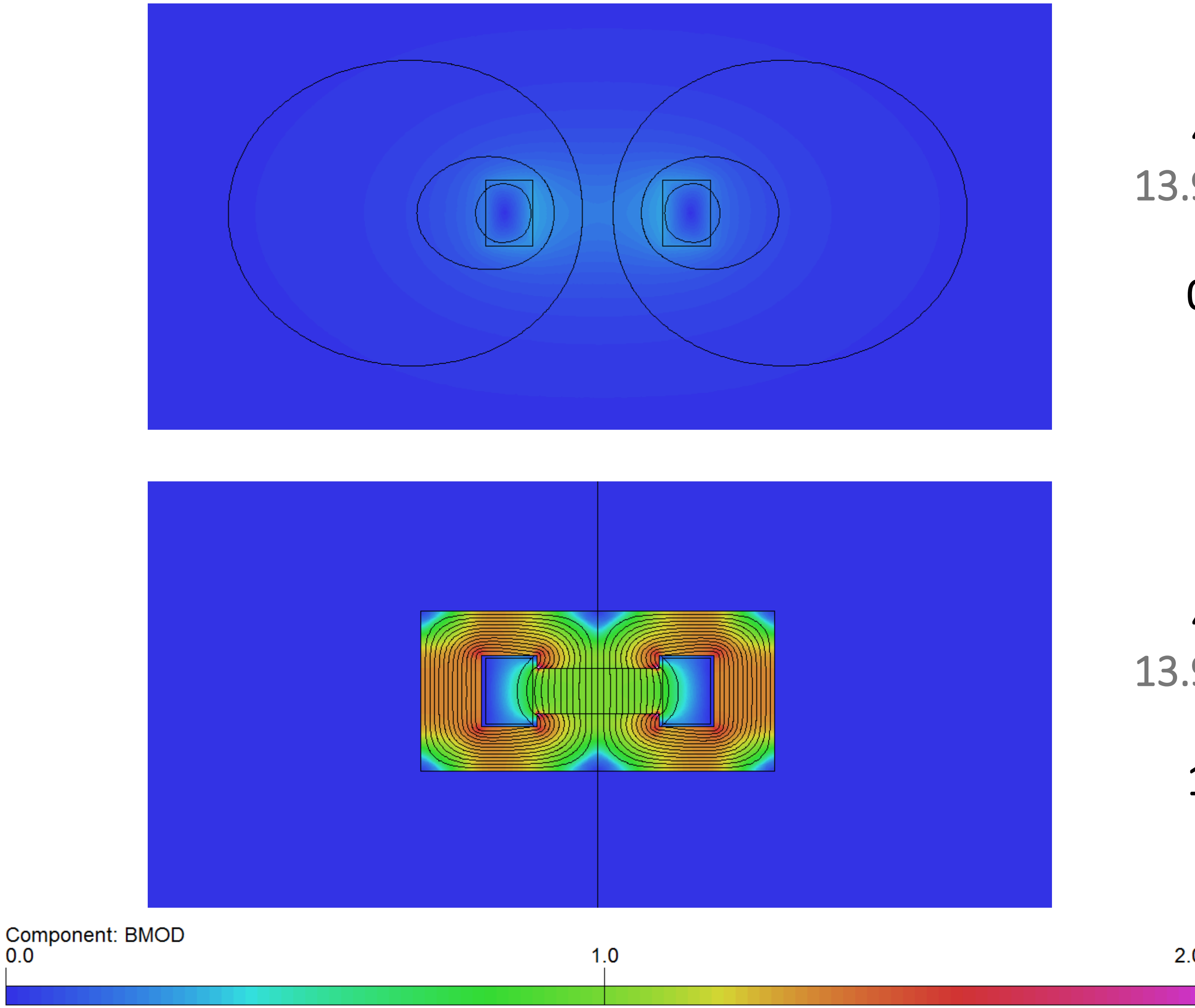
Thought experiment

Let's make a thought experiment, simulating in 2D two busbars without (top figure) / with (bottom figure) iron





This is the situation with 40 kA in each busbar



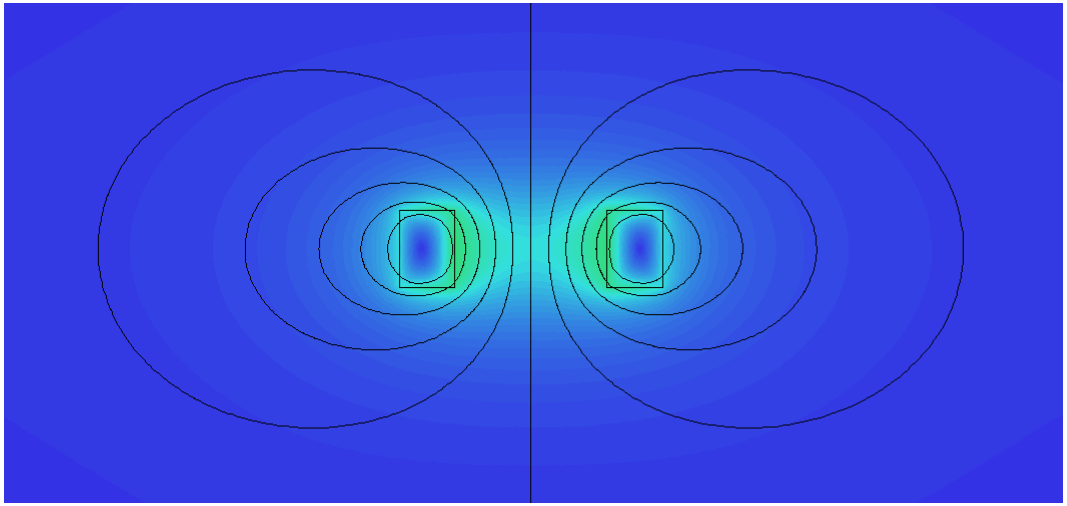
40 kA
13.9 A/mm^2

0.20 T

40 kA
13.9 A/mm^2

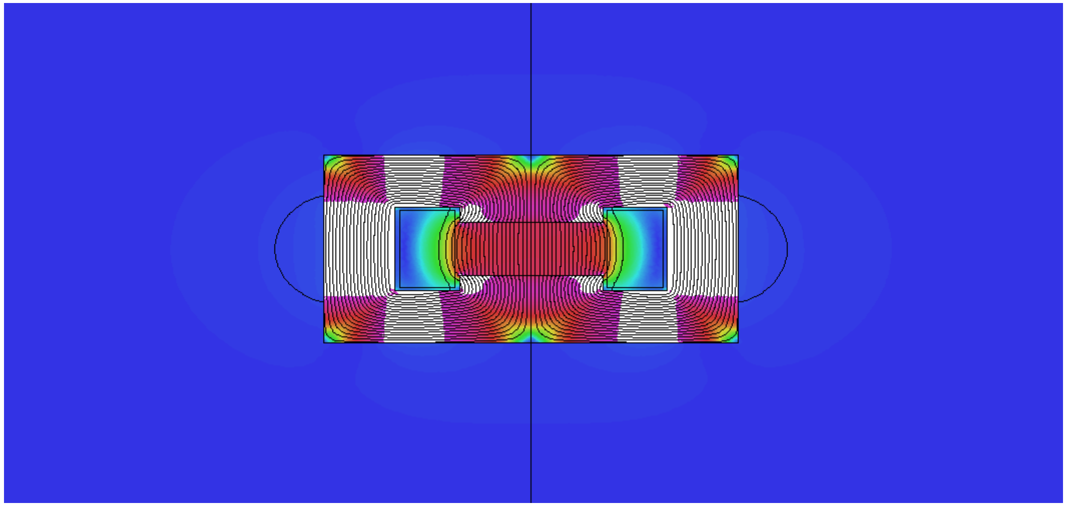
1.00 T

This is the situation if we double the Ampere-turns: 80 kA instead of 40 kA in each busbar



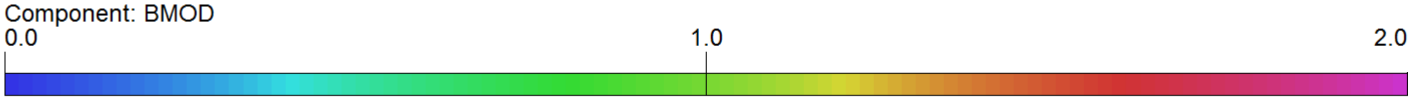
80 kA
27.7 A/mm^2

0.41 T

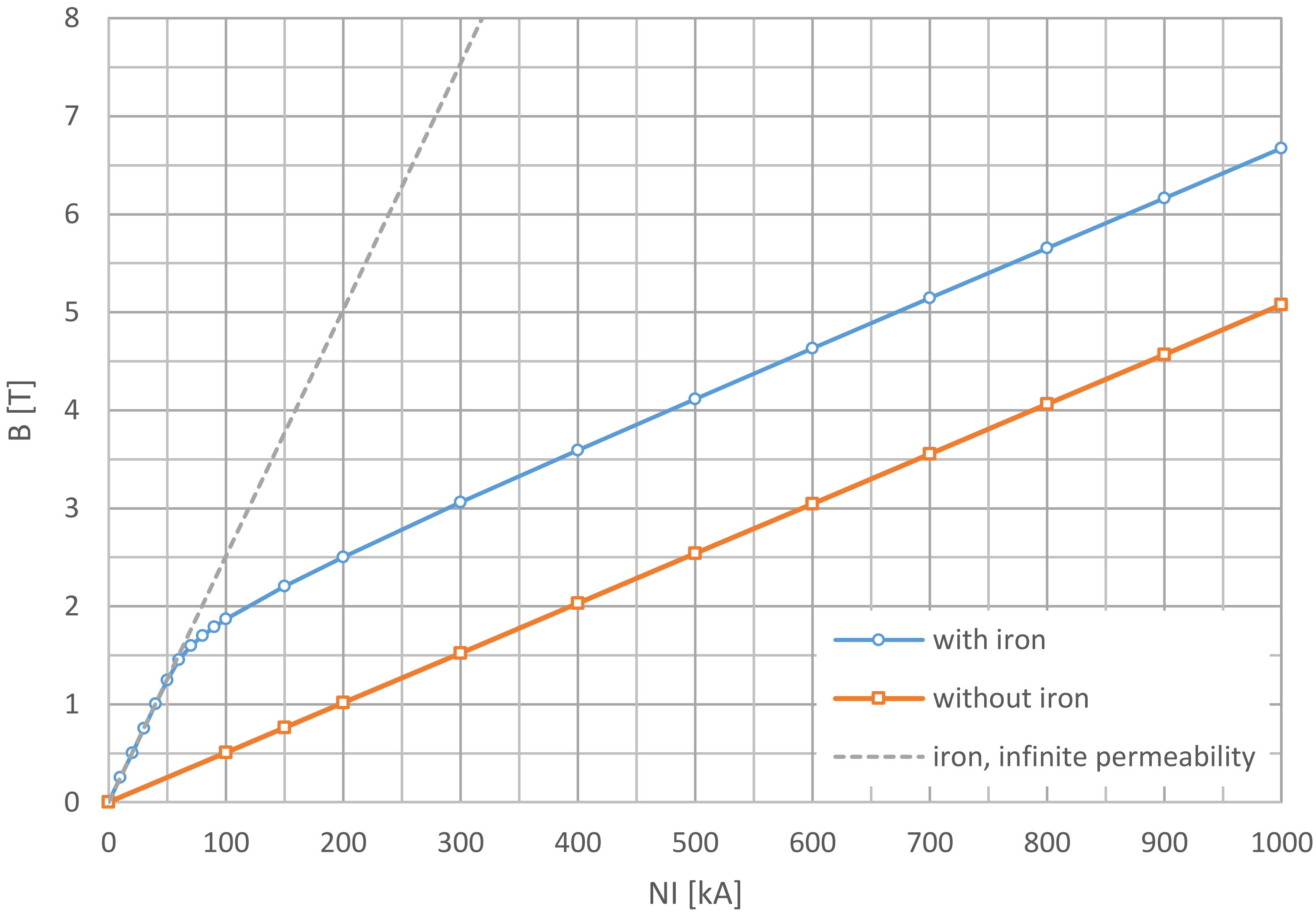


80 kA
27.7 A/mm^2

1.70 T



These two curves are the transfer functions – B field vs. current – for the two cases



In this though experiment, the field quality is quite different with / without iron

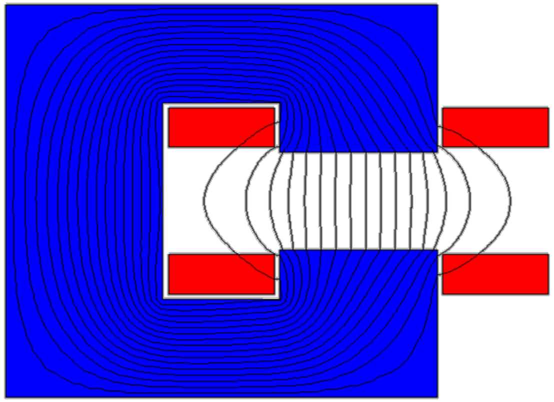
	b_3	b_5	b_7
without iron, 40 kA	401.9	10.1	0.0
without iron, 80 kA	401.9	10.1	0.0
with iron, 40 kA	-16.7	-6.2	-0.9
with iron, 80 kA	-38.5	-10.6	-0.9
with iron, 500 kA	120.4	0.6	-0.1

(harmonics in units of 10^{-4} at 17 mm radius)

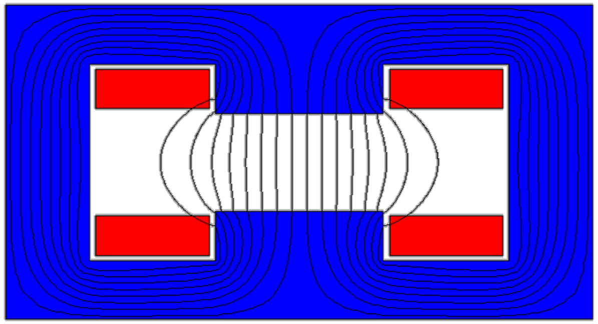
- 4a -

Basics for the design of resistive magnets 2D

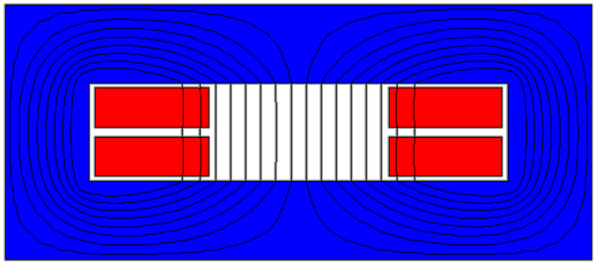
These are the most common types of resistive dipoles



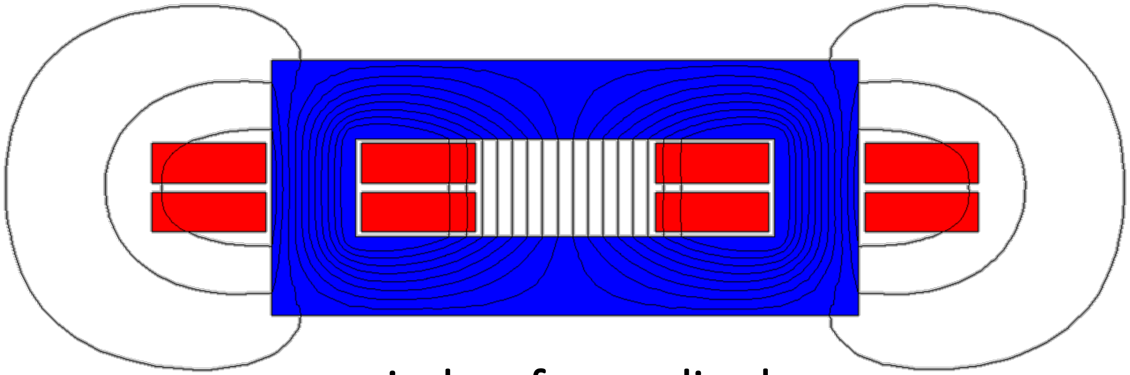
“C” dipole



“H” dipole

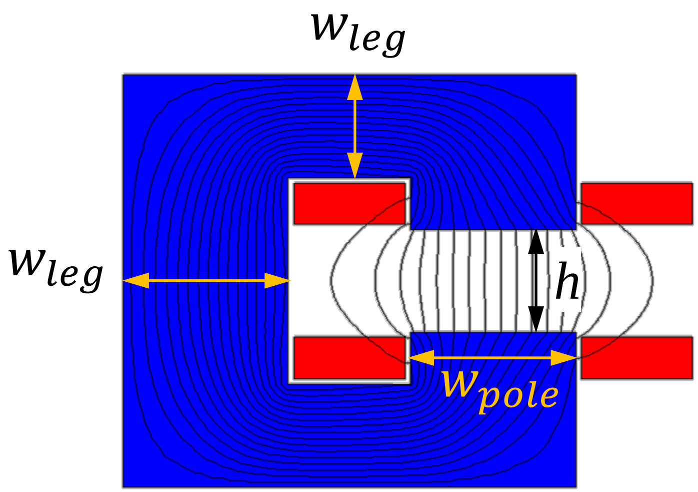


window frame dipole
(“O” dipole)



window frame dipole
(“O” dipole)

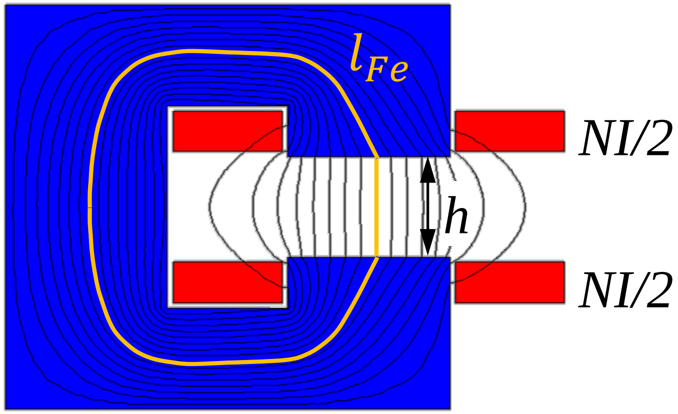
The magnetic circuit is dimensioned so that the pole is wide enough for field quality, and there is enough room for the flux in the return legs



$$w_{pole} \cong w_{GFR} + 2.5h$$

$$B_{leg} \cong B_{gap} \frac{w_{pole} + 1.2h}{w_{leg}}$$

The Ampere-turns are a linear function of the gap and of the B field



$$NI = \oint \vec{H} \cdot \vec{dl} = \frac{B_{Fe}}{\mu_0 \mu_r} \cdot l_{Fe} + \frac{B_{gap}}{\mu_0} \cdot h \cong \frac{B_{gap} h}{\mu_0}$$

$$NI = \frac{Bh}{\eta \mu_0} \qquad \eta = \frac{1}{1 + \frac{1}{\mu_r} \frac{l_{Fe}}{h}}$$

The same can be solved using magnetic reluctances and Hopkinson's law, which is a parallel of Ohm's law

$$\mathcal{R} = \frac{\mathrm{NI}}{\Phi}$$

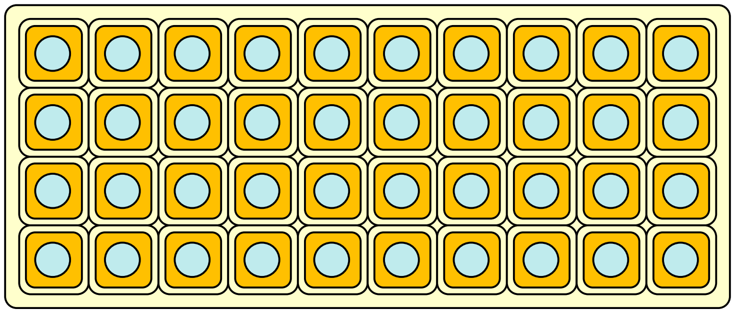
$$\mathrm{R} = \frac{\mathrm{V}}{I}$$

$$\mathcal{R} = \frac{l}{\mu_0 \mu_r A}$$

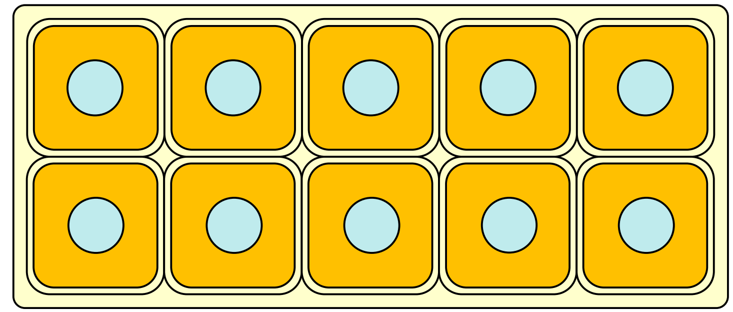
$$\mathrm{R} = \frac{l}{\sigma S}$$

$$\eta = \frac{1}{1 + \frac{\mathcal{R}_{Fe}}{\mathcal{R}_{gap}}}$$

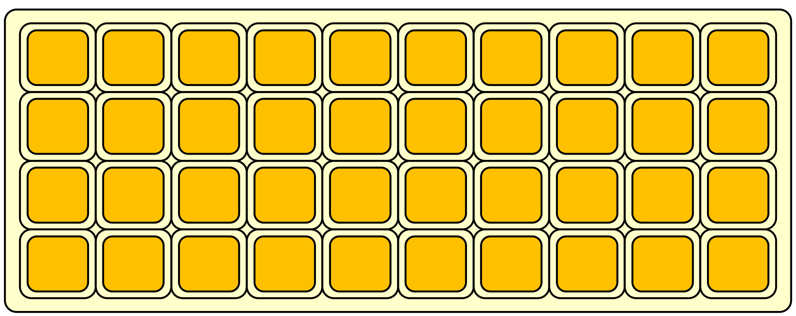
The same Ampere-turns can be provided by different coils, for example 10 kA can be arranged as



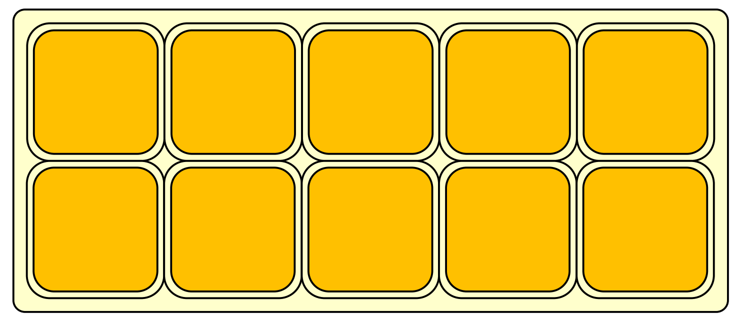
250 A × 40 turns, water cooled



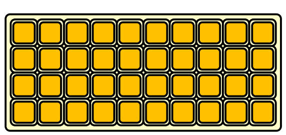
1000 A × 10 turns, water cooled



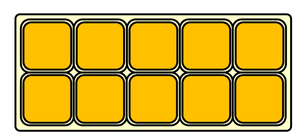
250 A × 40 turns, air cooled (outside)



1000 A × 10 turns, air cooled (outside)



250 A × 40 turns, air cooled (outside)



1000 A × 10 turns, air cooled (outside)

If the magnet is not dc, then an rms power / current has to be considered, for the most demanding duty cycle

$$P_{rms} = RI_{rms}^2 = \frac{1}{T}\int_0^T R[I(t)]^2 dt$$

for a pure sine wave $$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$$

for a linear ramp from 0 $$I_{rms} = \frac{I_{peak}}{\sqrt{3}}$$

These are common formulae useful to compute the main electric parameters of a resistive dipole

Ampere-turns total $$NI = \frac{Bh}{\eta \mu_0}$$

Resistance per m length $$R_u = \frac{2\rho}{A_{cond}} = \frac{2\rho j}{NI}$$

Power per m length $$P_u = 2\rho j NI = 2\rho j^2 A_{cond} = \frac{2\rho j Bh}{\eta \mu_0}$$

Inductance per m length $$L_u \cong \frac{\mu_0 N^2 (w_{pole} + 1.2h)}{h}$$

The table describes the field quality for the different layouts of these examples

	C-shaped	H-shaped	O-shaped
b_2	1.4	0	0
b_3	-88.2	-87.0	0.2
b_4	0.7	0	0
b_5	-31.6	-31.4	-0.1
b_6	0.1	0	0
b_7	-3.8	-3.8	-0.1
b_8	0.0	0	0
b_9	0.0	0.0	0.0

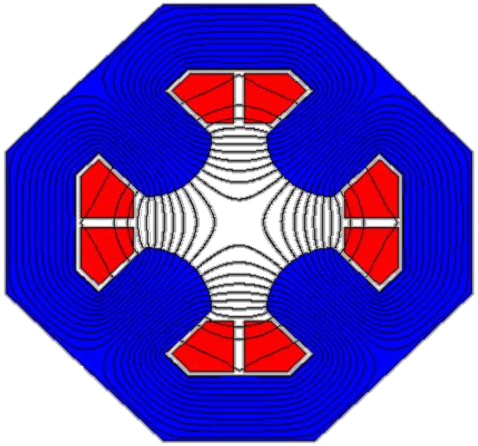
multipoles in units of 10^{-4} at R = 17 mm

NI = 20 kA

h = 50 mm

w_{pole} = 80 mm

These are the most common types of resistive quadrupoles



standard
quadrupole

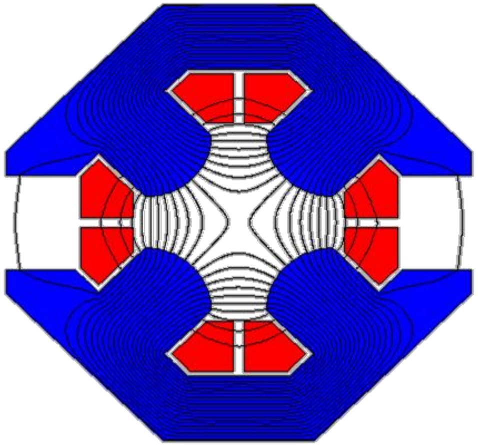
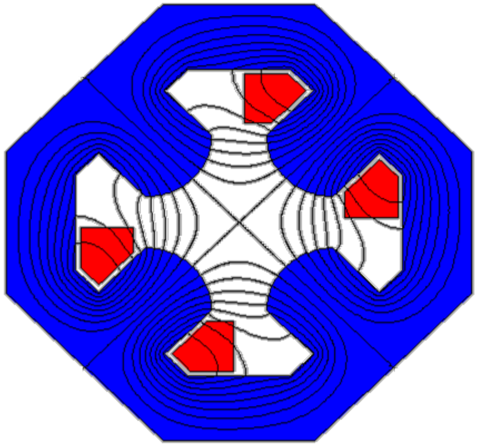


figure-of-8
quadrupole



quadrupole
with half the coils

These are useful formulae for standard resistive quadrupoles

Pole tip field $$B_{pole} = Gr$$

Ampere-turns per pole $$NI = \frac{Gr^2}{2\eta\mu_0}$$

Resistance per m length total (4 quadrants) $$R_u = \frac{8\rho}{A_{cond}} = \frac{8\rho j}{NI}$$

Power per m length $$P_u = \frac{4\rho j G r^2}{\eta\mu_0}$$

The *ideal* poles for dipole, quadrupole, sextupole, etc. are lines of constant scalar potential

dipole

$\rho \sin(\theta) = \pm h/2$ $y = \pm h/2$ straight line

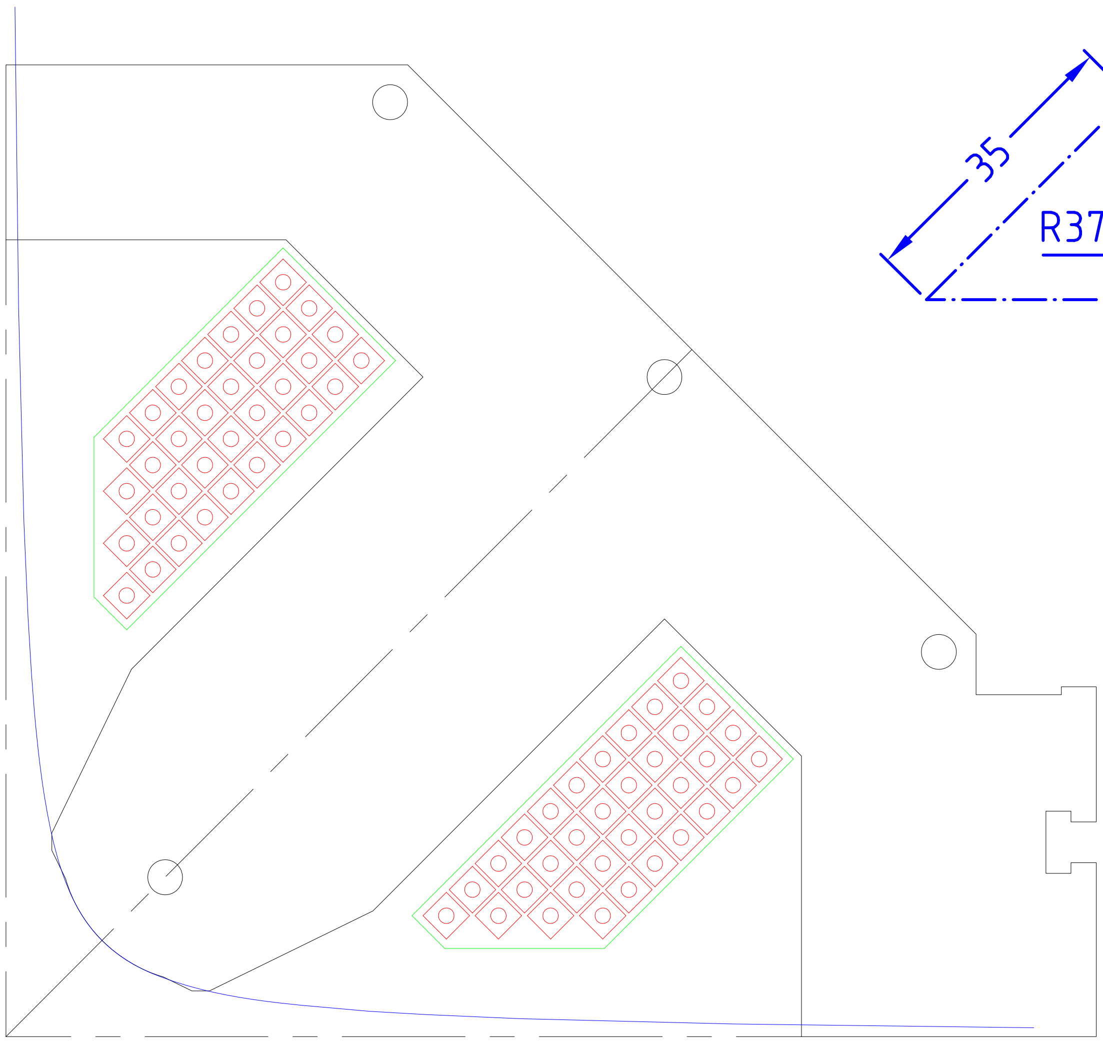
quadrupole

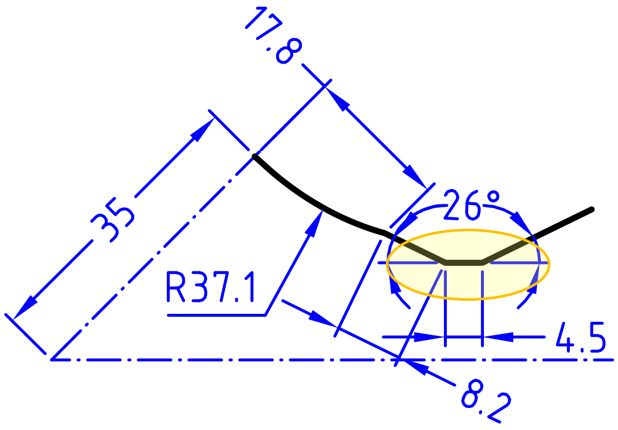
$\rho^2 \sin(2\theta) = \pm r^2$ $2xy = \pm r^2$ hyperbola

sextupole

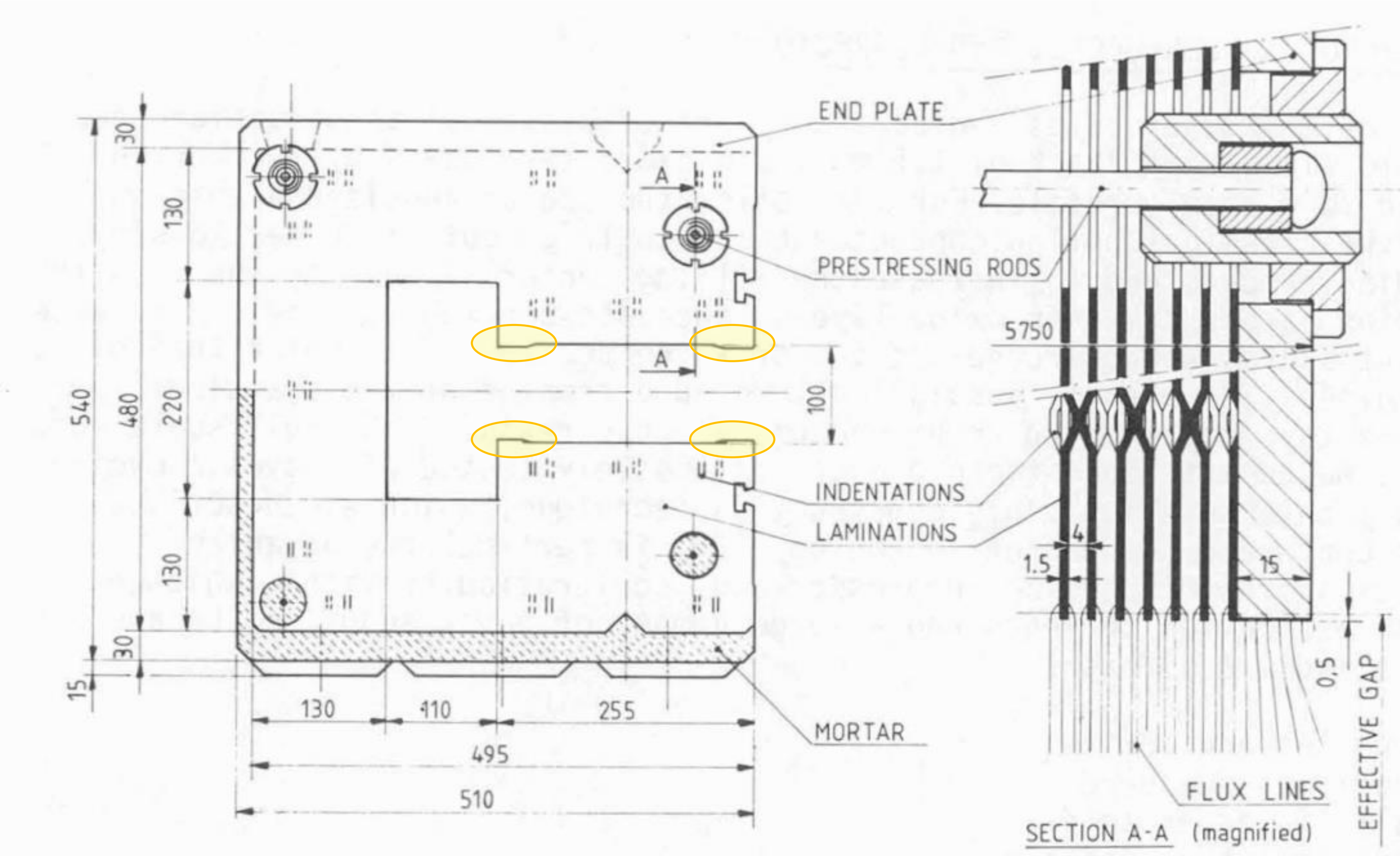
$\rho^3 \sin(3\theta) = \pm r^3$ $3x^2 y - y^3 = \pm r^3$

This is the real pole used for example in the SESAME quadrupoles vs. the theoretical hyperbola





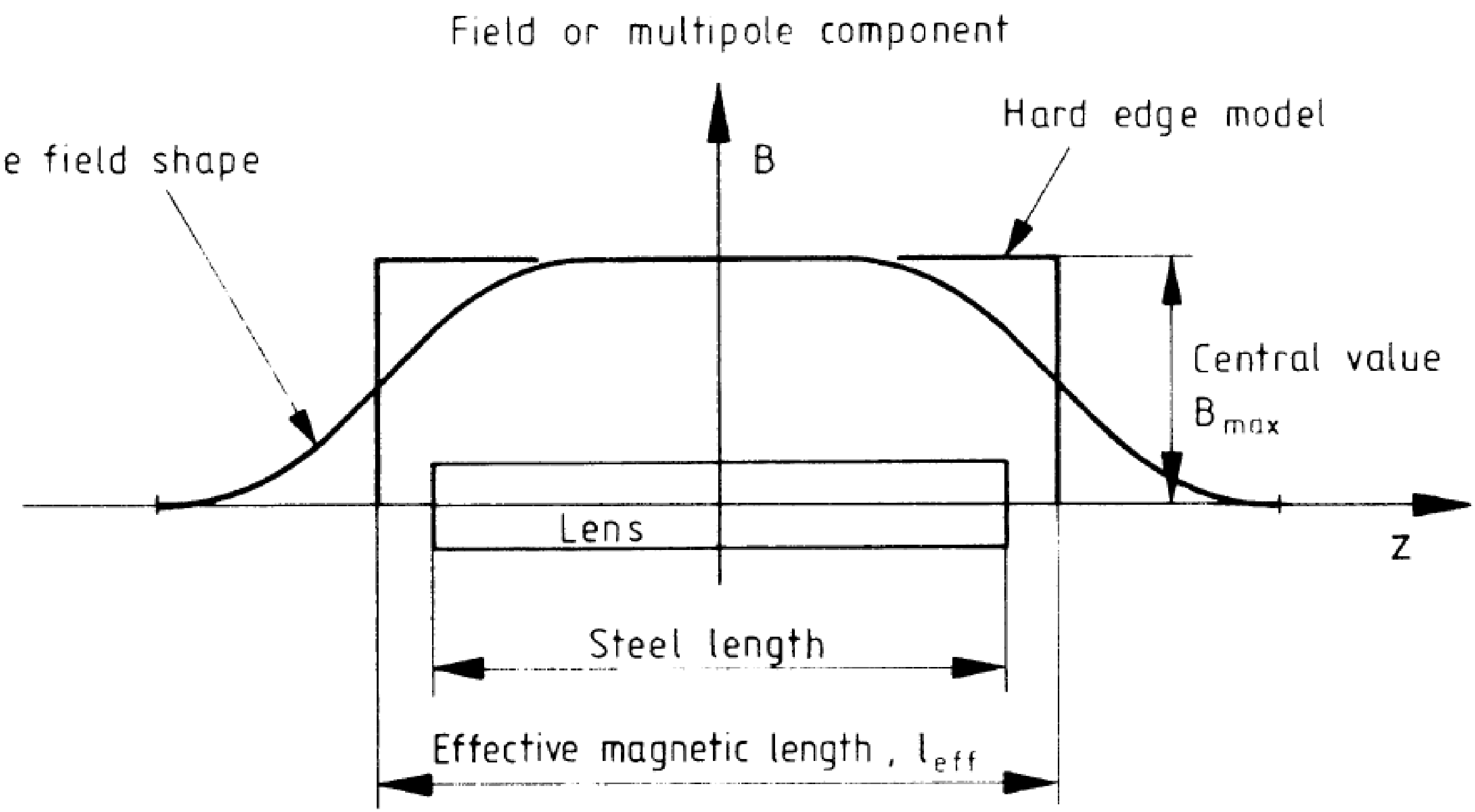
This is the lamination of the LEP main bending magnets, with the pole shims well visible



- 4b -

Basics for the design of resistive magnets 3D

In 3D, the longitudinal dimension of the magnet is described by a magnetic length



$$l_m B_0 = \int_{-\infty}^{\infty} B(z)dz$$

The magnetic length can be estimated at first order with simple formulae

$$l_m > l_{Fe}$$

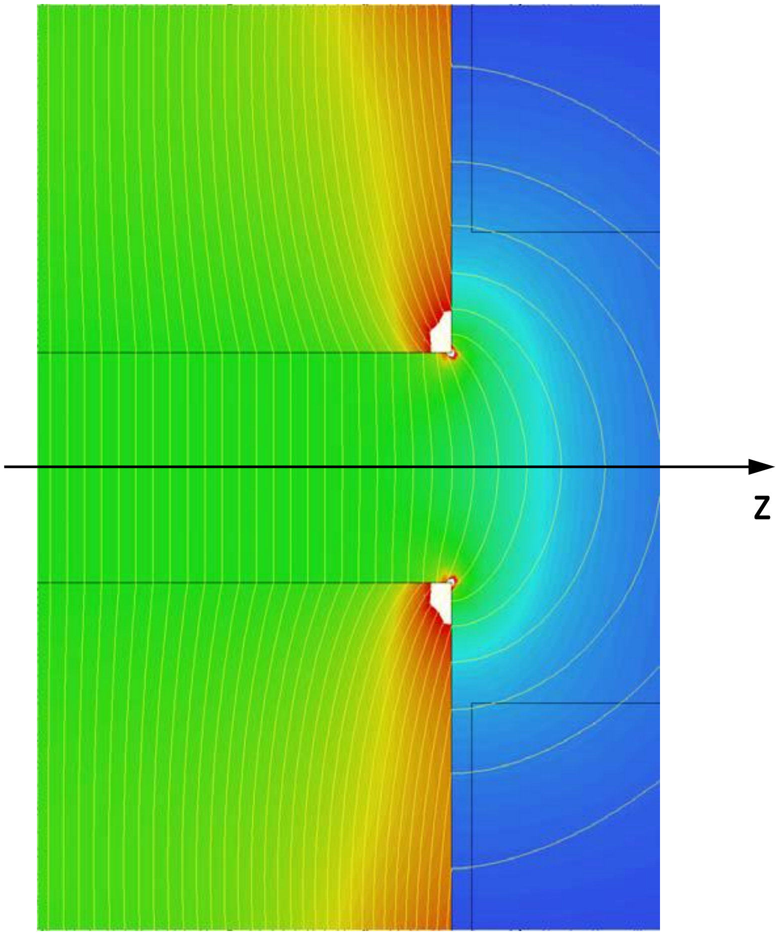
dipole

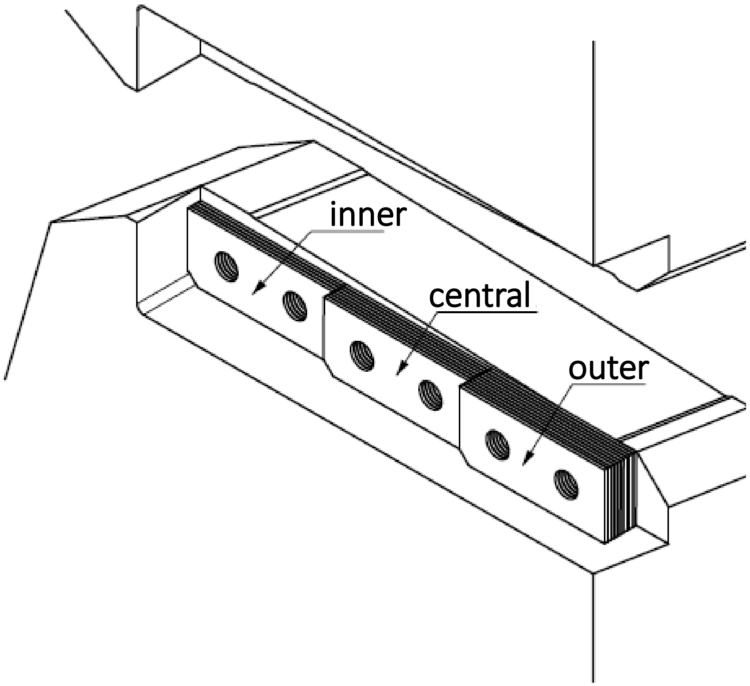
$$l_m \cong l_{Fe} + h$$

quadrupole

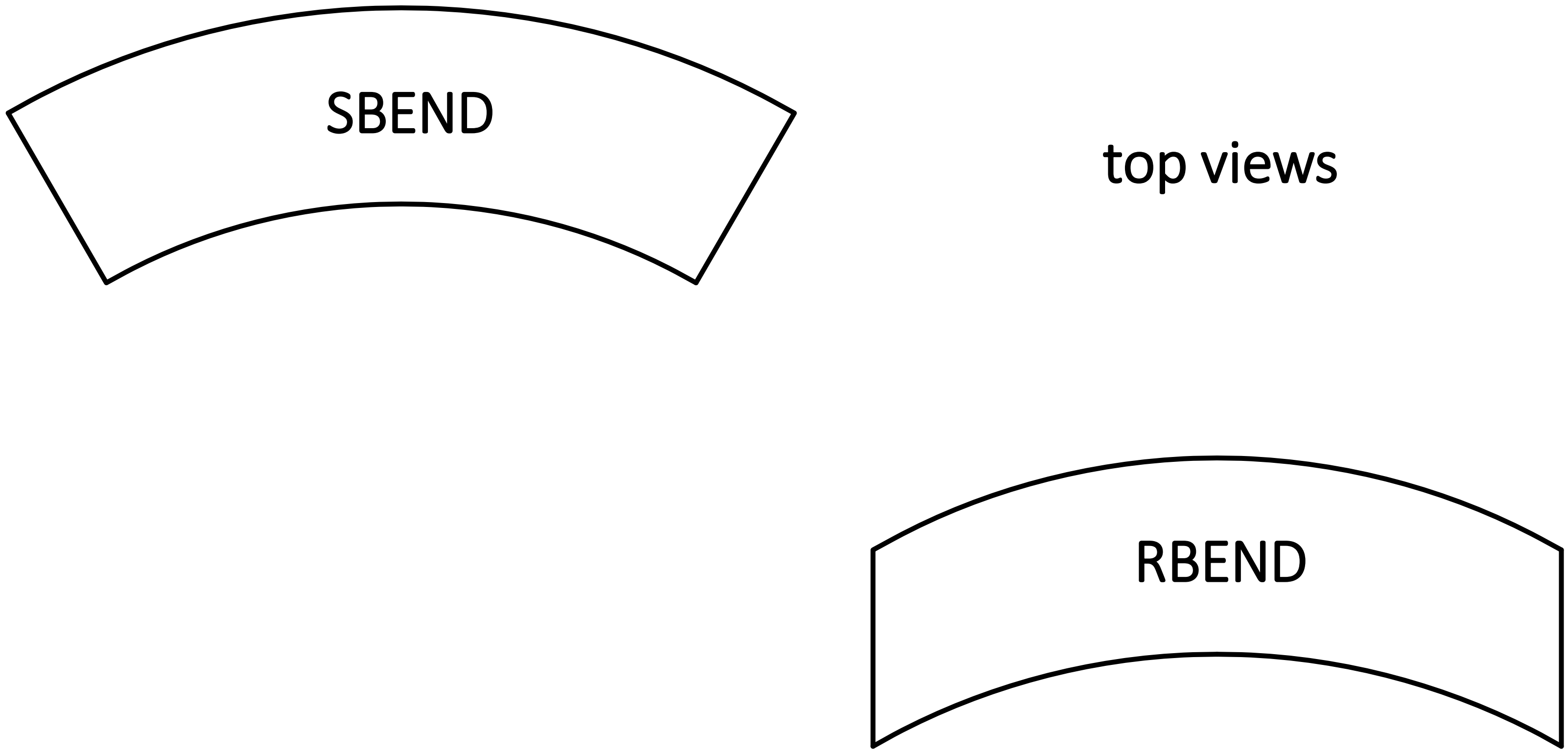
$$l_m \cong l_{Fe} + 0.80r$$

There are many different options to terminate the pole ends, depending on the type of magnet, its field level, etc.

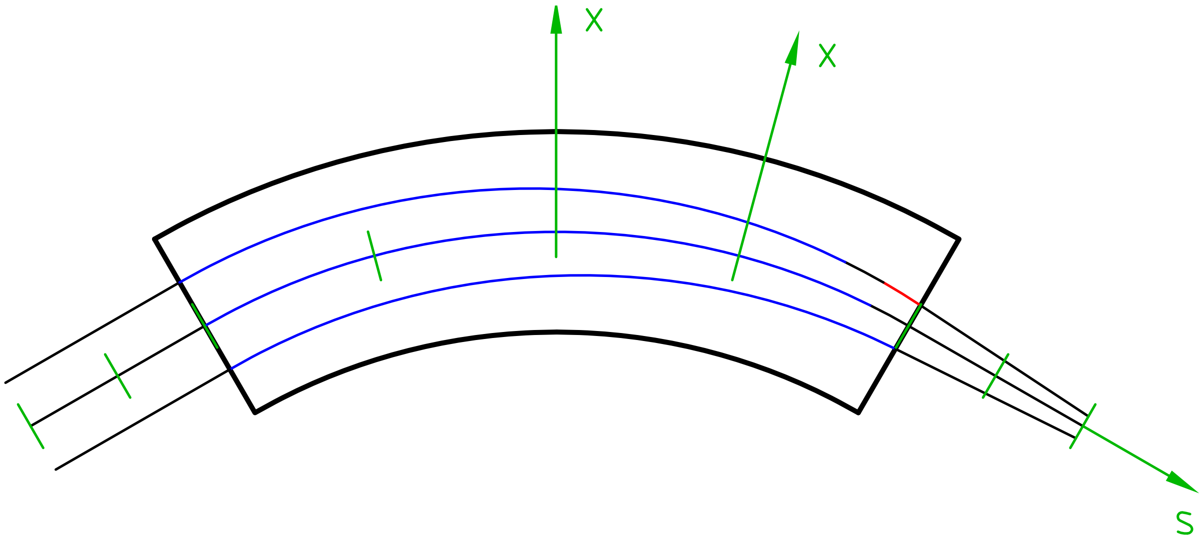


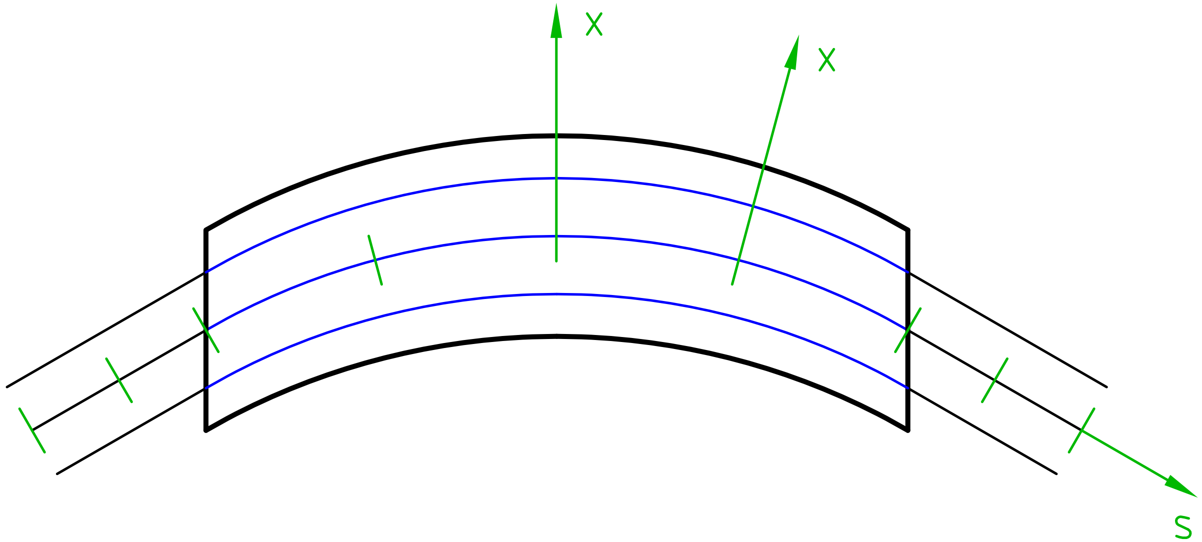


Usually two dipole elements are found in lattice codes: the sector dipole (SBEND) and the parallel faces dipole (RBEND)



The two types of dipoles are slightly different in terms of focusing, for a geometric effect



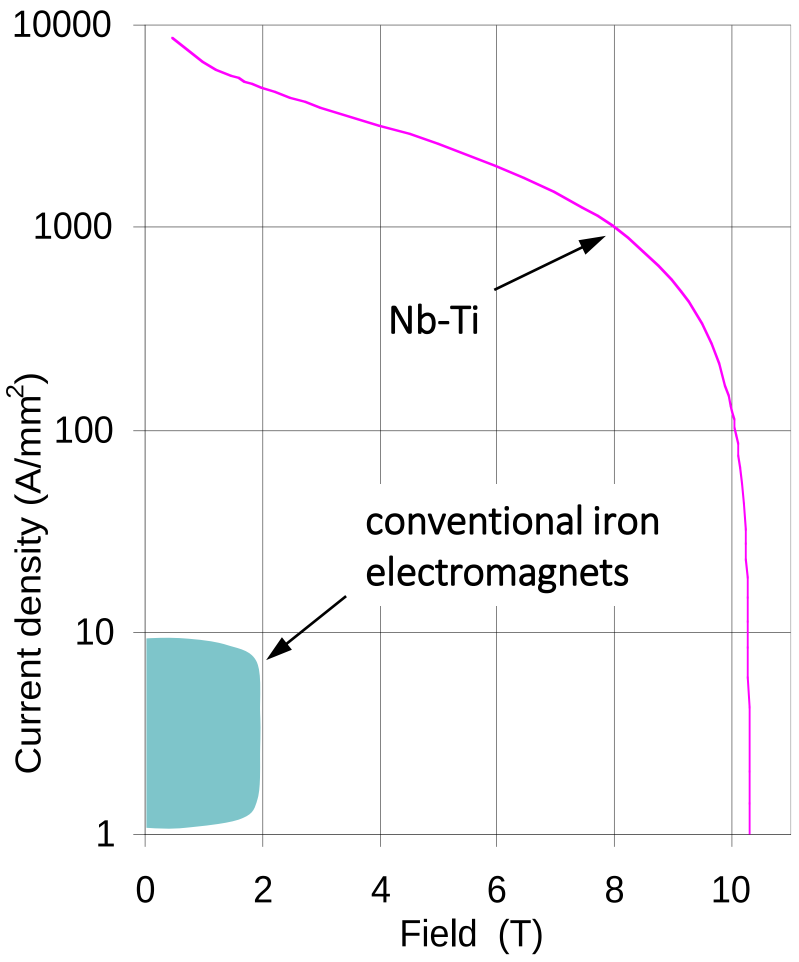


- 5 -

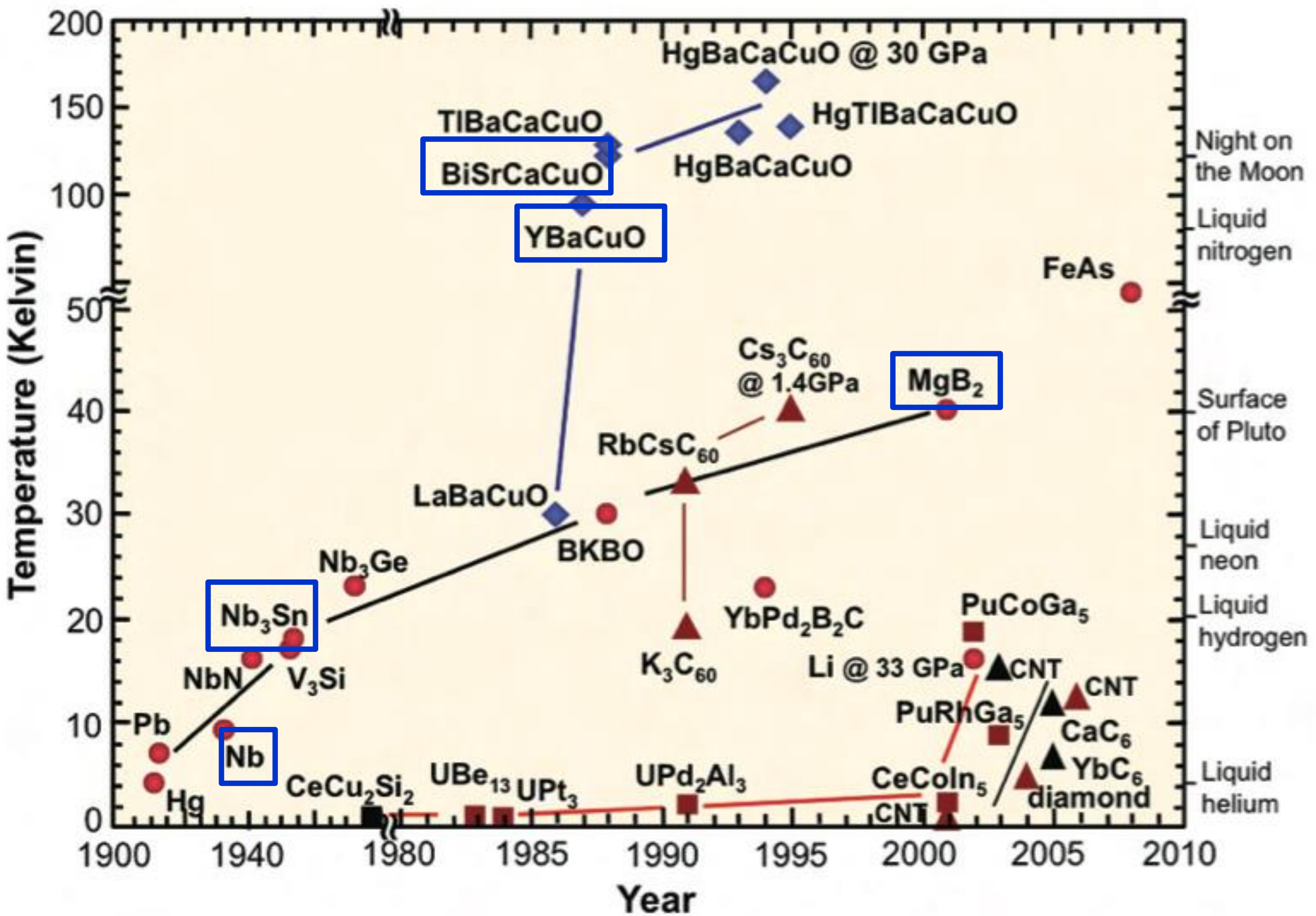
A glimpse on the design of superconducting magnets

(thanks to Luca Bottura
for the material of the slides)

Superconductivity makes possible large accelerators with fields well above 2 T



This is a history chart of superconductors, starting with Hg all the way to HTS (High Temperature Superconductors)



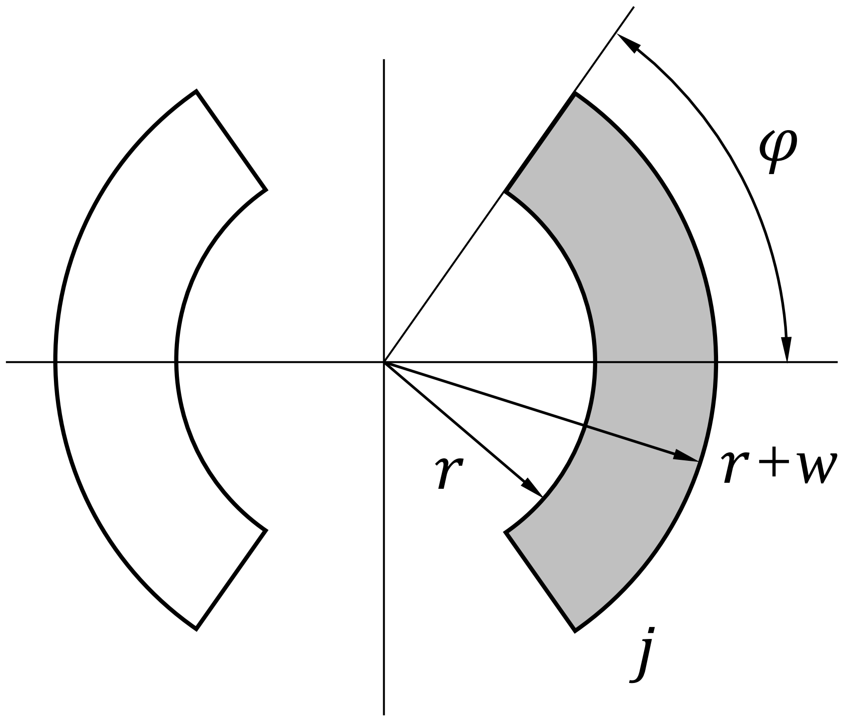
This is a summary of (somehow) practical superconductors

	LTS			HTS	
material	Nb-Ti	Nb_3Sn	MgB_2	YBCO	BSCCO
year of discovery	1961	1954	2001	1987	1988
T_c [K]	9.2	18.2	39	≈93	95 / 108
B_c [T]	14.5	≈30	36…74	120…250	≈200

The field in the aperture can be derived using Biot-Savart law (in 2D)

$$B_\theta = \frac{\mu_0 I}{2\pi\rho}$$

Biot-Savart law for an infinite wire



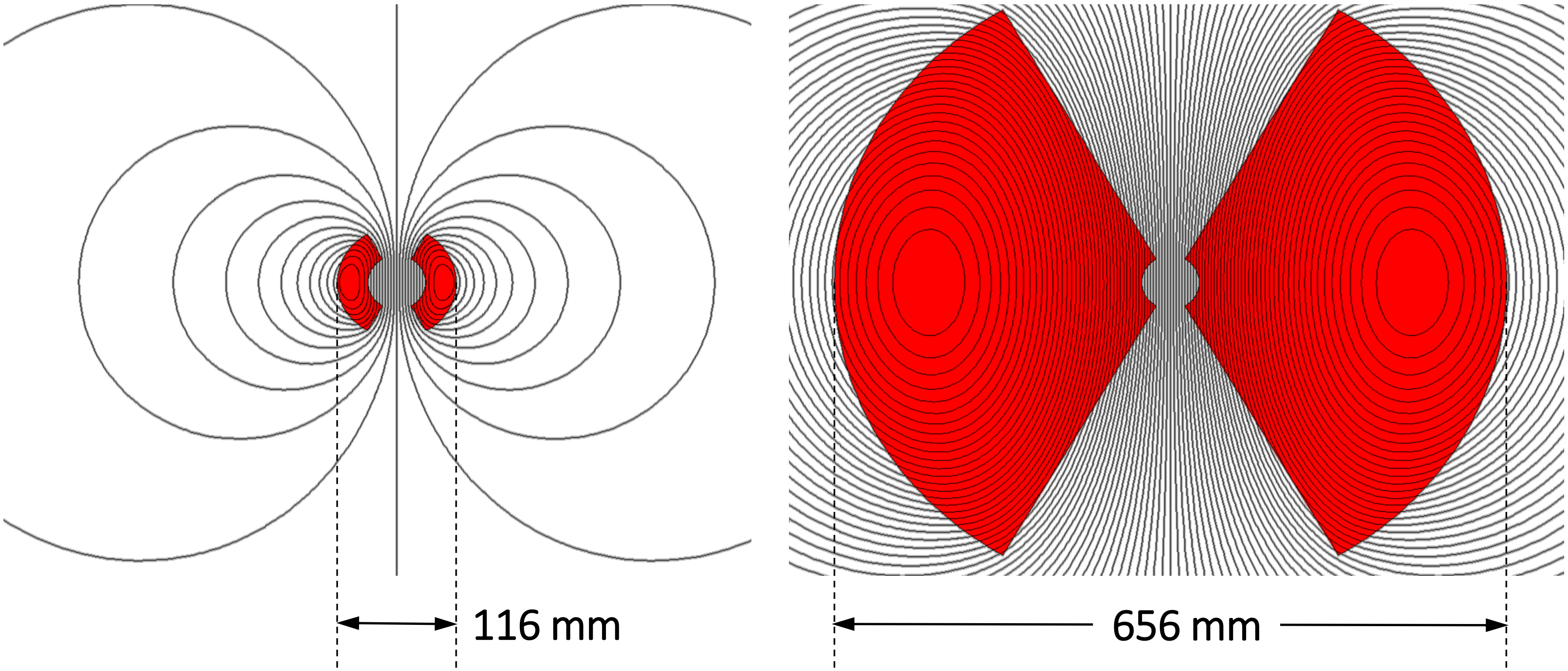
$$B = \frac{2\mu_0 \sin\varphi}{\pi} jw$$

for a sector coil

$$B = \frac{\sqrt{3}\mu_0}{\pi} jw$$

for a 60 deg sector coil

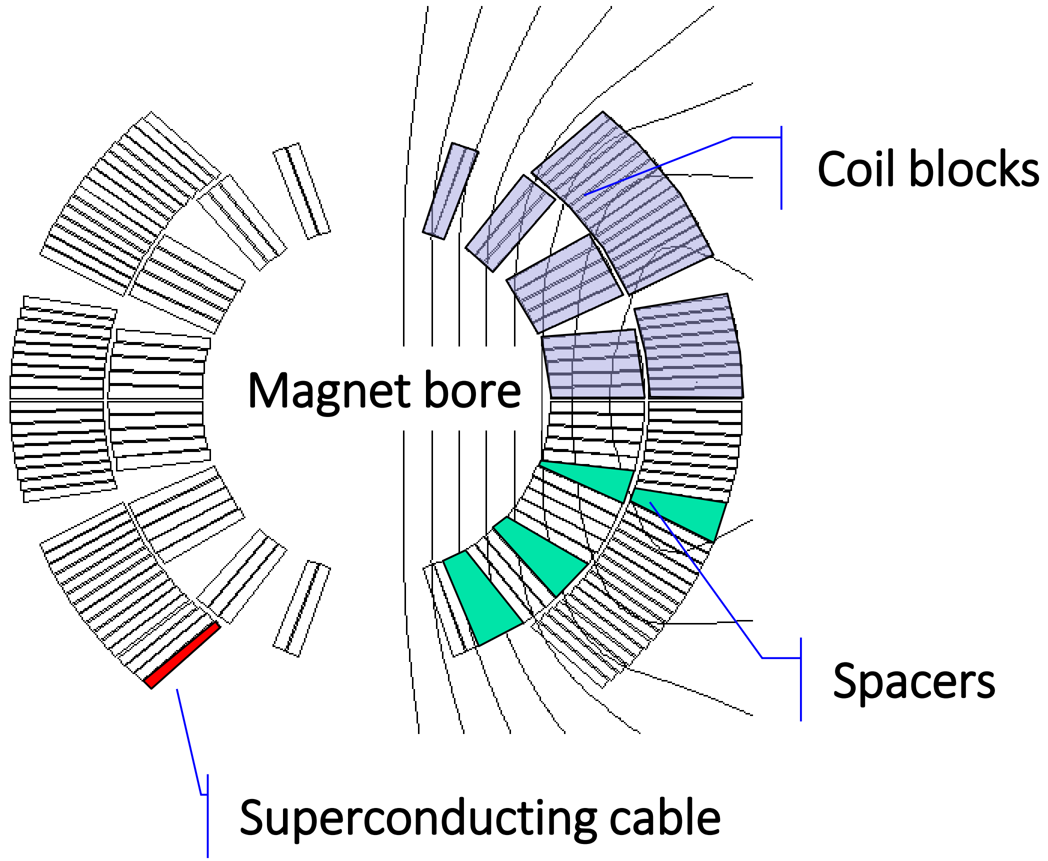
This is how it would look like one aperture of the LHC dipoles at 8.3 T, with two different current densities (without iron)



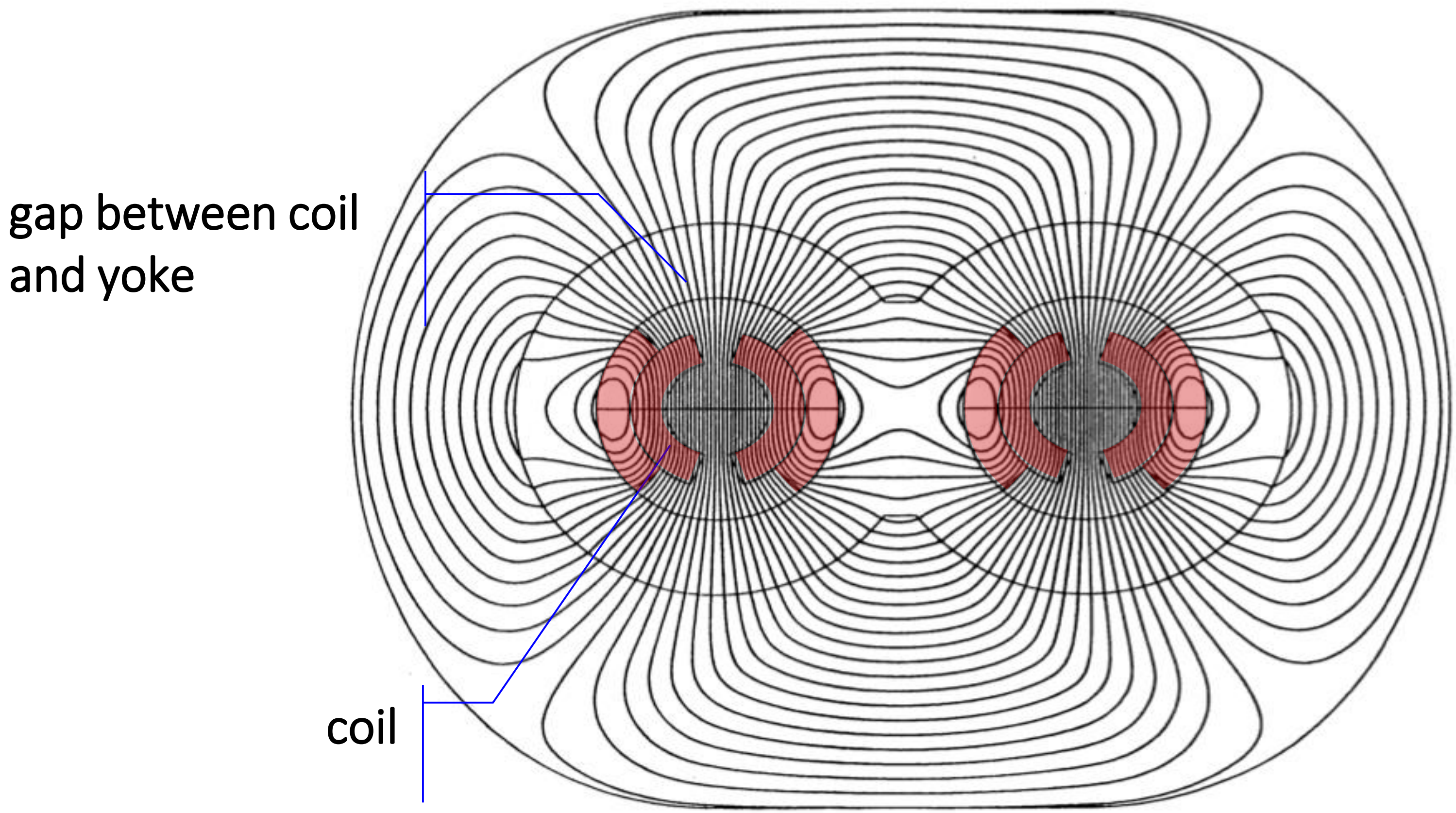
$j = 400\ A/mm^2$
w = 30 mm
NI = 1.2 MA
P = 14.9 MW/m (if Cu at room temp.)

$j = 40\ A/mm^2$
w = 300 mm
NI = 4.5 MA
P = 6.2 MW/m (if Cu at room temp.)

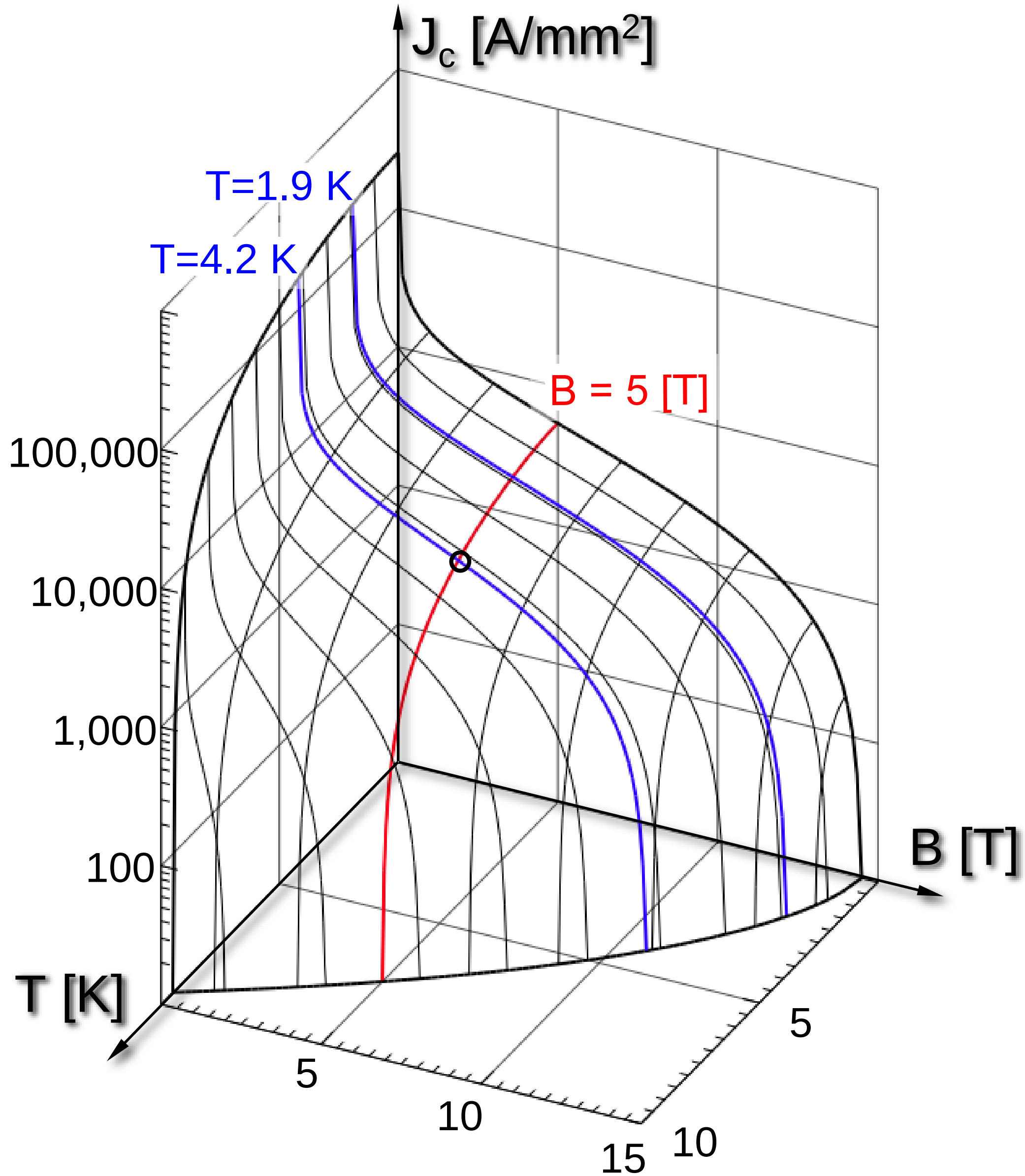
This is the actual coil of the LHC main dipoles (one aperture), showing the position of the superconducting cables



Around the coils, iron is used to close the magnetic circuit

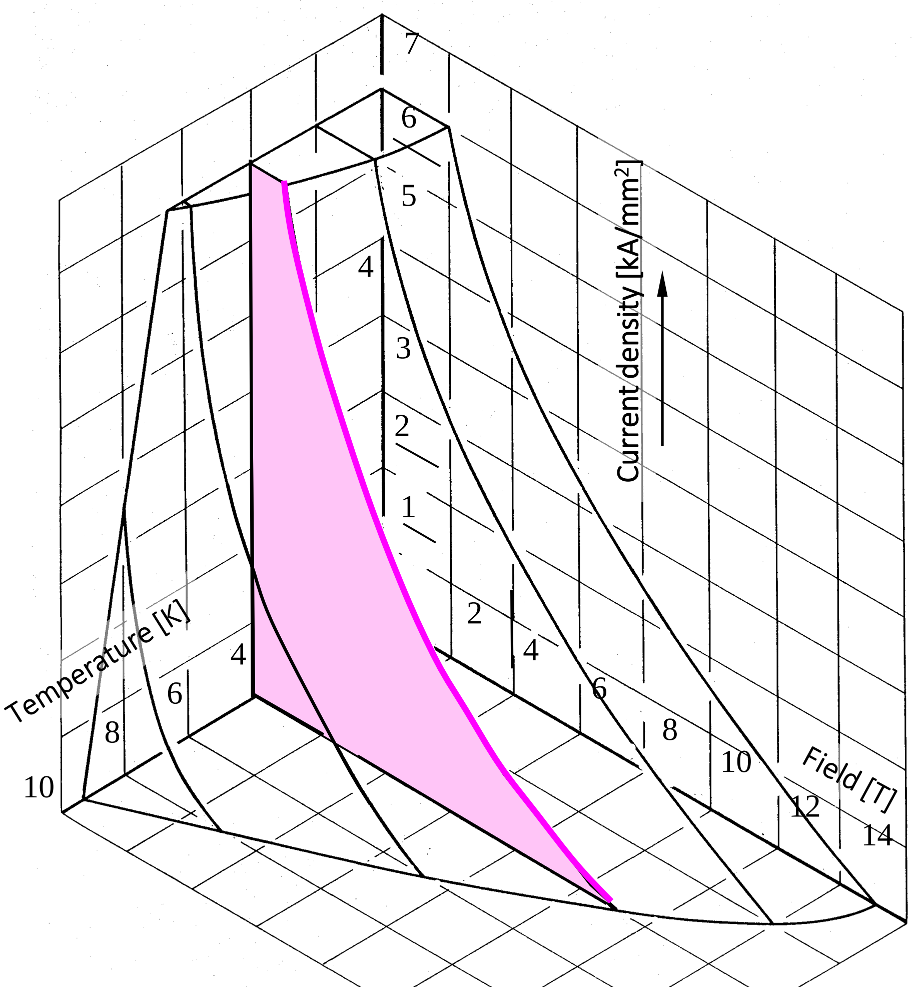


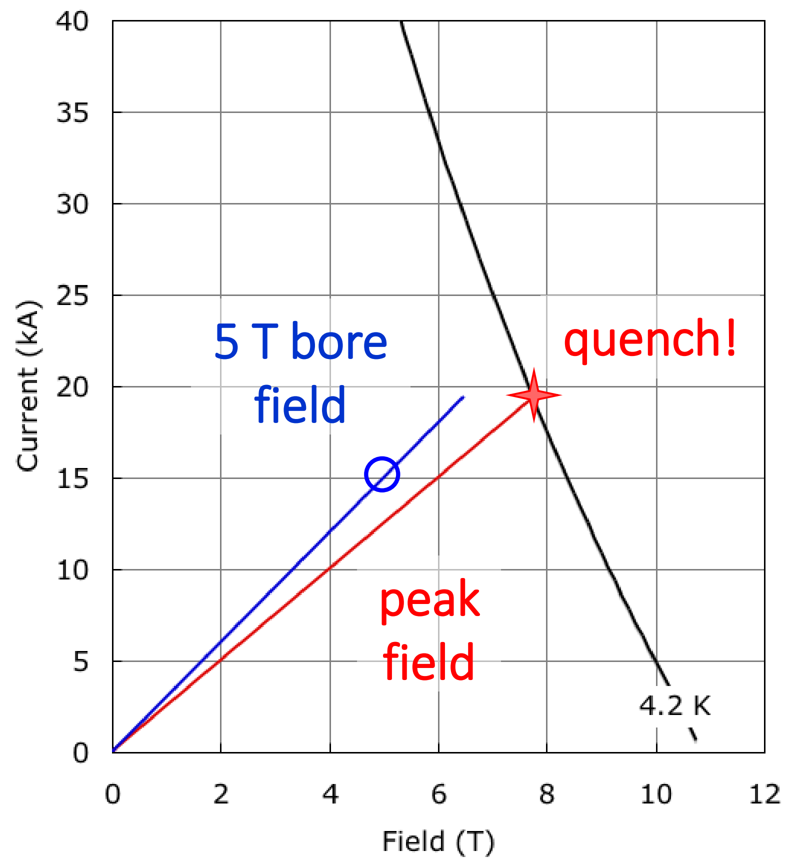
The current density is high – though finite – and it depends on the temperature and the field



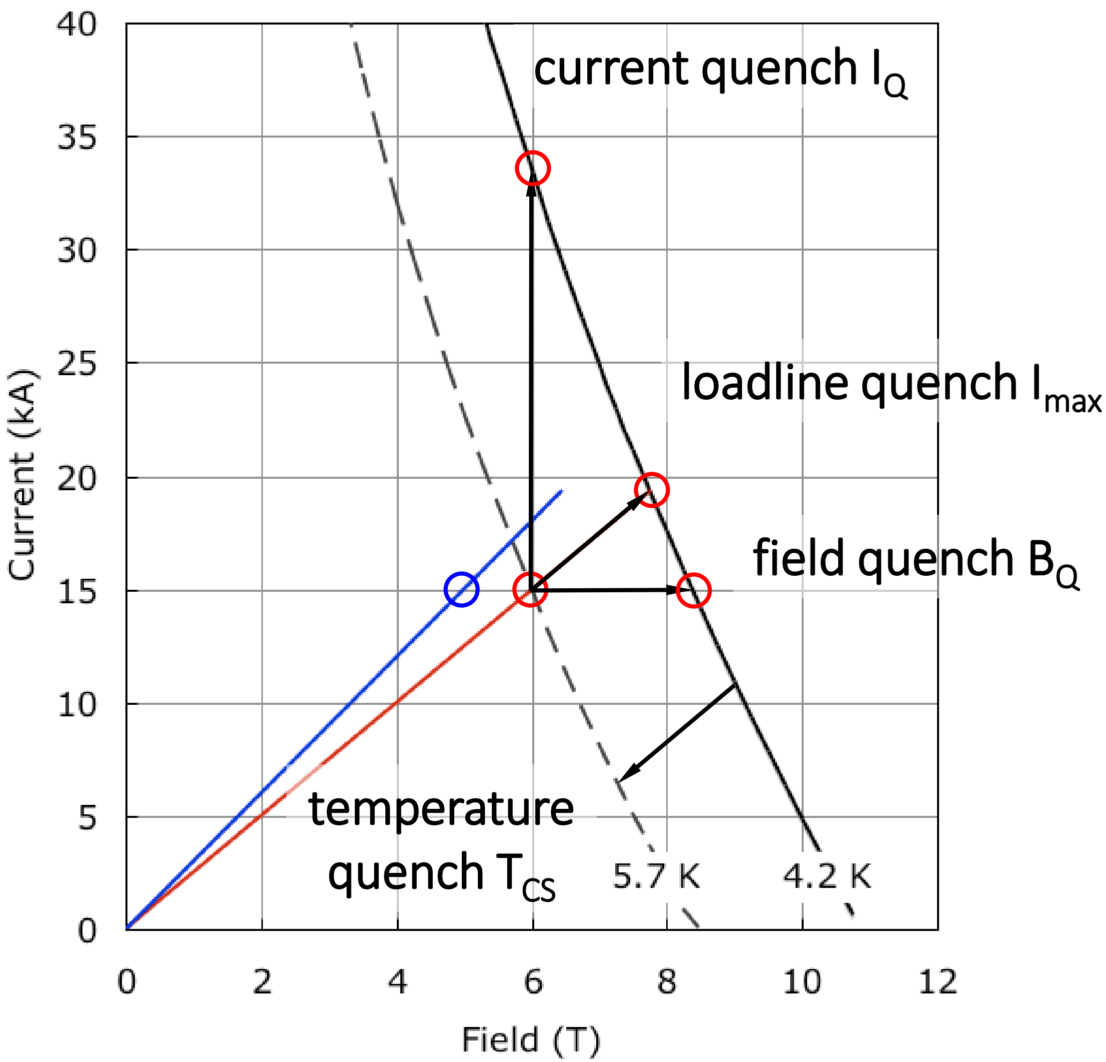
The maximum achievable field (on paper) depends on the amount of conductor and on the superconductor's critical line

Nb-Ti critical surface —— $I_C = J_C \times A_{SC}$ ——→ Nb-Ti critical current I_C (B)



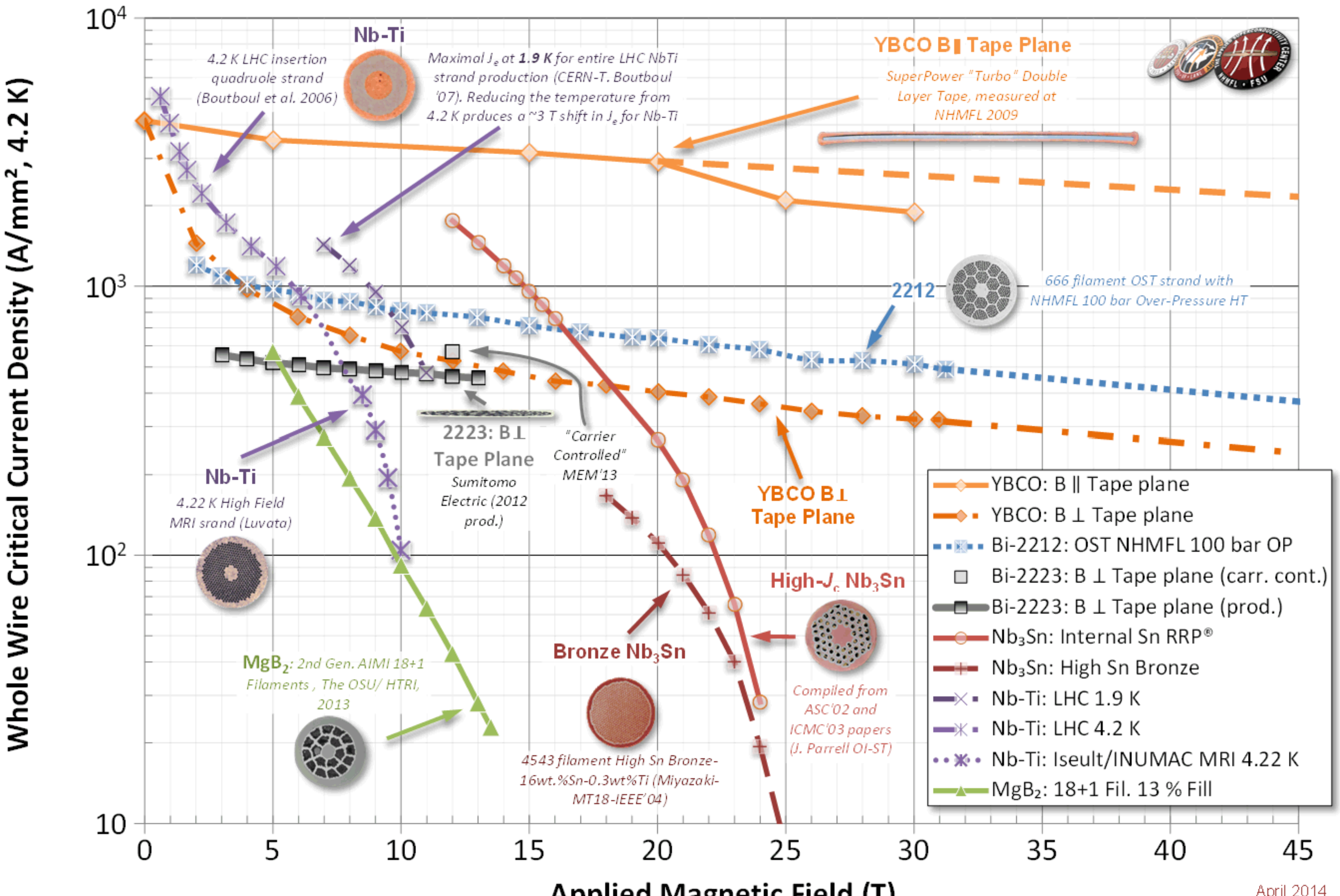


In practical operation, margins are needed with respect to this ultimate limit

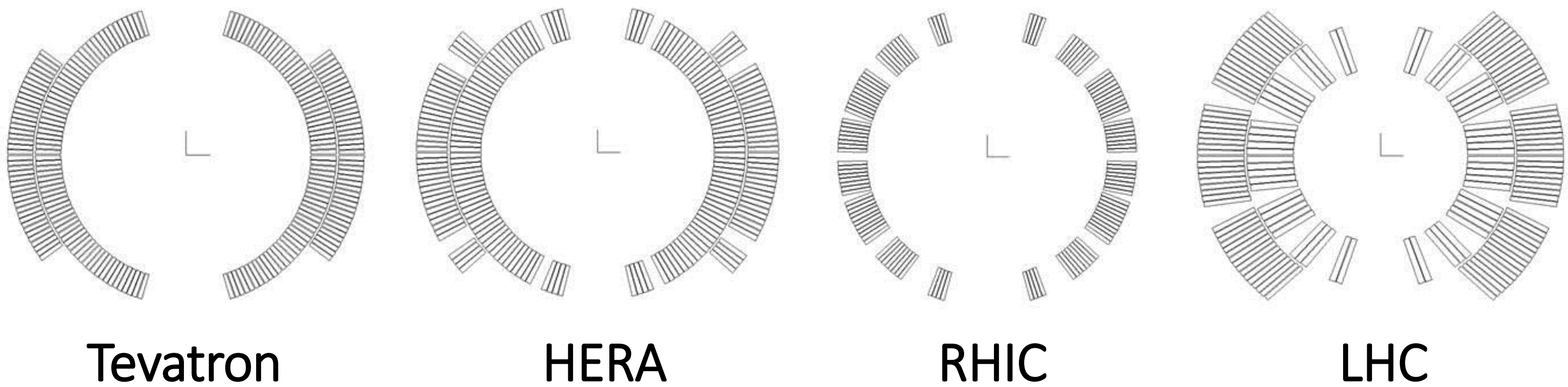


This is the best (Apr. 2014) critical current for several superconductors

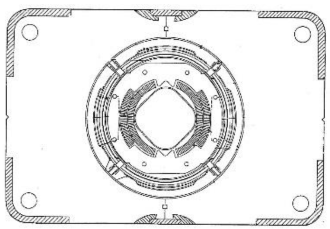
Applied Superconductivity Center at NHMFL



The coil cross sections of several superconducting dipoles show a certain evolution; all were (are) based on Nb-Ti

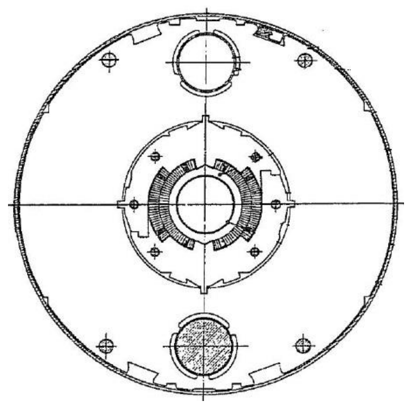


Different choices were made for the iron, the mechanical structure and the operating temperature



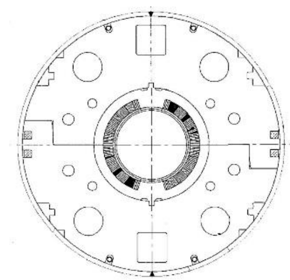
Tevatron

76 mm bore
B = 4.3 T
T = 4.2 K
first beam 1983



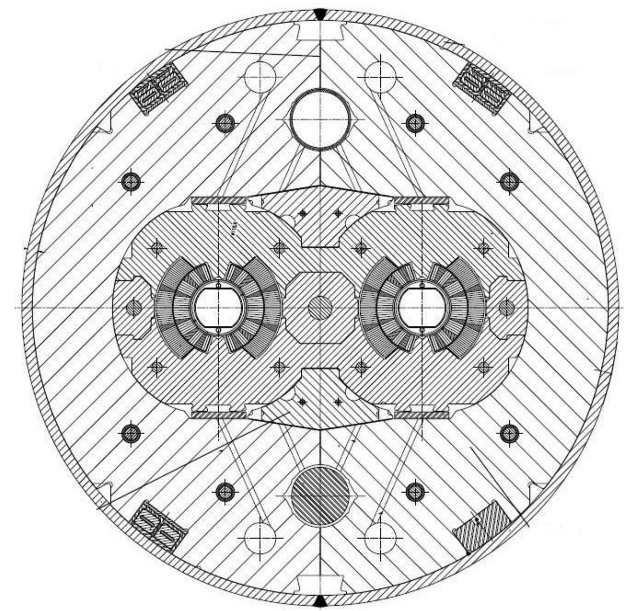
HERA

75 mm bore
B = 5.0 T
T = 4.5 K
first beam 1991



RHIC

80 mm bore
B = 3.5 T
T = 4.3-4.6 K
first beam 2000



LHC

56 mm bore
B = 8.3 T
T = 1.9 K
first beam 2008

These are the same magnets in the respective tunnels

Tevatron @ FNAL
(Chicago, IL, USA)

RHIC @ BNL
(Upton, NY, USA)

HERA @ DESY
(Hamburg, D)

LHC @ CERN
(Geneva, CH/FR)