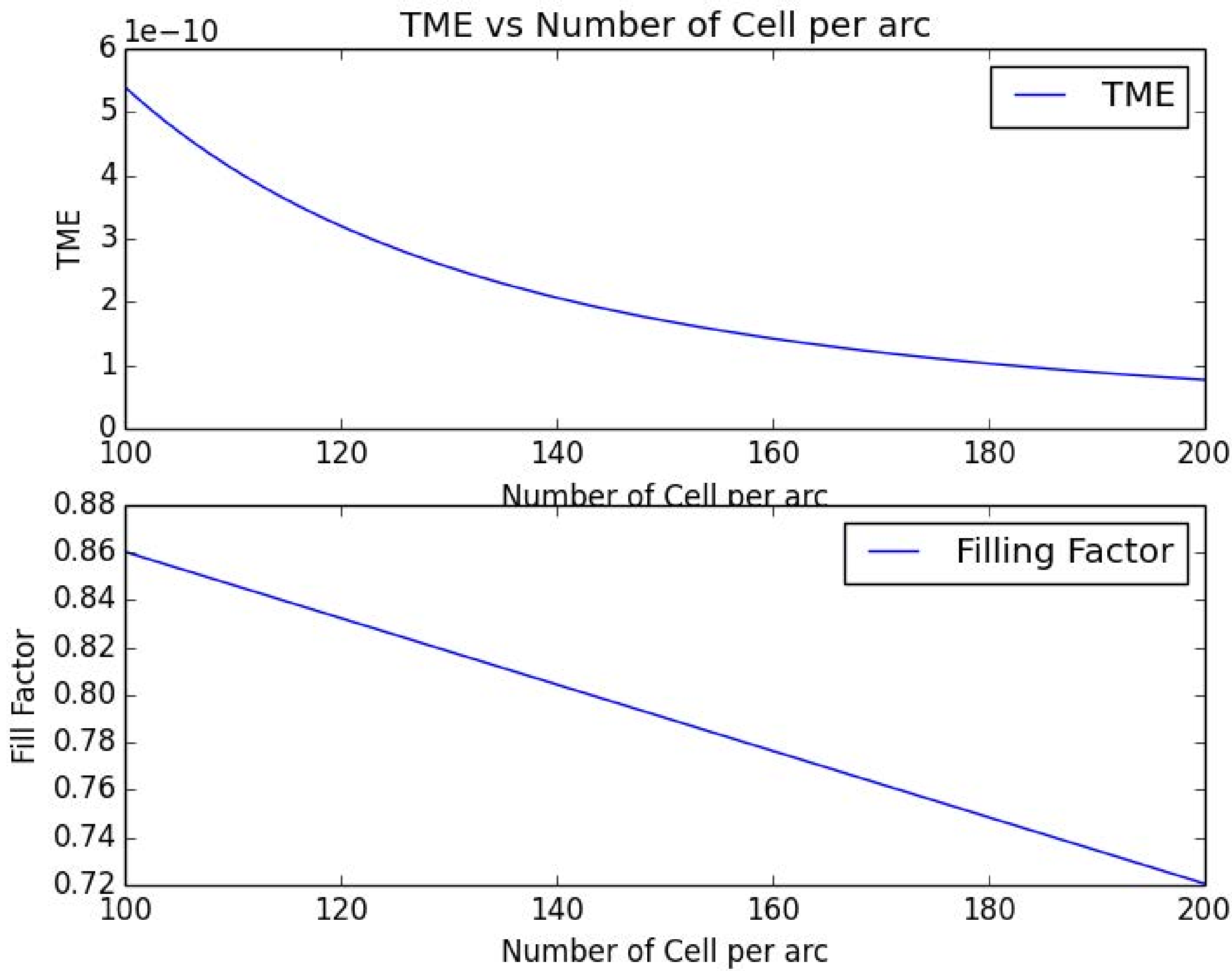
```
import numpy as np
import matplotlib.pyplot as plt
import pymadx
import os

NCell = []#Array for Number of Cells
FF = []#Array for Fill Factor (Length of Dipoles/Length of Cell)
EX = []#Array for Theoretical Minimum Emittance


for N in np.linspace(100,200,101): #For loop to run over different number of Cells per arc

#########################################Defines Lattice
Input###############################################

    L = 6800 #Length of one Arc
    dL = L/N #Length of Cell (Length of Arc / Number of Cells per arc)
    Angle = 6.28318530718/(12*N*4) #Bend angle per Dipole (Total # Dipoles = 12*N*4 as 4 Dipoles per cell and 12
arcs)

    LQuadD = 1.5 #Length of Quad
    LQuadF = LQuadD/2 #Half Quad length as Cell is defined half way through
    LSext = 0.5 #Gap for Sext
    LDrift = 0.5 #Length of Drift
    K1 = 0.01   #Quad Coeff starting piont

    LDipole = (dL-3*LQuadD-2*LSext-8*LDrift)/4.0 #Calculates length of Dipole given other Parameters
    bheam = pymadx.Builder.Beam('e-',175)

    ArcCell = pymadx.Builder.Machine()
    ArcCell.AddBeam(bheam)

    ArcCell.AddQuadrupole('FQ',LQuadF, K1)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddDipole('B','sbend',LDipole,Angle)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddDipole('B','sbend',LDipole,Angle)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddQuadrupole('DQ',LQuadD, -K1)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddDipole('B','sbend',LDipole,Angle)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddDipole('B','sbend',LDipole,Angle)
    ArcCell.AddDrift('D',LDrift)
    ArcCell.AddQuadrupole('FQ',LQuadF, K1)

#############################################################################################
##########################
```

```
    ArcCell.Write('Lattice_Length_variations')#Writes Lattice input to file, Madx then import this using CALL.



    os.system('madx' ' ' '< MasterMadx.madx')#Runs the Madx File



    Twiss_File = pymadx.Tfs('lattice_parameters.tfs') #opens the Twiss output file

#################Working out parameters############################

    S = Twiss_File.GetColumn("S")
    Betx = Twiss_File.GetColumn("BETX")
    Bety = Twiss_File.GetColumn("BETY")
    Alfx = Twiss_File.GetColumn("ALFX")
    Alfy = Twiss_File.GetColumn("ALFY")
    ############################################################ Calc for TME ######################
    bx = Betx[0]
    ax = Alfx[0]


    cg = 3.832e-13
    gamma=342465.8

    gx = (1 + ax*ax)/bx

    theta = 2 * Angle

    l = 4* LDipole

    ex = cg * (gamma**2)* (theta**3)* ((gx*l)/20.0 - ax/4.0 + bx/(3.0*l)) # Formula from book photo in document

    ff = l/dL

    NCell.append(N)
    FF.append(ff)
    EX.append(ex)


######################################################### Push files to folder so that they don't interfer with next run #############

    os.system('rm' ' ' 'Lattice_Length_variations_sequence.madx')
    os.system('rm' ' ' 'Lattice_Length_variations_components.madx')
    os.system('rm' ' ' 'Lattice_Length_variations_beam.madx')
    os.system('rm' ' ' 'Lattice_Length_variations.madx')
    os.system('rm' ' ' 'lattice_parameters.tfs')

plt.figure(1)
plt.subplot(211)
plt.plot(NCell,EX, label = 'TME')
plt.xlabel('Number of Cell per arc')
plt.ylabel('TME')
```

```
plt.title('TME vs Number of Cell per arc')
plt.legend()
plt.subplot(212)
plt.plot(NCell,FF, label = 'Filling Factor')
plt.xlabel('Number of Cell per arc')
plt.ylabel('Fill Factor')
#plt.title('Fill Factor vs Number of Cell per arc')
plt.legend()
plt.savefig('/Users/tomvaughan/Documents/PhD/FCC_Project/Lattice/untitled folder/Testgraph', format = 'pdf')
plt.clf()
```

```
//Madx Master FIle FCC-ee


TITLE,'FCC-ee Master FIle';

BEAM, PARTICLE=Electron, ENERGY=176.0, EX = 2E-9, EY = 2E-12;



CALL,FILE='Lattice_Length_variations.madx';



MATCH,SEQUENCE=lattice;
CONSTRAINT,SEQUENCE=lattice,RANGE=#E,MUX=0.25,MUY = 0.25;
VARY,NAME=FQ->K1,STEP=1E-6;
VARY,NAME=DQ->K1,STEP=1E-6;
LMDIF,CALLS=100,TOLERANCE=1E-20;
ENDMATCH;


TWISS,SAVE, FILE=Lattice_Parameters.tfs;
```



TME vs Number of Cell per arc
1e−10
6
5
4
3
2
1
0
TME
TME
100
120
140
160
180
200
Number of Cell per arc
0.88
0.86
0.84
0.82
0.80
0.78
0.76
0.74
0.72
Fill Factor
Filling Factor
100
120
140
160
180
200
Number of Cell per arc

is small compared to the bending radius and so the condition $s/R \ll 1$ virtually always holds. Only the initial values of the beta function may now be varied in order to minimize the emittance. For simplicity we will neglect weak focusing, which in general only has a very small effect on the emittance. From the point of view of beam focusing, the dipole magnet effectively behaves like a drift region, the transfer matrix $\mathbf{M}$ of which is given in in (3.72). Using the initial values $\beta_0$ and $\alpha_0$, (3.149) yields the transformation

$$\begin{pmatrix} \beta(s) & -\alpha(s) \\ -\alpha(s) & \gamma(s) \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 & -\alpha_0 \\ -\alpha_0 & \gamma_0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix}, \tag{6.51}$$

from which we directly obtain the form of the optical functions:

$$\begin{aligned} \beta(s) &= \beta_0 - 2\alpha_0 s + \gamma_0 s^2 \\ \alpha(s) &= \alpha_0 - \gamma_0 s \\ \gamma(s) &= \gamma_0 = \text{const.} \end{aligned} \tag{6.52}$$

Using the dispersion function (6.50) and the beta function (6.52), equation (6.48) allows us to specify the function $\mathcal{H}(s)$, important for calculating the emittance:

$$\begin{aligned} \mathcal{H}(s) &= \gamma(s)D^2(s) + 2\alpha(s)D(s)D'(s) + \beta(s)D'^2(s) \\ &= \frac{1}{R^2}\left(\frac{\gamma_0}{4}s^4 - \alpha_0 s^3 + \beta_0 s^2\right). \end{aligned} \tag{6.53}$$

If we again assume that all the bending magnets in the ring are identical and that the damping number $J_x \approx 1$, it then follows from (6.48) and (6.53) that

$$\varepsilon_x = C_\gamma \frac{\gamma^2}{R}\frac{1}{l}\int_0^l \mathcal{H}(s)\,ds = C_\gamma \gamma^2 \left(\frac{l}{R}\right)^3 \left(\frac{\gamma_0 l}{20} - \frac{\alpha_0}{4} + \frac{\beta_0}{3l}\right) \tag{6.54}$$

with

$$C_\gamma = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc} = 3.832 \times 10^{-13}\ \text{m}.$$

Recognising that the ratio $l/R = \Theta$ corresponds to the bending angle per dipole magnet, we may write the emittance in the form

$$\boxed{\varepsilon_x = C_\gamma \gamma^2 \Theta^3 \left(\frac{\gamma_0 l}{20} - \frac{\alpha_0}{4} + \frac{\beta_0}{3l}\right).} \tag{6.55}$$

Since $\Theta$ enters this expression raised to the third power, it is clearly better to use many periodic cells with short bending magnets, rather than fewer cells with relatively long dipoles, if the aim is to keep the emittance small. The relation $\varepsilon_x \propto \Theta^3$ holds for all possible periodic magnet structures.

Since the magnet structure and the beam energy are fixed, the emittance in (6.55) is now only a function of the initial values $\beta_0$ and $\alpha_0$ of the optical