Synchrotron Radiation Group: Contributions Week 5

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1 Synchrotron Radiation as function of FODO periods

Firstly, a brief revision of the important equations to answer how the number of FODO periods influences the synchrotron radiation.

The power radiated (and hence to be compensated by the RF cavities) on a circular orbit is given by the short equation

$$P(\text{kW}) = 88.46\frac{E^4(\text{GeV})I(\text{A})}{\rho(\text{m})}, \tag{1.1}$$

where E is the energy of the accelerated electron, I is the current and ρ denotes the bending radius. The product of the bending radius and the magnetic field B is fixed via

$$B\rho(\text{Tm}) = 3.3356 \times pc(\text{GeV}). \tag{1.2}$$

This means that for a given energy, e.g. 175GeV, and a fixed luminosity, the radiated power depends only on ρ or B respectively as they are related via eq. (1.2).

Let us now work with the basic design of the FCC-lattice, only assuming arcs and longs straight sections (LSS). There are 12 of each, arcs are 6.8km, LSSs 1.5km long. The bending radius is simply the circumference of a circle only consisting of the bending magnets of the collider divided by 2π:

$$\rho = \frac{circumference \times F}{2\pi}. \tag{1.3}$$

For a fixed length of arcs and LSSs, the filling factor F determines ρ or the other way round. Simplified, arcs will consist of FODO periods. The filling factor can then be varied by changing the ratio of dipoles to quadrupoles and drift (later on sextupoles, kickers etc.), i.e. the length of dipoles per FODO in relation to the total length of one FODO cell, where the number of FODO cells per arc N_{FODO} is equal to the arc length divided by the FODO length. The filling factor can be expressed as

$$F = \frac{total\ length\ dipoles}{total\ circumference} = \frac{6.8 \times length\ dipoles\ in\ FODO}{(6.8 + 1.5) \times total\ length\ FODO}. \tag{1.4}$$

For a fixed length of the lattice, arcs and so on, the filling factor depends on the ratio of dipole to other components in a FODO cell (eq. (1.4)) and then via (1.3) to the bending radius. If we fix the filling factor of one FODO

cell in addition and only scale the entire cell in order to vary the number of FODO cells, the bending radius and as a result the synchrotron radiation is not affected.

For a fixed lattice length, FODO cell proportions and B-field, the bending radius is constant as well. As a result the radiated power remains constant is not a function of the number of FODO cells.

2 The fan of radiation within a dipole

Assuming a FODO lattice similar as in the Christmas exercises, one 50m long FODO cell has 4 dipoles at 10m length each. This results in 1632 FODO cells in 12×6.8km of arcs, and hence 6528 dipoles in total. Finally, this means that the bending angle θ_b per dipole magnet amounts to only about $0.055°$, i.e. **dipole sections are at first approximation almost straight.**

At the target energy of 175GeV, the Lorentz factor γ is of order 10^5. This leads to strongly focused emission in forward-direction with a divergence of a few microradiens:

$$\theta_d iv, c = \gamma^{-1}, \tag{2.1}$$

where this relation only holds at the critical frequency ω_c.

The exact angular spectrum can be calculated via the power emitted per solid angle. For a γ as large as in our case, the expression is dominated by a term proportional to $(1 - \beta\cos(\theta))^{-3}$, where θ is the angle between observer and propagation direction of the particle. A polar plot confirms our initial calculations of the divergence: **the angular profile exhibits a narrowly emitted radiation burst in forward direction with transverse symmetry.**