Magnets for Accelerators – conventional and (some) super-conducting.

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Course Philosophy

Provide an introduction to magnet technology in particle accelerators for:

i)   r**oom-temperature, static** (d.c.) **electro-magnets**, and

ii)   an initial overview of the **low-temperature super-conducting (LTS) magnets**, concentrating on electro-magnetic issues.

To introduce students to the use of the FEA code OPERA 2D – to enable on-going design work in the next 6 weeks.

Contents – lectures 1&2.

1. **DC Magnets-design and construction:**

a) **Introduction:**

* Nomenclature;

* Dipole, quadrupole and sextupole magnets;

* 'Higher order' magnets.

b) **Magneto-statics in free space (no ferromagnetic materials or currents):**

* Maxwell's 2 magneto-static equations;

* Solutions in two dimensions with scalar potential (no currents);

* Cylindrical harmonic in two dimensions (trigonometric formulation);

* Field lines and potential for dipole, quadrupole, sextupole;

* Significance of vector potential in 2D.

Contents (cont.)

c) Adding ferromagnetic poles:

•Ideal pole shapes for dipole, quad and sextupole;

•Field harmonics-symmetry constraints and significance;

d) The addition of currents:

•   Ampere-turns in dipole, quad and sextupole.

•Coil economic optimisation-capital/running costs.

e) The magnetic circuit:

•Steel requirements-permeability and coercivity.

•Backleg and coil geometry- 'C', 'H' and 'window frame' designs.

•Classical solution to end and side geometries – the Rogowsky roll-off.

Contents (cont.)

f) An introduction to super-conducting magnets:

- Basic concepts;

- Materials for and design of coils;

- External cold steel.

g) Magnet design using f.e.a. software:

•FEA techniques - Modern codes- OPERA 2D; OPERA 3D.

•Judgement of magnet suitability in design.

•Magnet ends-computation and design.

h) Some examples of recent magnet engineering;

i) Appendix relating specifically to super-conductivity:

- Provides more information on some issues;

- Further information needed for the tutorial.

a) Introduction.

Nomenclature.

Magnetic Field: (the magneto-motive force produced by electric currents)

      symbol is $\underline{\mathbf{H}}$ (as a vector);

      units are Amps/metre in S.I units (Oersteds in cgs);

Magnetic Induction or **Flux Density:** (the density of magnetic flux driven through a medium by the magnetic field)

      symbol is $\underline{\mathbf{B}}$ (as a vector);

      units are Tesla (Webers/m^2 in mks, Gauss in cgs);

Note: induction is frequently referred to as "Magnetic Field".

Permeability of free space:

      symbol is μ_0 ;

      units are Henries/metre;

Permeability (abbreviation of **relative permeability**):

      symbol is μ;

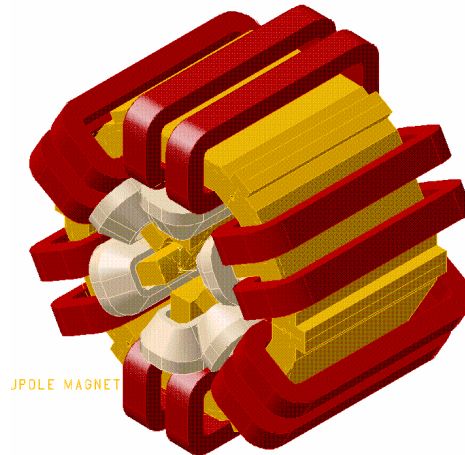   the quantity is dimensionless;

Magnet types

Dipoles to bend the beam:
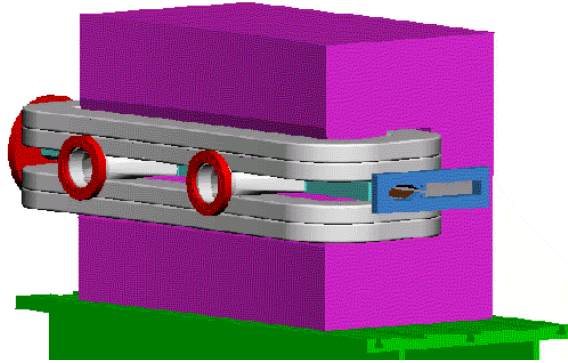
Quadrupoles to focus it:
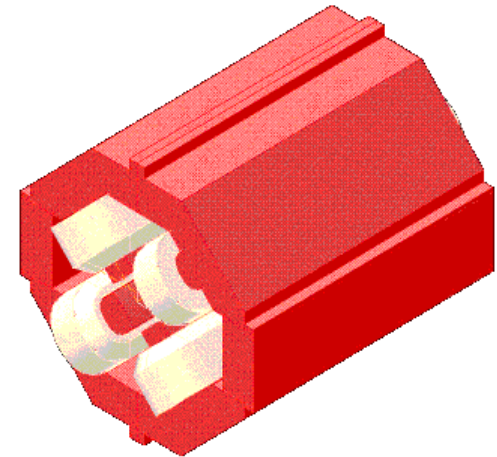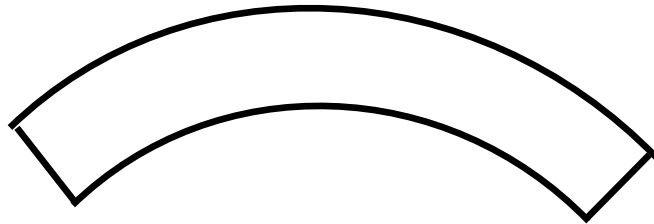
Sextupoles to correct chromaticity:

JPOLE MAGNET

We shall establish a formal approach to describing these magnets.
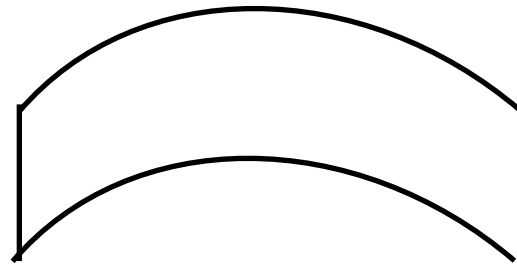
Magnets - dipoles

To bend the beam uniformly, dipoles need to produce a field that is constant across the aperture.

But at the ends they can be either:

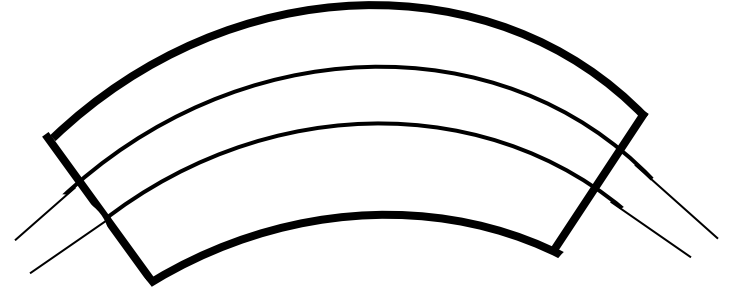**Sector dipole**

Parallel ended dipole.

They have different focusing effect on the beam;
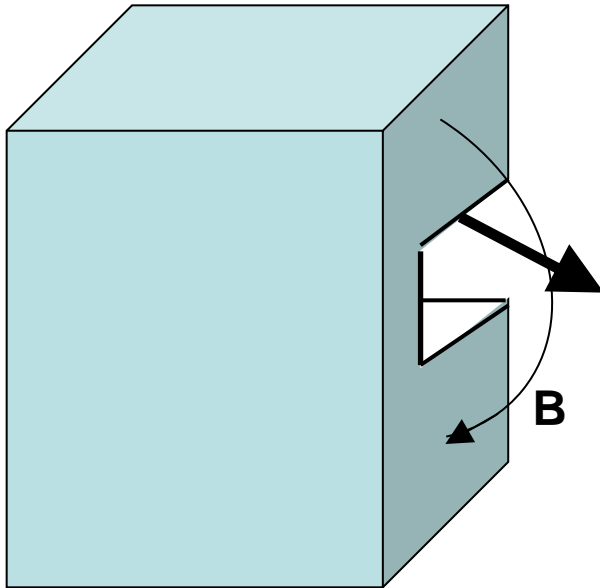(their curved nature is to save material and has no effect on beam focusing).
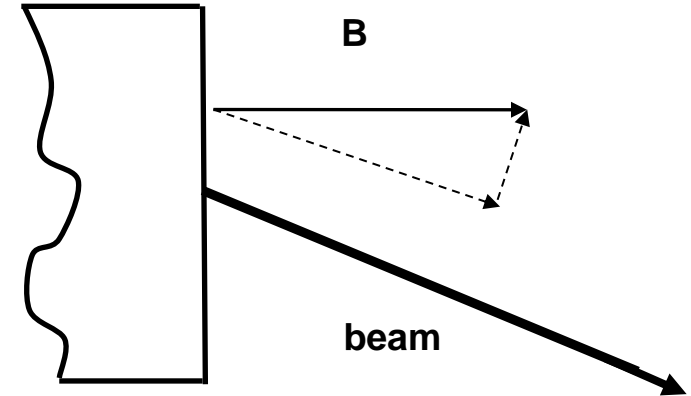
Dipole end focusing

Sector dipoles focus horizontally :

The end field in a parallel ended
dipole focuses vertically :

Off the vertical
centre line, the
field component
normal to the
beam direction
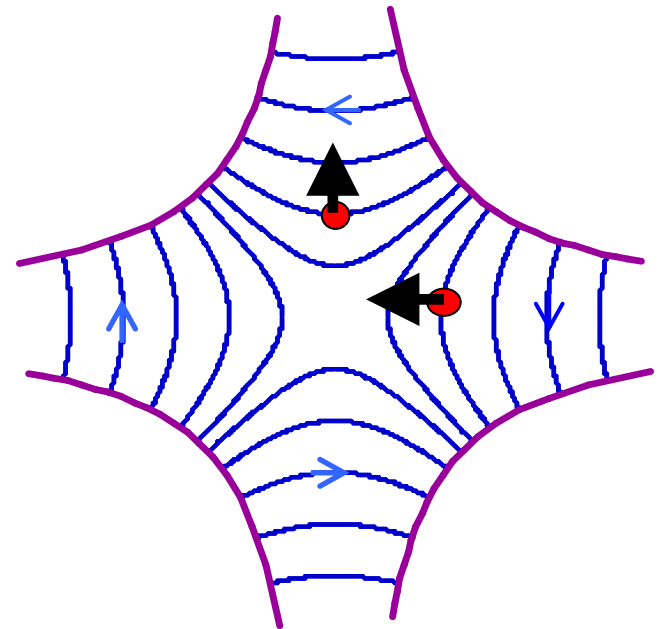produces a
vertical
focusing force.

B

B

beam

Magnets - quadrupoles

Quadrupoles produce a linear field variation across the beam.

Field is zero at the 'magnetic centre' so that 'on-axis' beam is not bent.

Note: beam that is radially focused is vertically defocused.
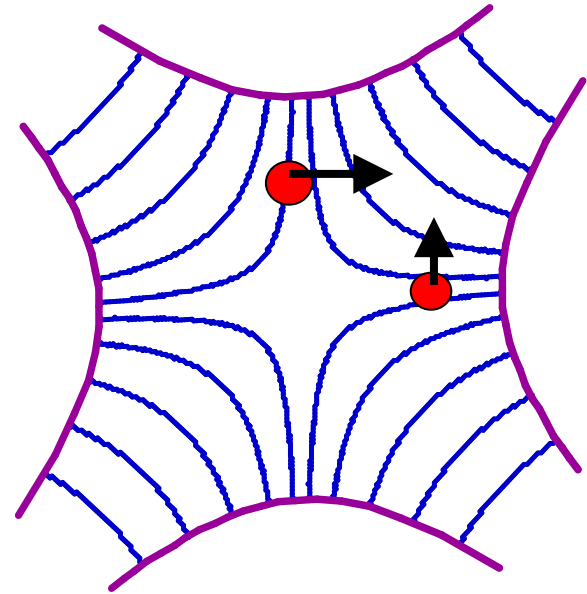
These are 'upright' quadrupoles.

'Skew' Quadrupoles.

Beam that has radial displacement (but not vertical) is deflected vertically;

horizontally centred beam with vertical displacement is deflected radially;

so skew quadrupoles couple horizontal and vertical transverse oscillations.
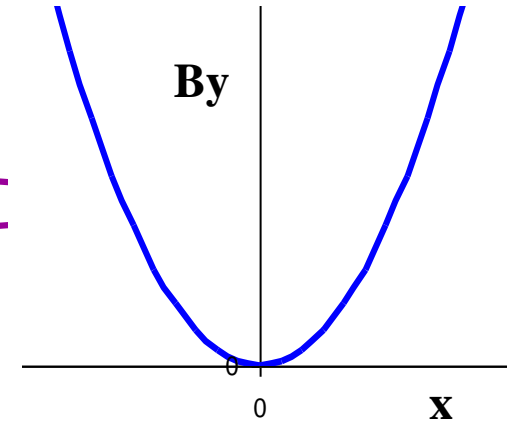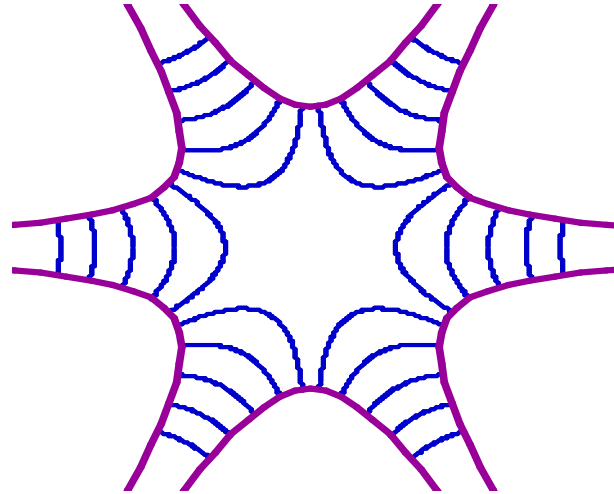
Magnets - sextupoles

In a sextupole, the field varies as the square of the displacement.

- off-momentum particles are incorrectly focused in quadrupoles (eg, high momentum particles with greater rigidity are under-focused), so transverse oscillation frequencies are modified - **chromaticity**;
- but off momentum particles circulate with a radial displacement (high momentum particles at larger x);
- so positive sextupole field corrects this effect – can reduce chromaticity to 0.
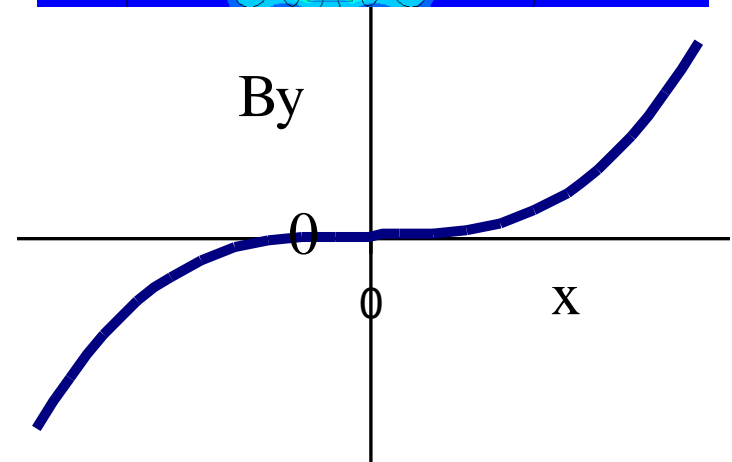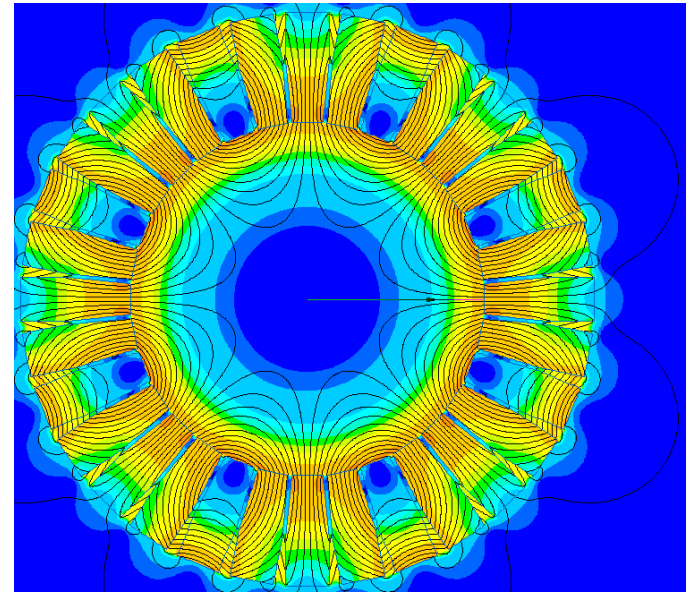
Magnets – 'higher orders'.

eg – Octupoles:

Effect?

$$By \propto x^3$$

Octupole field induces 'Landau damping' :

• introduces tune-spread as a function of oscillation amplitude;

•de-coheres the oscillations;

•reduces coupling.

By

0

0   x

b) Magneto-statics in free space

No currents, no steel - Maxwell's static equations in free space:

$$\underline{\nabla}.\underline{B} = 0 \ ;$$

$$\underline{\nabla} \wedge \underline{H} = \underline{i} \ ;$$

In the absence of currents: $\underline{i} = 0.$

Then we can put: $\underline{B} = - \underline{\nabla}\phi$

So that: $\underline{\nabla}^2\phi = 0$ (Laplace's equation).

Taking the two dimensional case (ie constant in the z direction) and solving for polar coordinates (r,θ):

$$\phi = (E+F \ \theta)(G+H \ \ln r) + \Sigma_{n=1}^{\infty} (J_n \ r^n \cos n\theta + K_n \ r^n \sin n\theta$$
$$+L_n \ r^{-n} \cos n \ \theta + M_n \ r^{-n} \sin n \ \theta)$$

In practical situations:

The scalar potential simplifies to:

$$\phi = \Sigma_n (J_n\, r^n \cos n\theta + K_n\, r^n \sin n\theta),$$

with n integral and J_n, K_n a function of geometry.

Giving components of flux density:

$$B_r = -\Sigma_n (n\, J_n\, r^{n-1} \cos n\theta + nK_n\, r^{n-1} \sin n\theta)$$
$$B_\theta = -\Sigma_n (-n\, J_n\, r^{n-1} \sin n\theta + nK_n\, r^{n-1} \cos n\theta)$$

Physical significance

This is an infinite series of cylindrical harmonics; they define the allowed distributions of **B** in 2 dimensions <u>in the absence of currents</u> within the domain of (r,θ).

Distributions not given by above are not physically realisable.

Coefficients J_n, K_n are determined by geometry (remote iron boundaries and current sources).

In Cartesian Coordinates

To obtain these equations in Cartesian coordinates, expand the equations for ϕ and differentiate to obtain flux densities;

$\cos 2\theta = \cos^2\theta - \sin^2\theta;$ $\cos 3\theta = \cos^3\theta - 3\cos\theta \sin^2\theta;$

$\sin 2\theta = 2 \sin\theta \cos\theta;$ $\sin 3\theta = 3\sin\theta \cos^2\theta - \sin^3\theta;$

$\cos 4\theta = \cos^4\theta + \sin^4\theta - 6 \cos^2\theta \sin^2\theta;$

$\sin 4\theta = 4 \sin\theta \cos^3\theta - 4 \sin^3\theta \cos\theta;$

etc (messy!);

$x = r \cos \theta;$ $y = r \sin \theta;$

and $B_x = - \partial\phi/\partial x;$ $B_y = - \partial\phi/\partial y$

n = 1 Dipole field!

Cylindrical:

$B_r = J_1 \cos \theta + K_1 \sin \theta;$

$B_\theta = -J_1 \sin \theta + K_1 \cos \theta;$

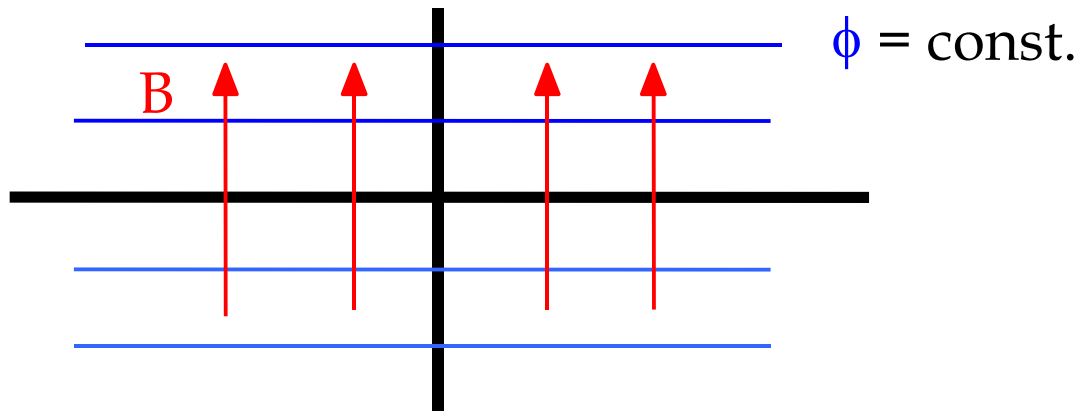$\phi = J_1 \, r \cos \theta + K_1 \, r \sin \theta.$

Cartesian:

$B_x = J_1$

$B_y = K_1$

$\phi = J_1 \, x + K_1 \, y$

So, $J_1 = 0$ gives vertical dipole field:

ϕ = const.

B

$K_1 = 0$ gives horizontal dipole field.

n = 2 Quadrupole field !

Cylindrical:

$B_r = 2 J_2 r \cos 2\theta + 2K_2 r \sin 2\theta;$

$B_\theta = -2J_2 r \sin 2\theta + 2K_2 r \cos 2\theta;$

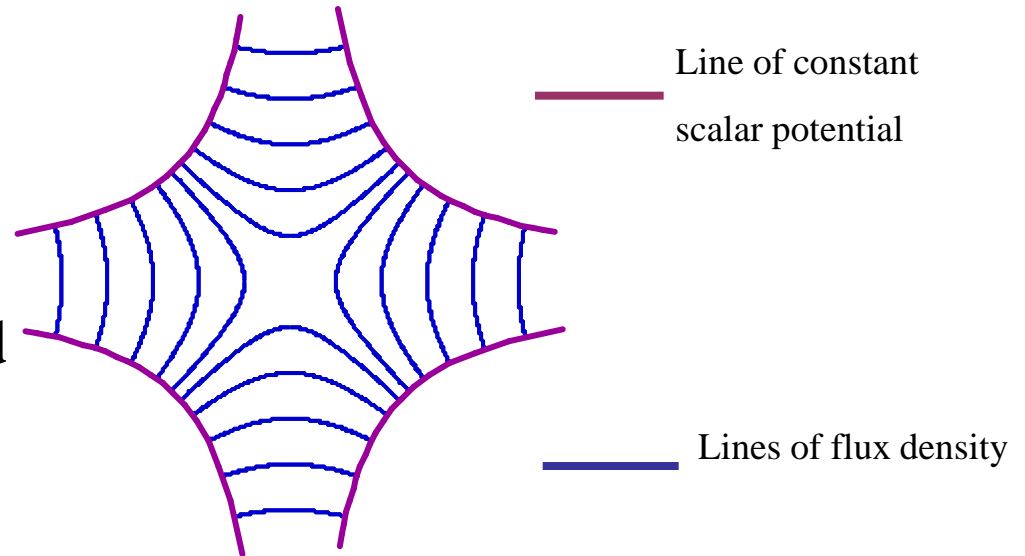$\phi = J_2 r 2 \cos 2\theta + K_2 r 2 \sin 2\theta;$

Cartesian:

$Bx = 2 (J_2 x + K_2 y)$

$B_y = 2 (-J_2 y + K_2 x)$

$\phi = J_2 (x^2 - y^2) + 2K_2 xy$

$J_2 = 0$ gives 'normal' or 'right' quadrupole field.

$K_2 = 0$ gives 'skew' quad fields (above rotated by $\pi/4$).

———— Line of constant scalar potential

———— Lines of flux density

n = 3 Sextupole field !

Cylindrical;

$B_r = 3 J_3 r^2 \cos 3\theta + 3K_3 r^2 \sin 3\theta;$

$B_\theta = -3J_3 r^2 \sin 3\theta + 3K_3 r^2 \cos 3\theta;$
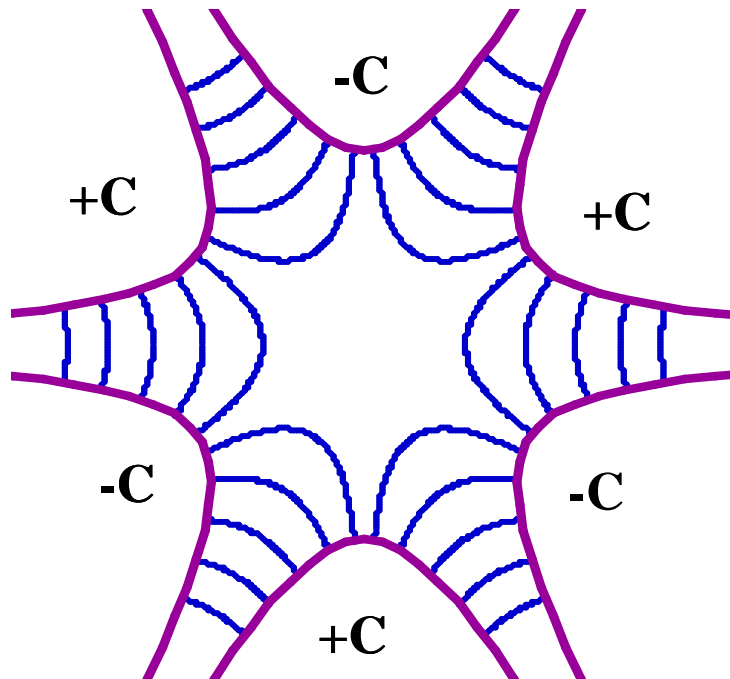
$\phi = J_3 r^3 \cos 3\theta + K_3 r^3 \sin 3\theta;$

Cartesian:

$B_x = 3\{J_3 (x^2-y^2) + 2K_3 yx\}$

$B_y = 3\{-2 J_3 xy + K_3(x^2-y^2)\}$

$\phi = J_3 (x^3-3y^2x) + K_3(3yx^2-y^3)$
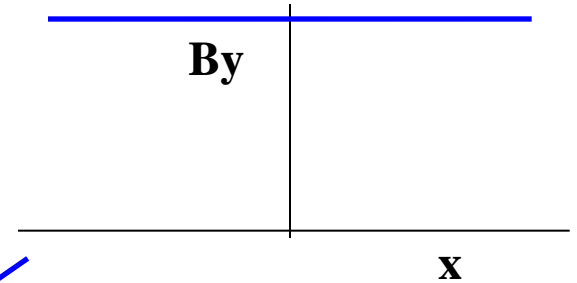
$J_3 = 0$ giving 'normal' or 'right' sextupole field.
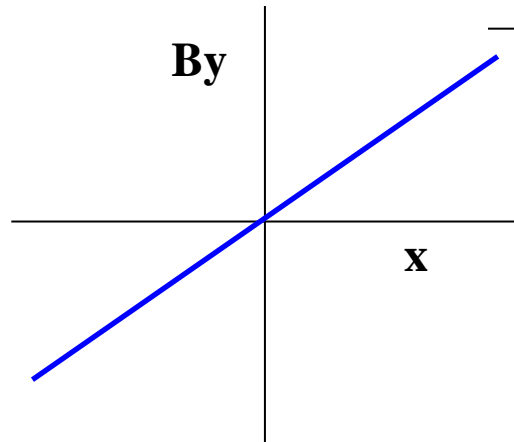

———— Line of constant scalar potential

———— Lines of flux density

Neil Marks, SRFC, ASTeC, Cockcroft Institute, University of Liverpool.   J.A.I, January 2015

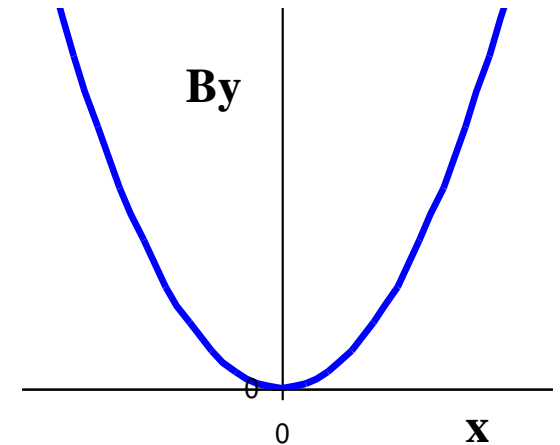# Summary; variation of B_y on x axis

Dipole; constant field:

Quad; linear variation:

Sextupole: quadratic variation:

Alternative notification (Most lattice codes)

$$B(x) = B\rho \sum_{n=0}^{\infty} \frac{k_n\, x^n}{n!}$$

magnet strengths are specified by the value of k_n; (normalised to the beam rigidity);

order n of k is different to the 'standard' notation:

dipole is	$n = 0$;
quad is	$n = 1$; etc.

k has units:

k_0 (dipole)	m^{-1};	
k_1 (quadrupole)	m^{-2};	etc.

Significance of vector potential in 2D.

We have: $\quad\quad\quad\quad\quad$ **B** = curl **A** $\quad\quad\quad$ (**A** is vector potential);

and $\quad\quad\quad\quad\quad\quad\quad$ div **A** = 0

Expanding: $\quad\quad\quad\quad$ **B** = curl **A** =

$(\partial A_z/\partial y - \partial A_y/\partial z)$ **i** + $(\partial A_x/\partial z - \partial A_z/\partial x)$ **j** + $(\partial A_y/\partial x - \partial A_x/\partial y)$ **k;**

where $\quad\quad\quad\quad\quad$ **i, j, k**, and unit vectors in x, y, z.

In 2 dimensions $\quad\quad\quad$ $B_z = 0$; $\quad\quad\quad\quad$ $\partial/\partial z = 0$;

So $\quad\quad\quad\quad\quad\quad\quad$ $A_x = A_y = 0$;

and $\quad\quad\quad\quad\quad\quad$ **B** = $(\partial A_z/\partial y)$ **i** $-$ $(\partial A_z/\partial x)$ **j**

A is in the z direction, normal to the 2 D problem.

Note: $\quad\quad\quad\quad\quad$ div **B** = $\partial^2 A_z/\partial x\,\partial y - \partial^2 A_z/\partial x\,\partial y = 0$;

Total flux between two points $\propto \Delta A$

In a two dimensional problem the magnetic flux between two points is proportional to the difference between the vector potentials at those points.

$$\Phi \propto (A_2 - A_1)$$
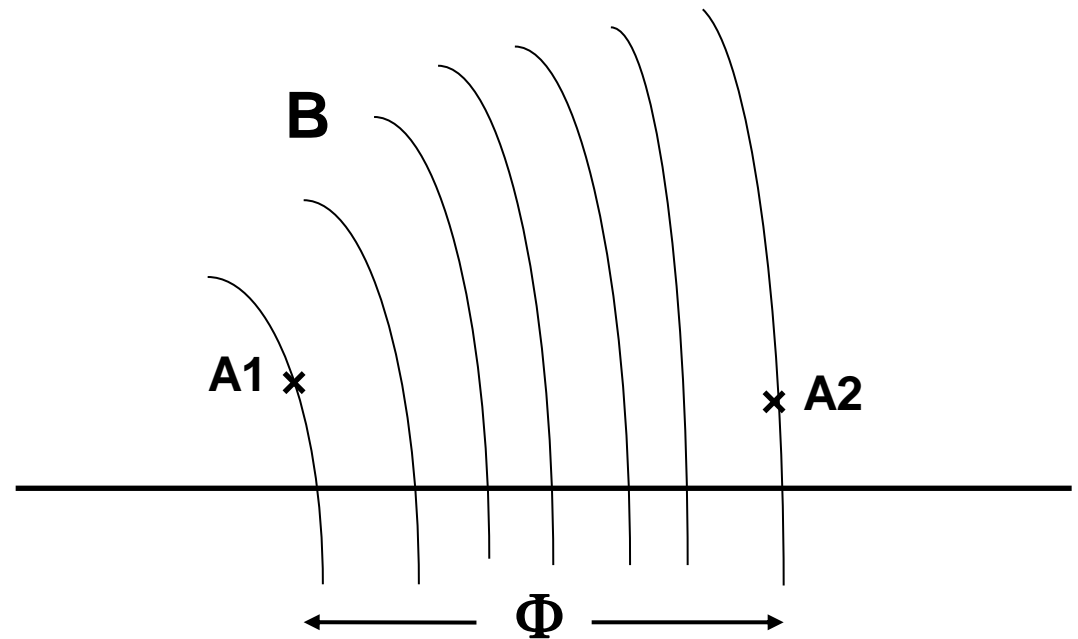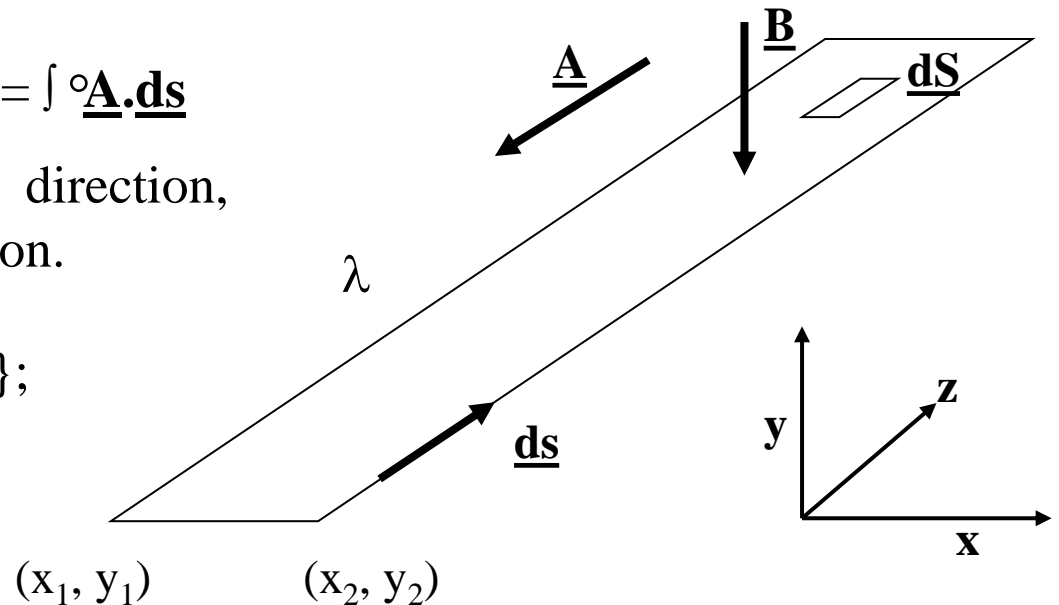

Proof on next slide.

Proof.

Consider a rectangular closed path, length λ in z direction at (x_1, y_1) and (x_2, y_2); apply Stokes' theorem:

$$\Phi = \int\int \underline{\mathbf{B}}.\underline{\mathbf{dS}} = \int\int (\operatorname{curl} \underline{\mathbf{A}}).\underline{\mathbf{dS}} = \oint \underline{\mathbf{A}}.\underline{\mathbf{ds}}$$

But A is exclusively in the z direction, and is constant in this direction.
So:
$$\int \underline{\mathbf{A}}.d\underline{\mathbf{s}} = \lambda \{ A(x_1,y_1) - A(x_2,y_2) \};$$
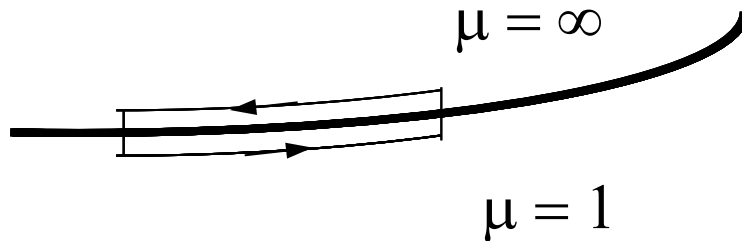

(x_1, y_1) (x_2, y_2)

$$\Phi = \lambda \{ A(x_1,y_1) - A(x_2,y_2) \};$$

c) Introduction of steel poles and yokes.

What is the perfect pole shape?

What is the ideal pole shape?

•Flux is normal to a ferromagnetic surface with infinite μ:

$\mu = \infty$

$\mu = 1$

curl H = 0

therefore ∫ H.ds = 0;

in steel H = 0;

therefore parallel H air = 0

therefore B is normal to surface.

•Flux is normal to lines of scalar potential, ($\underline{\mathbf{B}} = - \underline{\nabla}\phi$);

•So the lines of scalar potential are the perfect pole shapes!

(but these are infinitely long!)

Equations for the ideal pole

Equations for Ideal (infinite) poles;

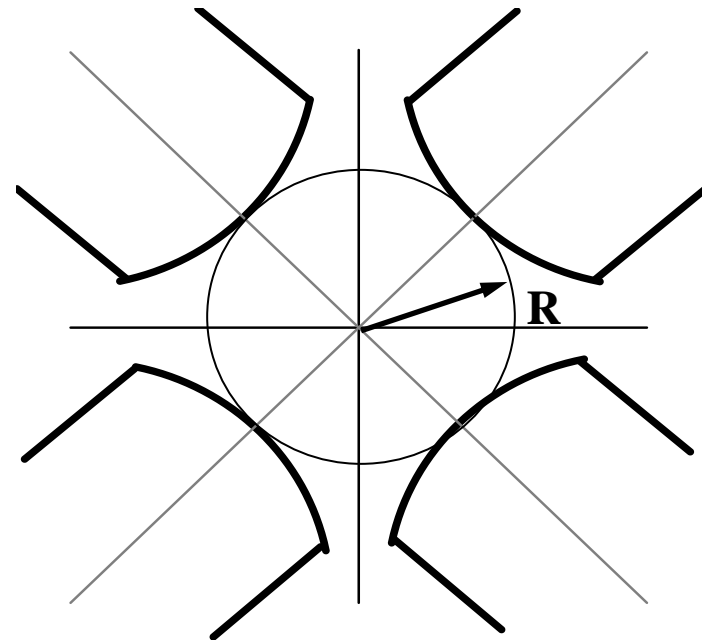$(J_n = 0)$ for **normal** (ie not skew) fields:

Dipole:

   $y = \pm g/2;$

(g is interpole gap).

Quadrupole:

   $xy = \pm R^2/2;$
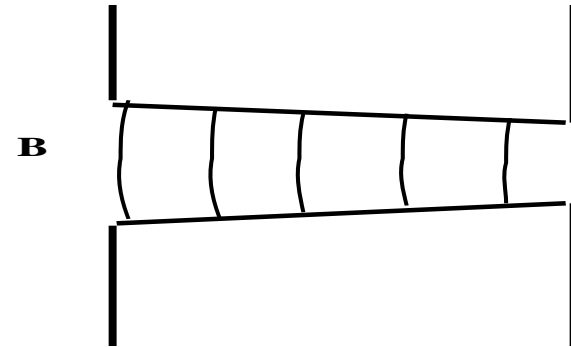
Sextupole:

   $3x^2y - y^3 = \pm R^3;$

Combined function (c.f.) magnets

'Combined Function Magnets' - often dipole and quadrupole field combined (but see next-but-one slide):

A quadrupole magnet with

physical centre shifted from

magnetic centre.

Characterised by 'field index' n,

+ve or -ve depending

on direction of gradient;

do not confuse with harmonic n!

B

$$n = -\left(\frac{\rho}{B_0}\right)\left(\frac{\partial B}{\partial x}\right),$$

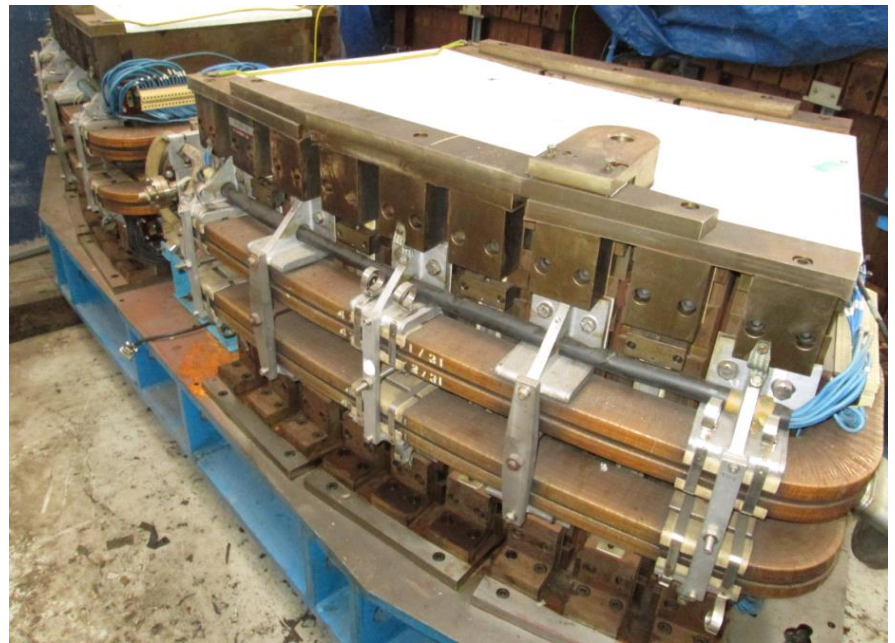ρ is radius of curvature of the beam;

B_o is central dipole field

Typical combined dipole & quadrupole

SRS Booster c.f. dipole

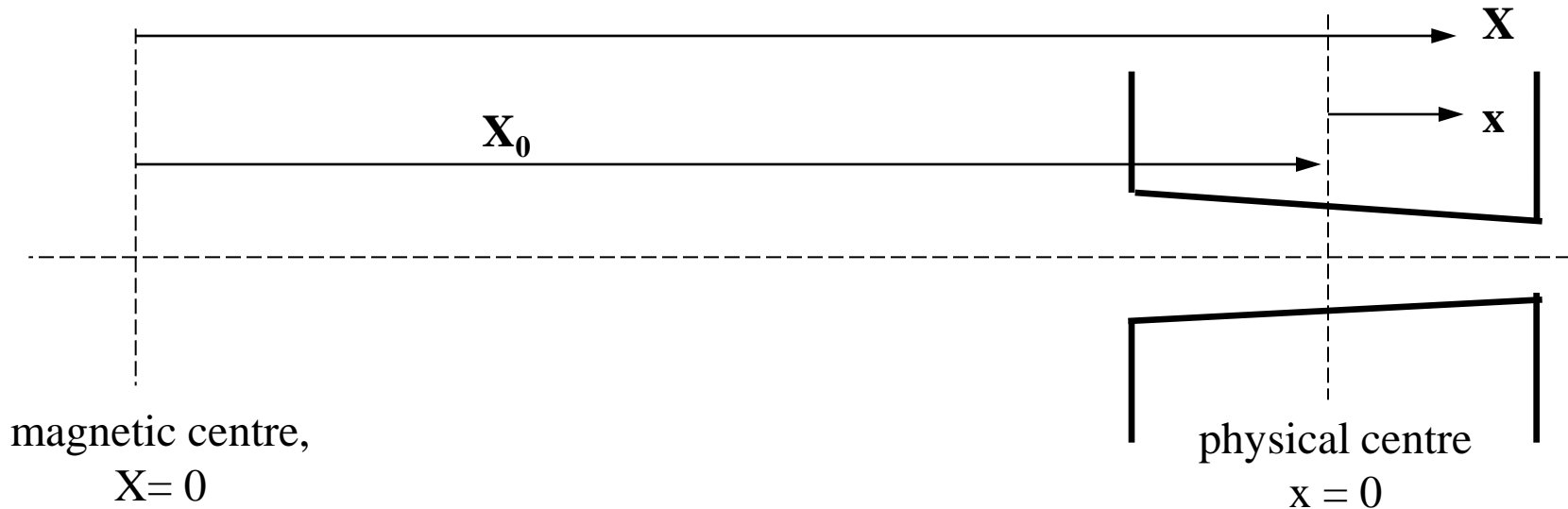'D' type +ve n.

'F' type -ve n

Combined function geometry

Combined function (dipole & quadrupole) magnet:

- beam is at physical centre
- flux density at beam   = B_0;
- gradient at beam   = $\partial B/\partial x$;
- magnetic centre is at B and X   =   0.
- separation magnetic to physical centre   = X_0

magnetic centre,
   X= 0

physical centre
   x = 0

Other combined function magnets.

Other combinations:

- •dipole, quadrupole and sextupole;
- •dipole & sextupole (for chromaticity control);
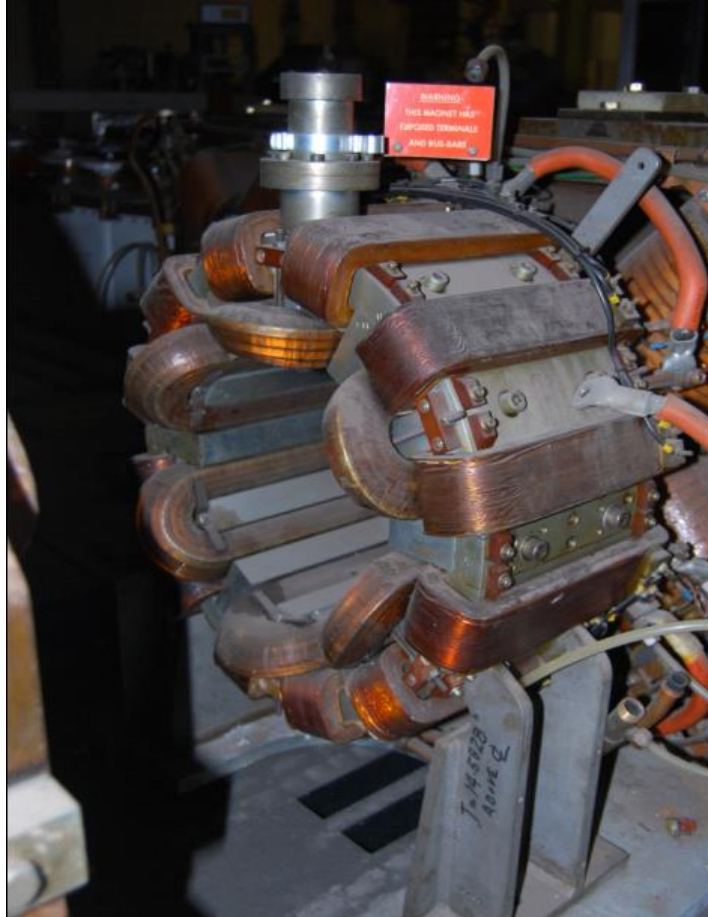- •dipole, skew quad, sextupole, octupole (at DL)

Generated by

- •pole shapes given by sum of correct scalar potentials

  - amplitudes built into pole geometry – not variable.

- •multiple coils mounted on the yoke

  - amplitudes independently varied by coil currents.

The SRS multipole magnet.
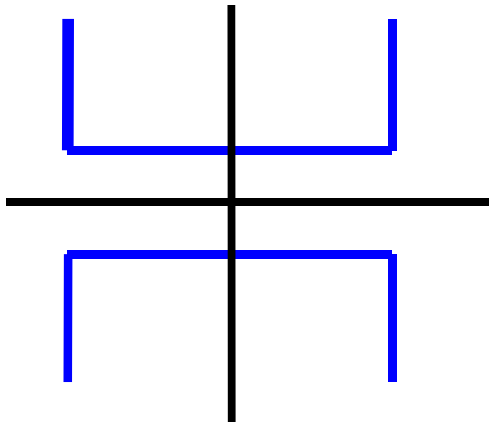

Could develop:
- vertical dipole
- horizontal dipole;
- upright quad;
- skew quad;
- sextupole;
- octupole;
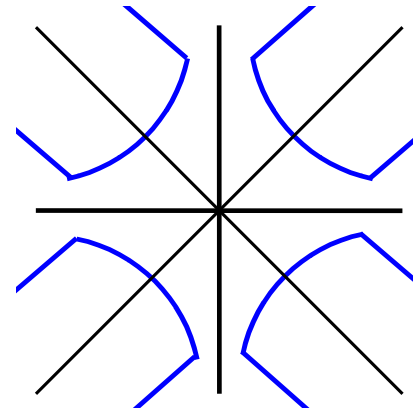- others.

The Practical Pole

Practically, poles are finite, **introducing errors**; these appear as higher harmonics which degrade the field distribution.

However, the iron geometries have certain symmetries that **restrict** the nature of these errors.

Dipole: Quadrupole:

Possible symmetries

Lines of symmetry:

	Dipole:	Quad
Pole orientation	$y = 0$;	$x = 0$; $y = 0$

determines whether pole

is normal or skew.

Additional symmetry $x = 0$; $y = \pm\, x$

imposed by pole edges.

The additional constraints imposed by the symmetrical pole
edges limits the values of n that have non zero coefficients

Dipole symmetries

Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	all $J_n = 0$;
Pole edges	$\phi(\theta) = \phi(\pi - \theta)$	K_n non-zero only for: $n = 1, 3, 5$, etc;

$+\phi$

$-\phi$

$+\phi$ $+\phi$
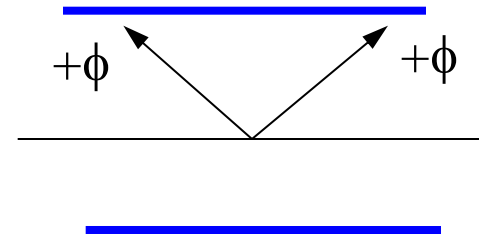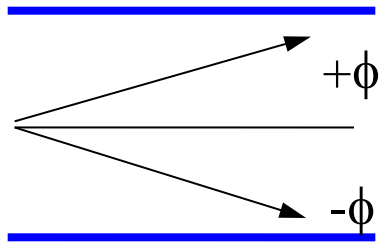
So, for a fully symmetric dipole, only 6, 10, 14 etc pole errors can be present.

Quadrupole symmetries

Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	All $J_n = 0$;
	$\phi(\theta) = -\phi(\pi -\theta)$	$K_n = 0$ all odd n;
Pole edges	$\phi(\theta) = \phi(\pi/2 -\theta)$	K_n non-zero only for: n = 2, 6, 10, etc;

So, a fully symmetric quadrupole, only 12, 20, 28 etc pole errors can be present.

Sextupole symmetries

Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$ $\phi(\theta) = -\phi(2\pi/3 - \theta)$ $\phi(\theta) = -\phi(4\pi/3 - \theta)$	All $J_n = 0$; $K_n = 0$ for all n not multiples of 3;
Pole edges	$\phi(\theta) = \phi(\pi/3 - \theta)$	K_n non-zero only for: n = 3, 9, 15, etc.

So, a fully symmetric sextupole, only 18, 30, 42 etc pole errors can be present.

Summary - 'Allowed' Harmonics

Summary of 'allowed harmonics' in <u>fully symmetric</u> magnets:

Fundamental geometry	'Allowed' harmonics
Dipole, n = 1	n = 3, 5, 7, ... (6 pole, 10 pole, etc.)
Quadrupole, n = 2	n = 6, 10, 14, ... (12 pole, 20 pole, etc.)
Sextupole, n = 3	n = 9, 15, 21, ... (18 pole, 30 pole, etc.)
Octupole, n = 4	n = 12, 20, 28, ... (24 pole, 40 pole, etc.)

d) Introducing currents in coils to generate the field.

Introduction of currents

Now for $\underline{j} \neq 0$ $\qquad$ $\underline{\nabla} \wedge \underline{H} = \underline{j}$;

To expand, use Stoke's Theorem:
for any vector $\underline{V}$ and a closed curve s
:

$$\int \underline{V}.\underline{ds} = \iint \text{curl } \underline{V}.\underline{dS}$$
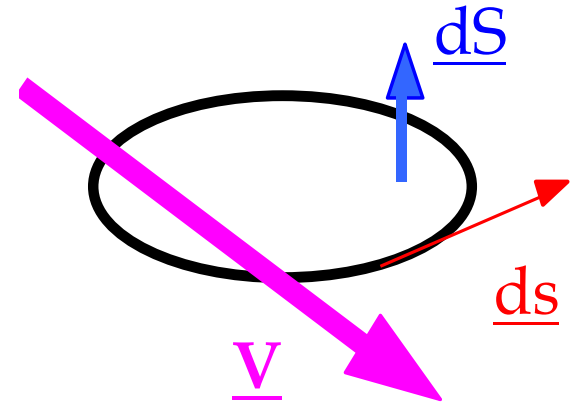
Apply this to: **curl H** $= \underline{j}$;

then in a magnetic circuit:

$$\boxed{\int \underline{H}.\underline{ds} = N \, I;}$$

N I (Ampere-turns) is total current cutting $\underline{S}$

Excitation current in a dipole

B is approx constant round the loop made up of λ and g, (but see below);

But in iron, $\mu \gg 1$,
and $H_{iron} = H_{air} / \mu$;

So
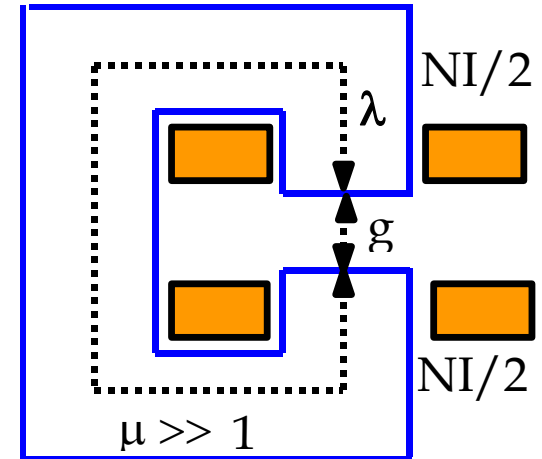
$$B_{air} = \mu_0 \, NI / (g + \lambda/\mu);$$

g, and λ/μ are the 'reluctance' of the gap and iron.

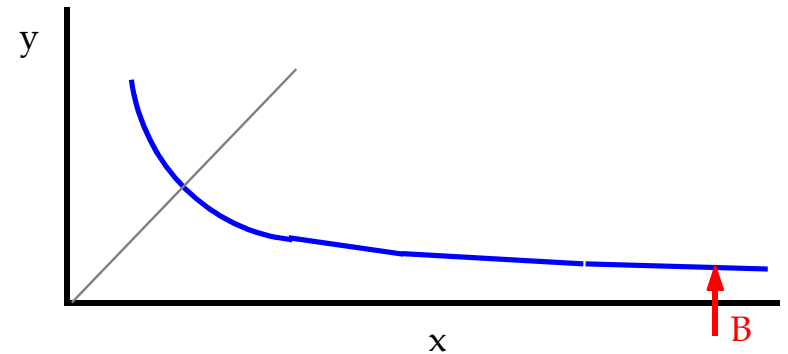Approximation ignoring iron reluctance ($\lambda/\mu \ll g$):

$$NI = B \, g / \mu_0$$

NI/2

λ

g

NI/2

$\mu \gg 1$

Excitation current in quad & sextupole

For quadrupoles and sextupoles, the required excitation can be calculated by considering fields and gap at large x. For example: **Quadrupole:**

Pole equation: $xy = R^2/2$

On x axes $B_Y = gx$;

where g is gradient (T/m).

At large x (to give vertical lines of B):

$$N I = (gx)(R^2/2x)/\mu_0$$

ie

$$N I = g R^2/2 \mu_0 \text{ (per pole)}.$$

The same method for a **Sextupole**, (coefficient g_S,), gives:

$$N I = g_S R^3/3 \mu_0 \text{ (per pole)}$$

General solution for magnets order n

In air (remote currents!), $\quad\quad\quad\quad$ $\mathbf{B} = \mu_0 \mathbf{H}$

$$\underline{\mathbf{B}} = - \underline{\nabla}\phi$$

Integrating over a limited path
(not circular) in air: $\quad\quad$ $N\,I = (\phi_1 - \phi_2)/\mu_o$
ϕ_1, ϕ_2 are the scalar potentials at two points in air.
Define $\phi = 0$ at magnet centre;
then potential at the pole is:

$$\mu_o\,NI$$

y

$\phi = \mu_0\,NI$

$\phi = 0$

Apply the general equations for magnetic
field harmonic order n for non-skew
magnets (all $Jn = 0$) giving:

$$N\,I = (1/n)\,(1/\mu_0)\,\{B_r/R^{(n-1)}\}\,R^{\,n}$$

Where:

$\quad\quad$ NI is excitation per pole;
$\quad\quad$ R is the inscribed radius (or half gap in a dipole);
$\quad\quad$ term in brackets $\{\}$ is magnet strength in T/m $^{(n-1)}$.
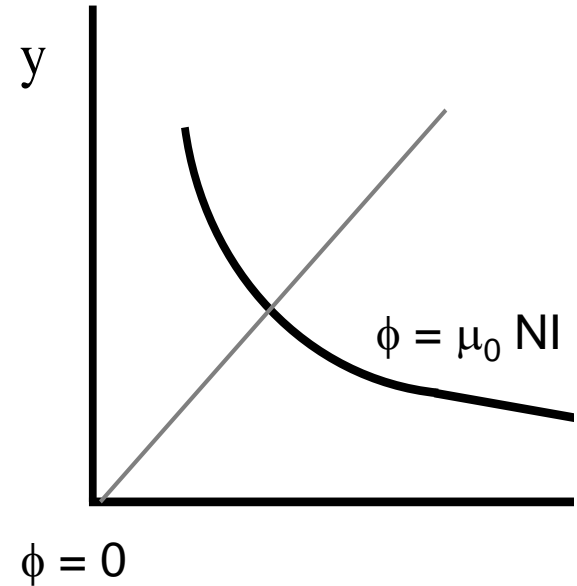
Coil geometry (room temp magnets)

Standard design is rectangular copper (or aluminium) conductor, with cooling water tube. Insulation is glass cloth and epoxy resin.

Amp-turns (NI) are determined, but total copper area (A_{copper}) and number of turns (N) are two degrees of freedom and need to be decided.
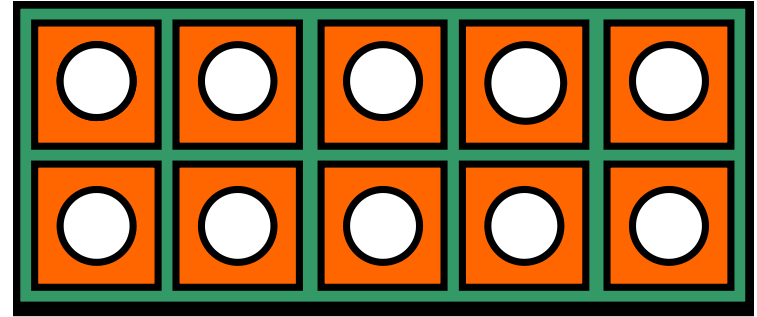
Heat generated in the coil is a function of the RMS current density:

$$j_{rms} = NI_{rms}/A_{copper}$$

Optimum j_{rms} determined from **economic** criteria.

I_{rms} depends on current waveform

With an arbitrary waveform of period τ, power generated in coil is:

$$W = \frac{R}{\tau} \int_0^{\tau} \{ I(t) \}^2 \, dt = R \, I_{rms}^2$$

In a DC magnet the $I_{rms} = I_{dc}$
For a pure a.c. sin-wave $I_{rms} = (1/\sqrt{2}) I_{peak}$
For a discontinuous waveform the integration is
over the whole of a single period.

A typical waveform for a booster synchrotron
is a biased sin wave):

$$I_{rms} = \sqrt{\{I_{dc}^2 + (1/2) I_{ac}^2\}}$$
If $I_{dc} = I_{ac} = (1/2) I_{peak}$
$I_{rms} = I_{dc} (\sqrt{(3/2)}$
$\quad\quad = I_{peak} (1/2)(\sqrt{(3/2)};$

I_{peak}

I_{ac}

I_{dc}

0
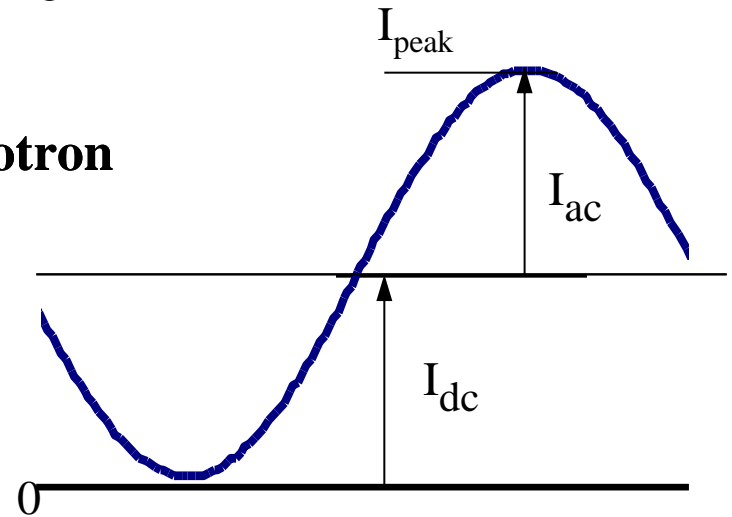
Current density (j_{rms})- optimisation

Advantages of low j_{rms}:
- **lower power loss** – power bill is decreased;
- **lower power loss** – power converter size is decreased;
- **less heat** dissipated into magnet tunnel.

Advantages of high j:
- **smaller coils;**
- **lower capital cost;**
- **smaller magnets.**

Chosen value of j_{rms} is an optimisation of magnet capital against power costs.
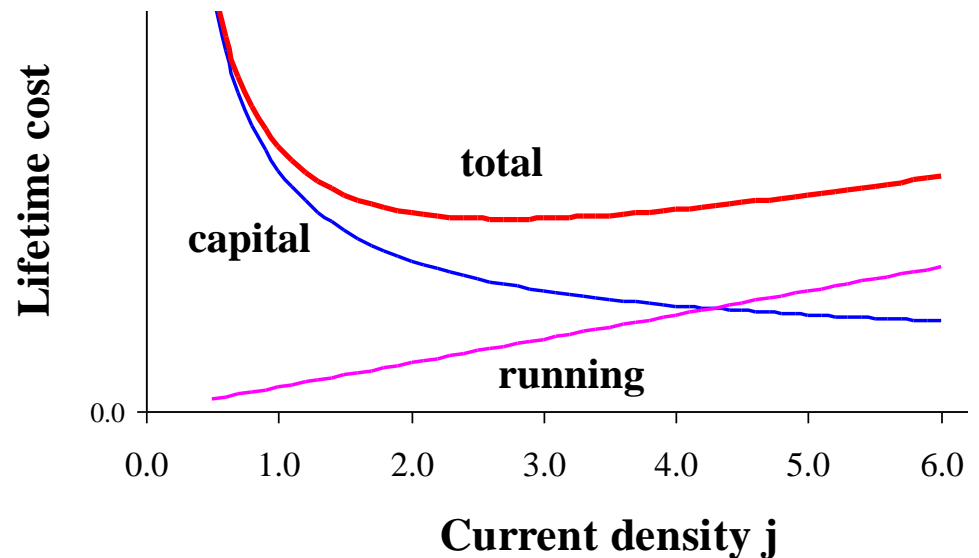
Number of turns, N

The value of number of turns (N) is chosen to match power supply and interconnection impedances.

Factors determining choice of N:

Large N (low current)	Small N (high current)
Small, neat terminals.	Large, bulky terminals
Thin interconnections- low cost and flexible.	Thick, expensive connections.
More insulation in coil; larger coil volume, increased assembly costs.	High percentage of copper in coil; more efficient use of available space;
High voltage power supply safety problems.	High current power supply. --greater losses.

Examples of typical turns/current

From the Diamond 3 GeV synchrotron source:

Dipole:

N (per magnet):	40;	
I max	1500	A;
Volts (circuit):	500	V.

Quadrupole:

N (per pole)	54;	
I max	200	A;
Volts (per magnet):	25	V.

Sextupole:

N (per pole)	48;	
I max	100	A;
Volts (per magnet)	25	V.

e) The Magnetic Circuit.

Permeability of low silicon steel

Neil Marks, SRFC, ASTeC, Cockcroft Institute, University of Liverpool.   J.A.I, January 2015
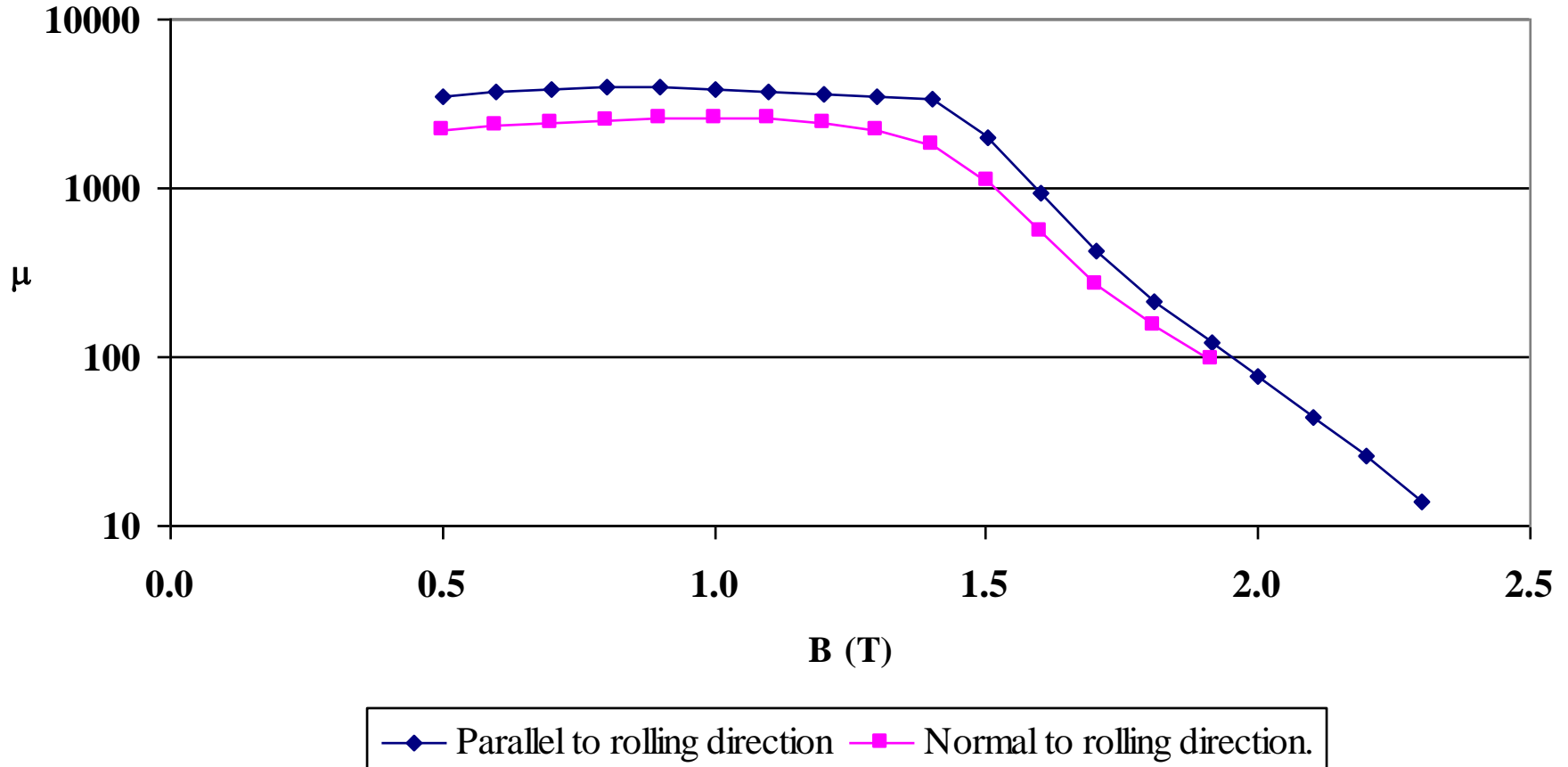
Flux at the pole and in the circuit.

Flux in the yoke includes the gap flux and stray flux, which extends (approx) one gap width on either side of the gap.

Approximate value for total flux in the back-leg of magnet length λ:

$$\Phi = B_{gap} (b + 2g) \lambda.$$

Width of backleg is chosen to limit B_{yoke} and hence maintain high μ.
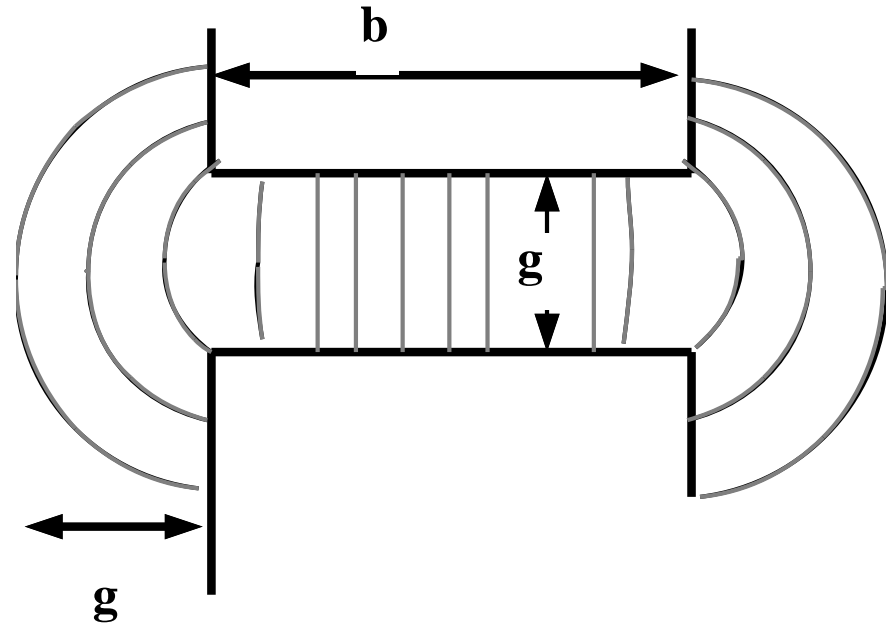
Note – fea codes give values of vector potential (Az); hence values of total flux can be obtained.

'Residual' fields

Residual field - the flux density in a gap at I = 0;

Remnant field B_R - value of B at H = 0;

Coercive force H_C - negative value of field at B = 0;

I = 0:   ∫ H.ds   = 0;

So: $(H_{steel}) \lambda + (H_{gap})g = 0$;

B_{gap} $= (\mu_0)(-H_{steel})(\lambda/g)$;

B_{gap} $\approx (\mu_0) (H_C)(\lambda/g)$;

Where: λ is path length in steel;

g is gap height.

Because of presence of gap, residual field is determined by coercive force H_C (A/m) and <u>not</u> remnant flux density B_R (Tesla).

B_R

$-H_C$

Fi
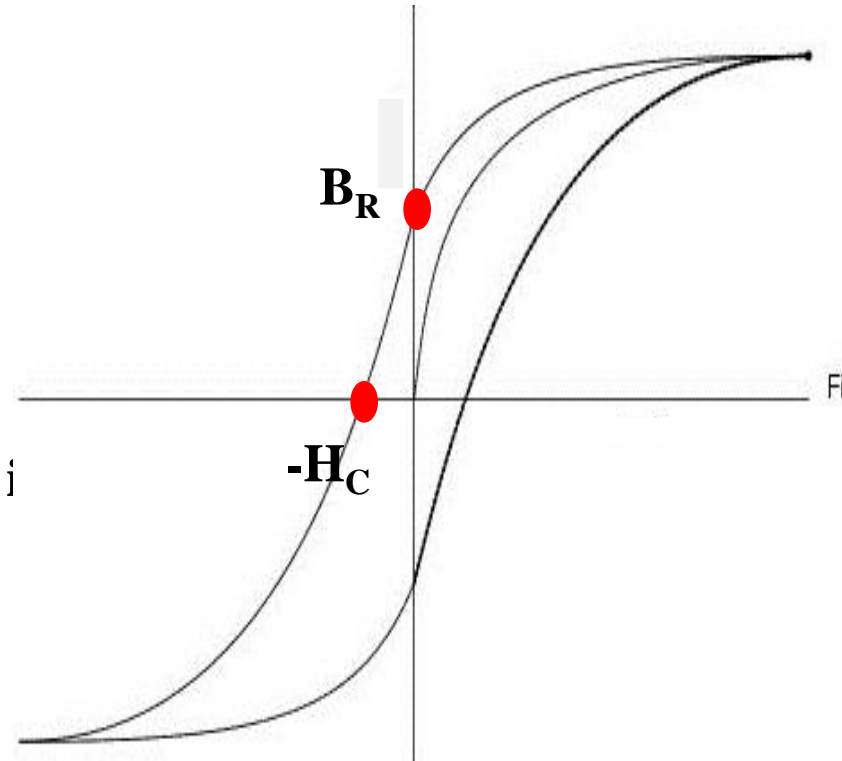
Magnet geometry

Dipoles can be 'C core' 'H core' or 'Window frame'

"C' Core:
Advantages:
  Easy access;
  Classic design;
Disadvantages:
  Pole shims needed;
 Asymmetric (small);
 Less rigid;
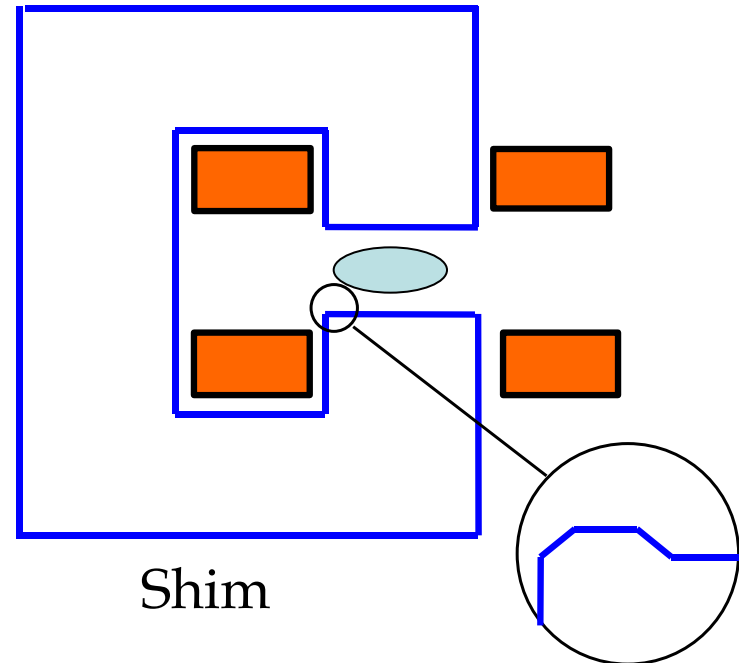
Shim

The 'shim' is a small, additional piece of fero-magnetic material added on each side of the two poles – it compensates for the finite cut-off of the pole, and is optimised to reduce the 6, 10, 14...... pole error harmonics.

A typical 'C' cored Dipole

Cross section of the Diamond storage ring dipole.
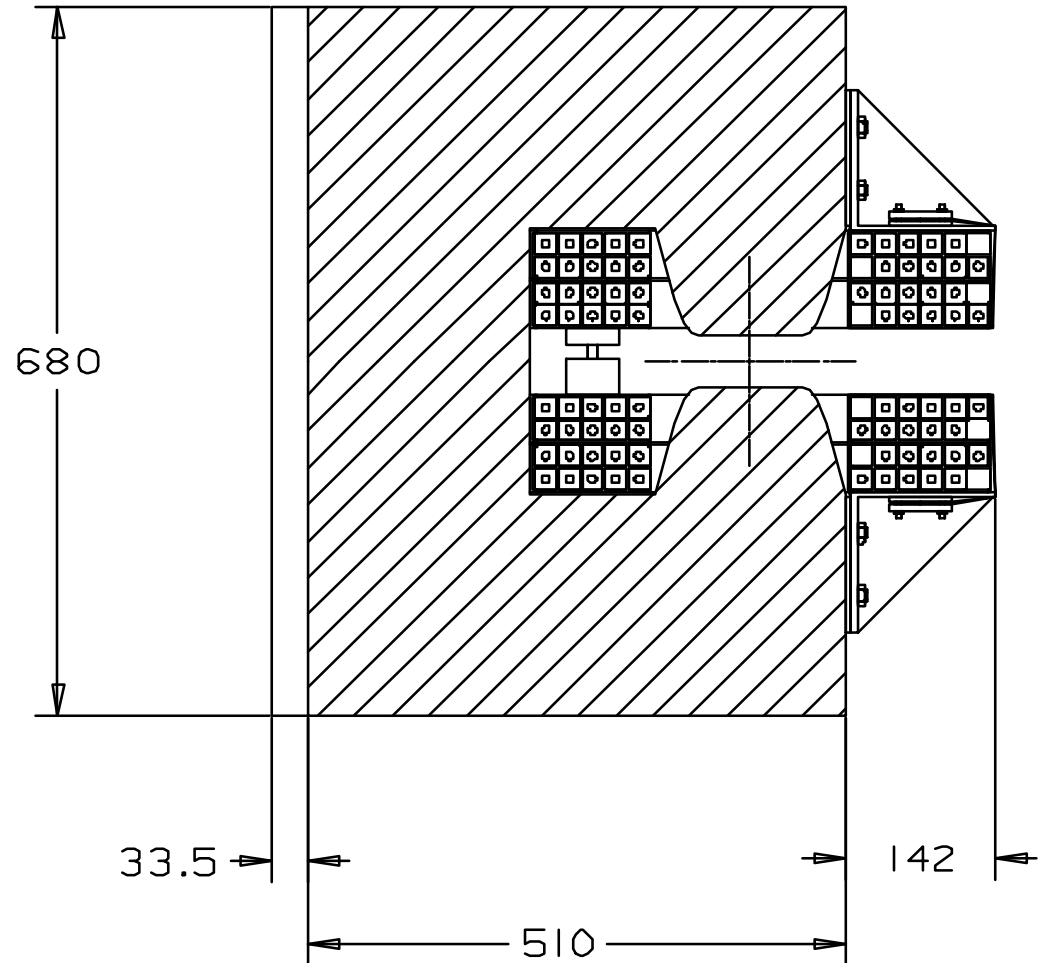
H core and window-frame magnets

"Window Frame'

Advantages:

    High quality field;

    No pole shim;

    Symmetric & rigid;

Disadvantages:

    Major access problems;

    Insulation thickness
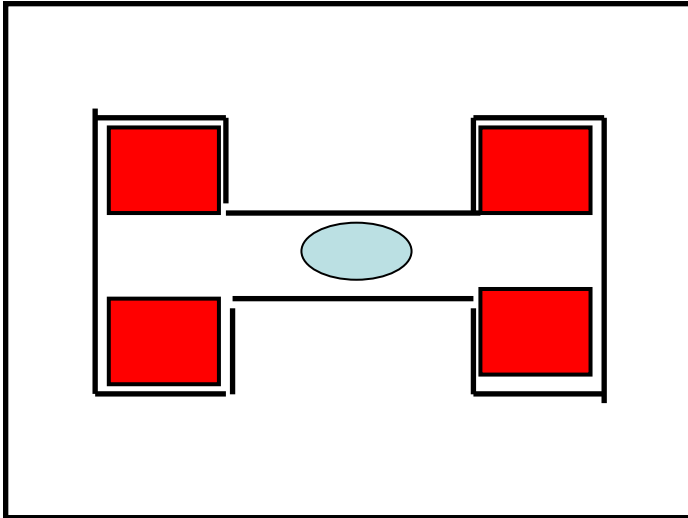
'H core':

Advantages:

    Symmetric;

    More rigid;

Disadvantages:

    Still needs shims;

    Access problems.
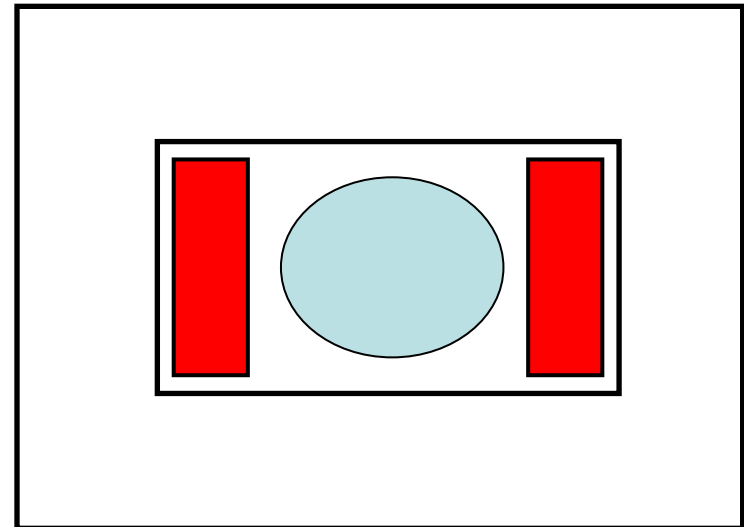
Window frame dipole

Providing the conductor is continuous to the steel 'window frame' surfaces (impossible because coil must be electrically insulated), and the steel has infinite μ, this magnet generates perfect dipole field.
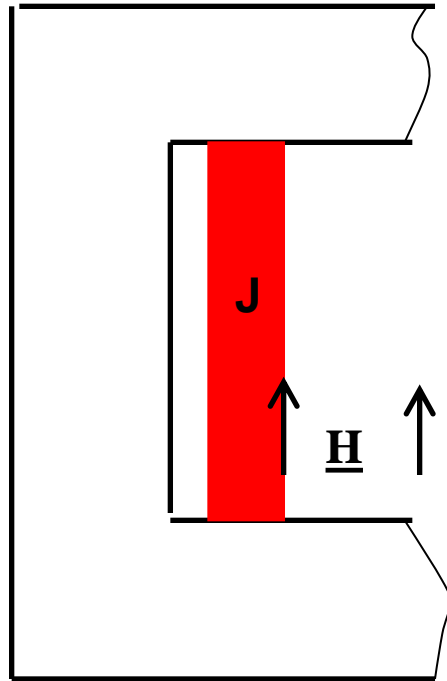
Providing current density J is uniform in conductor:

• **H** is uniform and vertical up outer face of conductor;

• **H** is uniform, vertical and with same value in the middle of the gap;
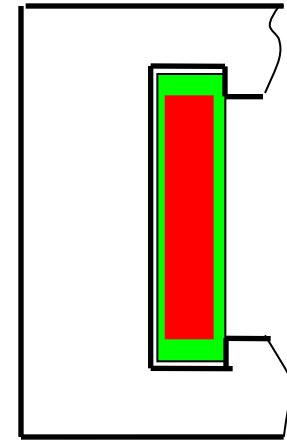
• → perfect dipole field.

The practical window frame dipole.

Insulation added to coil:
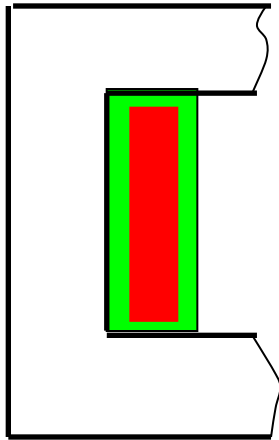
B increases close to coil insulation surface
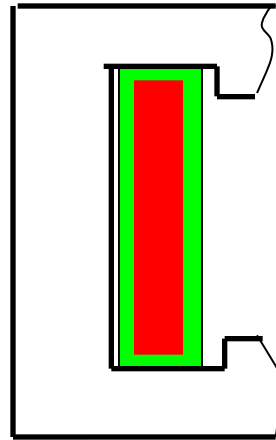
B decrease close to coil insulation surface

best compromise

An open-sided Quadrupole.

'Diamond' storage ring quadrupole.

The yoke support pieces in the horizontal plane need to provide space for beam-lines and are not ferro-magnetic.

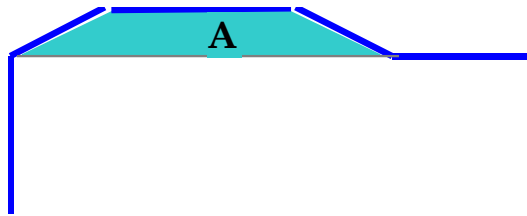Error harmonics include n = 4 (octupole) a finite permeability error.
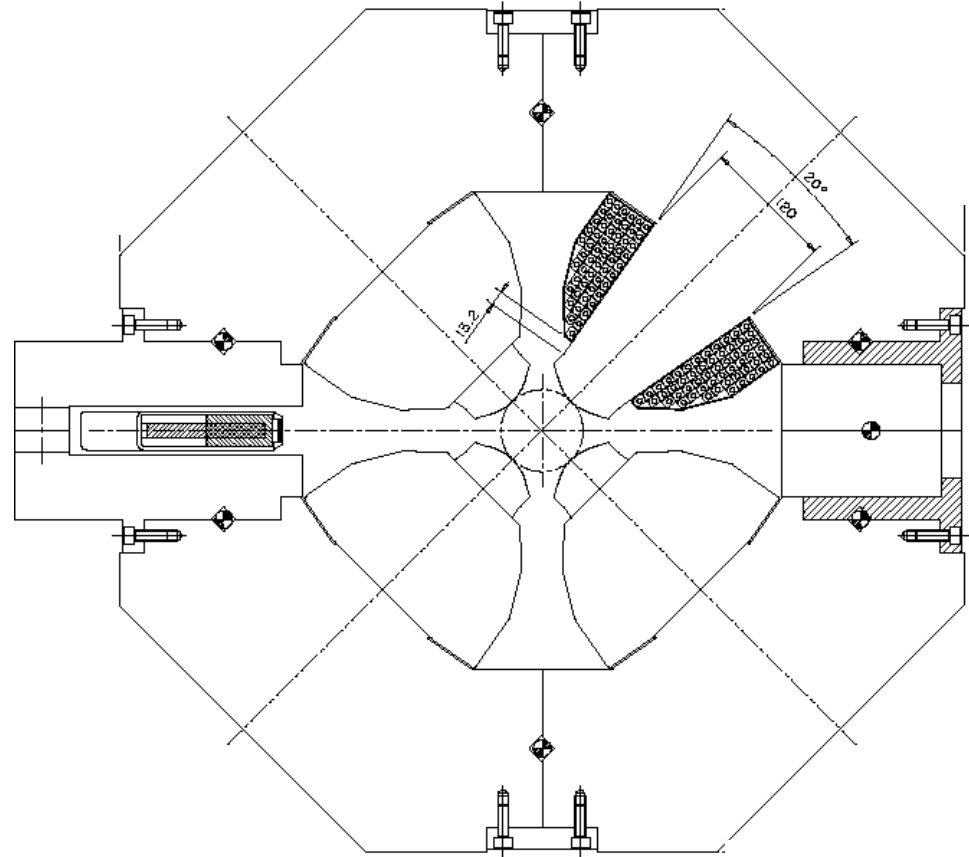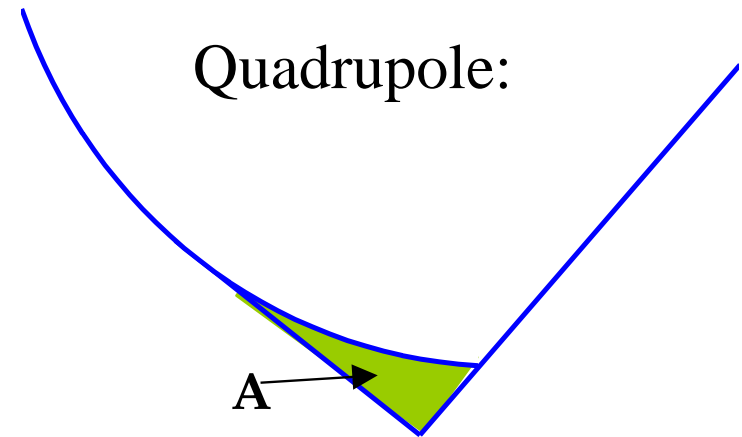
Typical pole designs

To compensate for the non-infinite pole, shims are added at the pole edges. The area and shape of the shims determine the amplitude of error harmonics which will be present.

Dipole:

A

Quadrupole:

A

The designer optimises the pole by 'predicting' the field resulting from a given pole geometry and then adjusting it to give the required quality.

When high fields are present, chamfer angles must be small, and tapering of poles may be necessary

Assessing adequate design I.

A <u>simple</u> judgement of field quality is given by plotting:

- **Dipole:** $\{B_y (x) - B_y (0)\}/B_Y (0)$ $(\Delta B(x)/B(0))$
- **Quad:**   variation in $dB_y (x)/dx$ $(\Delta g(x)/g(0))$
- **6poles:**   variation in $d^2 B_y(x)/dx^2$ $(\Delta g_2(x)/g_2(0))$

'Typical' acceptable variation inside 'good field' region:

$$\Delta B(x)/B(0) \quad \leq \quad 0.01\%$$

$$\Delta g(x)/g(0) \quad \leq \quad 0.1\%$$

$$\Delta g_2(x)/g_2(0) \quad \leq \quad 1.0\%$$

Such criteria often used at the beginning of a project; sometimes adequate for completion; but see next slide!

Assessing adequate design II

The expansion of $B_y(x)_{y=0}$ is a Taylor series:

$$B_y(x) = \sum_{n=1}^{\infty} \{b_n x^{(n-1)}\}$$

$$= \quad b_1 \qquad + b_2 x \qquad + b_3 x^2 + \dots$$

$$\quad\quad\text{dipole} \qquad \text{quad} \qquad\qquad \text{sextupole}$$

Also $\qquad\qquad \partial B_y(x)/\partial x = b_2 + 2\, b_3 x + \dots\dots$ etc.

quad gradient $\quad g \equiv b_2 = \partial B_y(x)/\partial x \qquad$ at $x = 0$;

sext. gradient $\quad g_s \equiv b_3 = (½) \, \partial^2 B_y(x)/\partial x^2 \quad$ at $x = 0$;

Lattice designers will be most interested in the Taylor coefficients – inserting data into lattice codes; judging resonance blow up, etc.

Note that minimising $\Delta B(x)/B(0)$, $\Delta g(x)/g(0)$, etc; does not necessarily provide the best optimum for lattice designers!

How do we terminate a pole end?

For a pole with B ≥ 1.2 T saturation and non-linear behaviour will result if a square end is used:
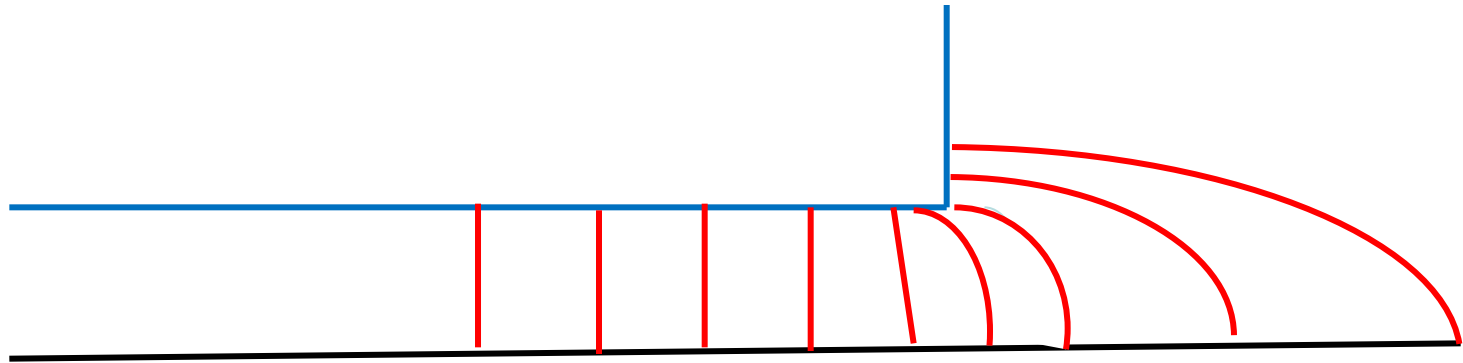
A smooth 'roll-off' is needed at pole edges (transverse); and at the magnet ends (in the 3rd dimension).
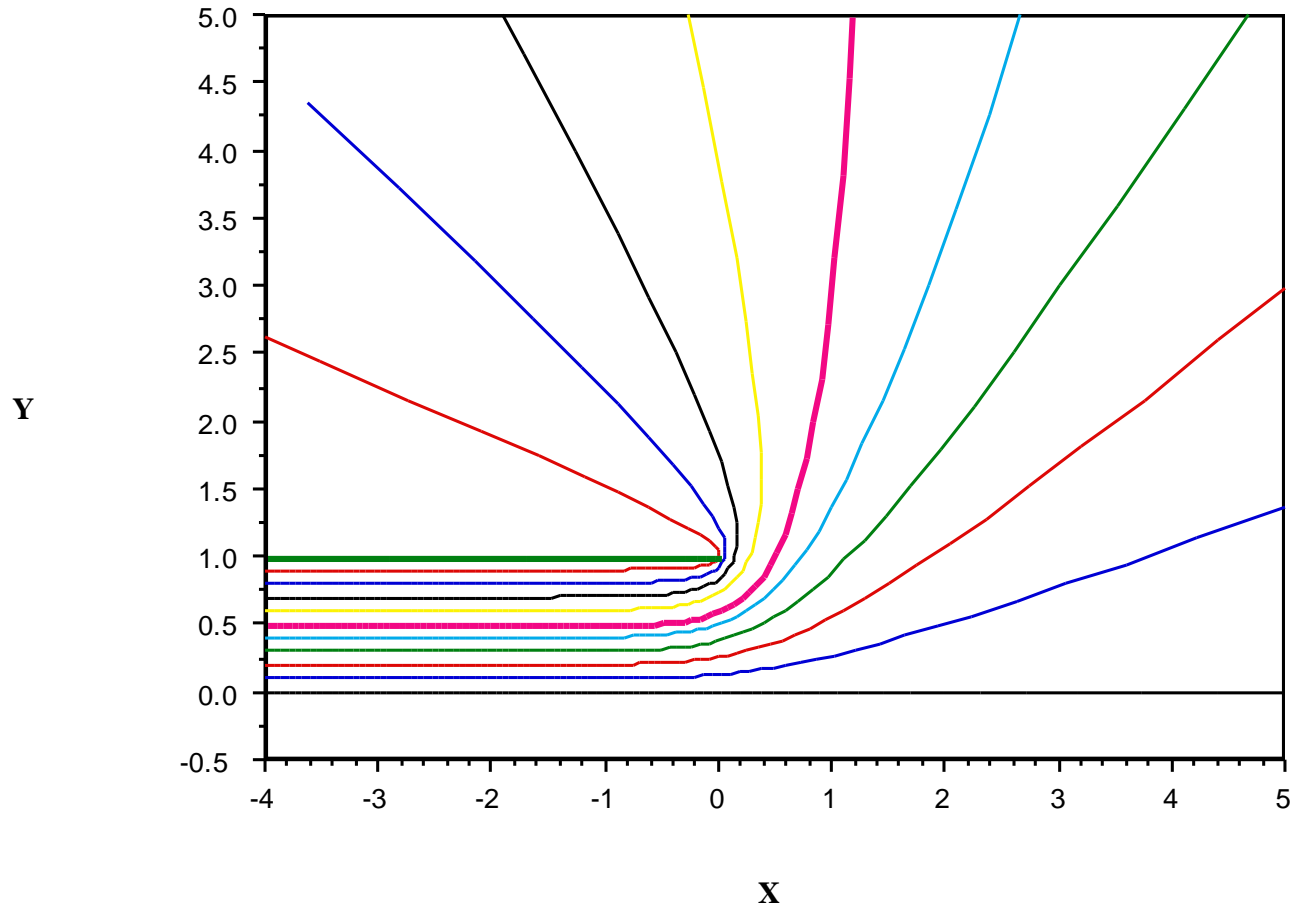
But what shape?

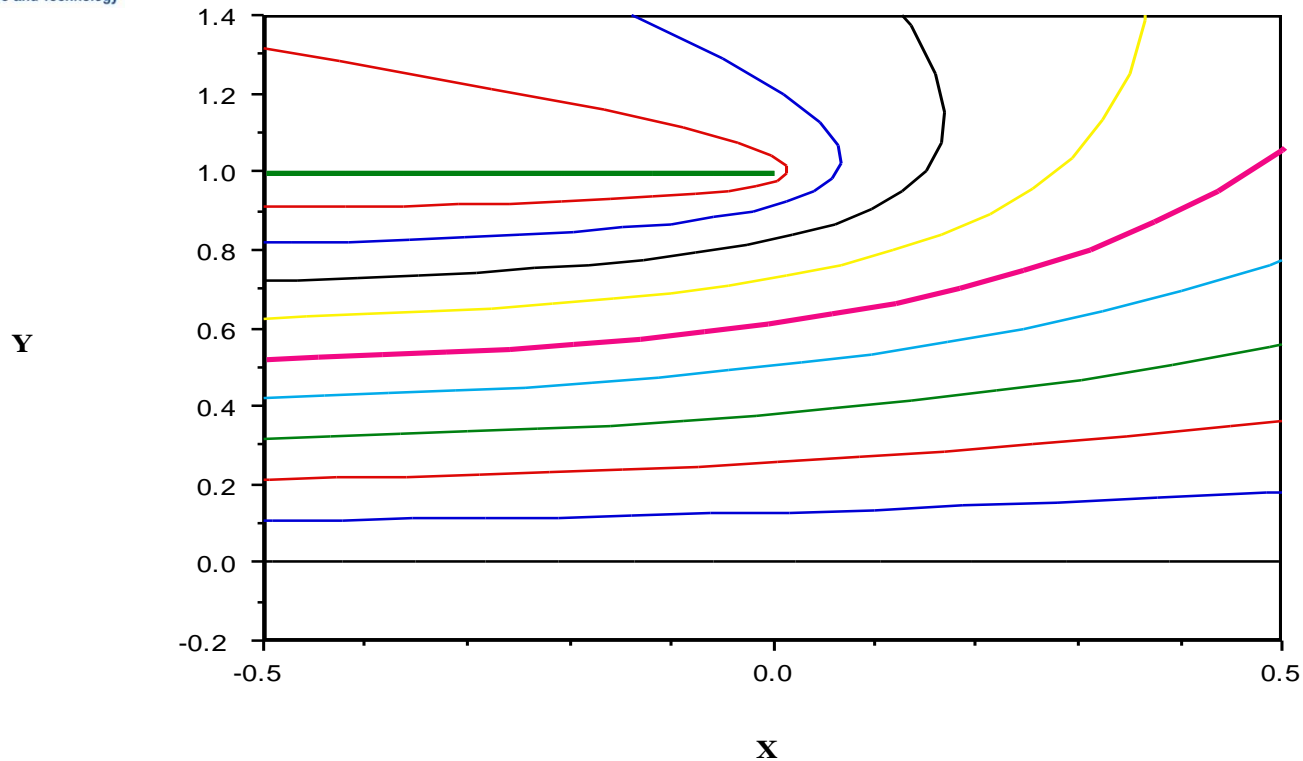Solution provided by Walter Rogowski – see next 4 slides.

How derived?

Rogowski calculated <u>electric potential lines</u> around a flat <u>capacitor plate</u>:

Blown-up version

The central heavy line is for $\emptyset = 0.5$.

Then Rogowski showed that this was the fastest changing line along which the field intensity was **monotonically decreasing**.

Then applied to magnet ends

Conclusion: Recall that a high μ steel surface is a line of constant scalar potential. Hence, a magnet pole end using the $\phi = 0.5$ potential line provides the maximum rate of **increase** in gap with a monotonic **decrease** in flux density at the surface ie no saturation.
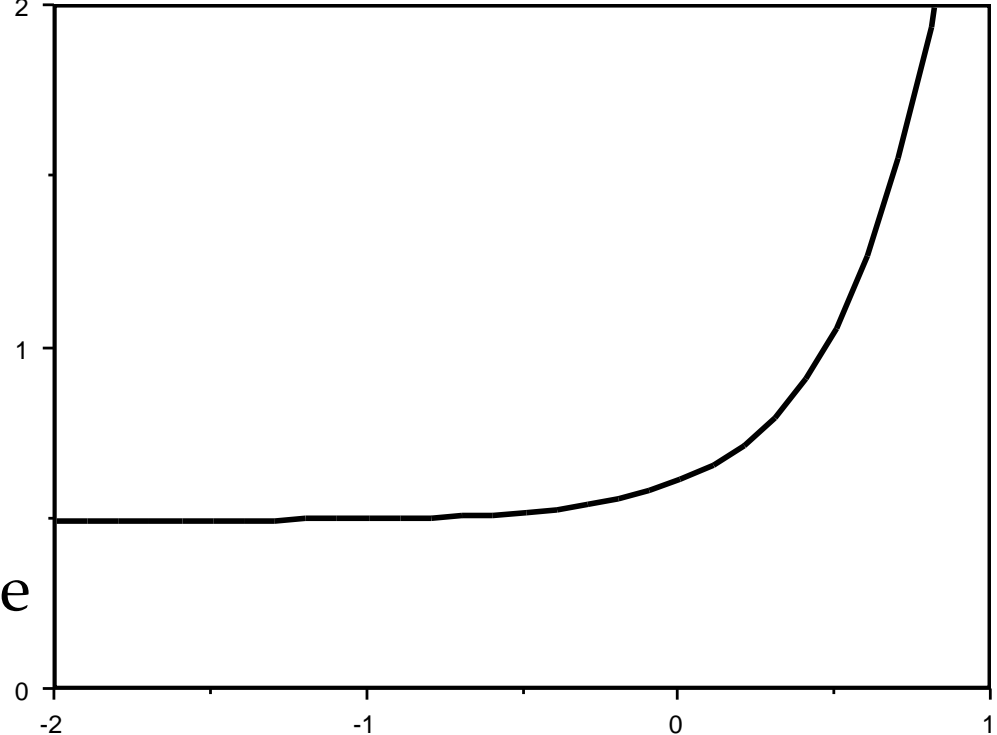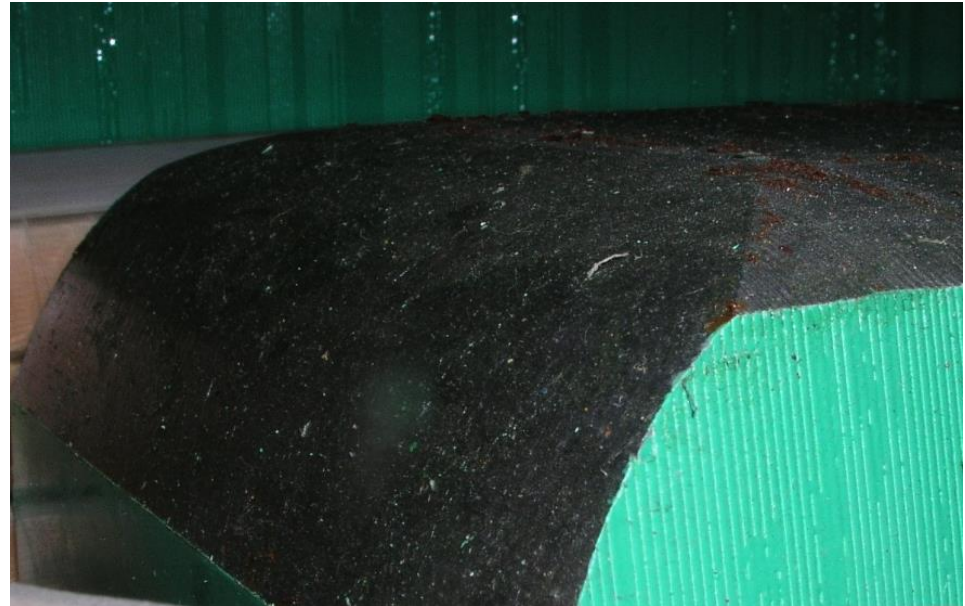
The equation

The 'Rogowski' roll-off:

Equation: $y = g/2 + (g/\pi\alpha) [\exp(\alpha\pi x/g) - 1];$

g/2 is dipole half gap;
y = 0 is centre line of gap;
**α** is a parameter controlling gradient at x = 0 (~1).

f) Super-conducting magnets;

Acknowledging and with thanks to Dr Martin Wilson; ex Oxford Instruments and CERN and consultant on LHC magnet design, for the use of diagrams.

For access to the overheads of Dr Wilson's four courses, see URLs at p 1 of the appendix.

LHC superconducting dipoles

'Cold bore' magnets – two beams and two coil sets in one cryostat – unusual.
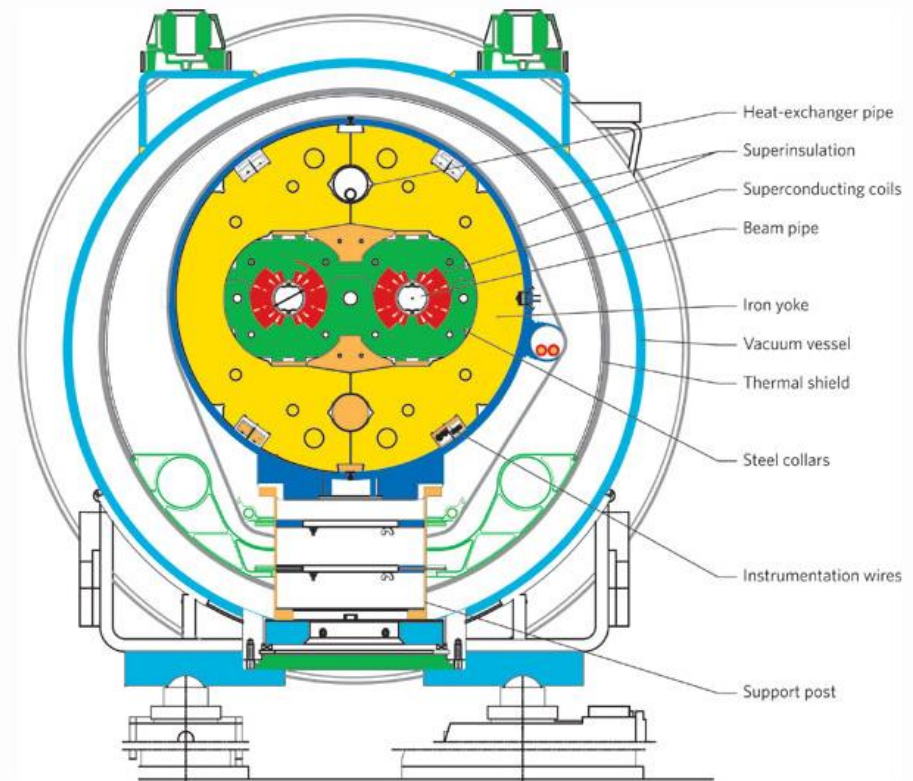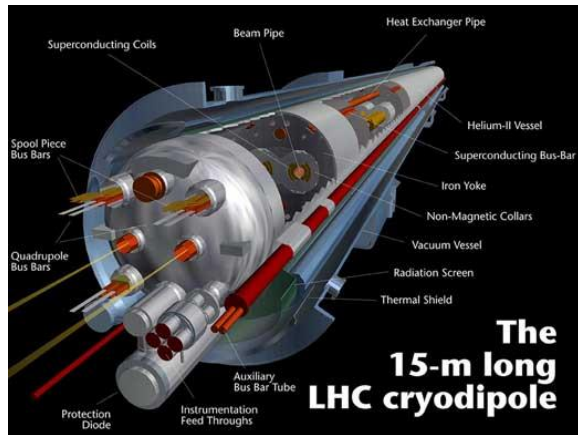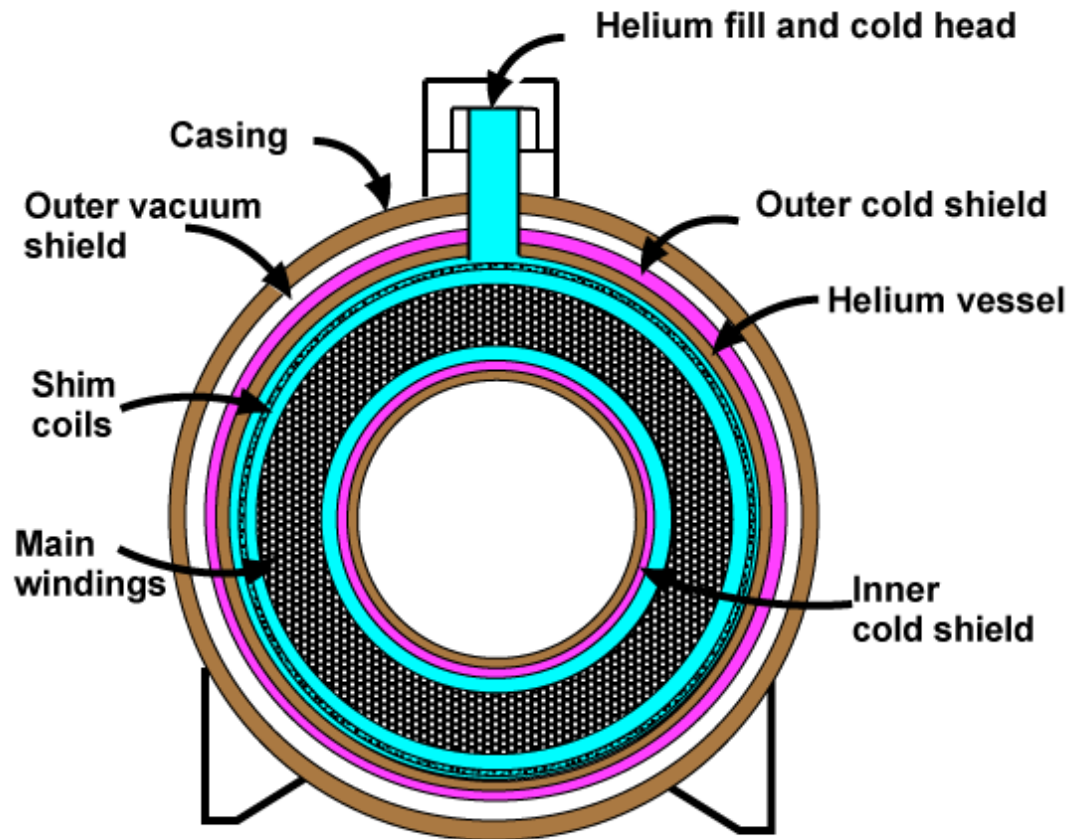

Unlike 'room-temperature' magnets, where the field distribution and amplitude is dominated by ferrous yokes, s.c. magnets are coil current dominated; steel provides external screening.

The LHC dipole; ultimate field >8T with LHe at 1.9K.

A more usual cross section
(warm bore beam pipe):

Helium fill and cold head

Casing

Outer vacuum shield

Outer cold shield

Helium vessel

Shim coils

Main windings

Inner cold shield

Representative cross-section of a typical superconducting magnet (designs vary). Liquid helium chambers are colored aqua. Active shielding coils (not pictured) are near the shim coils at the two scanner ends.

The 'cold shield' between room temperature and LHe can also include a LN_2 layer and, in that case, multiple vacuum layers.

Differences between room temp and s.c. magnets (for <u>typical</u> magnets).

	Room Temp	**Super conducting**
Coil position:	Outside poles, remote from beam; material is economic choice;	Around beam; the coil **material** is of paramount importance and determines performance;
Flux density distribution:	Strongly determined by pole iron geometry;	Completely determined by coil geometry;
Steel:	Poles above and below beam, with yoke etc;	Remote from beam, placed externally to limit stray fields;
Magnetic forces:	Seldom a problem;	Massive; major mechanical design problem;
Coil current density (J):	Determined by economic criteria;	Determined by s.c. material performance – major design issue;
Coil cooling:	Needed – demineralised water, but seldom a problem;	Major issue of cryogenic design – all parts of coils **must** be in good thermal contact with LHe.

Most commonly Low Temperature Super-conductors (LTS):

NbTi:

- the 'standard' material used during the last 40 years;
- critical temperature (0 field and 0 current density): $T_c = 9.2$ K;
- critical field (0 K and 0 current density): $B_c = 10.5$ T;
- is ductile.

Nb$_3$Sn:

- used to get to higher field;
- $T_c = 18.3$ K;
- $B_c = 20.5$ T;
- **is brittle** – ie non-ductile;

Used at 4.2 K (boiling point of LHe at 1 atmosphere), or lower temp for higher field (eg, the LHC at 1.9K to develop > 8 T) by operating the cryostat at < 1 atmosphere.

- coils have to be wound with separate layers of Nb and Sn and then the alloy formed at the interface with heat treatment.

Both are type 2 superconductors – B penetrates coil!

See p2 of appendix for explanation.

Flux density, current density and temperature of s.c. material.

The s.c. state of the coil material is determined by the values of **flux density B**, **current density J** and **temperature** θ at the super-conductor.

The diagram is for NbTi; material that is within the solid figure, is superconducting. Any part of the coil that is not within these bounds is '**normal**'. When this occurs, the coil '**quenches**'.
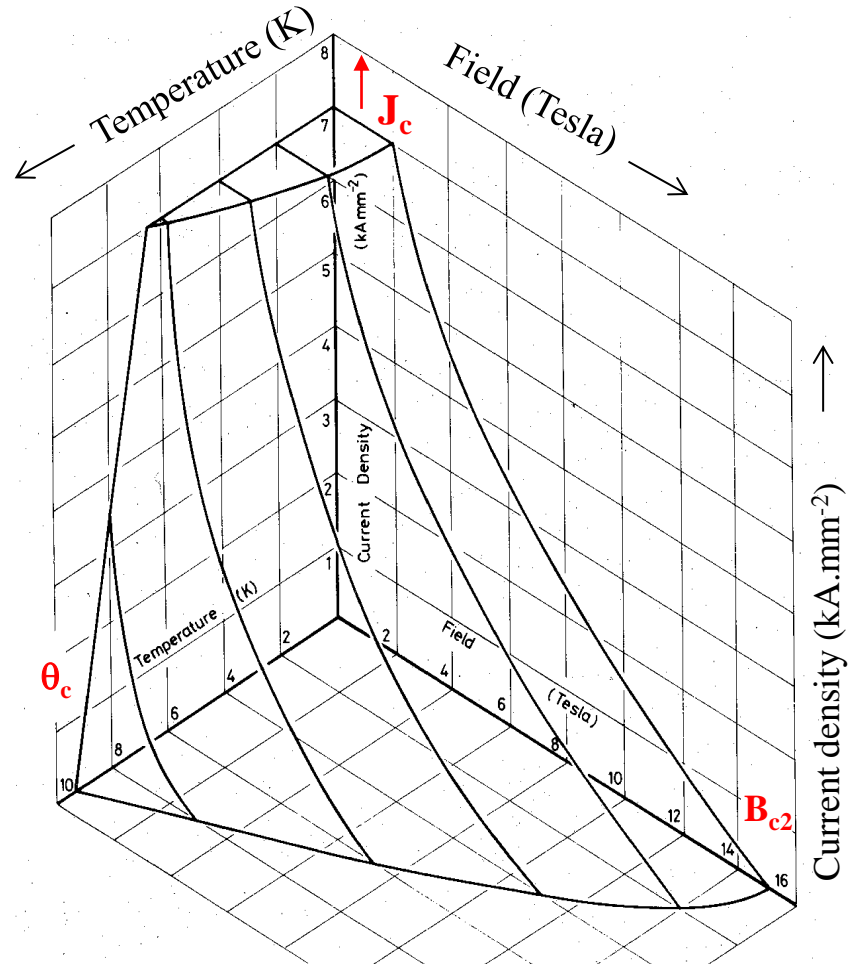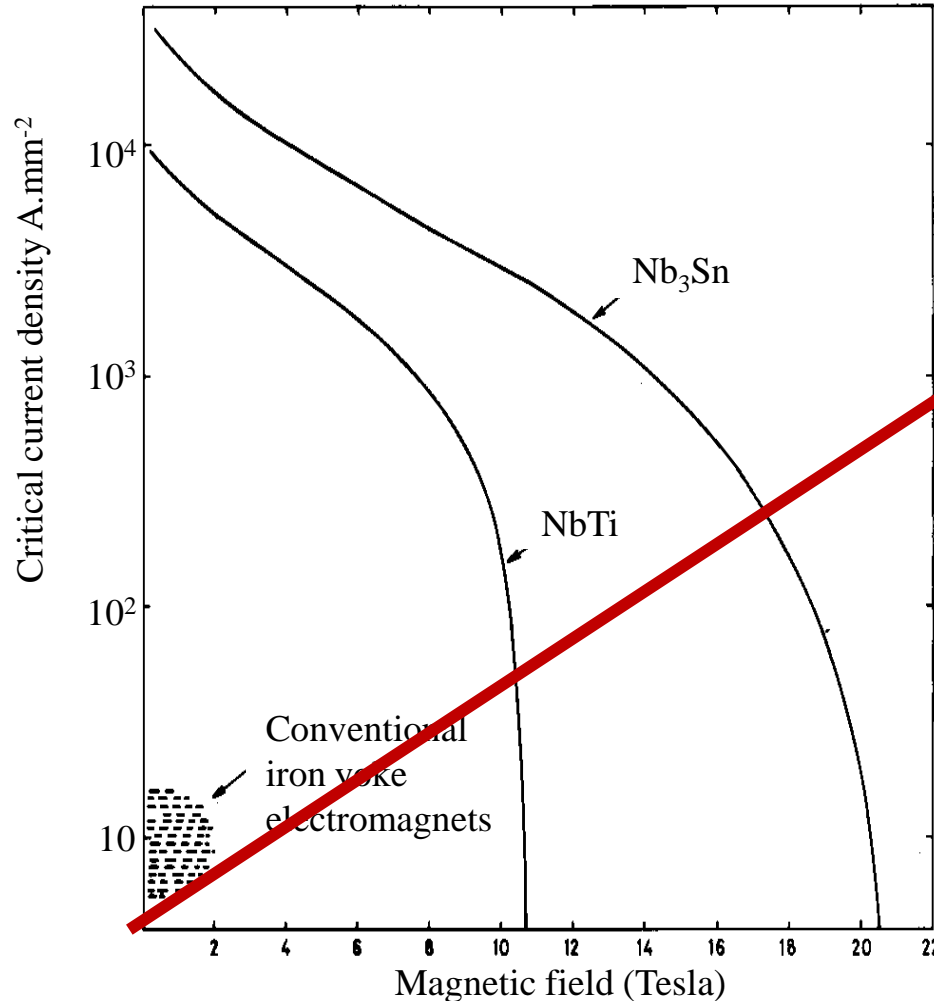
Critical flux density vs temperature.

With current density J =0.
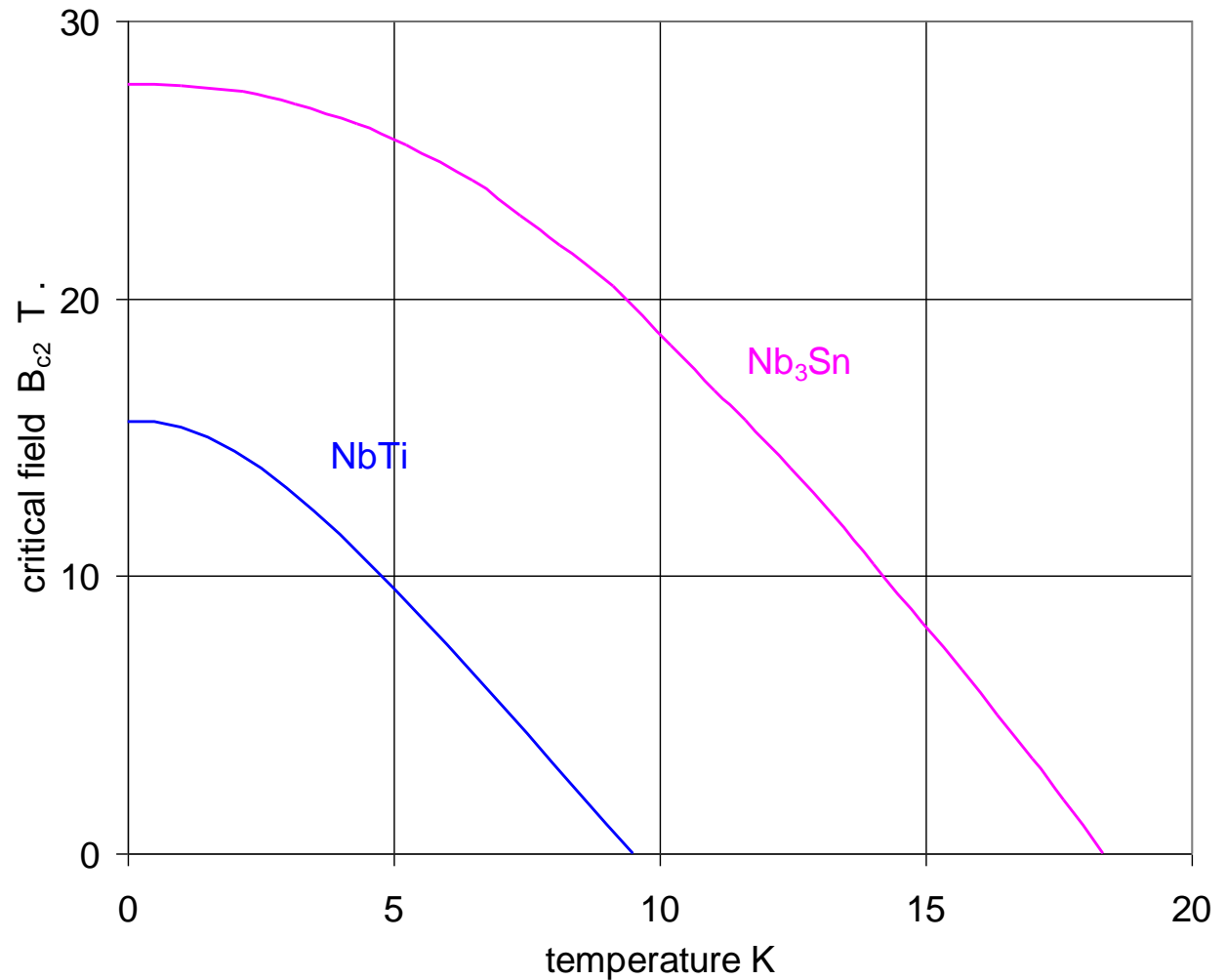
Critical values of J vs B for NbTi and Nb$_3$Sn at 4.2K.

Load line:
Defines the ratio between current density and flux density for a particular model.

It is usual to operate as circa 80% of the upper limit.

If non-linear steel is present, the line will curve upwards.

Critical values of NbTi and Nb₃Sn at 4.2 and 1.9 K.

From:

'**Limits of NbTi and Nb₃Sn for High Field Accelerator Magnets**'; A. Godeke et al:

LBNL-62138_Conf.pdf

Note: that the design is based on operation at 80% of the critical values. This is standard procedure.

see also P3 of appendix
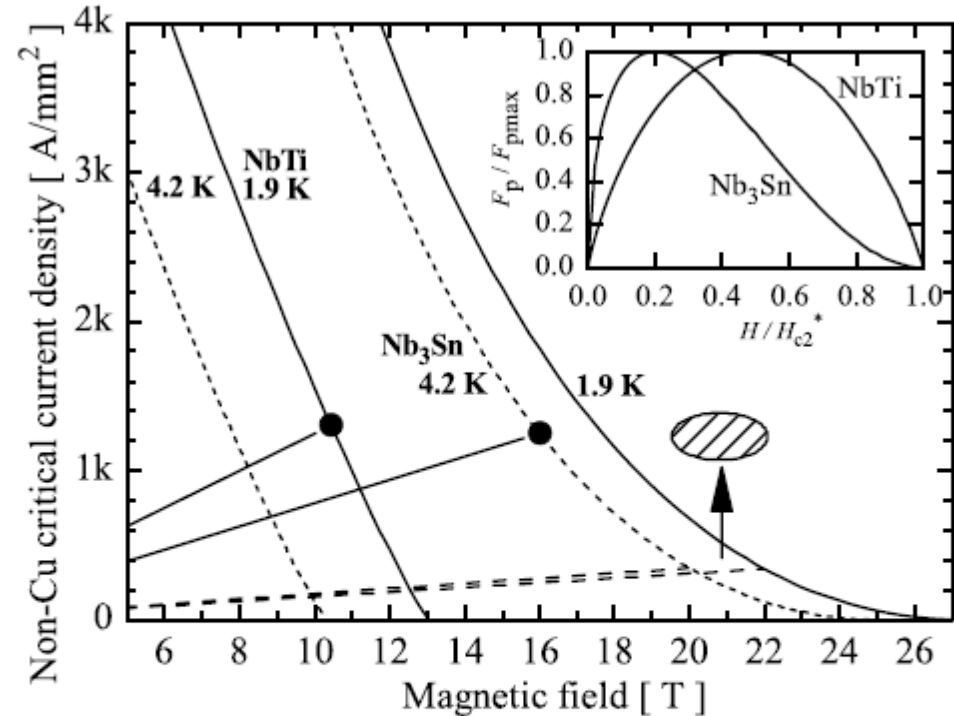
Fig. 2. Critical current density as function of magnetic field at 1.9 and 4.2 K in NbTi and Nb₃Sn. Included are the load-lines for record magnets and for a hypothetical Nb₃Sn dipole magnet achieving 80% of its intrinsic limitation. The inset shows the normalized pinning force as function of reduced magnetic field.

S.C. Material 'stabilisation'.

If there is any 'disturbance' (movement at μm level; μJ heat deposited….) to the s.c. when operating, minute parts of the material '**go normal**'. Until that segment returns to s.c. state, it **MUST NOT** conduct current (**high resistivity when normal** – **excessive heat generated**).

So, the s.c matrix of very thin 'cables' is 'stabilised' by enclosing it in a copper mass. The Cu cross-section is typically equal to or x2 greater than the s.c. material.

A super-conducting cable.
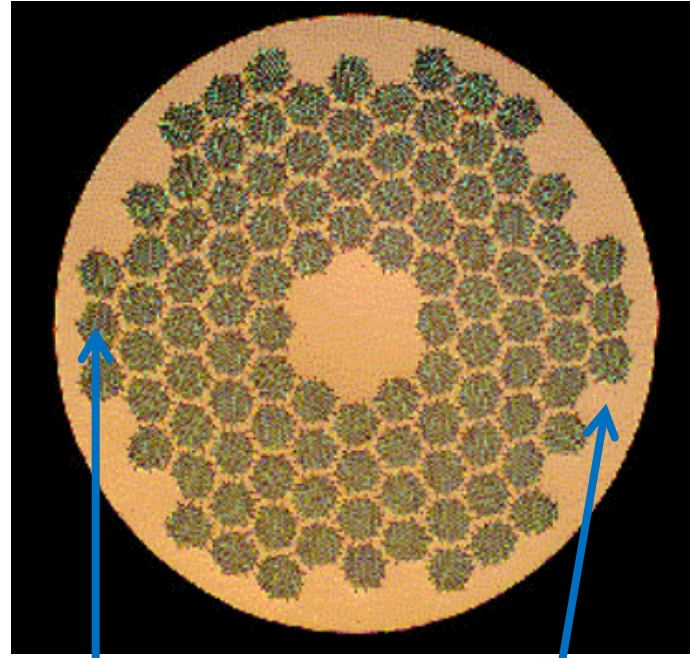
s.c. 'strands'   Copper stabiliser

For further information on the structure of s.c. cables see appendix p 4; and restraining forces see appendix p 5.

'Engineering' current density

Coil excitation - use the 'engineering' current density J_{eng}
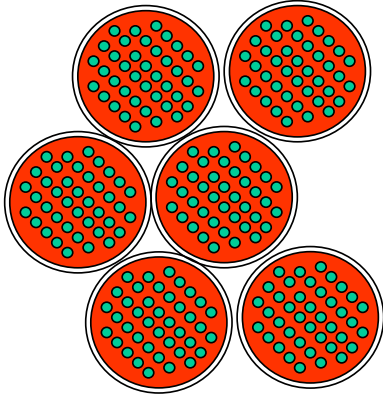
$$J_{eng} = \frac{current}{unit\ cell\ area} = J_{supercon} \times \lambda_{metal} \times \lambda_{winding}$$

Where:

$J_{supercon}$ is the current density in the J vs B data;

λ_{metal} is the ratio s.c to matrix cross section;

$\lambda_{winding}$ is the ratio matrix to total cross section area;

for NbTi $\lambda_{metal} = 0.4$ to 0.25

for Nb_3Sn $\lambda_{metal} \sim 0.3$

And $\lambda_{winding} \sim 0.7$ to 0.8

So typically J_{eng} is only 25% to 35% of $J_{material}$

Dipole field from overlapping cylinders

curl H = J;

A cylinder with current density J:

at radius r:

$B = \mu_0 H = \mu_0 J r/2$ (using Stoke's theorem).
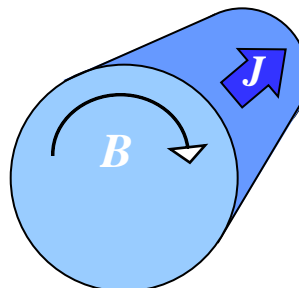
$$B = \frac{\mu_o J r}{2}$$

- two cylinders with opposite current densities, pushed together;
- where they overlap, currents cancel out;
- zero current in the aperture;
- fields in the aperture:

$$B_y = \frac{\mu_o J}{2}\left(r_1 cos\theta_1 - r_2 cos\theta_2\right) = \frac{\mu_o J t}{2}$$

$$B_x = \frac{\mu_o J}{2}\left(-r_1 sin\theta_1 + r_2 sin\theta_2\right) = 0$$
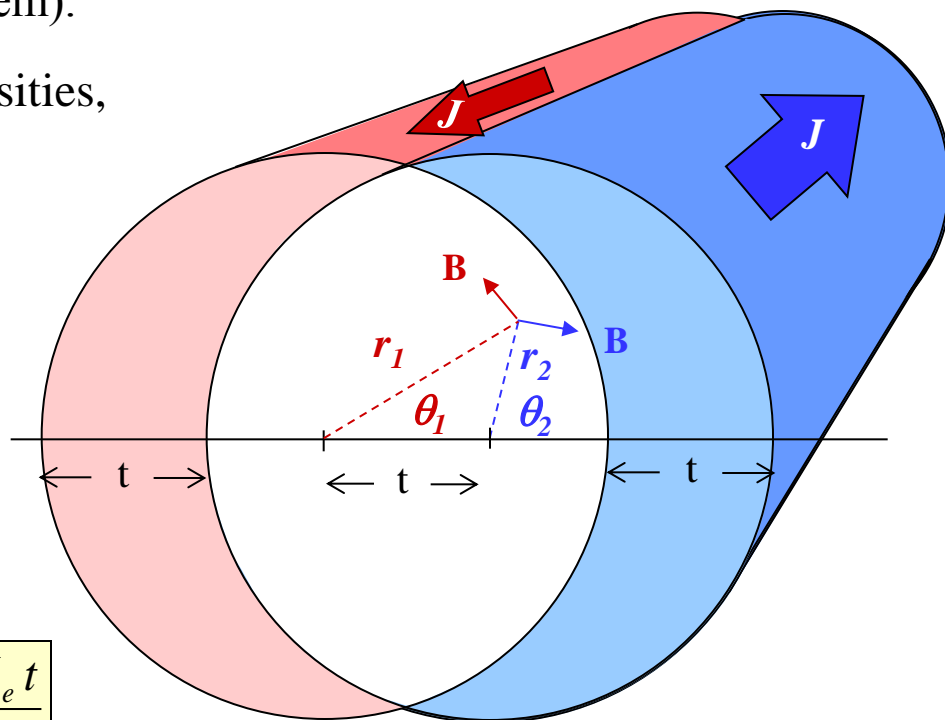
Thus, a perfect dipole field

$$B_y = \frac{\mu_o J_e t}{2}$$

Coil geometry for dipole field.

The intersecting cylinders give perfect dipole field but do not fit well into a circular aperture.

A cos θ distribution around a internal circular arc is also suitable.

But see the coil in the LHC dipoles; a quasi overlapping cylinder pair:
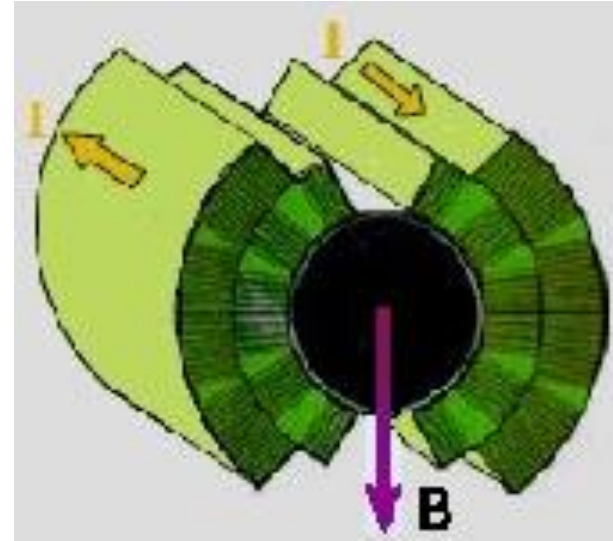
LHC coil cross section.

Beam vac vessel diameter: 56 mm;

Coil outer diameter: 120 mm.

g) Magnet design using finite element analysis (F.E.A.) codes

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Computer codes.

A number of computer codes are available;

eg the Vector Fields codes -'OPERA 2D and 3D'. These have:

- finite elements with variable triangular mesh;
- multiple iterations to simulate steel non-linearity;
- extensive pre and post processors;
- compatibility with many platforms and P.C. o.s.

Technique is iterative:

- calculate flux generated by a defined geometry;
- adjust the geometry until required distribution is achieved.

Design **Procedures** – **OPERA 2D**.

Pre-processor:

The model is set-up in 2D using a GUI (graphics user's interface) to define 'regions':

- steel regions;
- coils (including current density);
- a 'background' region which defines the   physical extent of the model;
- the symmetry constraints on the boundaries;
- the permeability for the steel (or use the pre-   programmed curve);
- mesh is generated and data saved.

Model of Diamond s.r. dipole
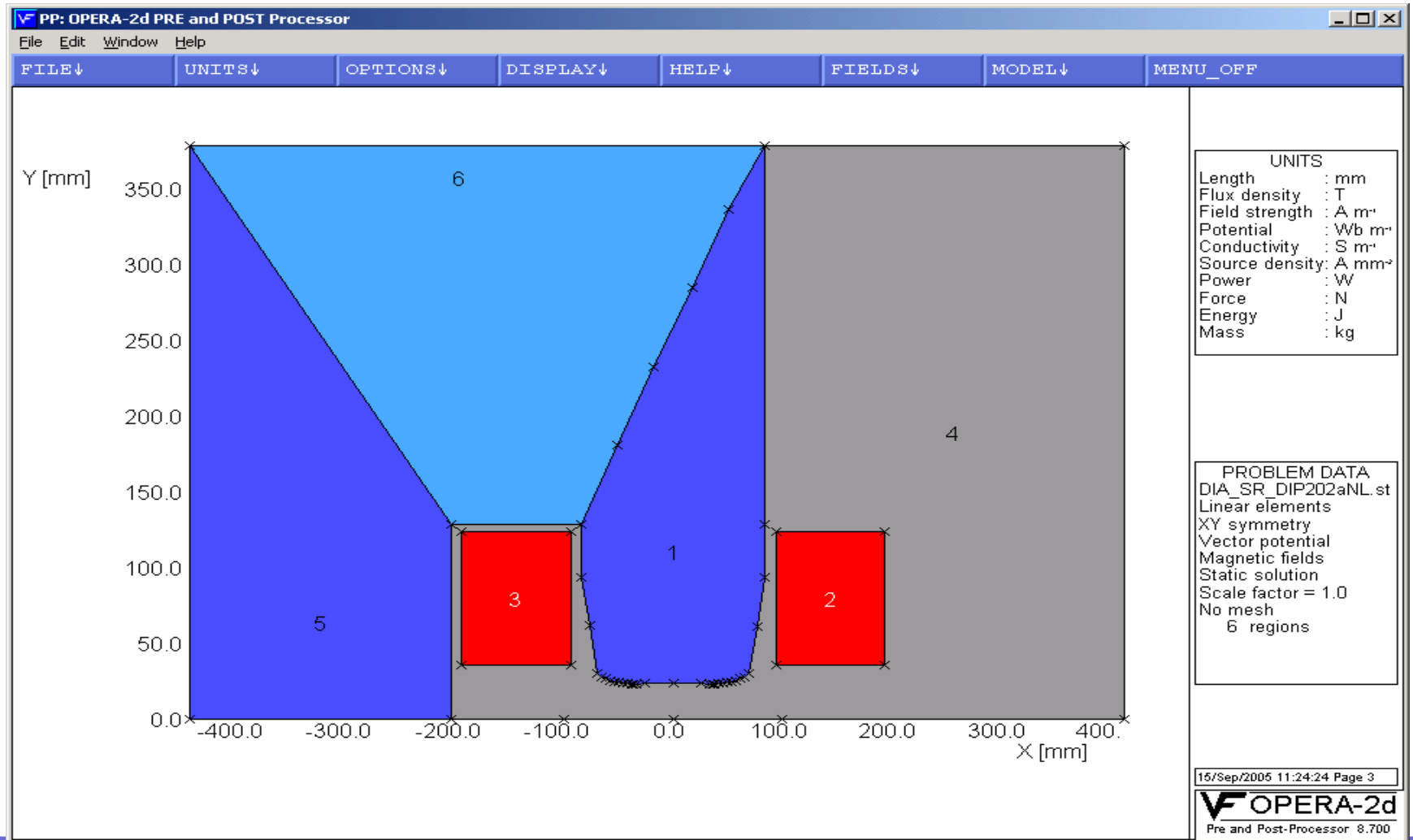
With mesh added
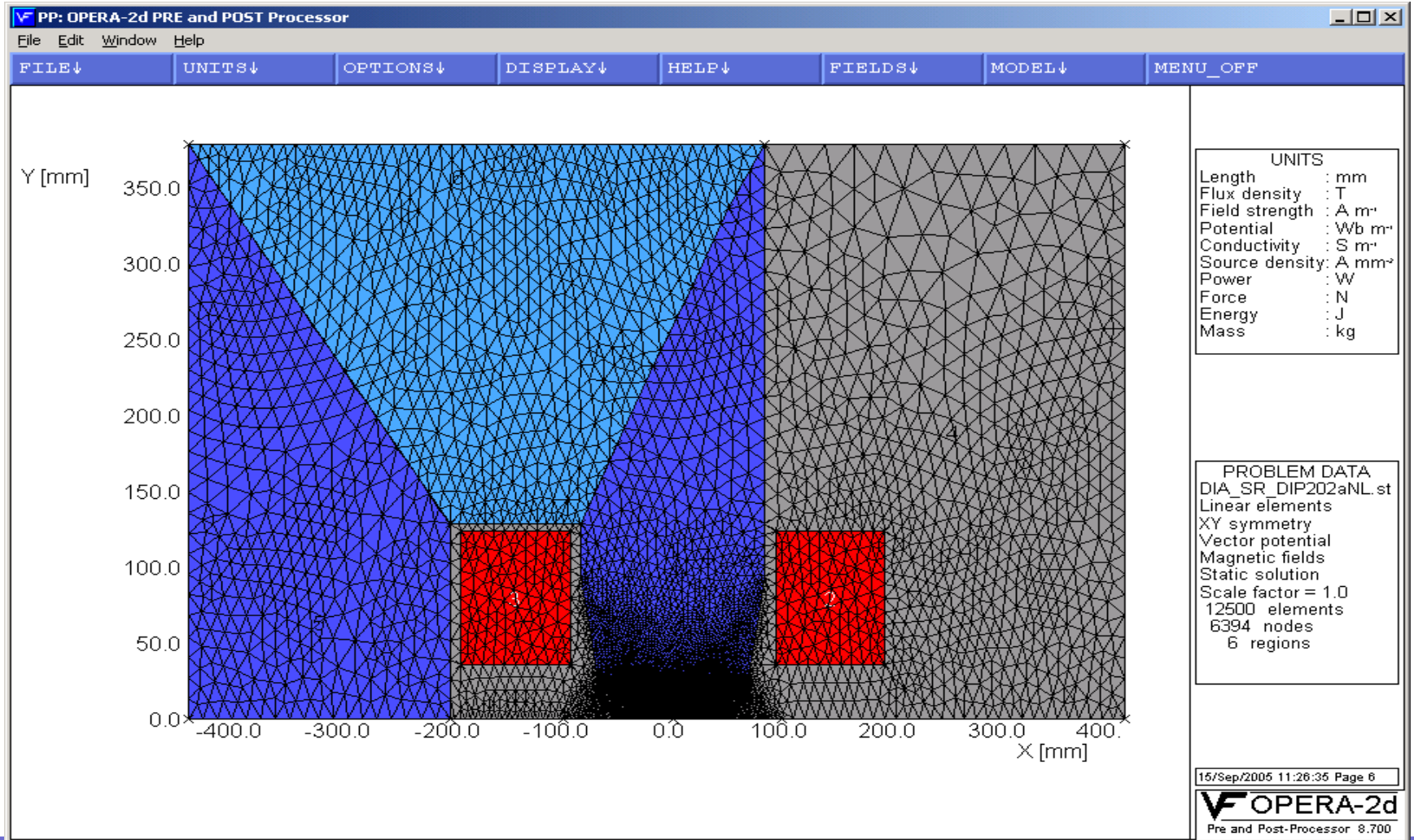
'Close-up' of pole region.

Pole profile, showing shim and 'Rogowski side roll-off' for Diamond 1.4 T dipole.:
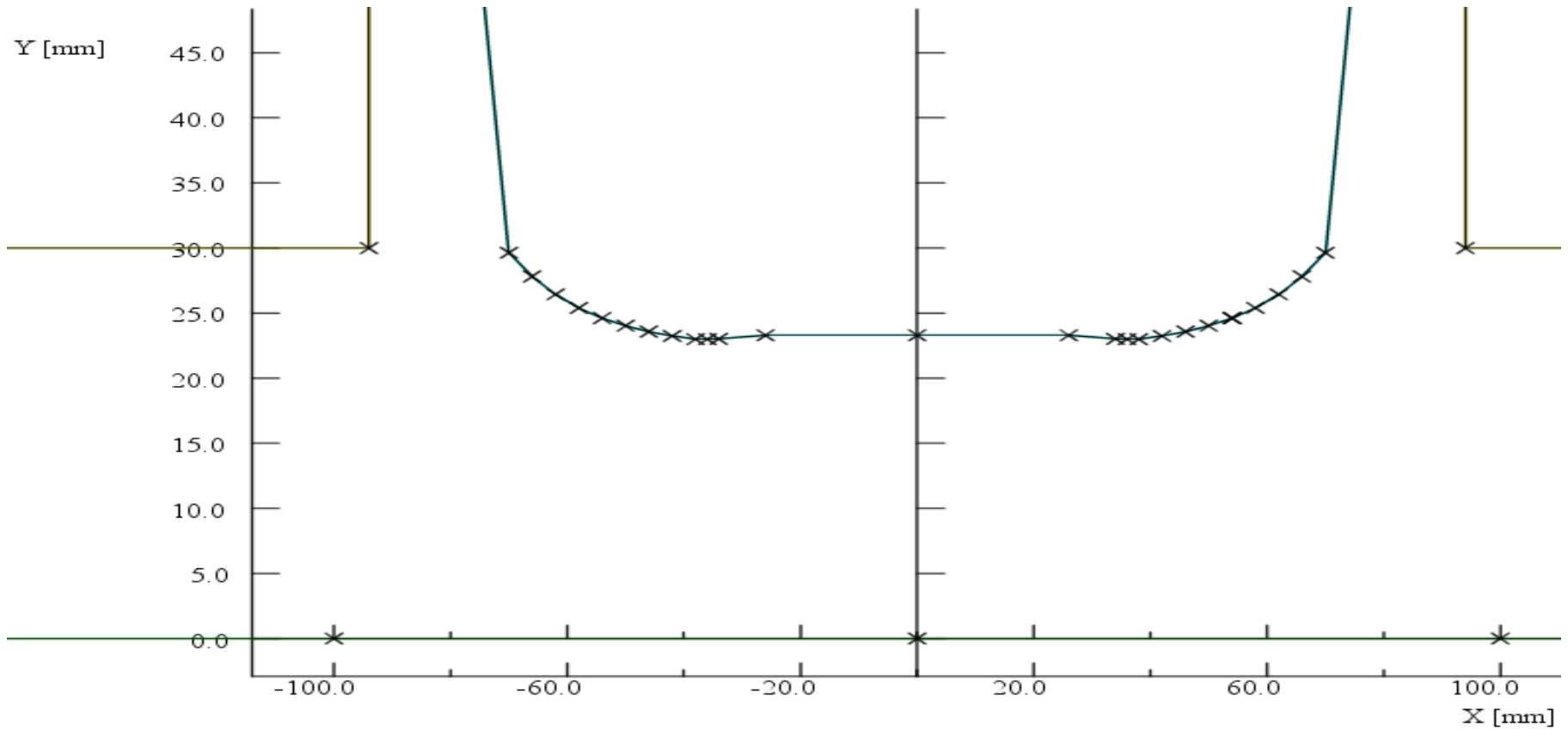
Diamond quadrupole model

Very preliminary model – fully symmetric around 4 axies.
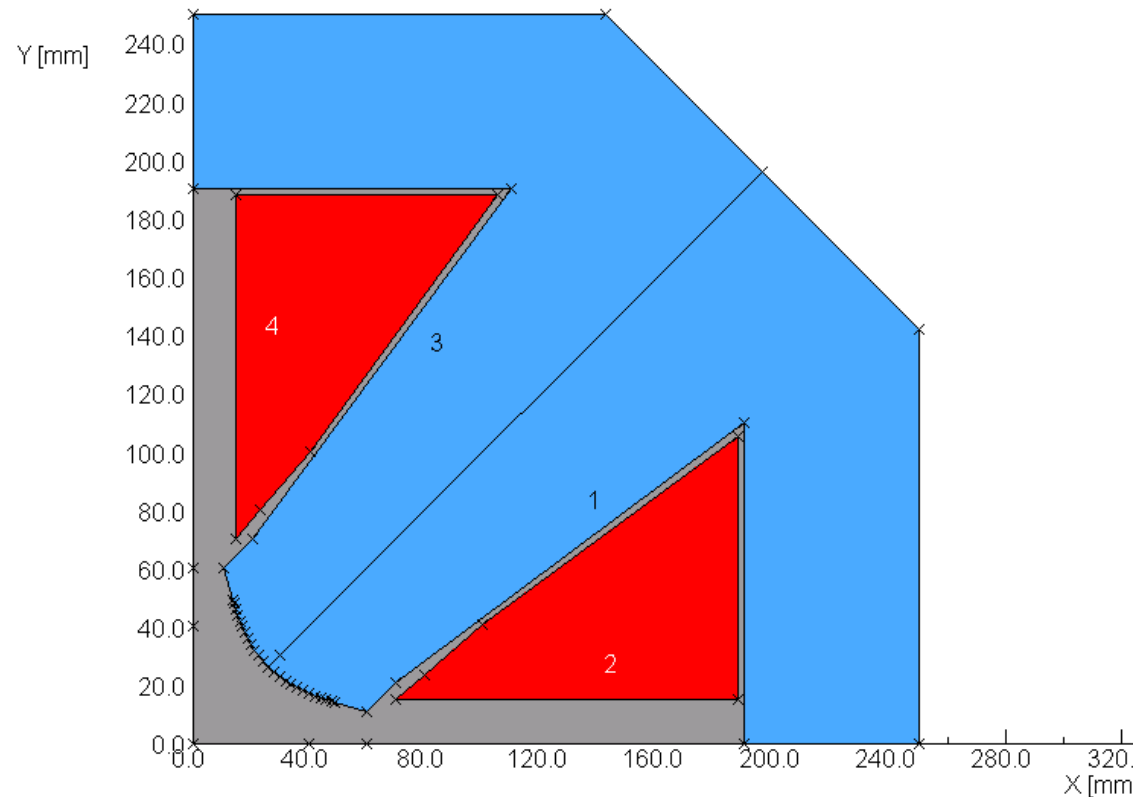
Note – one eighth of quadrupole <u>could</u> be used with opposite symmetries defined on horizontal and y = x axis.

Calculation.

'Solver':

either:

- linear which uses a predefined constant permeability for a single calculation, or

- non-linear, which is iterative with steel permeability set according to B in steel calculated on previous iteration.

Data Display – OPERA 2D.

Post-processor:

uses pre-processor model for many options for displaying field amplitude and quality:

- field lines;
- graphs;
- contours;
- gradients;
- harmonics (from a Fourier analysis around a pre-defined circle – valuable to lattice designers).

2 D Dipole field homogeneity on x axis
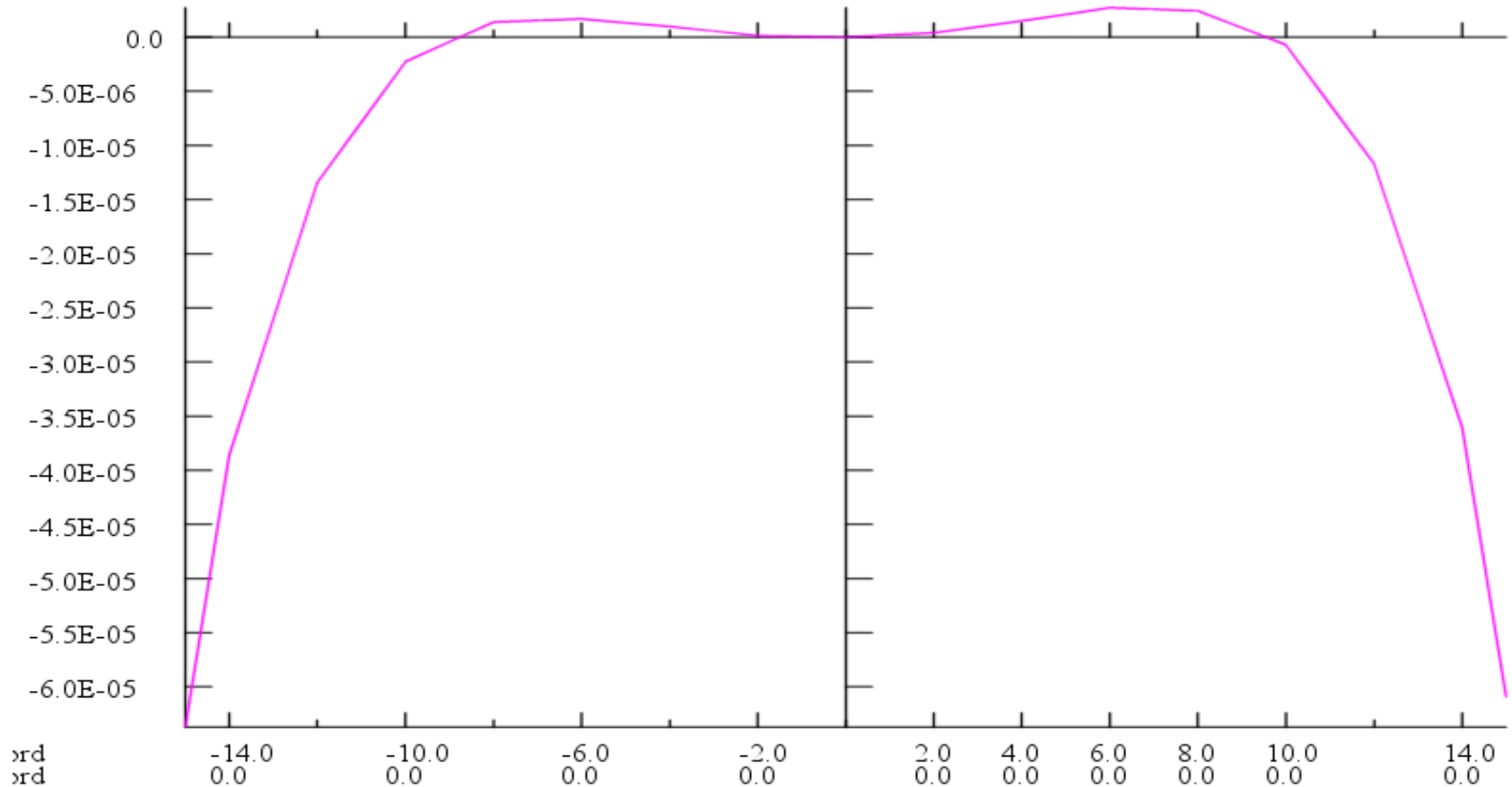
Diamond s.r. dipole: $\Delta B/B = \{By(x) - B(0,0)\}/B(0,0)$;
typically $\pm 1:10^4$ within the 'good field region' of $-12\text{mm} \leq x \leq +12\text{ mm}$..

2 D Flux density distribution in a dipole.
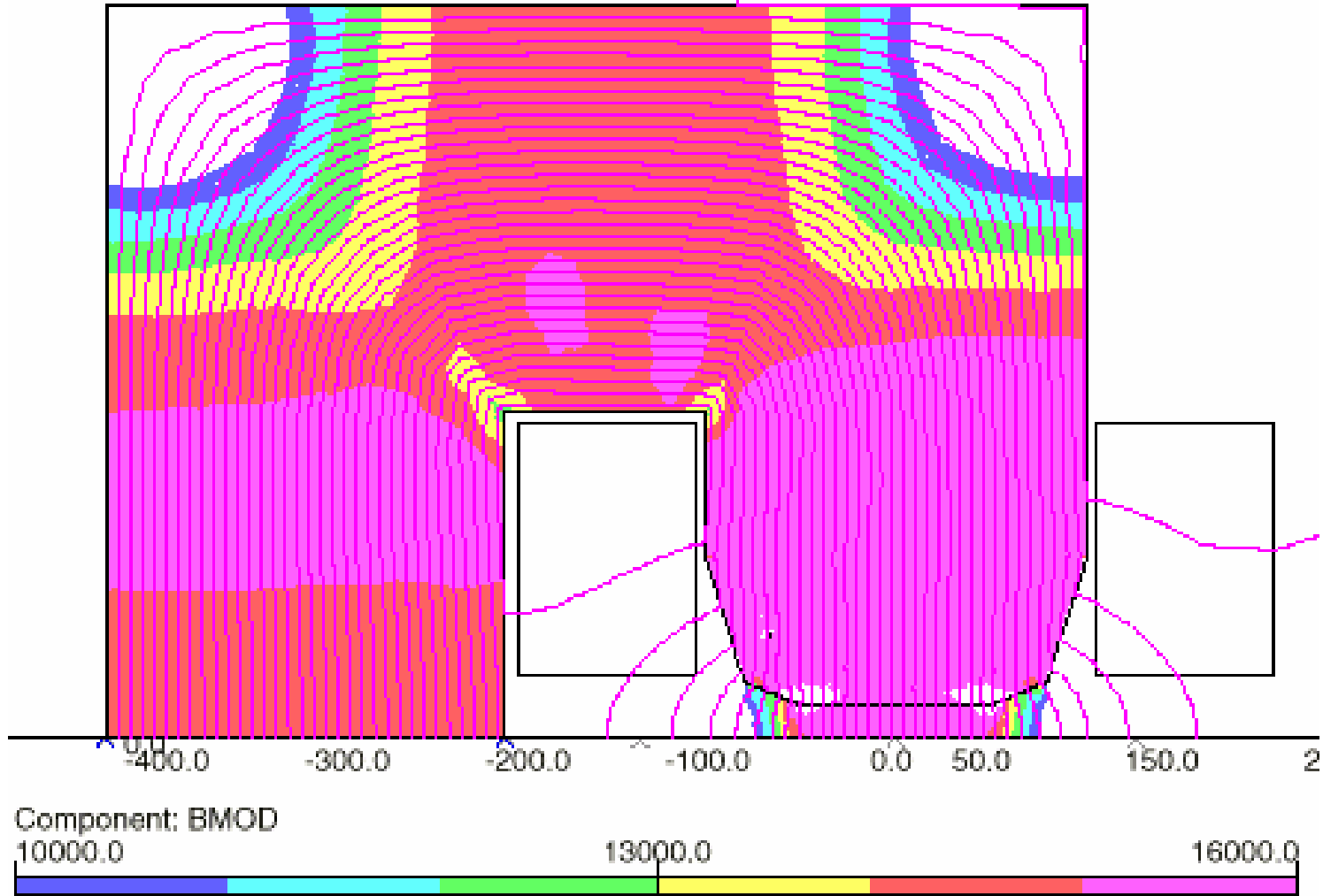
Component: BMOD

10000.0   13000.0   16000.0

2 D Dipole field homogeneity in gap

Transverse (x,y) plane in Diamond s.r. dipole;
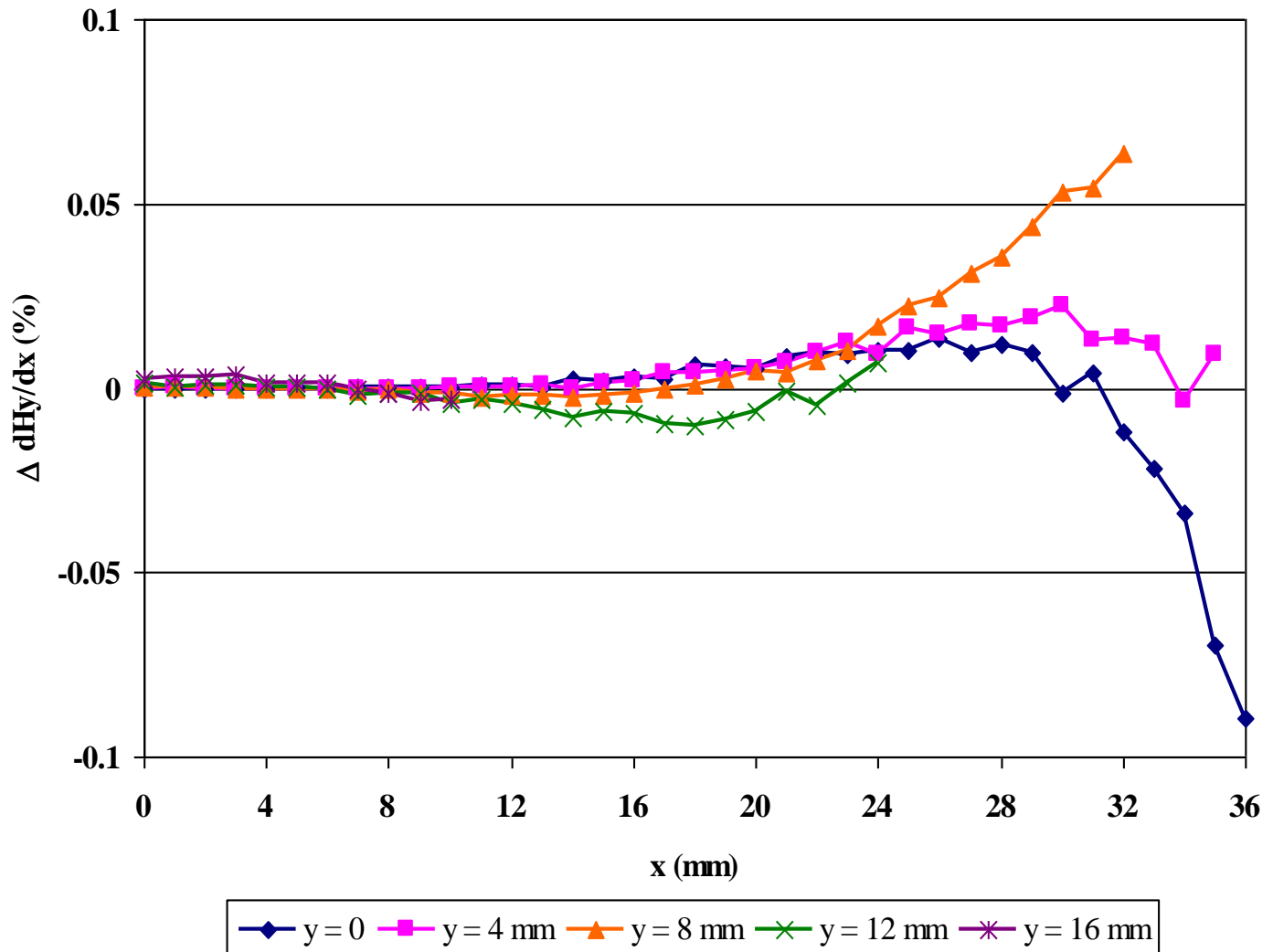
contours are ±0.01%

required good field region:
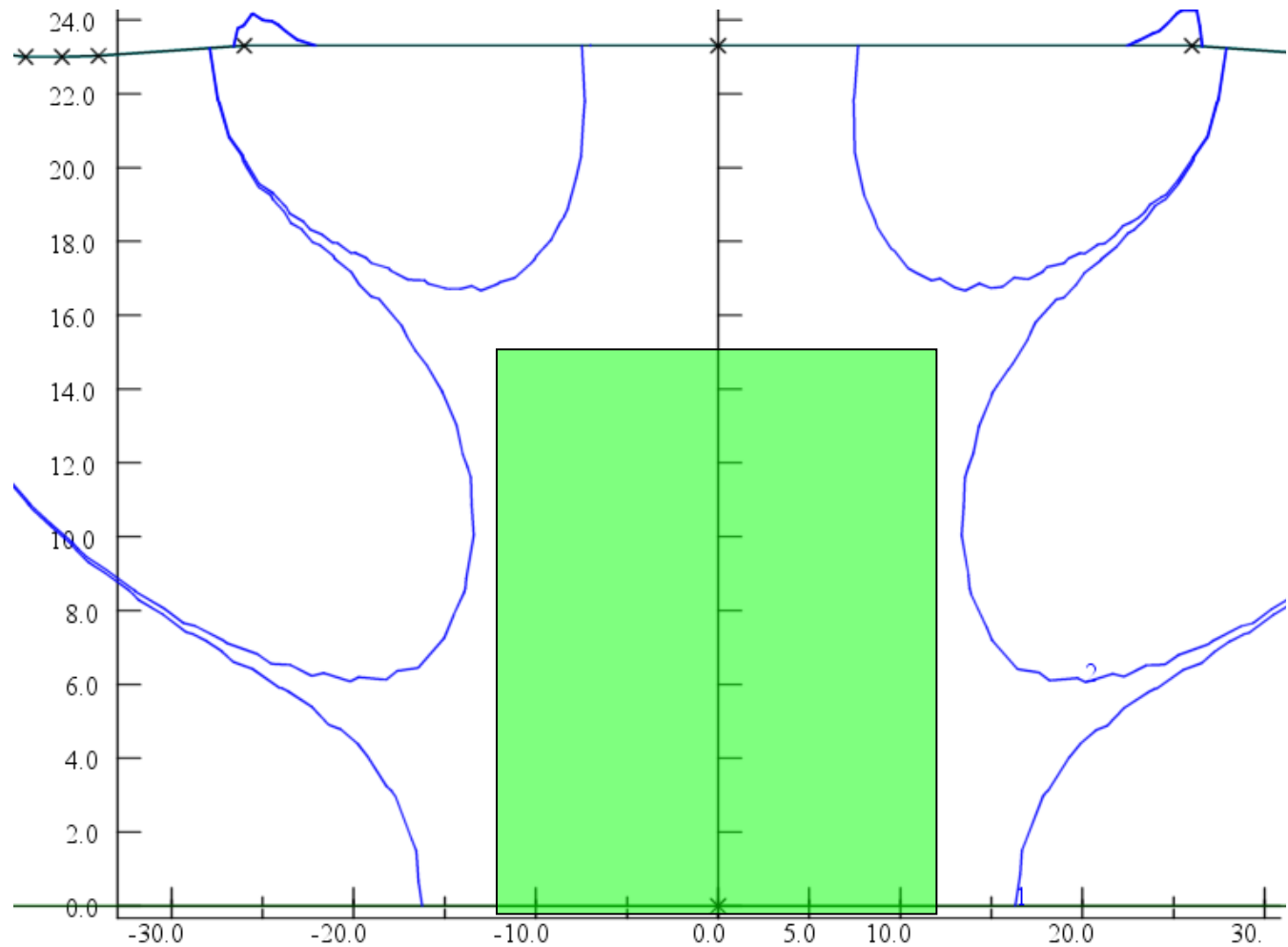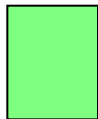
2 D Assessment of quad gradient.

Diamond WM quadrupole:

graph is <u>percentage</u> variation in dBy/dx vs x at different values of y.
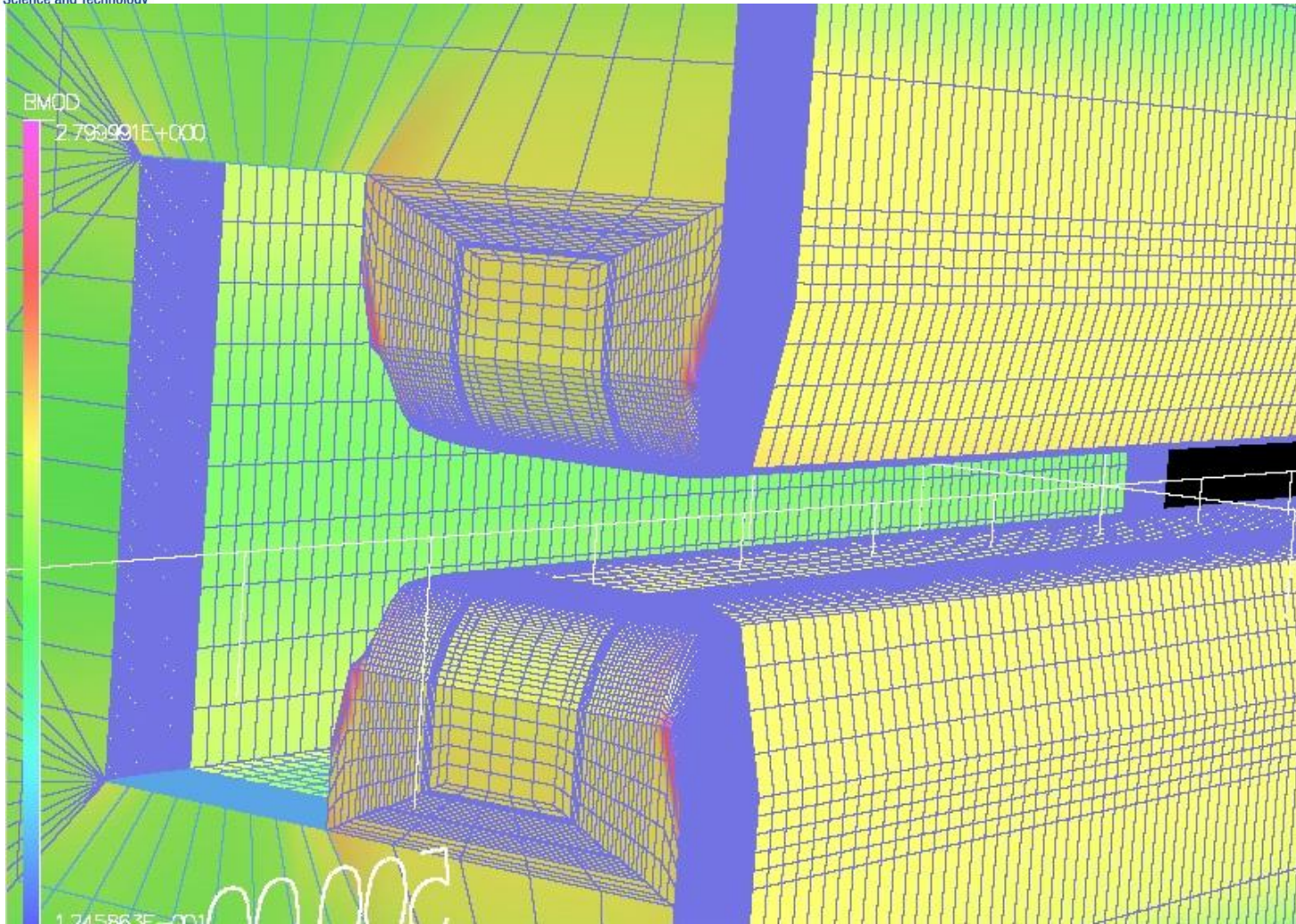
Gradient quality is to be ± 0.1 % or better to x = 36 mm.

Legend: y = 0, y = 4 mm, y = 8 mm, y = 12 mm, y = 16 mm

Y-axis: Δ dHy/dx (%)
X-axis: x (mm)

OPERA 3D model of Diamond dipole.

Harmonics indicate magnet quality

The amplitude and phase of the harmonic components in a magnet provide an assessment:

- when accelerator physicists are calculating beam behaviour in a lattice;
- when designs are judged for suitability;
- when the manufactured magnet is measured;
- to judge acceptability of a manufactured magnet.

Measurement of a magnet after manufacture will be discussed in the section on measurements.

The third dimension – magnet ends.

Fringe flux will be present at the magnet ends so beam deflection continues <u>beyond</u> magnet end:
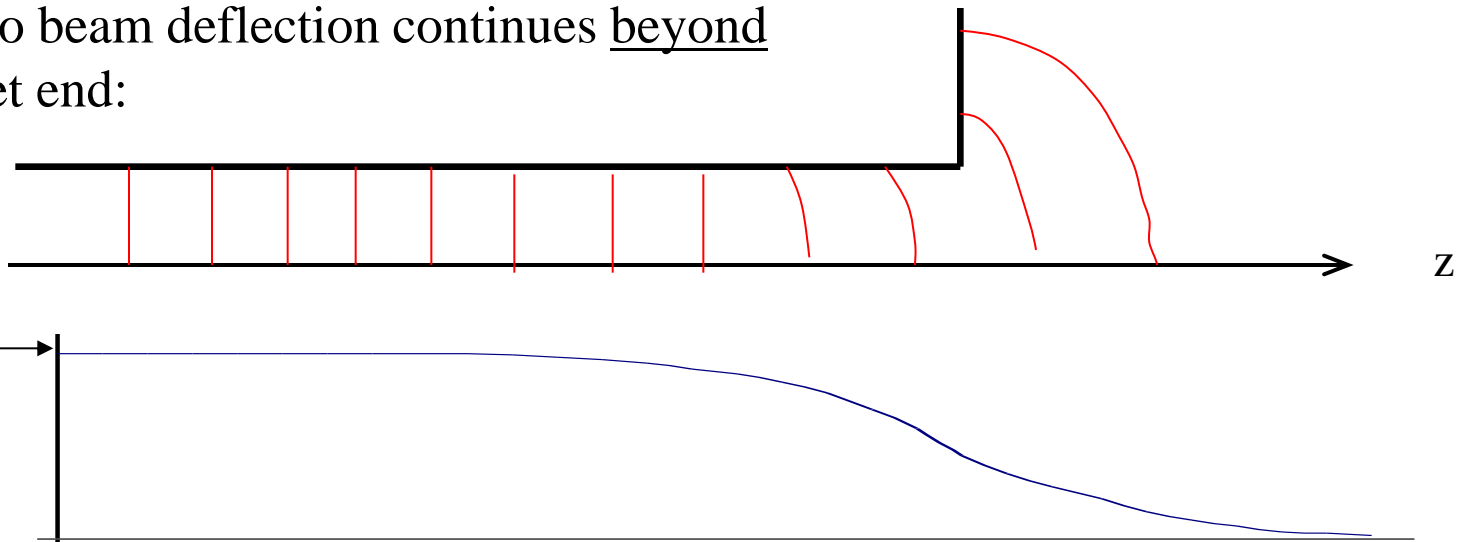
The magnet's strength is given by $\int B_y(z)\,dz$ along the magnet, the integration including the fringe field at each end;

The **'magnetic length'** is defined as $(1/B_0)(\int B_y(z)\,dz)$ over the same integration path, where B_0 is the field at the azimuthal centre.
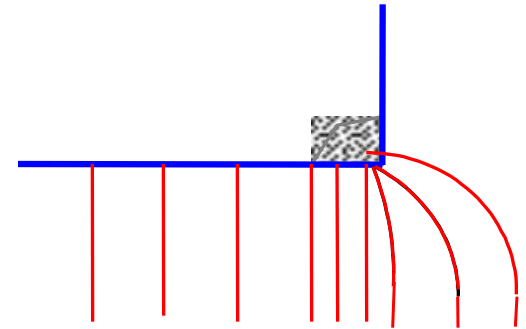
Magnet End Fields and Geometry.

It is necessary to terminate the magnet in a controlled way:

•to define the length (strength);

•to prevent saturation in a sharp corner (see diagram);

•to maintain length constant with x, y;

•to prevent flux entering normal
        to lamination (ac).

The end of the magnet is therefore
'chamfered' (a Rogowski roll-off if
high field), increasing the gap
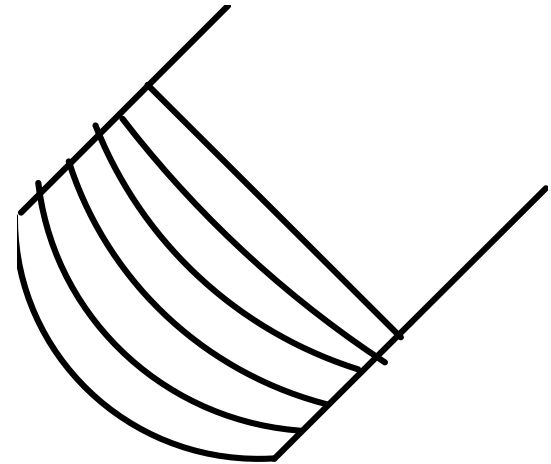(or inscribed radius) and lowering the
field as the end is approached.
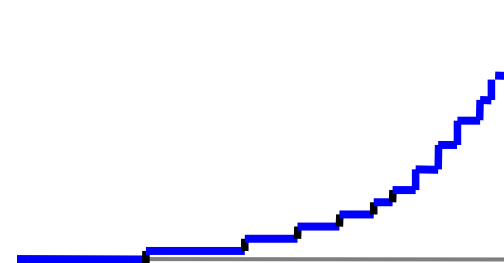
Pole profile adjustment

As the gap is increased, the size (area) of the shim is increased, to give *some* control of the field quality at the lower field. This is far from perfect!
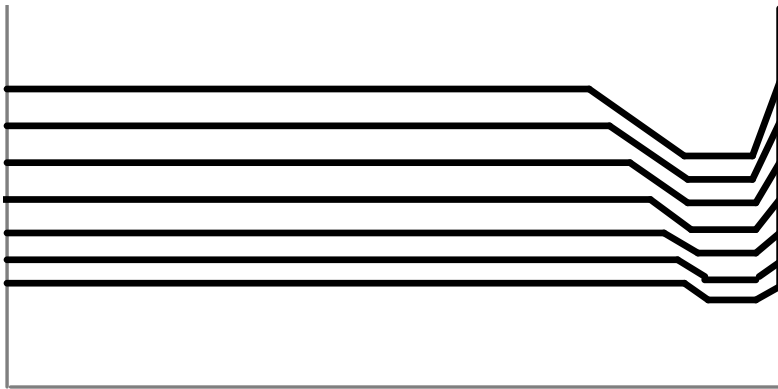
Transverse adjustment at end of dipole

Transverse adjustment at end of quadrupole
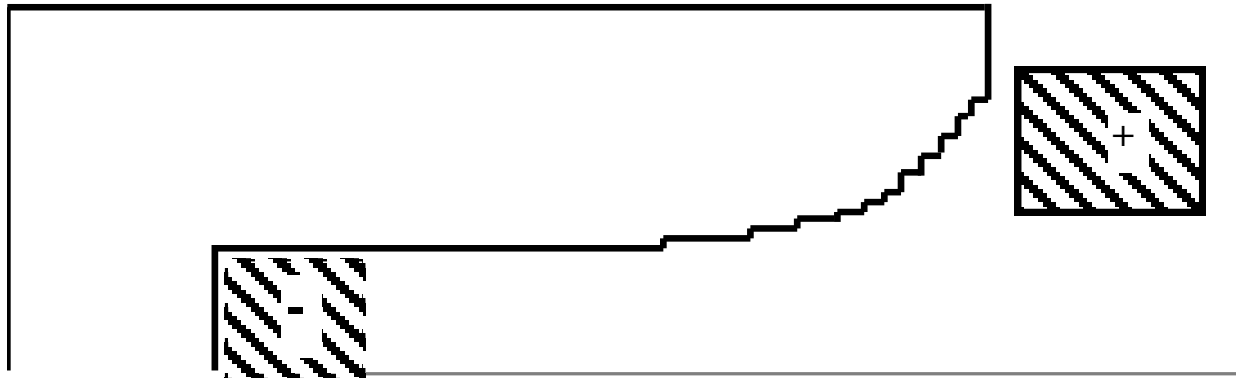
The NINA magnet ends

Calculation of end effects with 2D codes.

FEA model in longitudinal plane, with correct end geometry (including coil), but 'idealised' return yoke:

This will establish the end distribution; a numerical integration will give the 'B' length.

Provided dBY/dz is not too large, single 'slices' in the transverse plane can be used to calculated the radial distribution as the gap increases. Again, numerical integration will give ∫ B.dl as a function of x.

This technique is less satisfactory with a quadrupole, but end effects are less critical with a quad.

End geometries - dipole

Simpler geometries can be used in some cases.

The Diamond dipoles have a Rogawski roll-off at the ends (as well as Rogawski roll-offs at each side of the pole).

See photographs to follow.

This give small negative sextupole field in the ends which will be compensated by adjustments of the strengths in adjacent sextupole magnets – this is possible because each sextupole will have int own individual power supply

h) Some examples of Diamond magnets.

Diamond Dipole

Diamond dipole ends

Diamond W quad end
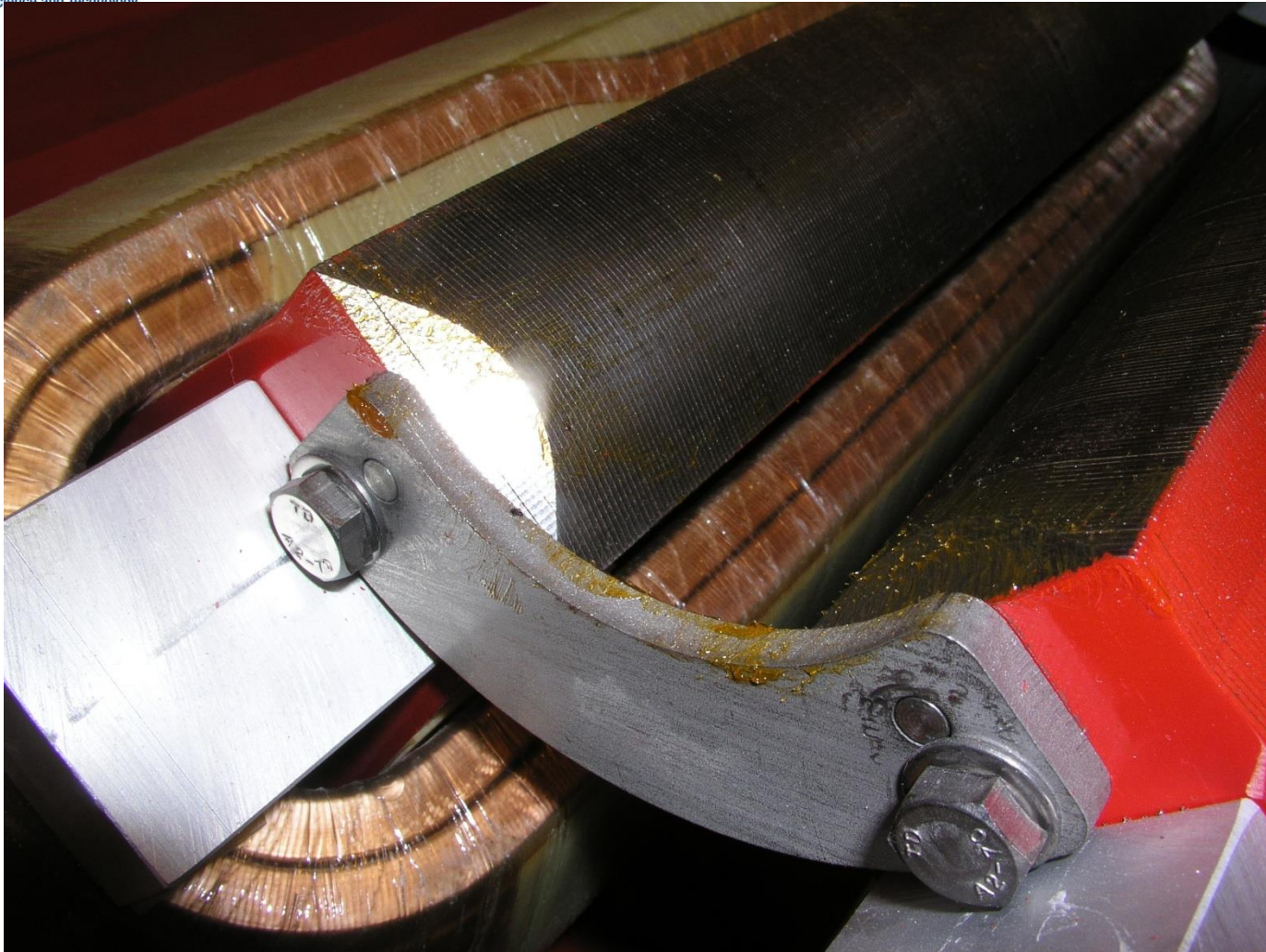
Sextupole ends

It is not usually necessary to chamfer sextupole ends (in a d.c. magnet). Diamond sextupole end:
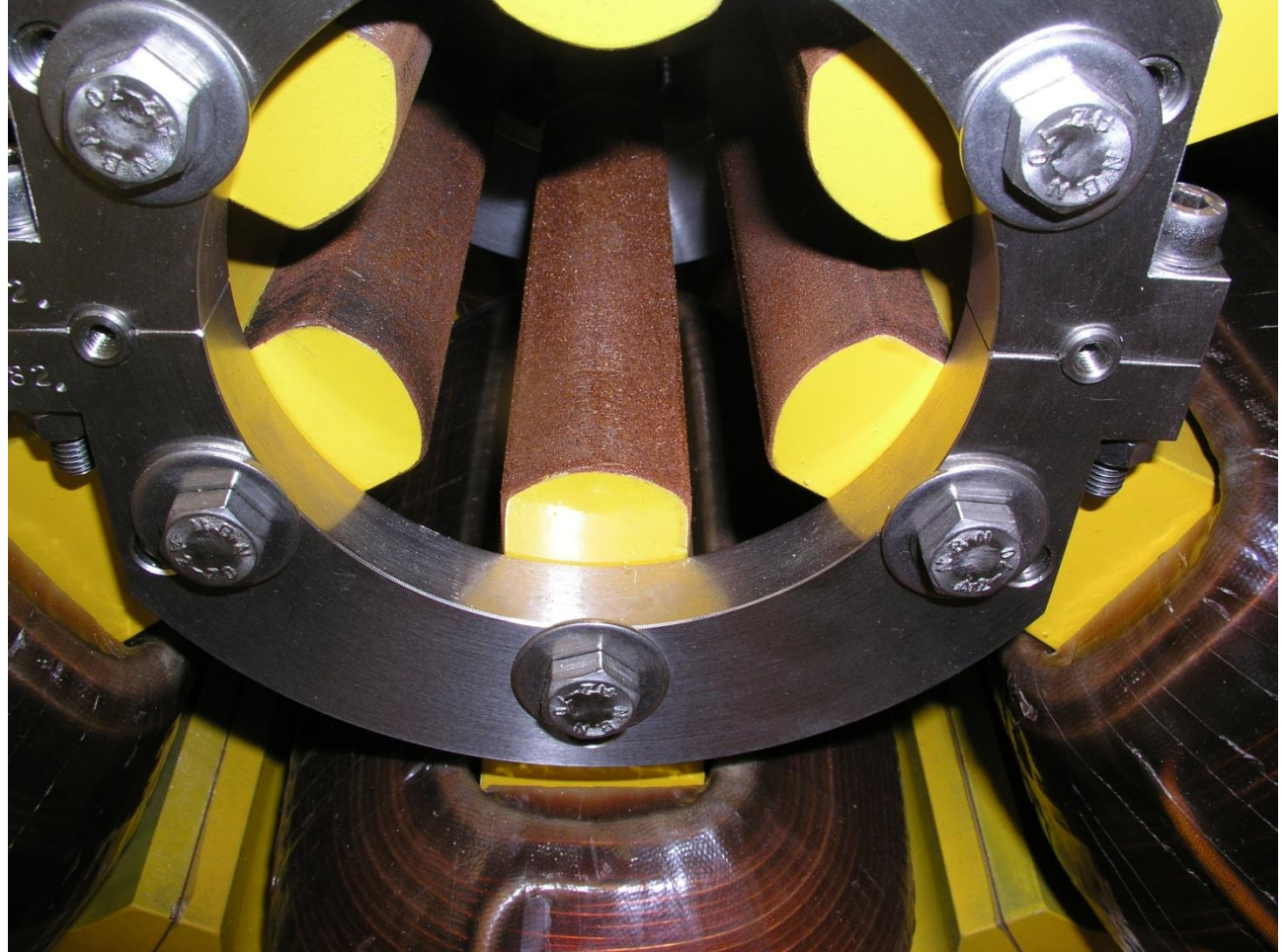
Sexy pics of sextupoles
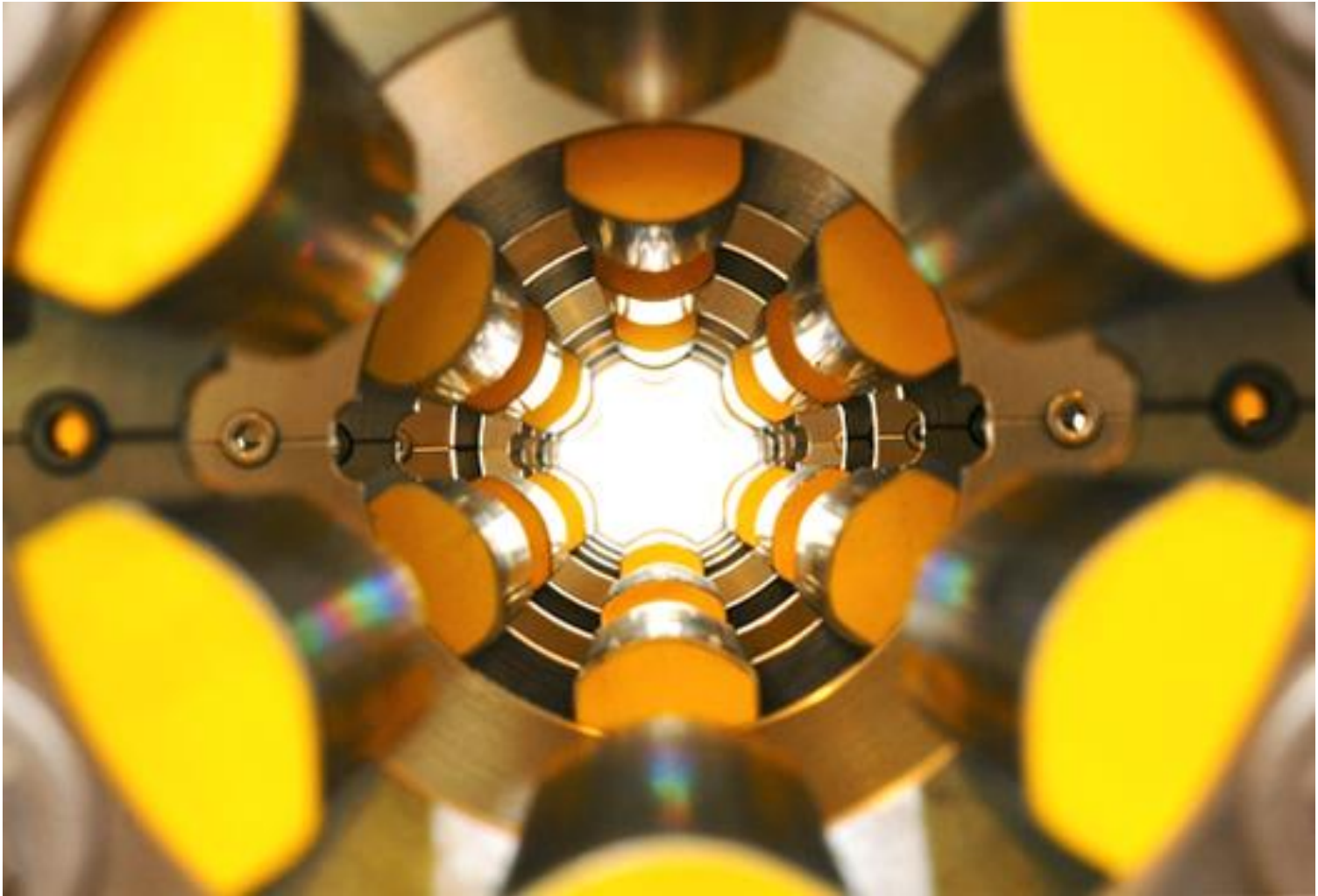
The End

I hope you found the magnets

ATTRACTIVE!

Neil Marks, SRFC, ASTeC, Cockcroft Institute, University of Liverpool.   J.A.I, January 2015