

Accelerator Design from Start to Finish

ACCELERATOR PHYSICS

MT 2015

E. J. N. Wilson

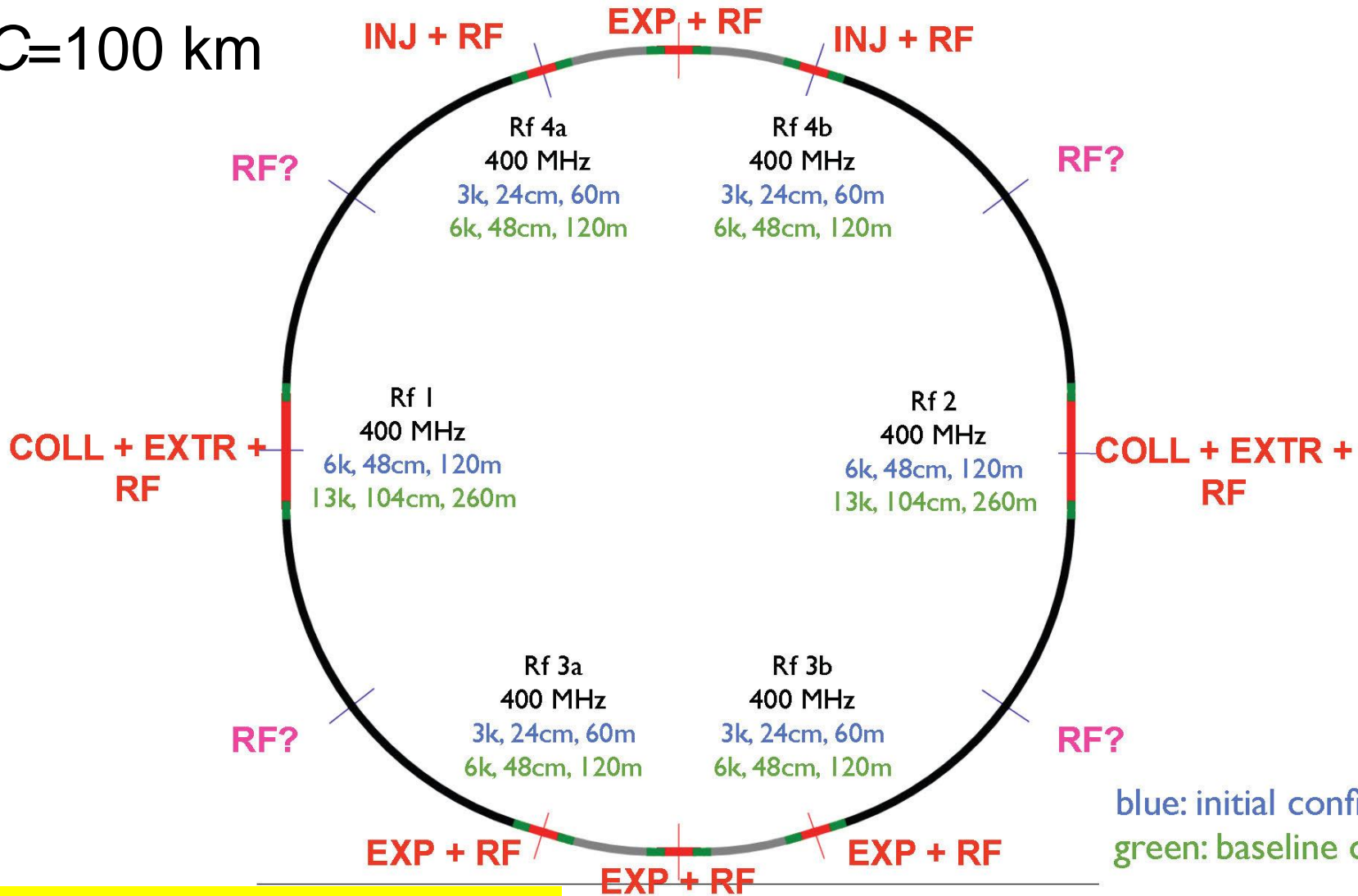
Putting "it" together



- The SPS Design Committee get down to business (1971)

FCC-ee preliminary layout

C=100 km

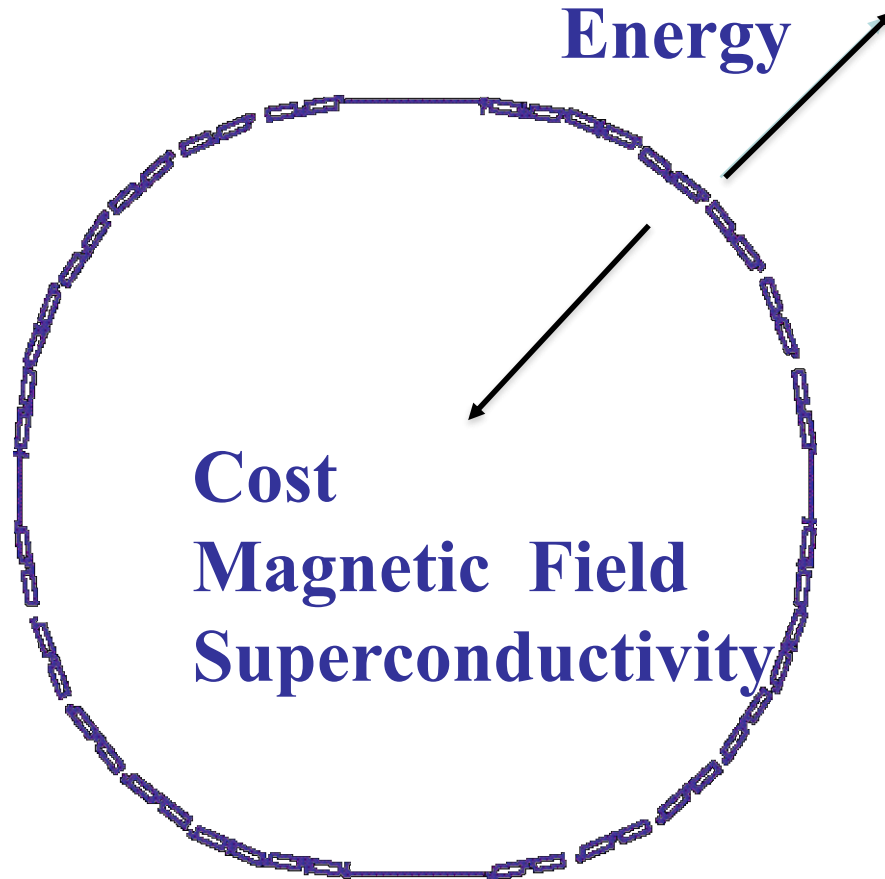


P. Lebrun, J. Osborne,
D. Schulte, U. Wienands

- ✓ consistent with *FCC-hh* layout
- ✓ RF staging scenario defined

The compromise between radius and magnetic field

Synchrotron Radiation





preliminary FCC-ee parameters

parameter	FCC-ee	LEP2
energy/beam	45 – 175 GeV	105 GeV
bunches/beam	50 – 60000	4
beam current	6.6 – 1450 mA	3 mA
hor. emittance	~2 nm	~22 nm
emittance ratio $\varepsilon_x/\varepsilon_y$	0.1%	1%
vert. IP beta function β_y^*	1 mm	50 mm
luminosity/IP	1.5-280 x 10 ³⁴ cm ⁻² s ⁻¹	0.0012 x 10 ³⁴ cm ⁻² s ⁻¹
energy loss/turn	0.03-7.55 GeV	3.34 GeV
synchrotron radiation power	100 MW	23 MW
RF voltage	0.3 – 11 GV	3.5 GV

- Large number of bunches at Z and WW and H requires **2 rings**.
- High luminosity means short beam lifetime (few mins) and requires continuous injection (**top up**).

Aperture compromise

Closer tolerances

Cost

Power consumption

Stored energy

(Tunnel diameter)

**Magnet
aperture**



Emittance

Injection steering

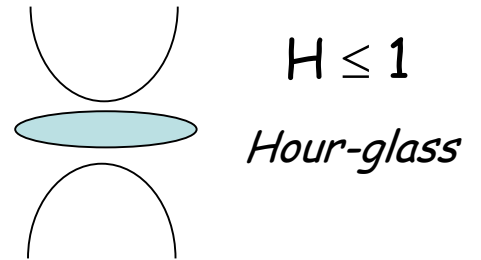
Space charge

Orbit errors

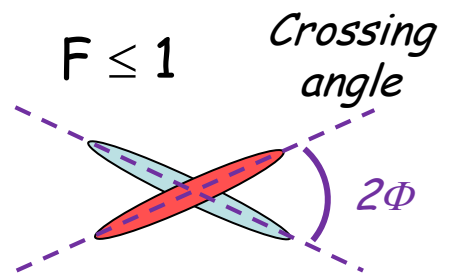
Dynamic aperture

luminosity scaling: larger E & ρ

$$e f k N = \text{beam current} \propto \frac{1}{E^4}$$



$$L = \frac{f k N^2}{4\pi \sigma_x \sigma_y} F H$$



$$\xi_y \propto \frac{\beta_y^* N}{E \sigma_x \sigma_y} \leq \xi_y^{\max}(E) \quad \text{Beam-beam parameter}$$

$$L \propto \frac{\rho P_{SR}}{E^3} \frac{\xi_y}{\beta_y^*}$$

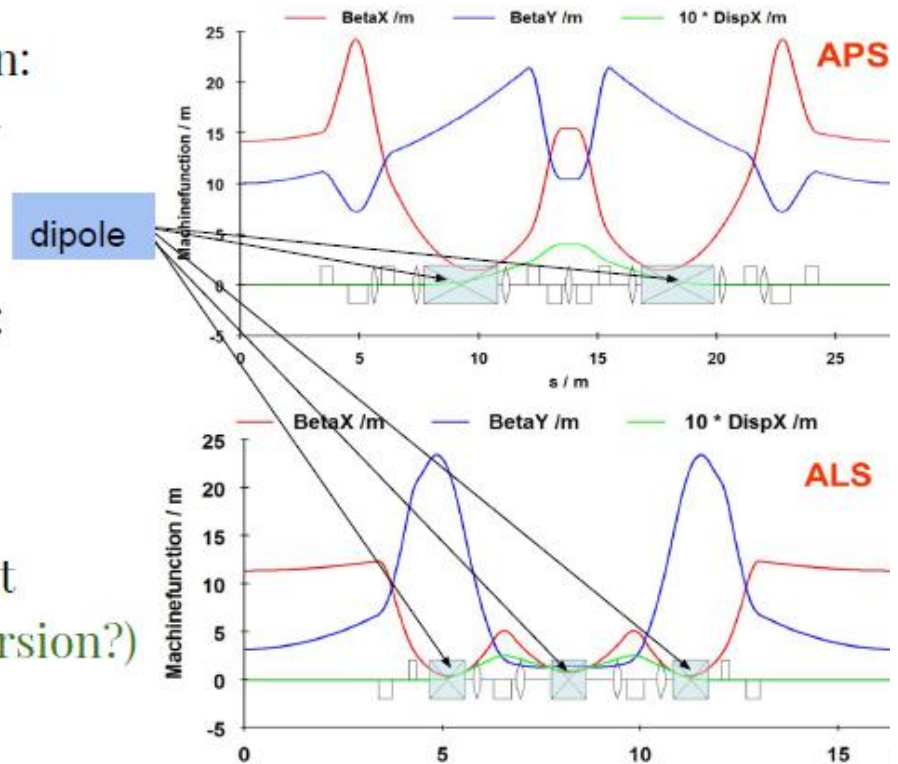
- σ = beam size
- k = no. bunches
- f = rev. frequency
- N = bunch population
- P_{SR} = synch. rad. power
- β^* = betatron fct at IP
(beam envelope)

Low emittance lattices

Examples of Low Emittance lattices

- DBA (Double Bend Achromat), used in:
 - ESRF, ELETTRA, APS, Diamond, SOLEIL...
- TBA (Triple Bend Achromat), used in:
 - ALS, SLS, PLS, ...

achromat: zero dispersion at long straight
can you think of why we want zero dispersion?)



Smooth approx. - choosing No. of periods

$$N\mu = 2\pi Q$$

$$\int \frac{ds}{\beta} = \int d\phi$$

$$\frac{2\pi R}{\bar{\beta}} = 2\pi Q$$

$$\therefore \bar{\beta} = \frac{R}{Q} \quad \left(= \frac{\lambda}{2\pi} \right)$$

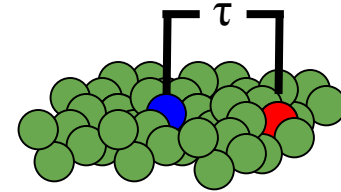
$$\gamma_{tr} \approx Q$$

$$\frac{1}{\gamma_{tr}^2} = \frac{\bar{D}}{R}$$

$$\therefore \bar{D} = \frac{R}{Q^2}$$

Radiation damping: Longitudinal plane (1/2)

- The synchronous particle is in the bunch centre; $\tau = \Delta s/c > 0$ is the time distance for an electron ahead of the synchronous particle
- Assuming changes in ε and τ occur slowly with respect to T_0 :



and on average in 1 turn:

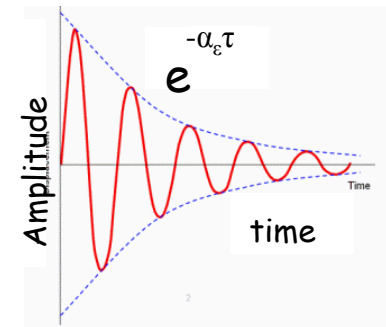
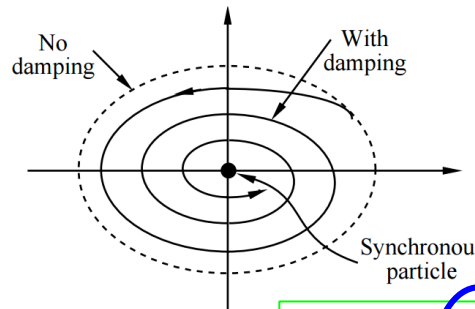
$$\frac{d\tau}{dt} = -\alpha \frac{\varepsilon}{E_0}$$

$$\frac{d\varepsilon}{dt} = \frac{eV(\tau) - U(\varepsilon)}{T_0}$$

ε and τ decrease **exponentially** with damping time τ_ε :

$$\varepsilon(t) = A e^{-\alpha_\varepsilon t} \cos(\Omega t - \phi)$$

$$\tau(t) = \frac{-\alpha}{E_0 \Omega} A e^{-\alpha_\varepsilon t} \sin(\Omega t - \phi)$$



$1/\tau_\varepsilon = \alpha_\varepsilon$: longitudinal damping factor

$$\alpha_\varepsilon = \frac{1}{2T_0} \frac{dU}{d\varepsilon}$$

let's calculate this!

$$\Omega^2 = \frac{e}{T_0} \dot{V}_0 \frac{\alpha}{E_0}$$

Radiation damping: Longitudinal plane (2/2)

- Rate of energy loss changes with energy because:

- it is itself a function of energy

- orbit deviates from reference orbit and there could be change in path length

$$U(\varepsilon) = \frac{1}{c} \oint P dl$$

: Integral of power radiated over time spent in bendings (both depend on energy of particle)

- P is function of E^2 and B^2 :

$$P = P_0 + \frac{2P_0}{E_0} \varepsilon$$

and

$$\frac{dU(\varepsilon)}{d\varepsilon} = \frac{1}{c} \oint \frac{2P_0}{E_0} ds = \frac{2U_0}{E_0}$$

(without taking into account path-lengthening)

Energy distribution of emitted photons

- Energy emitted in quanta; each quantum carries energy $u = \hbar\omega$;

– $n(u)$: number of photons emitted with energy in $u, u+du$

$$N = \int n(u) du = \frac{15\sqrt{3}}{8} \frac{P}{u_c} \text{ time}$$

– $u n(u)$: energy of photons emitted

$$\langle u \rangle = \frac{\int u n(u) du}{N} = \frac{P}{N} = \frac{8}{15\sqrt{3}} u_c$$

with energy in $u, u+du$

$$\langle u^2 \rangle = \frac{\int u^2 n(u) du}{N} = \frac{11}{27} u_c^2$$

- Total number of photons emitted per

Quantum fluctuations of synchrotron

osci $A^2 = \varepsilon^2 + \left(\frac{U_s \omega_s}{\alpha}\right)^2 \tau^2$

- Invariant longitudinal o $\varepsilon \rightarrow \varepsilon - u \quad \tau \rightarrow \tau$

- When a photon of energy u is emitted: $\frac{d \langle A^2 \rangle}{dt} = -\frac{2 \langle A^2 \rangle}{\tau_\varepsilon} + \langle N_\gamma \langle u^2 \rangle_\gamma \rangle$ *radiation damping* *quantum excitation*

- and the change of A^2 is: $\langle A^2 \rangle = \frac{\tau_\varepsilon}{2} \langle N_\gamma \langle u^2 \rangle_\gamma \rangle$

$$\sigma_\varepsilon^2 = \langle \varepsilon^2 \rangle = \frac{\langle A^2 \rangle}{2} \quad \frac{\sigma_\varepsilon^2}{E_0^2} = \frac{55}{64\sqrt{3}} \frac{\hbar}{mc} \frac{\gamma^2}{\rho}$$

- Average longitudinal for synchrotron with separated function magnets invariant decreases \swarrow

exponentially with damping time τ_ε and

Summary of radiation integrals

$$I_1 = \oint \frac{D}{\rho} ds$$

Momentum compaction factor

$$I_2 = \oint \frac{ds}{\rho^2}$$

$$\alpha = \frac{I_1}{2\pi R}$$

$$I_3 = \oint \frac{ds}{|\rho^3|}$$

Energy loss per turn

$$I_4 = \oint \frac{D}{\rho} \left(2k + \frac{1}{\rho^2} \right) ds$$

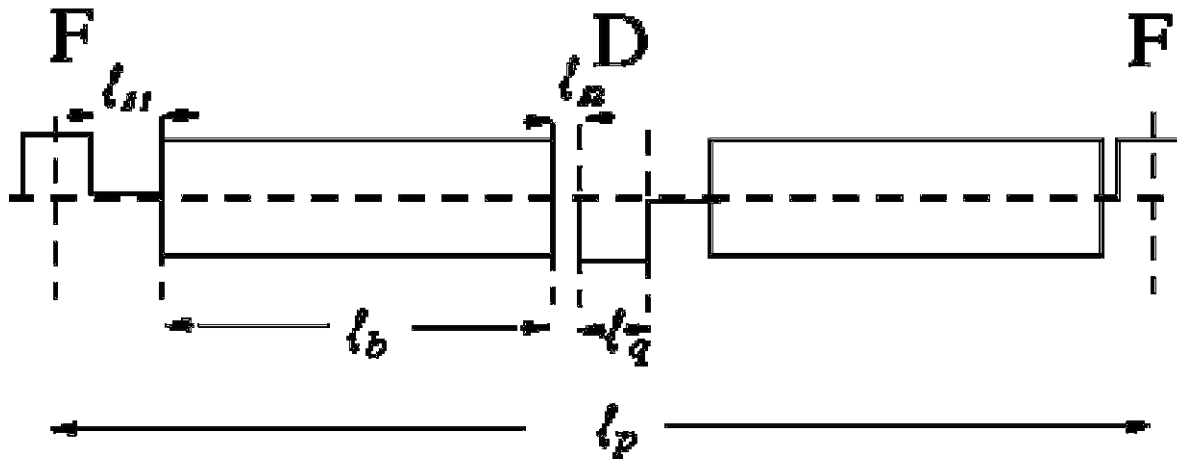
$$U_0 = \frac{1}{2\pi} C_\gamma E^4 \cdot I_2$$

$$I_5 = \oint \frac{\mathcal{H}}{|\rho^3|} ds$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\text{m}}{\text{GeV}^3} \right]$$

Period geometry

- Everything must add up for the ring

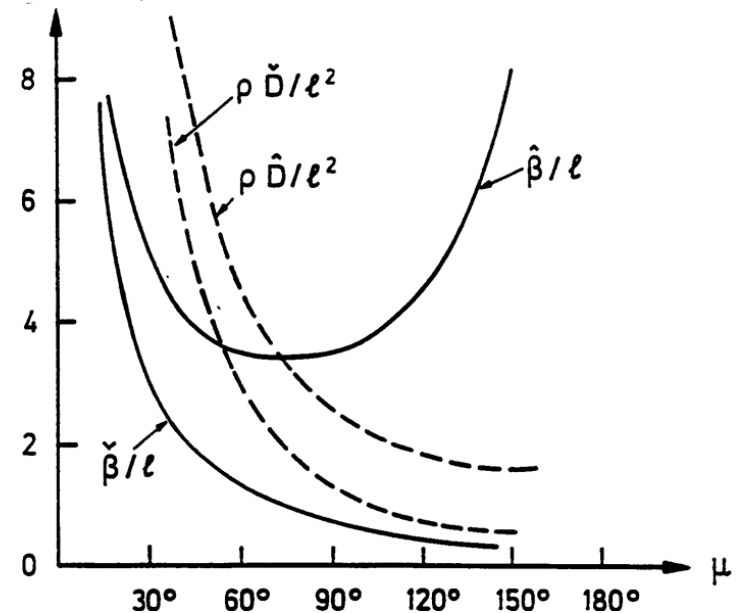


$$l_p = 2(l_b + l_{s1} + l_{s2} + l_q)$$

$$2\pi R = N_p l_p$$

$$2\pi(B\rho) = N_b l_b B = 2\pi(3.3356 p [GeV/c])$$

- The beta at the F quadrupole which defines the scale of the apertures goes through a minimum at about 70 deg/cell.
- Other considerations which might lead to close to 90 degrees per cell are
 - Sensitivity to closed orbit errors
 - Ease of locating correctors
 - Schemes for correcting the chromaticity in the arcs without exciting resonances



The lattice and insertions

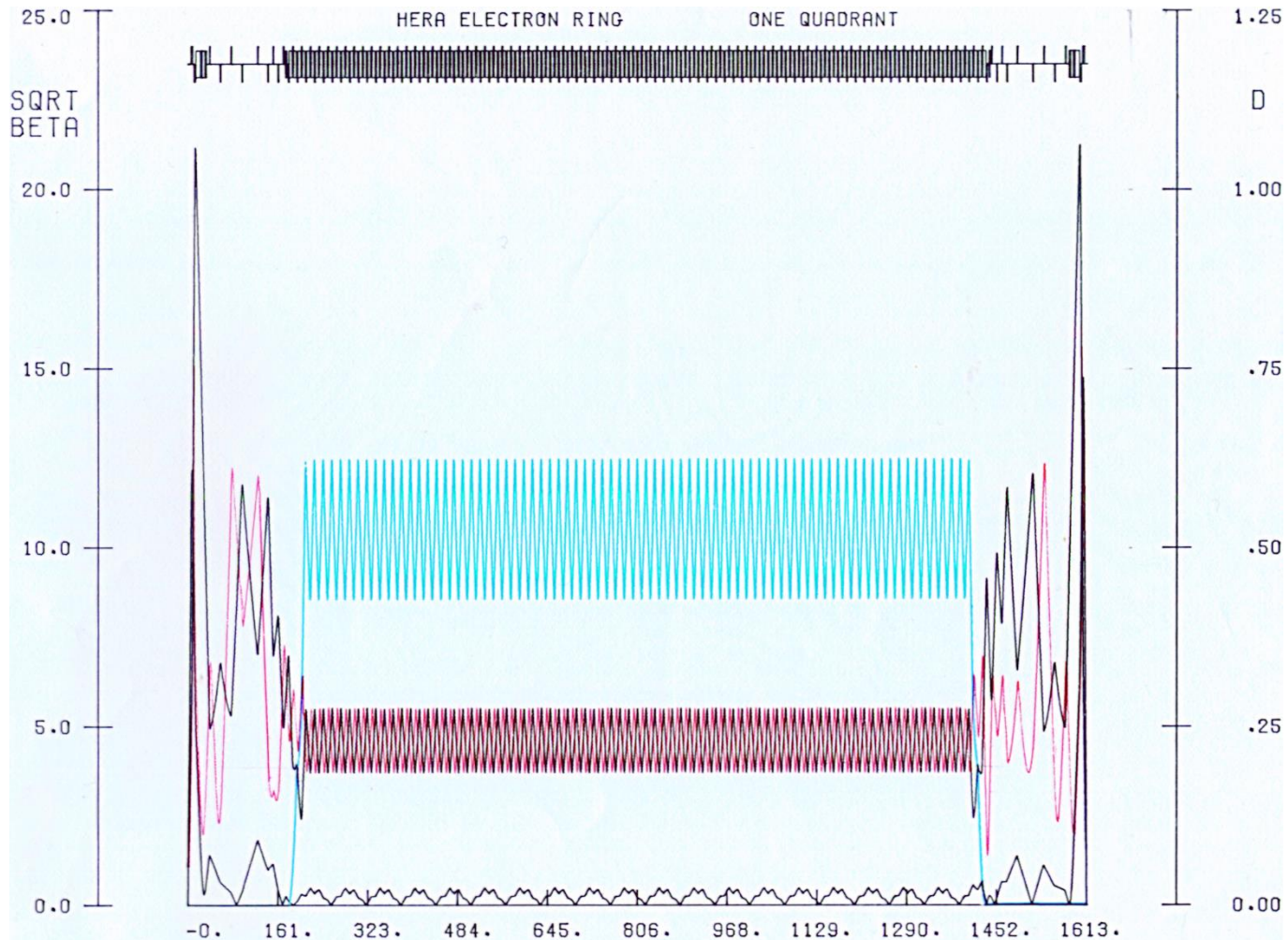
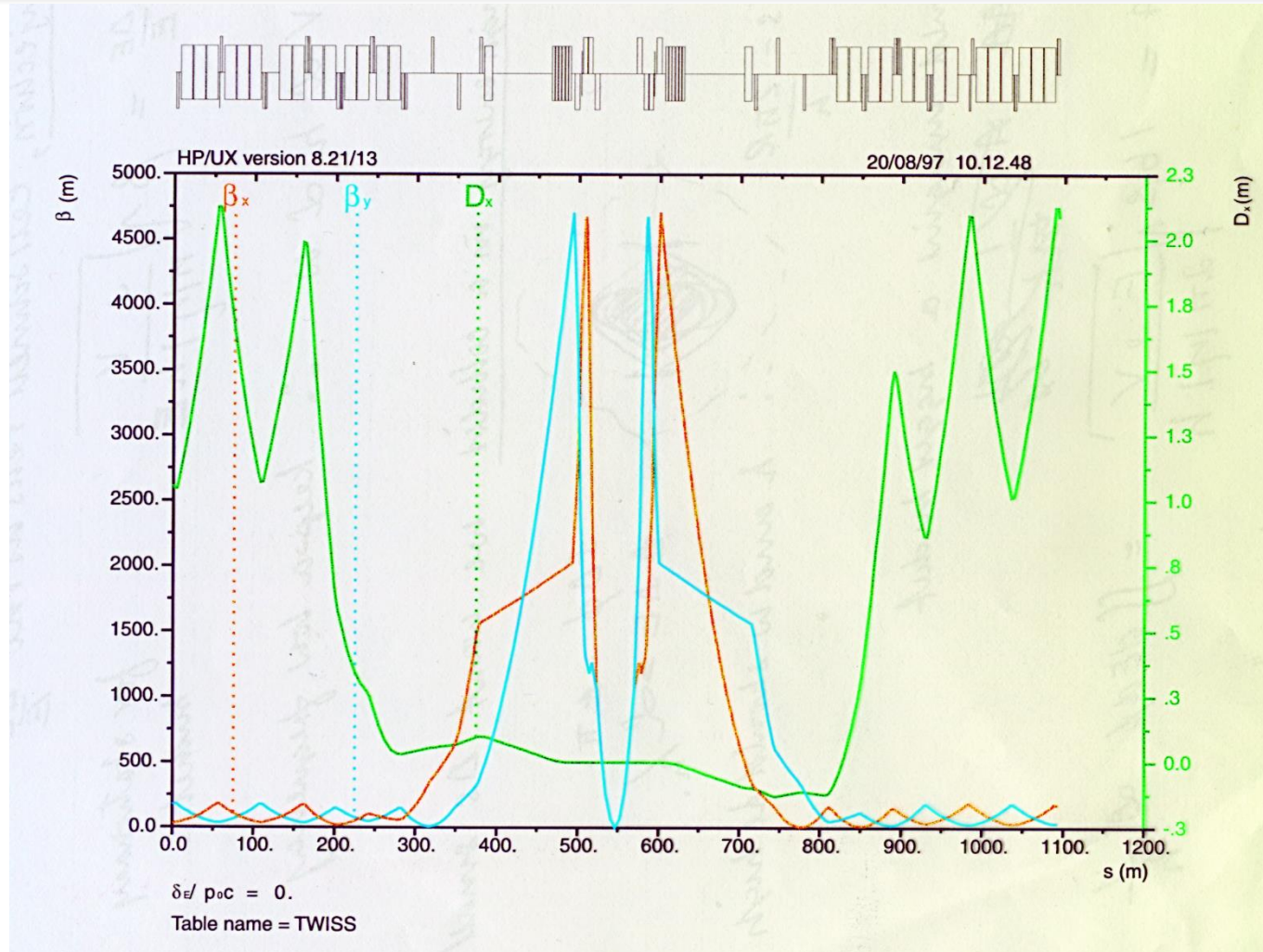


Fig. 3.1 - Electron Lattice with beta functions and dispersion
(β_z in red, β_x in black, D_z in black, D_x in blue)

Insertions



Correction of chromaticity

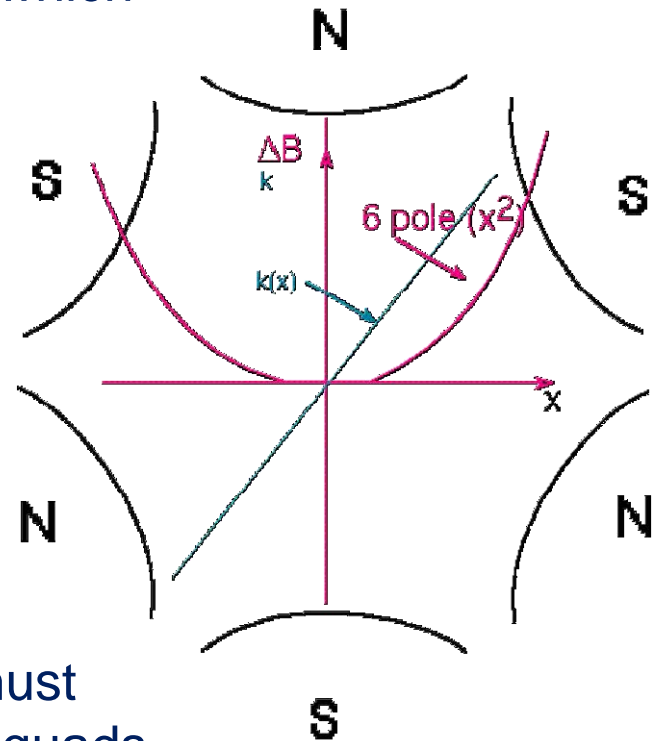
- Parabolic field of a 6 pole is really a gradient which rises linearly with x
- If x is the product of momentum error and dispersion

$$\Delta k = \frac{B'' D \Delta p}{(B\rho) p}$$

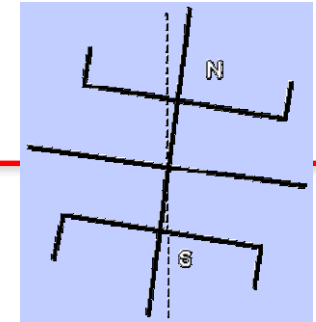
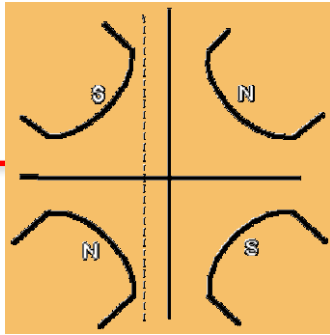
- The effect of all this extra focusing cancels **chromaticity**

$$\Delta Q = \left[\frac{1}{4\pi} \int \frac{B''(s)\beta(s)D(s)ds}{(B\rho)} \right] \frac{dp}{p}$$

- Because gradient is opposite in v plane we must have two sets of opposite polarity at F and D quads where betas are different



Sources of distortion

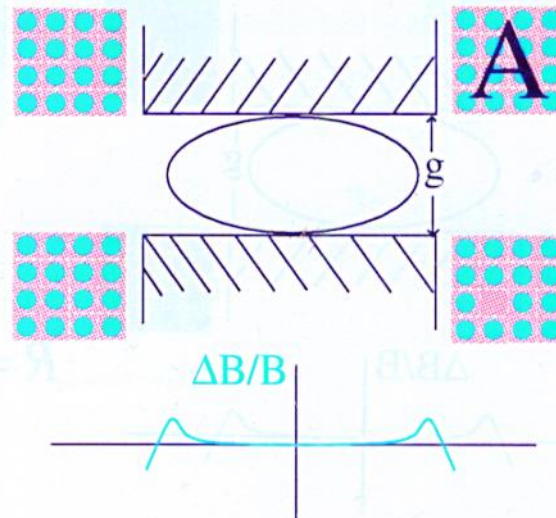


Δy Table 1 Sources of Closed Orbit Distortion Δ

Type of element	Source of kick	r.m.s. value	$\langle \Delta B / (B\rho) \rangle_{rms}$	plane
Gradient magnet	Displacement	$\langle \Delta y \rangle$	$k_i l_i \langle \Delta y \rangle$	x, z
Bending magnet (bending angle = θ_i)	Tilt	$\langle \Delta \rangle$	$\theta_i \langle \Delta \rangle$	z
Bending magnet	Field error	$\langle \Delta B / B \rangle$	$\theta_i \langle \Delta B / B \rangle$	x
Straight sections (length = d_i)	Stray field	$\langle \Delta B_s \rangle$	$d_i \langle \Delta B_s \rangle / (B\rho)_{inj}$	x, z

Magnet design

$$\Sigma Bl = 2\pi(B\rho) = 2\pi(3.3356\rho c)$$



$$NI = \frac{gB}{\mu_0}$$

$$R = \frac{2lN^2\rho}{A}$$

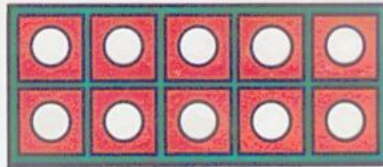
$$\text{Power} = I^2 R = \frac{g^2 B^2 2l\rho}{\mu_0^2 A}$$

Power for given $Bl \propto B$

Magnet configurations

Coil Design/Geometry

Standard design is rectangular copper (or aluminium) conductor, with cooling water tube. Insulation is glass cloth and epoxy resin.



Amp-turns (NI) are determined, but total copper area (A_{copper}) and number of turns (n) need to be decided.

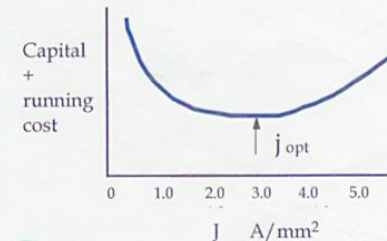
Copper Area. Current density $j = N/A_{\text{copper}}$

Optimum j determined from economic criteria.

Some fraction of the magnet capital costs (coil & yoke materials, plus assembly, testing and transport) vary (roughly) as $1/j$. Operational costs (price of electrical power over the life of the accelerator) vary as j . So total cost of building and running magnet 'amortised' over life of machine is:

$$\mathcal{E} = K + C/j + Rj$$

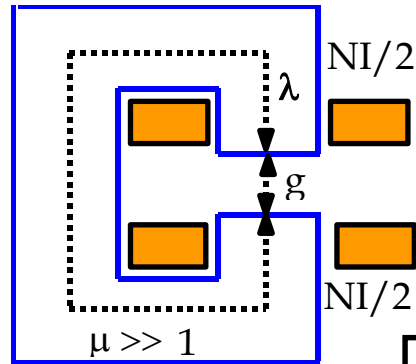
Values of K , C , R and j_{opt} depend on design, manufacturer, policy, country, etc. Values of 3 to 5 A/mm² for j_{opt} are typical.



Magnet cross sections

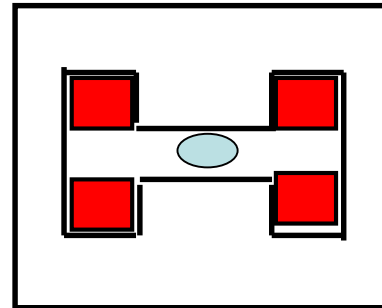
"C" Core:

- Easy access
- Less rig



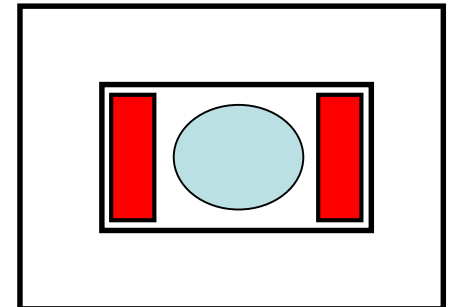
'H core':

- Symmetric;
- More rigid;
- Access problems.

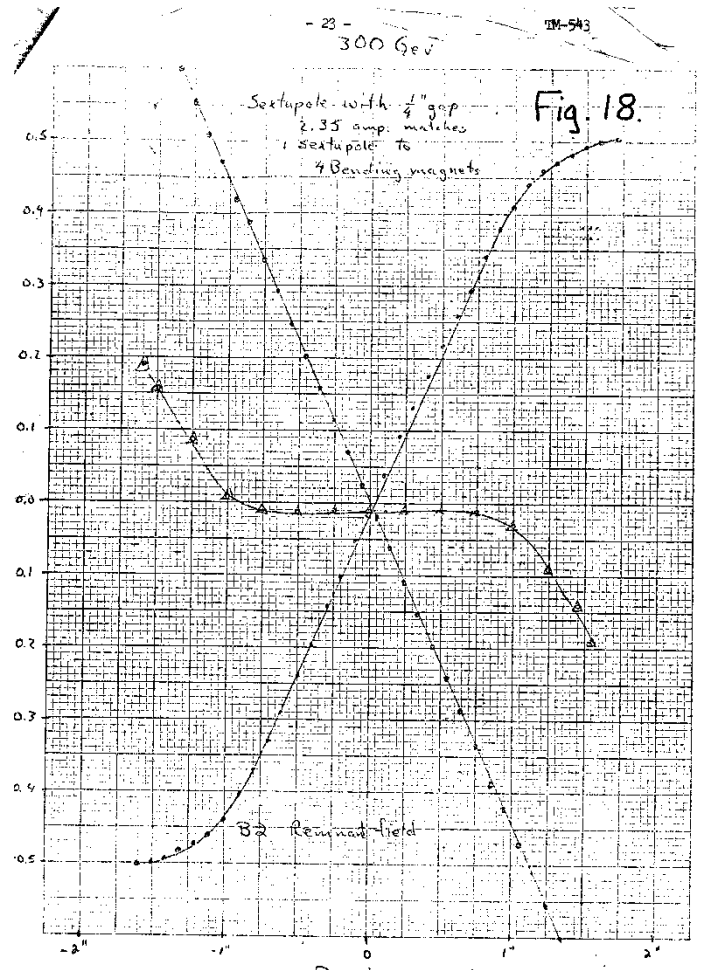


"Window Frame"

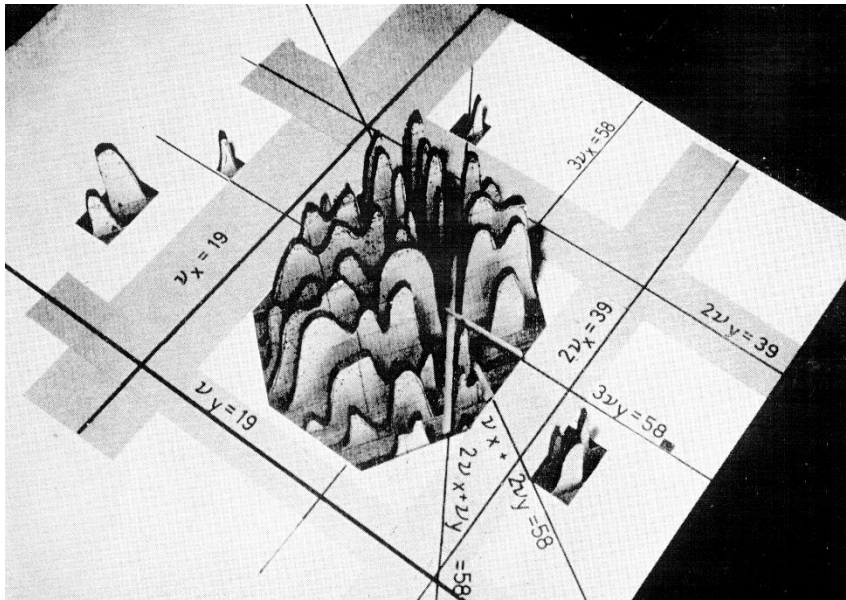
- High quality field;
- Major access problems
- Insulation thickness



How not to measure magnets

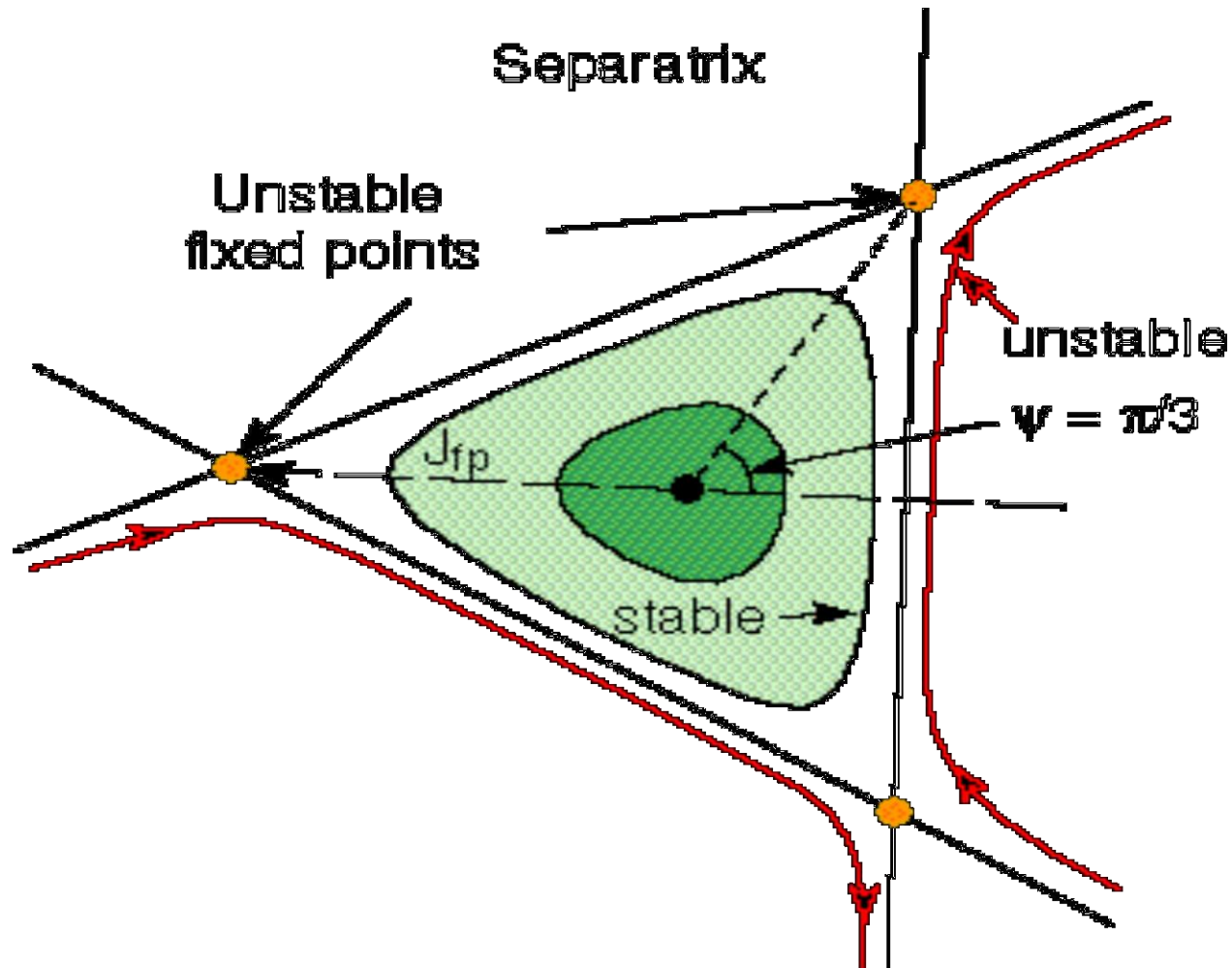


INJECTION STUDIES AT FNAL



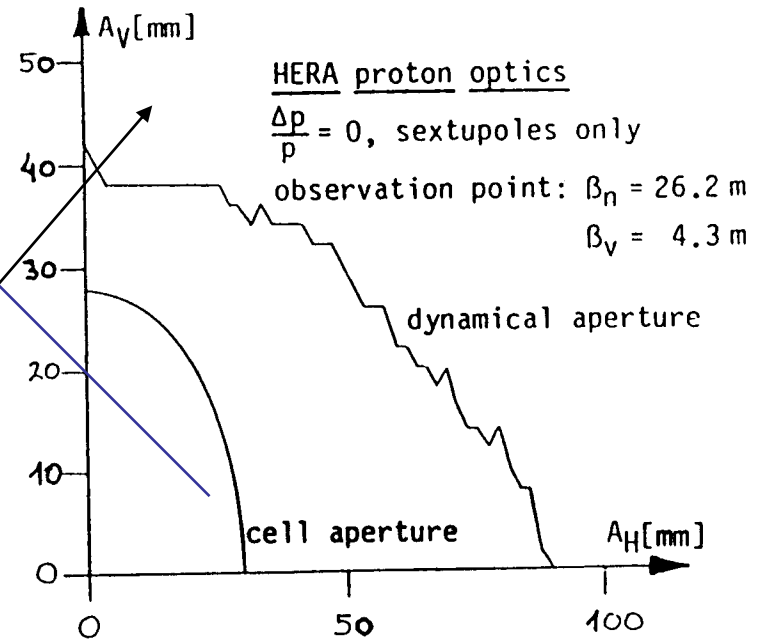
- Remanent sextupole in the FNAL main ring caused serious beam loss due to non-linear resonances.
- This was exacerbated by magnet ripple.
- A three dimensional hill and dale model spanning the Q (or ν) diagram

Magnet tolerances v. aperture (dynamic)



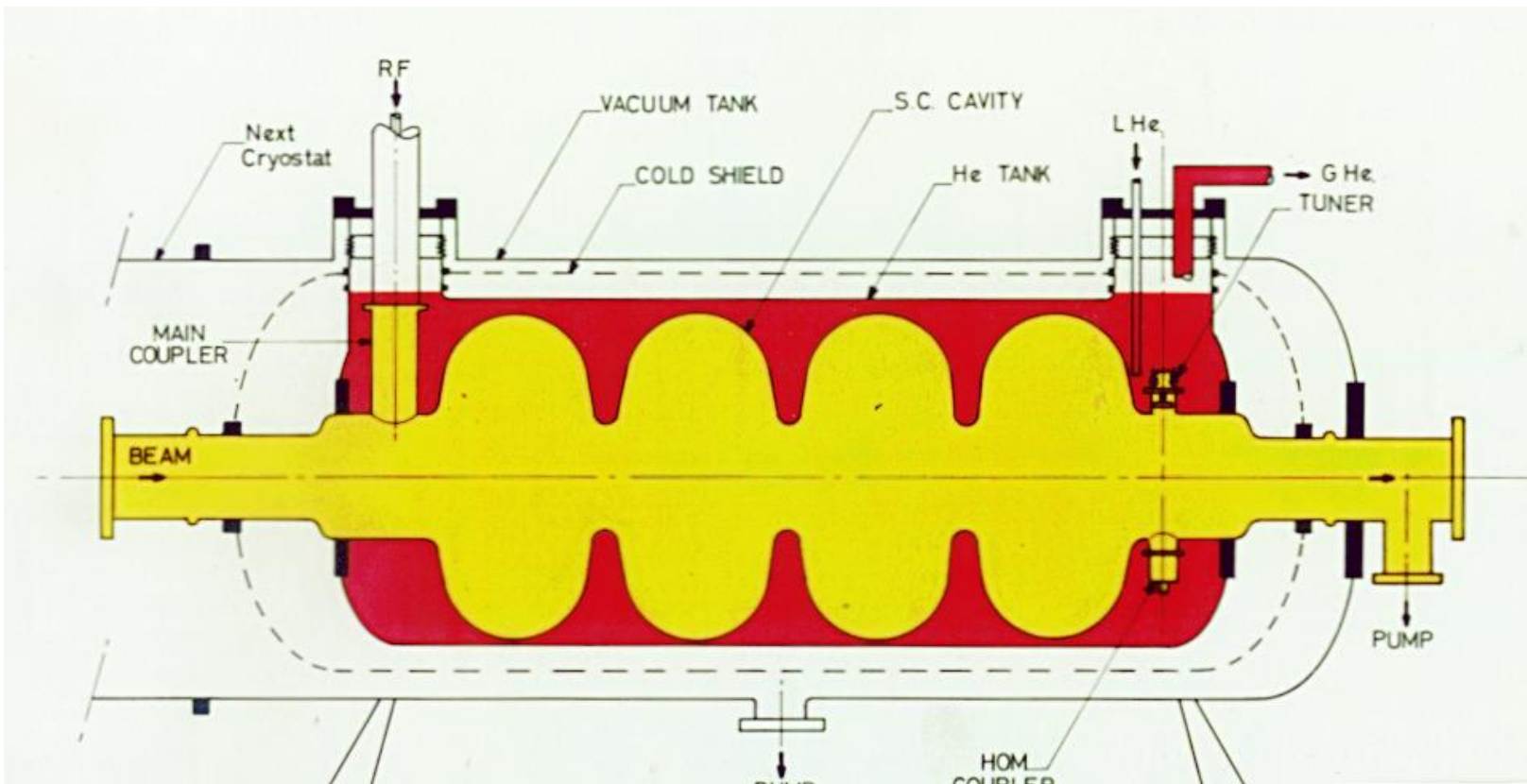
Dynamic Aperture

- Particles launched into simulator with different betatron amplitudes and region in which they turns is compared with the magnet aperture



RF System

- constraint is Voltage per meter and MW of power
- pressure from need to provide a good acceleration rate or large bucket (synchrotron emission in lepton machines)



Synchrotron motion

⌘ This is a biased rigid pendulum

$$\ddot{\phi} = -\frac{2\pi V_0 h \eta f^2}{E_0 \beta^2 \gamma} (\sin \phi - \sin \phi_s)$$

⌘ Synchrotron frequency

$$f_s = \sqrt{\frac{|\eta| h V_0 \cos \phi_s}{2\pi E_0 \beta^2 \gamma}} f .$$

⌘ Synchrotron “tune

$$f_s = \sqrt{\frac{|\eta| h V_0 \cos \phi_s}{2\pi E_0 \beta^2 \gamma}} f .$$

⌘ Should be less than 0.05

$$Q_s = \frac{f_s}{f} = \sqrt{\frac{|\eta| h V_0 \cos \phi_s}{2\pi E_0 \beta^2 \gamma}} .$$

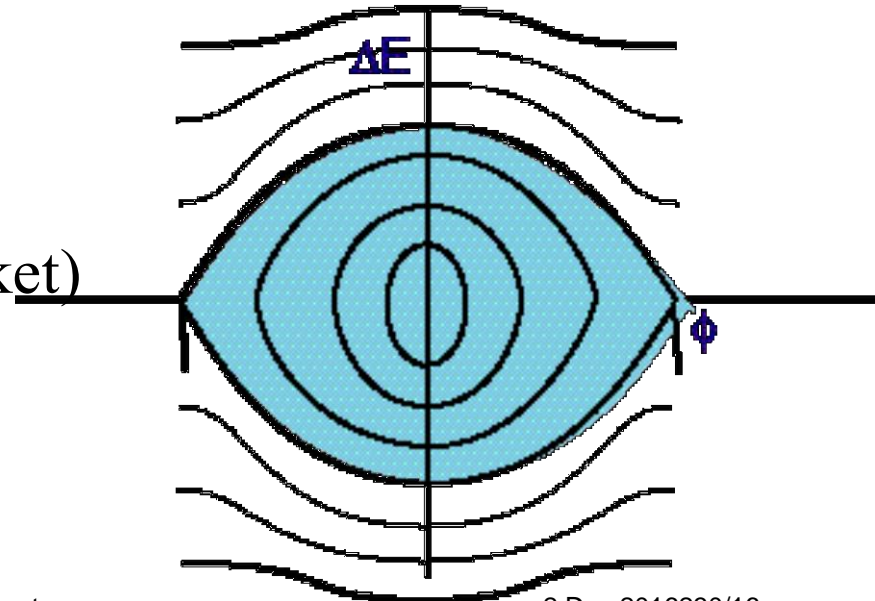
Rf volts to damp instabilities

- During collisions, in order to use the Keil Schnell criterion to combat instabilities we must have enough voltage to reach a threshold value of :

$$\frac{\Delta E}{E} = \beta \sqrt{\frac{eV}{\pi|\eta|hE}}$$

$$V \propto h \propto \omega_{rf}$$

(stationary bucket)



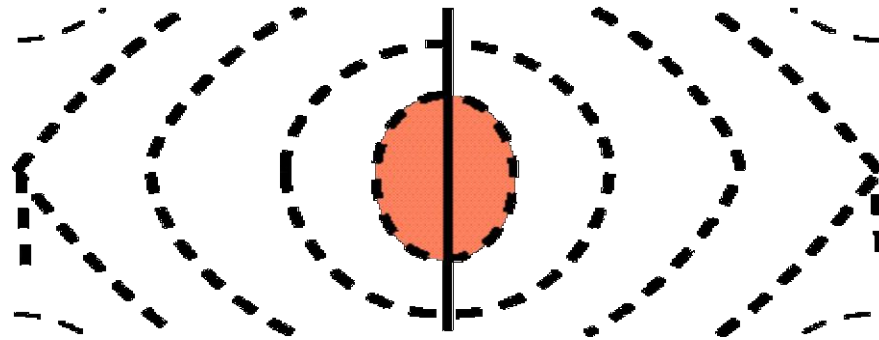
Organising the design work

- **A. Lattice**
- 1. Establish and update a parameter list
- 2. Choose a lattice <http://doc.cern.ch/yellowrep/2005/2005-012/p55.pdf>
- 3. Decide phase advance per cell
- 5. Decide period geometry
- 4. Calculate max and min beta and dispersion
- 6. Calculate radiation integrals
- 7. Acceptance required
- **B. Errors and corrections**
- http://preprints.cern.ch/cgi-bin/setlink?base=cernrep&categ=Yellow_Report&id=95-06_v1
- 8. Correction of chromaticity
- **C. Magnet and power supply**
- http://preprints.cern.ch/cgi-bin/setlink?base=cernrep&categ=Yellow_Report&id=92-05
- 9. The magnet aperture - the most expensive component
- 10. Calculating magnet stored energy
- **D. RF**
- <http://preprints.cern.ch/cernrep/2005/2005-003/2005-003.html>
- 11. Choice of RF frequency (scaling)
- 12. Choice of RF voltage (injection)
- 13. Bucket size for capture and acceleration
- **E. Collective effects**
- 14. Instability thresholds
- <http://doc.cern.ch/yellowrep/2005/2005-012/p139.pdf>



Short bunches needed for collisions

- When colliding bunches, we want a short bunch



- If h is small, the bucket area must be much bigger

$$A = \iint dE d\phi = 16\beta \sqrt{\frac{EeV}{2\pi|\eta|h}} \propto \frac{h_{snug}}{h}$$

$$V \propto h^3 \quad \text{and} \quad \text{Power} \propto h^6$$

The moment of truth!

Adams, waiting for the first beam in the SPS, asks his team if they have remembered everything.

